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ANALYTICAL QUALIFICATION OF SYSTEM
IDENTIFICATION (MODAL ANALYSIS)
CODES FOR USE IN THE DYNAMIC
TESTING OF NUCLEAR POWER
PLANT STRUCTURES

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I ABSTRACT

In this report we describe the analytical evaluation of two particular system identification codes used at Lawrence Livermore Laboratory. Both codes are eigenparameter identification codes, however, one uses a time domain approach while the other a frequency domain approach. The evaluation was accomplished by analytically generating several time history signals in which the true modal parameters were known. These time histories ranged from widely spaced modes with spacing factors of 100 percent to closely spaced modes with spacing factors of 6 percent. These signals were then polluted with various levels of simulated measurement noise and the ability of our computer codes to extract the parameters from this noisy data was evaluated. We were unable to detect any substantial difference in the performance between the time domain and frequency domain approach. We also found that the performance of the codes was not a function of the modal spacing (at least down to a spacing factor of 6 percent). Finally we found that performance of the codes was clearly a function of the signal to noise ratio.

II INTRODUCTION

Many physical phenomena are characterized by the fact that they can be (ideally) described by a set of 2nd order differential equations. The homogeneous solutions to these equations are given by a finite sum of damped sinusoidal terms. These terms contain constants which are commonly called damping, frequency, and residue system parameters. System identification is the process of identifying these parameters from the locus, or graph, of the solutions. The scientists and engineers at Lawrence Livermore Laboratory have developed codes which are capable of accurately and efficiently identifying these system parameters. These codes have been used for several years in such diverse areas as geophysical probing, antenna aperture syntheses, ultra fast streaking cameras, data characterization and compression, optical systems resolution, calibration of

[*] See section VI, equation (3).

fibre-optic cables, radar target identification, and EEG monitoring [Ref 1-5].

Recently, at the request of the United States Nuclear Regulatory Commission (NRC), we have modified these codes so that they may be used as a tool in the analysis and testing of Nuclear Power Plant (NPP) structures, components, and systems. For the most part, these modifications have consisted of adding post processing codes, or subroutines, to our existing codes so that the system (or modal) parameters may be used to display mode shapes and generate transfer functions.

In addition to these modifications we have also analytically qualified the performance of our codes. That is to say, we have examined the accuracy of our codes in the presence of (simulated) measurement noise. This report discusses the methods used and the results obtained.

Certainly, other systems identification and modal analysis codes are available, however, to the best of our knowledge, none have been thoroughly qualified and documented. Our evaluation process is important for two reasons. First, it obviously serves as a guide to the accuracy of our codes which will be made available as a tool to the NRC. Secondly, it serves as a benchmark case from which other system identification codes can be judged (be it better or worse). It appears obvious to us that if a Nuclear Power Plant, or some subsystem therein, is to be evaluated using a system identification, or modal analysis, approach then the performance of the codes being used must be known.

III THE IDENTIFICATION PROCESS

In this section we present a heuristic description of the identification process. The purpose of this presentation is not to become embroiled with the mathematical and programming details but, instead, to present a general discussion of the identification codes and techniques. Our identification codes have come to be known as the PARET codes. The name PARET is an acronym for PARameter Estimation Technique and originally referred to a particular time domain system identification technique which used Prony's algorithm. This technique was first implemented (at LLL) by Lager et. al. [Ref 8] in a code called SEMPEX which was improved upon and eventually replaced by a more versatile code called PARET. During early discussions between LLL and the NRC the overall project was christened

with the name PARET. This was a somewhat unfortunate choice because it tended to confuse the overall project with the name of a particular time domain code, whereas the total program also makes use of the frequency domain analysis code FOURIER/TRANSF. Therefore, when it is necessary, to avoid confusion, we will refer to the PARET code as *time domain PARET (TDP)* and the FOURIER/TRANSF code as *frequency domain PARET (FDP)*.

As already noted, the PARET process uses two complementary approaches (one in the time domain and the other in the frequency domain) and, as will be discussed later in this report, both yield comparable results. Basically, each method assumes that a structure is set in motion with an impulse excitation and then allowed to vibrate freely*. This free vibrational motion, known as the *impulse response*, contains valuable information concerning the inherent properties and dynamic behavior of the structure. The TDP approach assumes that this motion can be described by a finite sum of damped sinusoids, i.e.

$$(1) \quad h(t) = \sum_{k=1}^N \hat{S}_k e^{-\sigma_k t} \sin(2\pi w_k t + \phi_k). \quad (\text{TDP}).$$

The Fourier transform of equation (1) is called the *transfer function* of the structure and is described mathematically as a finite sum of Lorentzian functions, i.e.

$$(2) \quad H(w) = \sum_{k=1}^N \frac{S_k}{\sigma_k + 2\pi i(w - w_k)} \quad (\text{FDP}).$$

The FDP approach works with equation (2). The set of coefficients $\{S_k, \sigma_k, w_k\}$ are known as parameters (or more properly modal parameters) of the structure. Specifically we have:

$S_k = \hat{S}_k e^{i\phi_k}$ is called the *kth residue* and is related to the strength of the *kth mode* and/or mode shape,
 σ_k .. is the *kth modal damping parameter*, and
 w_k .. is the *kth modal frequency parameter*.

[*] For the sake of discussion, we will only consider impulses excitation in this report. The more general case of non-impulsive excitation is discussed in reference 7.

Basically, the PARET process is to excite a structure and record the resulting time history motions at several selected locations. Assuming that the structure behaves linearly, these time histories should indeed be described by a form of equation (1) in the time domain and/or equation (2) in the frequency domain. They are then fed into a computer and the PARET codes choose the "best" set of coefficients $\{S_k, \sigma_k, w_k\}$ such that equations (1) and/or (2) will match, or describe, these measured time histories.

IV PERFORMANCE EVALUATION METHODOLOGY

In order to use the PARET codes (or any other system identification code) with confidence it is necessary to qualify their performance. That is to say, we must evaluate their ability to estimate the modal parameters of a structure using time histories contaminated with "measurement" noise. To *analytically* accomplish this goal we generated simulated time histories using given modal parameters. Since the true parameters are known we are able to compare them to the code results and, consequently, obtain a measure of their accuracy. In this study we qualify the codes performance on three different types of time history signals, namely

- (1) a single mode time history,
- (2) widely spaced modal time histories, and
- (3) closely spaced modal time histories.

The results of this qualification are presented in the next three sections. In this section we discuss the performance evaluation methodology. That is to say, we discuss the method by which we evaluated the performance of the PARET codes for estimating modal frequencies and damping given data contaminated with "measurement" noise. The performance was found as a function of the level of the noise and is quantified in the fashion of Smith and Lager [ref 9] by computing the statistical bias and variability (standard deviation) in the parameter estimates produced by the PARET codes.

The performance is computed by applying the PARET codes to an ensemble of waveforms that are identical with the exception that each is polluted by a different sequence of random noise. The standard deviation of the noise is kept constant for the ensemble. This was accomplished by choosing an ensemble size of 10 and making 10 copies of the simulated response waveform. Each of the copies was then

contaminated by adding white Gaussian noise with a fixed standard deviation. The PARET codes were then used to estimate the parameters for each of the waveforms. The variability in the parameter estimates is expressed in terms of the unbiased standard deviation σ according to the equation

$$\sigma^2 = \frac{1}{N-1} \sum_{i=1}^N (x_i - \mu)^2$$

where μ is the mean value of the estimates over the ensemble and is given by

$$\mu = \frac{1}{N} \sum_{i=1}^N x_i$$

where x_i is the parameter value estimated from the i th waveform, and $N = 10$ is the number of waveforms in the ensemble.

For each of the parameters the bias b in the estimates was computed using the formula

$$b = x_t - \mu$$

where x_t is the true value for the parameter.

The noise added to each waveform in the ensemble was from a Gaussian distribution with zero mean and standard deviation σ_n . This was chosen because it is representative of the noise from measuring instruments and also for its computational convenience. The values were computed by making use of the Central Limit Theorem and summing the numbers from a uniform distribution according to the equation

$$g(\sigma_n) = \sigma_n \sum_{i=1}^{12} (\eta_i - .5)$$

where $g(\sigma_n)$ is the value from a Gaussian distribution with standard deviation σ_n and η_i is the value from a distribution uniform over the interval 0 to 1. The standard

deviation of the noise σ_n was chosen as a fraction of the peak of the waveform, i.e.

$$\sigma_n = F y_{\max}$$

where F is the fraction of the peak and $y_{\max}$ is the maximum value of the waveform $y(t)$ (highest positive peak). For the remainder of this report we will define the signal-to-noise ratio as

$$S/N = 1/F$$

which is the ratio of the waveform peak to the standard deviation of the noise. This definition of S/N was chosen over the more rigorous definition of

$$S/N = \frac{P(y)}{P(n)},$$

where $P(y)$ is the power in waveform $y(t)$ and $P(n)$ is the power in the noise $n(t)$, because this power ratio is not independent of the time duration over which it is computed. For times after the waveform has decayed to zero (but the noise has not) this S/N can be made to decrease arbitrarily by extending the time for computing the power. Our definition of S/N circumvents this difficulty. Also it is our feeling that measurement instruments tend to produce a noise level that is more a function of the peak value rather than the actual power within the waveform because the range setting is usually chosen to have the peak yield a near full scale reading.

V PERFORMANCE FOR SINGLE MODE WAVEFORM

The qualification of PARET for a single mode waveform was performed using the following simulated response data

$$h(t) = S_1 e^{-\sigma_1 t} \sin(2\pi w_1 t)$$

where:

$$S_1 = 1$$

$$\begin{aligned}\sigma_1 &= 2, \\ w_1 &= 4.\end{aligned}$$

This waveform was analytically generated on the computer and the resulting data placed in a disk file for subsequent processing by the PARET codes. In the absence of noise these codes were able to identify the modal parameters to within the disk transfer accuracy. That is to say, to within the accuracy of which the data was written to the disk file (6 significant figures).

To qualify the performance of our codes in the presence of noise we polluted the pure waveform with various levels of noise. As mentioned earlier, our measure of signal to noise level is defined as the peak value of the (pure) signal waveform divided by the standard deviation of the added noise waveform. In this study we used signal to noise levels of 100, 70, 50, 40, 20, and 10. Examples of these

Figure V.1 Noise polluted waveforms.

waveforms are illustrated in Figure V.1.

For each signal to noise level we generated an ensemble of 10 polluted waveforms. The codes were then used on each and the results statistically evaluated. In Figures V.2 and V.3 we show typical results (S/N level=40) for both the TDP and FDP. In these figures, the solid line represents the (simulated) experimental data and the dashed line is the

Figure V.2 Typical results ($S/N=40$) using TDP.

PARET fit to this data based on the estimated parameters.

In Figure V.4 we show the data scatter vs S/N level for

Figure V.3 Typical results (S/N=40) using FDP.

the modal damping parameter. Although this particular graph was constructed using the FDP results, it is representative of the TDP results as well. In this figure the dotted line represents the true value. Also in this figure we have indicated the mean and standard deviation of the scatter for each S/N level. These results can be more conveniently expressed in terms of the percent bias and coefficient of variation. The percent bias is defined as

$$\left| \frac{\text{true value} - \text{mean value}}{\text{true value}} \right| \times 100,$$

and the coefficient of variation (CV) as

$$\frac{\text{standard deviation}}{\text{true value}}$$

Shown in Figure V.5 are graphs of the percent bias and CV for the frequency parameter (X-FDP, O-TDP). Similar results are shown in Figure V.6 for the damping parameter. As can be appreciated from these figures, the accuracy of the codes improves with increasing S/N levels. However, even with the most severe noise pollution the codes were able to obtain the frequency to within .6 percent bias ($CV < .025$) and the damping to within 4 percent bias ($CV < .2$).

Figure V.4 Scatter for modal damping

Figure V.5 Performance of PARET estimating frequency

Figure V.6 Performance of PARET estimating damping

VI PERFORMANCE FOR MULTI (WIDELY SPACED) MODES

The next step in our qualification procedure was to determine the performance of our codes on multi-modal data. To obtain this data we used the SAP4 finite element code and modeled an 8 story building shown schematically in

Figure VI.1. Multi-story bldg modeled by SAP4.

Figure VI.1*. This type of structure was chosen because it possesses several modes that are reasonably well separated, or widely spaced. Shown in Table VI.1 are the first 5 modal

Mode	Freq (Hz.)	Damp (1/sec)
1	1.0174	.320
2	3.0176	.948
3	4.9150	1.544
4	6.6450	2.086
5	8.1487	2.560

Table VI.1 Modal parameters predicted by SAP4.

[*] Node 2 is located at ground level and node 1 is a dummy node used by the SAP4 code to define a coordinate system.

frequency and damping parameters predicted by SAP4. As our measure of the spacing between modes we will define the percent spacing factor (PSF) between the two frequencies w_n and w_m as

$$(3) \quad \text{PSF}(n,m) = \frac{2|w_n - w_m|}{w_n + w_m} \times 100.$$

Based on this definition we have the following percent spacing factors for this SAP4 model

PSF(1,2)	99.1
PSF(2,3)	47.8
PSF(3,4)	29.9
PSF(4,5)	20.3

To obtain the time histories (simulated experimental data) at each node of the structure we subjected its base to a horizontal (x-direction) triangular acceleration pulse of total duration .02 sec. This is shown in Figure VI.2a. For all practical purposes, this pulse may be considered an impulse excitation for this structure. To elaborate further, in Figure VI.2b we show the Fourier transform, or frequency content, of this pulse and as can be seen its spectrum is essentially level out past the fifth modal frequency.

In Figure VI.3 we show the resulting "impulse" response time histories (along with a log plot of their Fourier transforms) at nodes 3, 6, and 10. As can be appreciated from this figure, the modes are most strongly defined at node 3. In other words, the first 5 modes are clearly present at this node. Therefore, we processed the time histories at this node with our PARET codes to obtain the first 5 modal frequencies and modal damping parameters. The results are shown in Table VI.2 for both the TDP and the FDP. The numbers in parentheses indicate the percent disagreement with the SAP4 results, which we mainly attribute to two factors. The first is the previously mentioned disk transfer accuracy. The second results from the slight inaccuracies in the Wilson- θ method used by SAP4 to generate the time histories.

We point out here that when dealing with a real, or more complex, structure it will not in general be possible to guarantee that any single node (or measurement location)

Figure VI.2. Excitation pulse in time (a) and frequency (b) domains.

will contain all the modal parameters. Therefore, it will usually be necessary to process the time histories from several locations along the structure to obtain all the modal damping and frequency parameters. With this fact in mind, our codes have been designed in such a way that they give us the option of processing the time histories from all the locations and averaging the results to obtain the desired parameters. Another benefit of processing all the time histories is that it allows us to obtain, and display, the mode shapes of the structure. When the PARET codes are run on the LLL computer system the mode shapes can be displayed in either of two ways. First is an animated display on the computer terminal CRT. The second method is to generate 16mm movies directly from the computer disk files. In Figure VI.4 we show selected frames from such a movie which illustrate the first 5 mode shapes of the multi-story structure.

Figure VI.3. Time histories and frequency content at selected nodes.

TDP		FDP	
frequency (Hz.)	damping (1/sec)	frequency (Hz.)	damping (1/sec)
1.0161 (0.13)	.320 (0.00)	1.0162 (0.12)	.319 (0.31)
3.0138 (0.13)	.948 (0.00)	3.0137 (0.13)	.924 (2.50)
4.9088 (0.13)	1.544 (0.00)	4.9083 (0.14)	1.552 (1.23)
6.6370 (0.12)	2.086 (0.00)	6.6370 (0.12)	2.107 (1.01)
8.1385 (0.13)	2.560 (0.00)	8.1375 (0.14)	2.585 (0.98)

Table VI.2. TDP and FDP modal parameters for multi-story bldg.

Figure VI.4. First 5 modes shapes of multi-story structure.

We next examined the performance of our codes on this multimode data in the presence of noise. To accomplish this study, we polluted the time history signal from node 3 with simulated measurement noise and generated several ensembles at various S/N levels (100, 70, 40, 20, and 10). Shown in

Figure VI.5 Time history signal at node 3 containing measurement noise

Figure VI.5 is a typical member of the $S/N=40$ ensemble. In Figure VI.6 we show a portion of the Fourier spectrum of this time history. This signal is clearly dominated by the first mode at 1.01 Hz. Because of the way in which we defined our S/N level we see that the S/N level for the second and third modes is greatly decreased (see insets in

Figure VI.6. Frequency content of typical ($S/N=40$) time history

Figure VI.6). Using the frequency domain plot in Figure VI.3 we determine that the S/N level is decreased by a factor of approximately 12 for mode 2 and 40 for mode 3 (e.g. $S/N=40$ for mode 1 results in $S/N=3.3$ for mode 2 and $S/N=1.0$ for mode 3). Since the modes are widely spaced they are essentially isolated and, consequently, have very little influence on each other. We, therefore, expected, and obtained results very similar to the single mode waveform study presented in section V. In Figures VI.7 to VI.9 we

show the results of this noise study on modes 1 through 3 respectively.

During this analysis we were pleasantly surprised to learn that PARET was able to perform remarkably well on signals with unrealistically strong noise pollution. (By unrealistically strong we mean much higher levels of noise that would ever be expected from measurements performed on a structure by a competent experimentalist). For example, on mode 2 we encountered S/N levels as low as .8 (more noise than signal) and as can be seen in Figure VI.8, PARET was able to estimate the frequency with an accuracy of better than 2.0 percent bias ($CV < .05$) and the damping with an accuracy better than 20 percent bias ($CV < .7$). On mode 3 we encountered S/N levels as low as .25 and obtained accuracies of less than 7 percent bias ($CV < .1$) for the frequency and 70 percent bias ($CV < 1.6$) for the damping. Although we would never recommend using PARET on such degraded signals, this performance is indeed satisfying.

[*] The FDP results actually resulted in improved performance (smaller percent bias and coefficient of variation). This was largely due to code improvements that were incorporated after the single mode study was performed.

Figure VI.7. Performance of PARET
estimating mode 1 parameters

Figure V1.8. Performance of PARET
estimating mode 2 parameters

Figure VI.9. Performance of PARET
estimating mode 3 parameters

VII PERFORMANCE FOR MULTI (CLOSELY SPACED) MODES

The final step in our qualification procedure was to evaluate the performance of PARET on closely spaced multimodal data. To obtain this data we again used the finite element code SAP4 and modeled the piping system which

Figure VII.1 Piping system modeled by SAP4.

is shown schematically in Figure VII.1. This particular structure was chosen because it contains the desired closely spaced modes. Listed in Table VII.1 are the first 8 modal

Mode	Freq (Hz.)	Damp (1/sec)
1	5.636	.354
2	7.234	.455
3	7.919	.498
4	8.585	.539
5	10.840	.681
6	12.630	.794
7	13.730	.863
8	14.590	.917

Table VII.1 Modal parameters predicted by SAP4

frequency and damping parameters as predicted by SAP4. The

percent spacing factors between these modes (see equation 3)

PSF(1,2)	24.8
PSF(2,3)	9.0
PSF(3,4)	8.1
PSF(4,5)	23.2
PSF(5,6)	15.3
PSF(6,7)	8.3
PSF(7,8)	6.1

Table VII.2 Modal percent spacing factors for piping system.

are given in Table VII.2.

To obtain the time histories (simulated experimental data) at each node of this structure we excited node 11 (see arrow in Figure VII.2) in the vertical (Y) direction with a short duration triangular force pulse. For the same reasons discussed in the previous section, this force pulse may be considered an impulse and, consequently, the resulting time history motions of the structure are (essentially) the impulse responses. Shown in Figures VII.3 - VII.5 are selected time histories and their Fourier transforms at nodes Z3, Y11, and X15*.

[*] All nodes are labeled with a letter and a number. The letter refers to the direction of the motion while the number designates that particular node

Figure VII.2 Impulse excitation location of piping system.

Figure VII.3. Time history and
Fourier transform at node Z.3.

Figure VII.4. Time history and
Fourier transform at node Y11.

Figure VII.5. Time history and
Fourier transform at node X15.

To obtain the modal frequency and damping parameters of this structure we used two different techniques. Both obtained excellent results. The first technique was to average two of the stronger waveforms (X14 and X24) and then process the resulting waveform using TDP. The second technique was to process all the waveforms using FDP and then average the resulting modal parameters. The results from both techniques are listed in Table VII.3 (numbers in

• It is to be noted here that TDP did not find the

parentheses indicate percent disagreement with SAP4 - see previous section).

TDP		FDP	
frequency (Hz.)	damping (1/sec)	frequency (Hz.)	damping (1/sec)
5.6362 (0.00)	.354 (0.00)	5.6362 (0.00)	.358 (1.13)
7.2341 (0.00)	.456 (0.22)	7.2342 (0.00)	.457 (0.44)
7.9181 (0.01)	.497 (0.20)	7.9182 (0.01)	.499 (0.20)
8.5847 (0.00)	.540 (0.19)	8.5872 (0.03)	.554 (2.78)
10.8400 (0.00)	.681 (0.00)	10.8402 (0.00)	.670 (1.62)
12.6330 (0.02)	.794 (0.00)	12.6332 (0.02)	.806 (1.51)
		13.7288 (0.00)	.860 (0.35)
14.5930 (0.02)	.918 (0.11)	14.5919 (0.01)	.921 (0.44)

Table VII.3. TDP and FDP
modal parameters for piping system.

From this analysis we also obtained the first 8 mode shapes which are illustrated in Figures VII.6 and VII.7. As mentioned in the previous section, PARET also produces animated 16 mm. movies which show the structure (from several different angles) vibrating in each of its pure modes. Often this is the only way that the mode shapes of a complex 3-dimensional structure can be appreciated. The illustrations in Figures VII.6 and VII.7 were generated using several frames from such a movie. Note that the dots in the figure represent the normal, or rest, position of the structure.

13.728 Hz. mode. This is because that particular motion was not present at the nodes processed (X14 and X24). This points out the fact that some care must be exercised when using this technique to make certain that the nodes selected do in fact contain all the information of interest.

Figure VII.6. Modes 1-4 of piping system via PARET.

Figure VII.7. Modes 5-8 of piping system via PARET.

We next examined the performance of PARET on this data in the presence of noise. To do this we polluted the time history signal from node X14 with simulated measurement noise and generated several ensembles at various S/N levels (100, 70, 40, 20, 10). Shown in Figure VII.8 is a typical member of the S/N=40 ensemble and in Figure VII.9 is a portion of its Fourier spectrum. We judge the codes based on their ability to extract modes 2 through 5 (7.23, 7.92, 8.58, and 10.84Hz.). Shown in Figures VII.10 to VII.13 are the results of this study. It is interesting to compare these results to those for the single mode study (Figures V.5 and V.6). For the strongest mode (7.9181 Hz.) we obtained accuracies which were on the order of two and one-half times better than the single mode study. Again, we attribute this fact to the minor code modifications and improvements that were made as the project progressed. For the other three modes the results were comparable to the single mode study. However, as can be appreciated from Figure VII.9 these modes are only half as strong as the 7.9181 Hz. mode. Consequently, if we reduce the S/N level accordingly (see comments in previous section) we will obtain comparable results for all the modes.

Figure VII.8. Typical member of S/N=40 ensemble.

Figure VII.9. Fourier spectrum
of typical $S/N=40$ ensemble member.

Figure VII.10. Performance of PARET
estimating mode 2 parameters.

FREQUENCY

DAMPING

Figure VII.11. Performance of PARET estimating mode 3 parameters.

FREQUENCY

DAMPING

Figure VII.12. Performance of PARET estimating mode 4 parameters.

Figure VII.13. Performance of PARET
estimating mode 5 parameters.

VIII SUMMARY AND CONCLUSIONS

In this report we have described the analytical qualification of the two system identification codes that will be made available to the NRC as part of this project. During this study we were mainly interested in answering three questions. (1) How well do these codes perform in the presence of noise? (2) Which code performs best, the time domain or frequency domain version? (3) How is the performance of the codes effected by the closeness of the modes?

We were unable to find any substantial difference in performance between the two codes. Consequently, we will not recommend the use of one over the other. This decision is left to the user and will, no doubt, be made based upon his/her own personal preference as to frequency or time domain analysis.

During this study we used three different types of time history signals. They were: single modal, widely spaced multi modal, and closely spaced multi modal data. The percent spacing of the multi modal data (see equation 3) ranged from 100 down to 6.1. Based upon the results of this study we conclude that the performance of the codes is not a function of the modal spacing. That is to say, they perform just as well for the closely spaced modal data as for the widely spaced ones.

We examined the performance of the codes on noise polluted time history signals. In other words, we examined the ability of the codes to extract the modal parameters from these time histories as a function of the signal to noise level. As previously noted, their performance was not a function of the spacing of the modes. Therefore, based on the "averaged" results of Sections V, VI, and VII we have constructed the data in Table VIII.1. The purpose of this table is to give a rough idea, or ballpark estimate, of the accuracy (in terms of percent bias and coefficient of variation) of the codes as a function of the signal to noise level.

S/N level	Frequency		Damping	
	PB	CV	PB	CV
100	.04	$3.2 \cdot 10^{-4}$	1.2	$2.5 \cdot 10^{-2}$
70	.04	$3.2 \cdot 10^{-4}$	1.5	$3.0 \cdot 10^{-2}$
40	.02	$5.3 \cdot 10^{-4}$	1.7	$5.0 \cdot 10^{-2}$
20	.04	$10. \cdot 10^{-4}$	2.3	$10. \cdot 10^{-2}$
10	.10	$21. \cdot 10^{-4}$	3.3	$20. \cdot 10^{-2}$

Figure VIII.1. Estimate of accuracy
of PARET codes vs S/N level.

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