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By

K.L. Wong

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Leakage of Runaway Electrons From Tokamaks

K. L. Wong

Plasma Physics Laboratory, Princeton University
Princeton, New Jersey 08544

ABSTRACT

Runaway electron orbits are calculated in a tokamak magnetic field. It is shown that these electrons tend to drift towards a larger major radius with a velocity $\vec{v}_R = qcE/B_0 \hat{R}$. This effect may be relevant to some recent experimental observations in tokamaks.

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Runaway electrons with energies in MeV range have been observed in many tokamak discharges.¹ These electrons mainly interact with the electric and magnetic fields because the plasma frictional drag on these particles is very small. The transport of these electrons has been experimentally investigated in large tokamaks² (PLT and PDX) in search of some mechanism which is related to the anomalous transport of the thermal electrons. In this note, we calculate the runaway electron orbits and show that they do not have closed drift surfaces, but rather tend to drift towards a larger major radius during acceleration in the ohmic electric field. The drift velocity is shown to be proportional to the accelerating electric field. This finding offers a possible explanation for the rapid reduction of hard x-ray emission observed in several lower hybrid wave current drive experiments in tokamaks.^{3,4,5}

Consider a runaway electron with parallel (with respect to the magnetic field) velocity $v_{\parallel}$ much larger than the perpendicular velocity $v_{\perp}$ so that $v = (v_{\parallel}^2 + v_{\perp}^2)^{1/2} \approx v_{\parallel}$. The electron will drift across the curved magnetic field line with a drift velocity

$$\vec{v}_d = \frac{\gamma m_0 c}{eB} \frac{v^2}{R} \frac{\vec{B} \times \vec{R}}{|\vec{B} \times \vec{R}|} \approx \frac{\gamma m_0 c}{eB_0} \frac{v^2}{R_0} \frac{\vec{B} \times \vec{R}}{|\vec{B} \times \vec{R}|} \quad (1)$$

where m_0 = electron rest mass
 c = speed of light

e = magnitude of electron charge (positive)

R = radius of curvature of magnetic field line which is also the distance to the major axis.

B = magnetic field strength $\propto 1/R$

R_0 = major radius of tokamak

B_0 = magnetic field at $R = R_0$

γ = $(1 - v^2/c^2)^{-1/2}$

The electron orbit does not stay on the magnetic surface due to this curvature drift. Figure 1 shows the projection of the electron orbit onto a fixed toroidal plane. For simplicity, let us assume concentric circular magnetic surfaces. Deviation from this assumption would introduce minor quantitative differences. The electron starting at point A ($\theta = 0$, $r = r_0$) drifts towards a smaller minor radius r when it is at the upper-half-plane. When it gets to the lower-half-plane, it starts to drift towards a larger r . As a zeroth order approximation, the electron moves on the circular drift surface with its center shifted outward by a distance $\Delta_0/2$ with respect to the center of the magnetic surface.¹ Since the electron is accelerated by an electric field E and the drift velocity $v_d = \gamma v^2$, one would expect the electron to drift a longer distance at the lower-half-plane despite the fact that it spends less time there ($t \propto v^{-1}$). As a consequence, the electron will end up at point B ($\theta = 0$, $r = r_0 + \Delta_1$) with $\Delta_1 > 0$. This mechanism bears some similarity with the well-known Ware-pinch⁶ effect on trapped particles. Both of them are

caused by the toroidal electric field. To calculate Δ_1 , let us write the radial component of the drift velocity as $\vec{v}_r = -v_d \sin \theta \hat{r}$

$$\text{and } \Delta_1 = \int_0^T v_r dt = - \int_0^T v_d \sin \theta dt, \quad (2)$$

where T is the time it takes to go from point A to point B.

Let s denote the distance travelled by the electron. Using the large-aspect-ratio approximation,

$$s = \theta (R_0^2 q^2 + r^2)^{1/2} \approx R_0 q \theta$$

$$\frac{d\theta}{dt} = \frac{d\theta}{ds} \frac{ds}{dt} = \frac{1}{R_0 q} v,$$

so that

$$dt = \frac{R_0 q}{v} d\theta, \quad (3)$$

where q is the safety factor. Substituting Eq. (3) into Eq. (1), we get

$$\begin{aligned} \Delta_1 &= - \int_0^{2\pi} v_d \sin \theta \frac{R_0 q}{v} d\theta \\ &= - \int_0^{2\pi} \frac{m_0 c}{e B_0} q \gamma v \sin \theta d\theta. \end{aligned} \quad (4)$$

Let $\gamma = \gamma_0$, $v = v_0$ at $\theta = 0$, from energy conservation, we have

$$\begin{aligned} \gamma m_0 c^2 &= \gamma_0 m_0 c^2 + e E R_0 q \theta \\ \gamma &= \gamma_0 + \frac{e E R_0 q}{m_0 c^2} \theta \end{aligned}$$

Therefore $\gamma v = \gamma c (1 - \gamma^{-2})^{1/2}$

$$= c (\gamma^2 - 1)^{1/2}$$

$$= \gamma_0 v_0 + \frac{e E R_0 q \theta}{m_0 v_0} \quad (5)$$

Obviously, $e E R_0 q \theta / m_0 v_0 \ll \gamma_0 v_0$ because the electron energy is much higher than the energy gain per transit around the torus. Substitute Eq. (5) into Eq. (4), we get

$$\Delta_1 = - \int_0^{2\pi} \frac{m_0 c}{e B_0} q (\gamma_0 v_0 + \frac{e E}{m_0} \frac{q R_0 \theta}{v_0}) \sin \theta d\theta$$

$$= 2\pi q^2 R_0 \frac{c}{v_0} \frac{E}{B_0} \quad (6)$$

Similarly, we can calculate Δ_0 by

$$\Delta_0 = - \int_0^\pi v_d \sin \theta \frac{R_0 q}{v} d\theta$$

$$= - 2 \frac{m_0 c}{e B_0} q \gamma_0 v_0 \quad (7)$$

Although $|\Delta_1/\Delta_0| = \pi e E R_0 q / \gamma_0 m_0 v_0^2 \ll 1$, there is a consistent tendency for the electron to drift outward along the major radius with a velocity

$$\vec{v}_R = \frac{\Delta_1}{T} \hat{R}$$

$$= \Delta_1 / (2\pi R_0 q / v_0) \hat{R}$$

$$= q c E / B_0 \hat{R} \quad (8)$$

It should be noted that the above calculation is valid only if the runaway electron stays close to the circular magnetic surface, i.e., $\Delta_0 \ll r_0$. Very energetic runaways can have Δ_0 comparable to r_0 . In fact, putting Δ_0 equals half the limiter radius can give a reasonable estimate on the maximum energy of runaway electrons that can exist in a tokamak. In addition to the outward drift $\vec{v}_R$, there is an $\vec{E} \times \vec{B}$ drift pinching towards the minor axis with the velocity

$$\vec{v}_E = \frac{\vec{E} \times \vec{B}}{B^2} c = - \frac{E |B_\theta|}{B^2} c \hat{r}, \quad (9)$$

which is less than $\vec{v}_R$ by the factor $|B_\theta|/qB \ll 1$ and therefore can be neglected.

The purpose of this calculation is to suggest an explanation for some recent observations in lower hybrid wave current drive experiments in tokamaks.^{3,4,5} Shortly after the injection of RF power, the loop voltage drops rapidly because part of the ohmic heating current originally carried by the thermal electrons is transferred to the energetic, collisionless electrons that resonate with the wave. Associated with the loop voltage change is the simultaneous drop ($t_x - t_v$) in the hard x-ray emission as shown in Fig. 2. The hard x-ray emission is expected to be proportional to the flux of runaway electrons Γ hitting the limiter:

$$\Gamma = n_Y V. \quad (10)$$

Obviously a decrease in loop voltage will slow down the runaway production rate, but the runaway electron density n_y can only change in time scale of its confinement time which is significantly longer than t_x . This implies that the velocity V in Eq. (10) is changing rapidly during the RF injection. It should be emphasized here that the outward drift $\vec{v}_R$ may not be the dominant transport mechanism for runaway electrons in tokamaks; in fact, there are strong evidence that magnetic field fluctuations can play an important role.² However, previous experiments¹ show that in low density discharges ($n \approx 1.2 \times 10^{13} \text{ cm}^{-3}$), the runaway electrons are better confined and the orbit-shift model correctly predicts many of their properties. It is well-known that the lower hybrid wave current drive experiments^{3,4,5} are successful only in the low density ($n < 8 \times 10^{12} \text{ cm}^{-3}$) slideaway regime. If the convective outward drift of the runaway electrons is the dominant transport mechanism, at least in the vicinity of the limiter, then $V \approx v_R \propto E \propto \text{loop voltage}$ which would explain the data shown in Fig. 2. The improvement of runaway confinement at low electric field also suggests the possibility of maintaining a low density, low loop voltage tokamak discharge for a long period of time as has been achieved recently in the ASDEX tokamak.⁷

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FIGURE CAPTIONS

Fig. 1. Projection of the electron orbit onto a toroidal plane. An accelerating electron starts at point "A" arrives at point "B" after one revolution around the magnetic axis.

Fig. 2. General features of the temporal evolution of (a) injected RF power, (b) toroidal loop voltage and (c) hard x-ray emission in typical lower hybrid wave current drive experiments (see references 3, 4 and 5).

81X1422

Major
Axis

Fig. 1

81X1421

Fig. 2