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ALO-3977-T1

A DEMONSTRATION SALT GRADIENT SOLAR POND

Fourth Semiannual Progress Report, November 1, 1978—June 30, 1979

By
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July 10, 1979

Work Performed Under Contract No. EG-77-S-04-3977

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MASTER

U.S. Department of Energy

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FOURTH SEMIANNUAL PROGRESS REPORT
to the
U.S. Department of Energy

[covering period 1 November 1978 - 30 June 1979]

"A Demonstration Salt Gradient Solar Pond"

D.O.E. Contract No. EG-77-S-04-3977

MASTER

10 July 1979

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Abstract

A thorough account is given of a solar pond study underway at the University of New Mexico since 1975. This report consists of the Ph.D. dissertation of Federica Zangrando entitled "Observation and Analysis of a Full-Scale Experimental Salt Gradient Solar Pond". The stability condition for a non-constant gradient, doubly-diffusive system is derived and a prescription for the routine detection of potential instabilities is presented. Techniques for handling unstable regions are described. Our conclusions so far strongly support the viability of the solar pond concept. Salt gradient ponds appear to have a very definite niche in the ecology of energy production.

PREFACE

Federica Zangrando's Ph.D. dissertation, which comprises the main body of this report, is the most comprehensive document yet to emerge from this country's research effort in salt-gradient solar ponds and is the result of over three years of single-minded, devoted effort by a very competent and energetic young physicist. This document is an account of how a full-scale solar pond was constructed, how problems were solved, a detailed thermal history, the results of heat extraction, and a discussion of maintenance requirements. In addition a detailed theoretical analysis of stability conditions for a salt-gradient pond is presented and the results compared with actual observations.

The general reader who wishes to get an introduction to pond technology might be advised to skip Chapter IV, Sections A and B, and, especially, Chapter V, which is a detailed mathematical analysis of the stability conditions. On the other hand, the reader who wants to know the results of the stability analysis need read only Section A of Chapter V. Table 5.2 summarizes the method of "Sublayer Stability Analysis" that Dr. Zangrando has developed, with some numerical examples. This analysis can be performed on a routine basis with an operating solar pond and can be easily computerized, as has been done with the UNM pond, using an MIK 11 minicomputer.

The principal new results to come from this work can be summarized as follows:

1. Salt gradient solar ponds ("ponds") are much more stable than previously thought. It was commonly thought that NaCl ponds were unstable. (Weinberger (1964) writes "A NaCl pond is unstable if energy is not removed from the bottom during periods of intense radiation.") In fact, it is possible to pump fluid in and out of the storage layer at high rates (60 gal/min) without disturbing the gradient.
2. The conditions for dynamical stability for actual ponds with non-constant gradients is the same as derived by Weinberger (1964) for a constant gradient (Chapter V). Checks on stability can be readily automated, with data provided (Appendix C).
3. Temperatures approaching 100°C can be reached in NaCl ponds. (The UNM record is 93°C, reached August 6, 1977. When the time to

decommission the UNM pond arrives, we plan to allow the pond to rise as high as possible, presumably to the point of boiling, about 103°C, in the storage layer.)

4. No internal partitions between convecting and non-convecting layers are needed, nor are they desirable. The interfaces between these two layers are quite stable. Indeed, a gradient in salinity cannot be maintained near an impermeable boundary as there would be no reservoir to supply salt to make up for that which diffuses away.

5. A thin convecting layer is inevitable on the surface of a pond. The reason is essentially because the surface is an impermeable boundary for salt diffusing up from below. The surface layer on the UNM pond tends to be at minimum about 10 cm. The presence of this surface layer is undesirable from the point of view of thermal efficiency, although it simplifies surface-cleaning operations.

6. Sloping walls in the region of the salt gradient are distinctly recommended against, based upon our experience with the UNM pond. The salt diffusing to the surface was 360 gm/m² day in the summer and 50 m/m² day in winter (see page 105), whereas a calculation done by M. Rothmeyer (1979) predicts 60 and 28 gm/m² day, respectively, for these values. The sloping walls of the UNM pond are very strongly suspected of being responsible for the excess salt transport (this would also contribute to a heat leak) by convection currents along the walls produced by downward distortions of the temperature and salinity isopleths. Work is currently underway to establish direct proof of the role played by this mechanism.

7. The average collecting efficiency for the UNM pond was 9% (see page 100) over the year November 1977-November 1978. The UNM pond was able to supply 71% of the needs of a "typical" 2000 ft² Albuquerque home during that period. The original pond design, based on the Rabl-Nielsen model (1975a), was to supply 100% of such needs. There seems to be reason to believe (see 6. above) that, with modification of the walls, this goal might have been met.

Now we come to the key question. "Is the salt-gradient solar pond concept economically useful in harnessing solar energy?" At this point the answer is a qualified "yes". It seems clear from the work reported here that the solar pond, although certainly not a panacea, may have a definite niche in the world's energy-producing ecology. What is that niche? The ideal situation in which a solar pond would fit best seems to have

the following specifications:

1. Salt and water cheap and plentiful. The water need not be pure or even potable. Sea water would be satisfactory. The water must only be free of suspended matter that could cause undue absorption of light.
2. Land available Even in an urban setting a pond might be welcome as a landscape element.
3. Little groundwater flow which could carry away heat stored in the ground.
4. A need for winter-summer averaging.
5. A need for low temperature heat $35^{\circ}\text{C} < T < 90^{\circ}\text{C}$.
6. The need for a pond large enough so that wall effects both on heat loss and salt transport would be negligible. A circular pond with sloping walls would have to have a diameter of over 150 m to reduce the salt loss due to wall convection to a level comparable to diffusion. Vertical, insulated walls could alleviate this problem, although they would add to the cost.

But why is the "yes" qualified? Because, so far, there are problems with ponds that hamper their usefulness in more general applications. The chief among these problems seems to be that of salt useage. Unless salt is very inexpensive and dilute saline solutions are easy to dispose of, the salt must somehow be returned to the bottom after it diffuses through the gradient to the surface. This problem is discussed in Section VII C. With a salt gradient pond there are two solutions: (a) expend some of the energy collected by the pond to condense the surface saline solution so that it may be reinjected into the storage layer, or (b) use a salt whose solubility is sufficiently dependent upon temperature so that as it diffuses into the less concentrated region where the temperature is less it finds itself supersaturated and "snows" salt crystals back down into the warmer bottom. This is called a "saturated pond" and is currently under study of the Desert Research Institute in Nevada (Tom Ochs) and in our Laboratory at UNM (Markus Rothmeyer).

Case (a), the recycling of surface salt back to the bottom, has been done at UNM by the "brute force" method of subsidiary evaporation ponds which consume sunlit area and are inefficient. To get an idea of the magnitude of this problem, we can calculate the minimum energy required to concentrate a 1% saline solution to a 20% solution, assuming we are

using reverse osmosis to perform this operation. This will give us the minimum energy because osmosis is a reversible process. If we make the unrealistic, but conservative, assumption that the solute is dilute, it is easy to show that the energy required per mole is

$$2RT \ln 20.$$

At 20°C, this works out to be 14.6 KJ/mole. Taking Markus Rothmeyer's (1979) summer figure of 1 mole/m² day, the minimum energy per m² per day to handle this problem is 14.6 KJ. Now in the summer we expect, in Albuquerque, to receive 30 MJ/m² day in insolation, which, even when taking account of the low efficiency, seems to mean that this problem should not be a major drain on the pond's resources. However, clearly, a "few" engineering problems need to be addressed!

Another problem that presently hampers the use of solar ponds, not unrelated to the salt problem, is that of minimizing the amount of human intervention required to keep a pond alive. On experimental ponds, such as ours at UNM, one can lavish large amounts of attention, but in the "real world" people must be paid well for their time. Again, the solution is one of engineering and the clear indication is that, for efficiency of human labor, one should make large ponds which are looked after by micro-processors.

Finally, then, it should be said that this report demonstrates that salt-gradient solar ponds do make sense and work well even in a climate of extremes such as New Mexico. The report also delineates the problems and offers some solutions. It is clear, as discussed in Chapter VIII, that more work needs to be done. We have passed another milestone along the road to "energy independence."

It seems appropriate, here, to acknowledge the beneficial influence of Professor Carl Nielsen on the New Mexico pond work. He was extremely helpful in the early stages of this project in his visits to New Mexico and in our visits to see his pond work in Ohio. His interest and support during this work helped contribute to its success.

Howard C. Bryant, 1979

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ACKNOWLEDGEMENTS

This research was conducted under Grant No. EG-77-S-04-3977 of the Department of Energy, Office of Conservation and Solar Applications, Solar Heating and Cooling R & D Branch, as part of a research program entitled: A Demonstration Salt Gradient Solar Pond, as well as under Grant No. ERB-161 of the New Mexico Energy Resources Board, as part of a research program entitled: Solar Ponds for Residential Heating.

The experimental facility was constructed on the grounds of the Department of Physics and Astronomy of the University of New Mexico; computations were performed on the CDC 6600 of the Los Alamos Scientific Laboratory.

The author wishes to express her thanks to the members of the dissertation committee, Dr. H.C. Bryant of the Department of Physics and Astronomy of the University of New Mexico (UNM), Dr. L. Bertram of Sandia Laboratories and the Department of Mechanical Engineering of UNM, Dr. V.H. Regener and Dr. C.P. Leavitt of the Department of Physics and Astronomy of UNM, and Dr. K.T. Feldman of the Department of Mechanical Engineering of UNM.

The author also wishes to acknowledge the support and cooperation received from the staff of the Department of Physics and Astronomy, UNM, as well as V.L. Moritz, Dr. T.D. Rupp and J. Rassouli.

OBSERVATION AND ANALYSIS
OF A FULL-SCALE EXPERIMENTAL
SALT GRADIENT SOLAR POND

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M.S., Physics, University of New Mexico, 1974
Ph.D., Physics, University of New Mexico, 1979

The objects of this research program were to study the physics of a double-diffusive system with strong, non-linear gradients as well as to analyse the operation and related engineering problems of a salt gradient solar pond to be used as a source of low temperature heat.

Double-diffusive systems in which two components with different molecular diffusivities contribute to the local density gradient have recently been the focus of interest as they exhibit motions unlike those of simple stratified fluids, and instabilities can arise even when the overall density is statically stable. Most research aimed at these systems studied the behavior of small constant gradients, as are found in oceans; little research has been done to understand the physics of systems with strongly varying fields. In a salt gradient stabilized solar pond, neither constant gradients nor constant properties can be assumed over the entire gradient region, as the temperature variation is large and the transport coefficients quite temperature sensitive.

Operation of a solar pond provided data indicating that the gradient breaks into small layers, generally localized to a small section of this zone. This suggests that stability is a function of local properties and local gradients only, instead of entire profiles. An analytical method was developed to predict the behavior of a fluid system with

localized constant properties, two diffusing components and arbitrary gradients in the steady state. In this analysis, classical, linear, constant gradients results are reproduced as a special case and numerical solutions for the non-constant gradients case are obtained by variational principles. The results, applied to actual solar pond data, predict sublayer instabilities accurately for a variety of gradient configurations. The results also confirm that the sublayer instability is the lowest mode of the whole layer solution. This approach, in addition, gives the scale of the sublayer which will overturn as well as general requirements for the optimal salinity gradient necessary to operate successfully. The sublayer stability analysis procedure developed allows for easy evaluation of solar pond stability in the field.

The previously derived one-dimensional steady-state model was modified to include heat losses to the ground and input reduction due to dirt trapped in the gradient region. Comparison of results to the temperature response of the pond, monitored during the first three years of operation and thus still in a transient state, yielded fair agreement.

The solar pond built at the University of New Mexico was designed to supply the full heating and hot water load of a 185m² house in Albuquerque. During the winter season of 1977-78, the pond was under continuous heat extraction to supply the equivalent space heating demand; for the remainder of 1978; the equivalent of the hot water requirement was extracted, while in the heating season of 1978-79 the pond was loaded with the total demand. No adverse effects were registered at any time, even though brine was purposefully reinjected in a manner that

maximizes disturbances in the storage region.

While the analysis of stability conditions was in progress, the pond was operated at very low salinity gradients. Instabilities which occurred when below the stability limit were corrected by fluid injection into the gradient, a previously untested procedure. This method to reestablish the gradient within the pond itself does not require massive transport of brine or loss of energy and led to the development of a new procedure for filling solar ponds, successfully tested in a 2,000m² pond.

Solar Pond, Analysis of
Solar Pond, Experimental
Solar Pond, Full-Scale
Salt Gradient Solar Pond, Stability of
Salt Gradient Solar Pond, Operation of

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CHAPTER I

INTRODUCTION

A salt gradient solar pond is a body of water in which a density gradient, positive downward, is maintained artificially to prevent convection; this gradient is generally established by the addition of salt. The light penetrating into the water is absorbed and a temperature gradient, also positive downward, is established as heat losses occur only by conduction and radiation. To maintain a non-convective regime, the salinity gradient must remain large enough to offset the buoyant effect of thermal expansion. As a non-saturated salinity gradient implies continuous salt migration to the surface, the imbalance in salt must be maintained artificially by replenishment of salt to the bottom and fresh water or low salinity brine to the surface. If a large convective layer is established or allowed to develop under the gradient region, it will act as thermal storage, insulated from the atmosphere by the gradient above.

The class of double-diffusive fluid systems, from oceans to stars, in which two components with different molecular diffusivities contribute to the local density gradient, has been the focus of great interest in the fields of fluid dynamics and astrophysics; these systems exhibit motions very different from those of simple stratified fluids, and instabilities can arise even when the overall density is statically stable. However, relatively little work has been done to understand and predict the physical behavior of systems with strongly varying fields, such as

temperature and salinity in a salt gradient stabilized solar pond. These have become a subject of interest in recent years due to the possibility of using them as low temperature solar collectors but, with few exceptions, research has been geared to the engineering aspects associated with operation and heat removal. The main research effort in this area is presented in Section A, while Section B lists some of the currently operating solar ponds, both for experimental and for demonstration purposes.

A. Historical Development

Natural occurrence of solar lakes was reported for the first time at the turn of the century. Von Kalecsinsky (1902) described unusual phenomena in the Medve Lagoon, Transylvania, where temperatures in excess of 70°C had been measured deep in the lake, while the surface temperature remained close to ambient. In his paper, von Kalecsinsky also suggested the establishment of artificial solar lakes to harness the energy of the sun for home and industrial purposes.

Similar conditions have been found in a few places around the world. In Antarctica for example, at latitude 77° south, Lake Vanda exhibits temperatures around 25°C at a depth of 60m, even though the surface is permanently covered by 3 to 4m of ice. Temperature scans taken at different locations show that isothermal layers are horizontal over the 8km length of the lake (Wilson and Wellman, 1962; Hoare, 1966). In Venezuela, at latitude 12° north, temperatures around 45°C have been measured just one meter below the surface of a lagoon in the Los Roques Islands (Hudec and Sonnefeld, 1974). Similar circumstances have been reported in the North-American continent with Hot Lake, Washington (Anderson, 1958) and

in Africa with Lake Mahaga, Uganda (Melack and Kilham, 1972).

Research on these lakes was in general conducted by biologists and limited to data collection for short intervals of time. Systematic analysis of the physical processes and long term behavior were never performed. These reports, however, have consistently shown a non-uniform vertical distribution of salts, commonly established by salt leaching at the bottom of the lake, while a river (or the sea) supplied fresh water (or sea water) to the surface. In this manner a natural salinity gradient is established and maintained, preventing convection. The density gradient thus established allows for a temperature gradient to be present, positive downward yet stable, and for characteristically higher temperatures at the bottom of the lake. The solar flux, which is mostly visible and ultraviolet, penetrates deeply into the pond. Since water is a poor conductor of heat and it is also opaque to infrared, the energy is effectively "trapped" within the lake's lower region.

The second reference for the possible use of solar ponds as solar collectors appears to be in 1948, when Dr. R. Block suggested a research program to the Israel National Research Council. It was only ten years later, however, that interest in the physical processes of solar ponds began to receive the funding necessary to implement the research. Weinberger (1964) analysed the one-dimensional heat conduction equation with internal heat generation and obtained predictions for the time dependent temperature distribution in the gradient region; he also analysed energy withdrawal effects and predicted pond efficiencies above 20%. Aspects of solar pond hydraulics were analysed by Elata and Levin (1965) who presented a criterion for withdrawal velocities of selected layers as a function of pond parameters, recognized the importance of

surface control against wind mixing and reported on the suggestion of the "falling pond" method to maintain the net migration of salt at zero. These results were also reported in a paper by Tabor and Matz (1965).

A team of scientists at the Uzbek Academy of Sciences has explored several items related to performance of solar ponds such as optical characteristics of magnesium chloride solutions (Usmanov et al., 1971), temperature dependence on depth (Eliseev et al., 1971), rates of evaporation (Usmanov et al., 1973a), optimistic efficiency predictions for shallow ponds (Eliseev et al., 1973) and brine withdrawal from a shallow experimental tank without creating mixing (Usmanov et al., 1973b).

The main thrust of the research, however, seemed to be directed more to the engineering of solar pond operation and to performance forecasting of large scale schemes. Israeli scientists had great interest in this concept for power production and the first experimental ponds began to operate around 1959. Tabor (1963) reports on several small tank experiments in which sodium chloride failed as a stabilizer even at small overall temperature differences. This stigma unfortunately remained until 1975, when ponds in the United States showed excellent performance with sodium chloride stabilization. The critical difference between these results is the geometrical configuration of the ponds. Tabor and co-workers were interested in the shallowest possible ponds for immediate use with a turbine. No storage was required hence any convective layer at the bottom of the pond meant a loss in efficiency by decreasing the operating temperature. In periods of high insolation, however, the temperature gradient near the bottom of such shallow ponds became so large that stabilization was impossible. This result was

apparently misunderstood by following workers who inferred that the failure of sodium chloride for shallow ponds implied failure as a stabilizer in general. Proximity to salt works allowed a varied choice of brine and subsequent experiments were performed with magnesium and potassium chlorides and other mixed end brines. Results seemed to be even less encouraging as everything that could have gone wrong, did. A 25m x 25m pond near the Dead Sea was plagued by surface convection, wall leakage and disintegration of the wooden walls. A second pond at the same location failed because of a porous bottom which resulted in an intermittent loss or gain of liquid, depending on the water logging of the adjacent ground. The third pond, located near Haifa, failed because of gas bubbling through the gradient; analysis of the soil showed rapid bacterial decomposition in the marshy ground below the pond (Tabor and Matz, 1965). Although negative, these results enlightened researchers on precautions to be taken before designing and installing new ponds. It is astonishing that even with so many problems temperatures as high as 96°C were registered for short periods. Around 1965, research in Israel was interrupted for about ten years, probably because, at the time, solar ponds could not compete with conventional power plants for electricity production.

Other proposals for experimental ponds came from Hirschmann (1970) who suggested converting some Chilean salt marshes into solar ponds and proposed a scheme to use solar ponds to preheat low salinity brine for desalination in addition to providing electricity to the plant (Hirschmann, 1965). Jain (1973) operated an experimental pond in India and extracted heat for four successive months; highest temperature obtained was 70°C. Chepurniy and Savage (1974) presented a finite-

difference computer simulation of a laboratory model to predict temperature response; Saulnier and co-workers (1975) maintained a small outside tank in operation for several runs of a few months, reaching a maximum bottom temperature of 82°C.

In 1974, Rabl and Nielsen (1975a) clearly suggested that solar ponds should be geared to heating instead of electricity production, because ponds could supply an inherent storage system at little additional cost. They proposed to include a large storage layer as a means to provide seasonal storage for winter home heating. This storage region would make the pond relatively insensitive to weekly fluctuations of the insolation and ambient temperature, while facilitating heat removal procedures, since this region would be large, homogeneous and convective. With this suggestion, sodium chloride was reinstated as the cheapest and most likely candidate for stabilization. Their one-dimensional derivation of the steady state temperature response of a solar pond with large storage predicted that solar ponds could operate successfully even in very high latitudes. Experimentation with a full-scale pond in Columbus, Ohio was very encouraging (Nielsen, 1976) as were results obtained in the same period at the University of New Mexico (Zangrando and Bryant, 1976). Following these successful experiments, a large solar pond was constructed in Minnisburg, Ohio; the pond is also stabilized by sodium chloride and will provide heating in the municipal recreation complex.

Renewed interest in solar pond operation in Israel appeared in the last three years, with proposals for experimental ponds, desalination plants powered by solar ponds (Tabor, 1975) and for turning a portion of the Dead Sea into a solar pond (Assaf, 1976). Electricity production

is still considered and experiments with small turbines are presently under way (Rambach, 1978).

Economic estimates have been directed at very large applications and are very favorable to heating applications (Zangrando and Bryant, 1977a; Styris et al., 1975, 1976); electricity production is also seriously considered (Drumheller et al., 1975), though uneconomical at present.

B. Present Solar Pond Facilities and Ongoing Research

A full-scale salt gradient solar pond has been in operation at the University of New Mexico (UNM) since 1975. The object of the research has been to study the long term physical behavior of this system when exposed to the environment, principally the stability of the gradient and the temperature response. In addition, research has been geared to establishment and modification of the gradient as well as evaluation of materials and equipment used.

Four ponds are presently in operation in Ohio. Two are under the supervision of Dr. C. Nielsen of the Ohio State University Physics Department. The first pond has provided information on operation, temperature response and development of the surface convective layer (Nielsen, 1976) while the new pond should provide extensive information on the heat transfer to the ground as well as the effect of a vertical reflecting wall to the total collecting efficiency (Nielsen, 1977). This pond will be used to provide hot wash water for a dairy adjacent to the site. The upper portion of the pond has been constructed with vertical walls to eliminate possible convection due to slanted walls in the gradient region. The third pond is in Wooster and has been in operation since 1975 also.

Several leakage problems have plagued this pond which was built with vertical reinforced walls and a fitted liner. The temperature regime has been fairly low due to the additional absorption of the greenhouse cover (Badger et al., 1977). The last and largest pond is in Miamisburg. The municipality built a 2,000m² pond to heat the public swimming pool and provide hot shower water in the summer, as well as some heat to the adjacent building in the winter (Wittenberg, 1978). Filling of this pond already provided valuable supportive information on a new filling technique developed at UNM (Zangrando and Bryant, 1977b).

As mentioned previously, experimental work has been revived in Israel with the construction of three solar ponds, one of which has been built to supply cooling and hot water for a resort hotel near the Dead Sea. This pond will use brine from the Dead Sea itself and it has been built deep enough to allow for a large storage convective layer at the bottom. The other ponds have been used to run experiments with secondary fluid turbines. Results appear to be very encouraging (Rambach, 1978). All ponds in Israel use salts other than sodium chloride for stabilization.

Any salt as well as any natural brine can be used to establish a solar pond; the criterion for selection of a particular salt should be based on the following factors:

1. It must be safe to handle and environmentally sound; disposal problems associated with toxicity and ground water contamination must be carefully evaluated.
2. It must be cheap and readily available; the proximity to a large supply of a particular salt will weigh favorably on the choice.
3. It must not reduce the optical transmission characteristics of water; salts which enhance organic growth, those which form

colloidal suspensions or simply color the water should be avoided.

4. The solubility should be strongly temperature dependent in order to reduce or eliminate diffusion; a saturated pond would have a stable gradient with no diffusion.

Sodium and magnesium chlorides satisfy only criteria 1, 2 and 3; other salts which have excellent temperature dependence do not satisfy the remaining necessary conditions. Salts which satisfy criteria 1,3 and 4 might be ideal as they would allow the pond to be in an entirely saturated state and therefore eliminate problems with diffusion. This fact alone may render them economically justifiable as there would be no replenishment necessary and no maintenance requirements. A laboratory study of sodium and potassium sulfate as well as potassium nitrate is under way at UNM with very encouraging results.

Finally, the alternative of viscous stabilization has been presented recently (Shaffer, 1976); in this approach, convection of the upper region is prevented by addition of primarily organic polymers to gel the pond and maintain it statically stable in the presence of a temperature gradient. Research in this area is proceeding with the testing of gelling substances in small laboratory tanks (Atwater, 1978).

CHAPTER II

THE UNM POND

The solar pond at the University of New Mexico (UNM) was built in the Fall of 1975 with dimensions: top diameter, 15m; depth, 2.5m; bank angle, 34° from horizontal; surface area, 175m^2 ; base area, 44m^2 ; average collecting area, 105m^2 . The average collecting area is defined as the area at a depth of 1m from the surface. As the pond walls are very slanted, much of the sunlight at the surface never reaches this depth because of absorption on the walls. This energy is therefore deposited in the gradient and not in the storage layer. Total capacity of the pond is 230m^3 , the average storage volume in the bottom convective layer is 130m^3 . Figure 2.1 is a schematic diagram of the pond.

The pit was excavated to about one half of the desired depth and the dirt removed raised the banks above the level of the surrounding ground for drainage. The walls were then smoothed and compacted and 5cm of sand added to the bottom to prevent possible liner perforation, as no insulation separates the liner from the walls. Although the sloping walls result in a large loss in volume compared to vertical walls, the former do not require retaining work. The circular shape of the pond was originally chosen to facilitate mathematical modelling of its behavior but it also maximizes the volume per wall area.

The liner was installed directly by pulling it into position. As the pond was circular while the liner was manufactured into one rectangular sheet, the excess of material from the corners was glued and used

FIGURE 2.1 - Schematic diagram of the UNM solar pond.

to line the subsidiary mixing pond, of dimensions: 8m x 5m x 0.5m. Two additional shallow evaporation ponds have been constructed on the site to evaporate the run-off brine from the surface of the solar pond and collect the diffused salt for recycling; their dimensions are 14m x 7m x 0.1m and 13m x 6.5m x 0.3m. Both evaporating ponds (1 and 2) and mixing pond are built at successively lower ground levels so that brine can be syphoned from the solar pond through to the mixing pond, where a pump must then be used to return the brine into the solar pond. Should pumping be necessary, independent connections and valving are provided and brine can also be removed or injected into the solar pond at any level.

The initial filling of the UNM pond is described in Chapter III; the desired final configuration was about 1m of gradient, the remainder 1.4m as storage. Originally, Rabl and Nielsen (1975a) suggested separating these two regions with a partition: if the partition were stiff, fresh water could be used in the storage region without problems due to buoyancy effects; if the partition were of thin plastic, brine at the maximum gradient concentration would be used. As the cost of a stiff partition is high and the installation is not trivial, a salty convective layer was chosen, with no partition at all. In this case the solar pond contains no physical partition but is still divided into two regions: the upper one (gradient) in which the salinity content changes from no salt at the surface to a maximum concentration at the "boundary"; the lower one (storage) which extends from this "boundary" down to the floor of the pond and which is homogeneous at the maximum concentration. An additional convective layer of much smaller size appears at the surface,

generally after a few days of operation; although undesirable, its presence seems inevitable at this point.

Temperature monitoring of the ground around the pond is accomplished with 36 thermocouples, buried at known depths in the ground below and in the side walls. See Figure 2.1 for their location. Two motor-driven thermocouples scan the depth of the pond to monitor the internal temperature distribution. The density distribution in the pond has been measured by weighing samples extracted from different levels. As this method provides only discrete data points, a conductivity probe has also been used to supply continuous output proportional to the local density. Although the accuracy of the information provided by the probe would be sufficient for operational requirements, it was marginal, though successful, for the analysis of stability conditions; these are described in Chapter V.

A black and white pyranometer has been installed near the pond to measure the total (direct and diffuse) radiation on a horizontal surface. The signal is displayed on a strip chart recorder with integrator.

Initially, all data obtained were recorded on strip charts or manually. Later, a data logger digitized all information received on a daily basis and produced a paper tape on which all data were stored. More recently, the data logger was connected directly to a computer for storage on floppy disks and direct analysis. A trailer adjacent to the pond houses all monitoring instrumentation except for the computer which is located in the main building. Details of the equipment and materials used at the UNM pond can be found in Appendix A.

Temperature, insolation and heat extraction data have been collected on a daily basis except for a few occasions; the density distribution,

which changes only slightly over long periods of time, has instead been monitored on a weekly basis, due to the lengthy sampling procedure.

Exceptions were times of gradient breakdown, when the density was sampled as necessary. The insolation data recorded near the pond are also supplemented by records from the Weather Service, located a few miles south of the University, which also provides all other daily climatological data. Connection to the computer allows for automatic daily data collection and immediate reduction; data will therefore be processed every day as soon as evaluating programs are ready.

CHAPTER III

ESTABLISHMENT AND MAINTENANCE OF THE GRADIENT

A. Filling Procedures

Establishment of a linear salinity gradient in an artificial solar pond can proceed in several ways, dependent on local conditions, size and time schedules. Three proposed ways are as follows:

1. Natural Diffusion - A layer of fresh water is superposed onto a layer of high salt concentration. The procedure would be to mix salt and water in the pond to produce the high salinity layer, then to add fresh water above. If the supply of fresh water is large, the mixing at the boundary will enhance salt redistribution. The salt migrates to the surface by diffusion and, if the top and bottom concentrations are kept constant, a salinity gradient will be established in time. This of course is the simplest but also the slowest process; it should only be considered if the pond is very large, of the order of the Great Salt Lake for example, or if starting time is unlimited.
2. Stacking - After mixing a high concentration solution in order to establish the storage layer, batches of progressively lower salinity brine are "stacked" one over the other until fresh water is added to the surface. This is the method used until present by most researchers in the field and was used originally at the UNM pond. For laboratory tanks and very small ponds, this method appears to be the most efficient way to establish a gradient. In

larger operating ponds, however, this method has at least two drawbacks. One is that a mixing tank is required and, when volumes are large, the additional cost and time could also be quite large. Two, the layers are continuously exposed to wind action throughout the filling process and mixing can occur beyond the desired amount. This method, therefore, requires continuous adjustment if climatic conditions are unfavorable.

The solar pond at UNM was filled using this technique for 28 consecutive hours in November 1975. A convective layer 20cm thick containing 15% sodium chloride by weight was established in the pond. Above this, fifteen layers about 4cm thick were pumped one over the other through a diffuser floating on the brine (Appendix A). Batches of brine were mixed in the subsidiary pond, starting at a 14% concentration, the correct volume was pumped into the solar pond and the brine in the mixing pond was diluted to a 13% concentration; the procedure was repeated until fresh water was added to the surface. At completion, the solar pond contained a gradient region 70cm thick, for a total depth of 90cm. The pond was not filled to capacity to allow experimentation with the gradient through the winter. In the Spring and Summer of 1976, the pond was progressively filled to capacity by injection of high salinity brine into the convective layer.

The following new method, instead, is the easiest and most expedient way to establish or modify a gradient. It was originally tested at the UNM pond (Zangrando and Bryant, 1976) to modify portions of the existing gradient and it has been successfully applied to establish the 1.3m thick gradient of the newly built solar pond in Miamisburg, Ohio, which

is currently the largest operational solar pond in the United States, with a surface area of 2,000m².

3. Redistribution - The pond is partially filled with high salinity brine and then fresh water is added through a diffuser which either moves continuously or steps through the upper portion of the existing solution. Steps of no more than 5cm are advised. The brine above the diffuser will be progressively diluted while the diffuser, as well as the water level, rise. Timing must be such that the diffuser reaches the surface at the predetermined final level of the pond. In the event the fresh water supply is insufficient to provide a good water flow in the diffuser, the fresh water can be added to the pond initially, floating on the high concentration layer. Using this as the fresh water reservoir, the procedure mentioned above can be performed in the same manner. The surface layer would shrink at the same rate as water would be pumped through the diffuser and injected below, but it would still remain at the top, floating on the partial gradient. A small amount of mixing may occur at the end of the process, leaving a surface convective layer; current experience with operating ponds shows that such layer, if initially absent, generally appears within two to three days from filling. The results would therefore be the same. The rate of redistribution appears to be quite flexible and depends on the capacity of the pumps used as well as the concentration gradient required. The entire operation can be performed automatically or manually, continuously or intermittently, without detrimental effects to the partial gradient. In the Miamisburg pond, for example,

redistribution was only performed during the daylight hours, for seven consecutive days.

Suppose, as the simplest example of redistribution, a vertical wall pond of area A and total proposed depth $D = L + H$, divided in a storage convective zone of depth L and concentration C_c and a linear gradient zone of depth H and average concentration $\frac{1}{2}C_c$ (to reach ideally a zero concentration at the surface). Suppose then that the pond is originally filled to a depth of $L + \frac{1}{2}H$ with brine at the maximum concentration C_c . All of the salt required for the establishment of the entire pond has been dissolved, but a volume $\frac{1}{2}HA$ of fresh water is still missing (the small volume of salt is neglected in this example).

If a diffuser is placed in the water at height L (the top of the final storage region) and raised at a constant rate, fresh water will be injected into the upper region of the pond, mix with the brine above to make a homogeneous layer and progressively dilute this shrinking region while the pond fills up to capacity D . The diffuser must therefore be raised at a rate proportional to the surface motion so that the diffuser reaches the surface at position D above the liner.

If the position of the diffuser at any time is y_d , starting from height L , while that of the surface is y_s , starting from height $L + \frac{1}{2}H$, then the problem can be stated as

$$y_d = c_1 y_s + c_2 \quad \text{with} \quad \begin{cases} \text{at } t=0 & \begin{cases} y_d = L \\ y_s = L + \frac{1}{2}H \end{cases} \\ \text{at } t=t_f & y_d = y_s = L + H \end{cases}$$

hence

$$y_d = 2y_s - (L + H) \quad \text{and} \quad \dot{y}_d = 2\dot{y}_s$$

where the dots refer to time derivatives and $\dot{y}_s$ is of course the flow into the diffuser divided by the area of the pond. At any time t , the volume of brine above the diffuser and the changes in mass and concentration at each point are, respectively,

$$V_a(t) = (y_s - y_d)A ,$$

$$\dot{C} = - \frac{C(t) \dot{y}_s A}{(y_s - y_d)A} ;$$

$$\dot{M} = - C(t) \dot{y}_s A ,$$

by substitution from above, the equation to be solved becomes

$$\frac{dC}{C} = \frac{-dy_s}{L + H - y_s} \quad \text{with} \quad \begin{cases} \text{at } t=0 & \begin{cases} C = C_c \\ y_s = L + \frac{1}{2}H \end{cases} \\ \text{at } t=t_f & \begin{cases} C = 0 \\ y_s = L + H \end{cases} \end{cases}$$

hence

$$C(x) = 2C_c \left(\frac{L + H - y_s}{H} \right) = 2C_c \left(1 - \frac{x_s}{H} \right) ,$$

where the vertical scale has been shifted up and $x_s = y_s - L$.

The total time required for this operation is determined only by the capacity of the pumps and the geometry of the pond. Although neither size, time scale nor rate of water addition affected the tests run at UNM and the filling at Miamisburg, the range of parameters tested so far is fairly small and is described in Appendix B.

The success of this method rests on the mixing which takes place in the volume above the diffuser, observed to be quite good. This can be seen most clearly in the data taken at the Miamisburg pond during the filling operation. Figure 3.1 presents subsequent specific gravity measurements taken while the diffuser was injecting water into the upper

FIGURE 3.1 - Specific gravity profiles in the Miamisburg solar pond during the filling operation. Reprinted with permission from the Miamisburg Solar Pond Group, Monsanto Laboratory, Miamisburg, Ohio; courtesy of L. Wittenberg.

Time sequence develops from the solid dot markers (shortly after the beginning of fresh water injection at a depth of about 1.3m from the desired final level) to the square markers (at the end of the operation). The region above the diffuser maintained an approximately constant salinity, at fixed time, except for small variations in the first 30cm above the diffuser. Concentration above the diffuser decreased progressively in the seven days of filling, leaving an almost linear gradient. The diffuser was stepped up by 5cm every 2 hours; the flowrate was 7.4l/s with an exit velocity of 0.6m/s.

zone. A single diffuser was used to cover the 2,000m² and each set of data shows a fairly constant concentration above the diffuser. The dashed curve represents the distribution at the end of filling. Details of equipment and rates used can be found in Appendix B.

B. Maintenance of the Gradient

As the working principle of a salt stabilized solar pond requires a salinity gradient, diffusion of salt from the high concentration region (storage layer) to the low concentration region (surface) takes place continuously. The tendency is therefore towards destruction of the gradient, unless salt is removed from the surface and replenished into the storage layer. This is accomplished by flushing the surface of the solar pond with fresh water or low salinity brine, collecting and evaporating the run-off and subsequently reinjecting the concentrated brine into the convective layer.

A different method has been suggested by Tabor and Matz (1965), denoted as the "falling pond" method. In this case hot brine is extracted from the convecting layer and passed through a flash evaporator to produce concentrate and steam. In this manner, water is extracted from the brine and the steam can be used, as an example, for electricity production. The concentrate, of smaller volume and higher salinity than the brine extracted originally, is returned to the bottom of the pond, lowering the total level; fresh water is then added to the top of the pond to bring the level back to the original position. If the drop in volume matches the diffusion rate, there is no net migration of salt and the gradient is maintained.

In the event undesired convection appears in the otherwise non-

convecting region, the gradient is observed to restore itself only if the vertical dimension of these layers is less than 3 or 4cm; otherwise redistribution is necessary. These instabilities can be corrected by using the same type of scanning injection described in the previous section, with greatly reduced water flow, as the object is not to add water or brine but to stir above or below the diffuser in order to reduce or increase the local salinity. The scanning rate does not have to remain constant and can be adjusted according to the degree of mixing required.

The procedure is to use cold, low salinity brine from the surface or hot, high salinity brine from the storage convective layer to inject into the particular region of gradient where instabilities have occurred. It has been observed that the starting position can be anywhere within the gradient but the procedure must terminate in an already existing convective layer (surface or storage layer), otherwise a new internal layer will appear; the stability analysis indicates that this effect is due to the residual dent left in the salinity profile.

Crude observation of the flow pattern around the diffuser shows a mixing region of about 15 to 20cm above or below the diffuser, depending on whether rising fresh water or sinking heavy brine is used as the diffuser fluid. This provides the mixing necessary to modify the gradient configuration. In both instances salt is effectively transported to the surface and this procedure should therefore be used sparingly, only to prevent breakdown or shift the position of gradient boundaries.

Experiments at the UNM facility suggest the following selection: if the gradient above the instability is strong, fresh water from the surface

of the pond is pumped through the scanning diffuser; if the gradient below is weak, high salinity brine from the convective layer is used. The mixing created by pumping brine into the solar pond, plus the internal circulation due to the difference in temperature and density between the brine at a particular level and the brine injected, do not further convection; instead they generally reestablish the gradient within one day. This observation appears to be in conflict with results obtained by Turner and Chen (1974) in laboratory tanks where small horizontal differences in density and fluid composition led to the establishment of convective layers separated by sharp interfaces.

In the few instances of complete gradient breakdown, hot brine from the convective layer has been syphoned to the auxiliary basin, mixed with more salt and pumped into the solar pond with the diffuser scanning from the top of the instability down to the convective layer, increasing the salinity of the entire pond. This procedure has been observed to drop the temperature of the convective layer by a few degrees only.

Two examples from data collected at the UNM facility before and after such redistribution are presented in Figure 3.2. Part (a) depicts the effect of redistributing high salinity brine taken directly from the storage convective layer; the scanner was placed at 45cm below the surface and was lowered to 75cm at the rate of 1cm/min. Temperature scans taken on the following day showed one convective layer about 3cm thick still present at 45cm; two days later, this layer disappeared spontaneously and the resulting salt distribution is shown by the dashed curve. As can be seen, salt was effectively shifted upwards, through mixing. The accuracy of the data is unfortunately not sufficient to compute a

FIGURE 3.2 - Effect of redistribution on the gradient. (a) Injection of high salinity brine from 45 to 75cm, with the diffuser moving downward at a rate of 1cm/min. (b) Injection of low salinity brine from 50cm up to the surface, at the rate of 1cm/min. The diffuser assembly is described in Appendix A and shown schematically in Figure A.1.

salt balance. Part (b) instead depicts the effect of redistributing low salinity brine taken from the surface of the solar pond; the scanner was placed at 50cm and raised to a few centimeters from the surface at the rate of 1cm/min. Once again, a small convective layer was left behind at about 50cm, but it disappeared spontaneously by March 18. In both instances, the volume of brine redistributed is small (4 to 6m³) yet the effect produced is quite noticeable. This technique has also been used successfully at the Ohio State pond (Nielsen, 1976).

CHAPTER IV

TEMPERATURE RESPONSE OF THE SOLAR POND

Predictions for the annual temperature response of a solar pond with a large storage layer have been presented by Rabl and Nielsen (1975a). Their one-dimensional, steady-state derivation, using sinusoidal insolation and ambient temperature, both with a period of one year, is applicable to a pond of infinite lateral extent and thus without edge losses. These assumptions allowed for a closed form solution which should be considered only as a guideline for possible temperature ranges as the results are an upper bound to the possible temperature response of actual operating ponds. Their derivation for the time dependent component of the storage layer temperature is rather tedious; simplification of this derivation by one additional approximation gives much simpler yet equivalent results. A second approximation can also be introduced easily to simulate the effect of edge losses and thus give a more realistic temperature prediction for operating ponds. This will be discussed in Section B, while data collected during three years of continuous operation will be presented in Section C.

A. Light Penetration

Water, with or without dissolved salts, is opaque to long wavelengths; this characteristic, which prevents the convective layer from radiating the energy absorbed, also prevents a portion of the sunlight impinging on the surface of the pond from reaching the convective layer

itself. Solar energy reaching the pond surface is partially reflected, absorbed and transmitted; absorption of the long wavelengths is an important effect as 27% of the total solar spectrum is absorbed within the first centimeter of water. The collection efficiency is therefore limited by the absorption of water in the surface and gradient layer.

Rabl and Nielsen (1975a; hereafter referred to as R&N) have shown that the insolation present at depth x can be approximated well by dividing the solar spectrum into four bands, each with an effective attenuation length. Thus

$$H(x) = H_s \tau \sum_{n=1}^4 \eta_n e^{-\mu_n x} \quad (4.1)$$

where H_s is the insolation at the surface, τ the transmission coefficient, μ_n the absorption coefficient for a given range of wavelengths and η_n a constant. The values reported are

$\eta_1 = 0.237$	$\mu_1 = 0.32 \cdot 10^{-3} \text{cm}^{-1}$		0.2 - 0.6 μm
$\eta_2 = 0.193$	$\mu_2 = 0.45 \cdot 10^{-2}$	for	0.6 - 0.75
$\eta_3 = 0.167$	$\mu_3 = 0.03$	wavelengths	0.75 - 0.9
$\eta_4 = 0.179$	$\mu_4 = 0.35$		0.9 - 1.2

with the remaining 22.4% of the input absorbed at the surface.

Bryant and Colbeck (1977) have shown that the fraction of light remaining after passing through a thickness x of clear water can be expressed as accurately, in the range 1cm to 10m, by

$$f(x) = a - b \ln x \quad (4.2)$$

where x is the depth of water in centimeters, $a = 0.73$ and $b = 0.08$.

This expression is of course not valid for $x < 1\text{cm}$. The similarity

between these two forms is shown in Figure 4.1 where $f(x)$ is compared with $H(x)/H_s$ and $\tau = 1$. Experimental data obtained at the UNM pond agree only approximately with these predictions. A solar cell scan without temperature compensation is superposed on Figure 4.1 and is presented for illustration purposes. In addition, compensated data from a scanning pyranometer are also presented, although these are not definitive. Light measurements are difficult to make accurately because of the large temperature gradient and the variation of solar spectrum as well as index of refraction with depth. A commercially produced submersible pyranometer will be used in future experiments. As a further complication, the two expressions presented above describe the transmission of rays normal to the surface of a perfectly clean sample. As the angle of incidence of sunlight is a function of time, both daily and annually, R&N suggest introducing effective absorption coefficients defined as $\bar{\mu}_n = \mu_n / \cos r$, where the $\cos r$ term refers to an average refractive angle computed from the position of the sun at 2 p.m. of equinox. This same correction could be applied to eq.(4.2).

B. The One-Dimensional, Steady-State Result

Neglecting daily variations, temperatures, insolation and heat removed from the solar pond can be taken as quantities consisting of the superposition of an average component and a sinusoidally varying component; all have a period of one year, but are out of phase with respect to the insolation. These can be expressed as

$$\begin{aligned} T &= \bar{T} + \tilde{T} \exp i(\omega t - \phi) & H &= \bar{H} + \tilde{H} \exp i(\omega t) \\ T_a &= \bar{T}_a + \tilde{T}_a \exp i(\omega t - \phi_a) & Q &= \bar{Q} + \tilde{Q} \exp i(\omega t - \phi_q) \end{aligned} \quad (4.3)$$

FIGURE 4.1 - Fraction of light remaining at depth x . The markers on the curves from eq.(4.1) and eq.(4.2) have been used only for clarity.

where T is the convective layer temperature, T_a the ambient temperature, H the insolation and Q the heat load. The bar represents average quantities while the tilde implies amplitudes of the sinusoidal components. This notation is similar to that presented by R&N. In their discussion, R&N analyse an infinite pond at steady-state which implies no net heat loss to the ground, then apply the results to finite size ponds. In addition, the heat requirement for winter heating is spread throughout the year, with $\tilde{Q} = \frac{1}{2}\bar{Q}$, which implies heat removal in the summer to compensate for the heat not taken out in the winter. This does not affect the average temperature calculation, but it can reduce the amplitude of the convective layer temperature, depending on the phase of the heat removal cycle chosen.

A much simpler expression for the predicted pond temperature can be reached by neglecting heat storage in the gradient region and in the ground. This simplification does not change the R&N result greatly, as most of the storage takes place in the large convective layer anyway.

With this assumption in mind, the equation to be solved is

$$H(x=l_i) = k_w \left. \frac{\partial v}{\partial x} \right|_{l_i} + Q + R \frac{dT}{dt} \quad (4.4)$$

where l_i is the thickness of the gradient from the surface, $v(x)$ the temperature distribution in the gradient, k_w the heat conductivity of the brine and $R = \rho_c C_v l_c$ with ρ_c as the density in the storage layer, C_v the specific heat and l_c the thickness of the storage convective layer. The insolation penetrating into the pond will be taken in the form of eq.(4.1).

By neglecting the heat stored in the gradient, the temperature

distribution must satisfy the conduction equation

$$\frac{\partial^2 v}{\partial x^2} = \frac{1}{k_w} \frac{dH}{dx}, \quad (4.5)$$

which can be integrated directly with boundary conditions: $v(x_0)=T_a$, $v(\ell_i)=T$. The first condition takes into account the possibility of a surface convective layer of depth x_0 which would absorb radiation but would not contribute any additional insulation. Integration of eq.(4.5) gives

$$\frac{\partial v}{\partial x} = \frac{H(x)}{k_w} + \frac{1}{\ell_i - x_0} (T - T_a - \frac{\tau F H_s}{k_w})$$

with

$$F = \sum_{n=1}^4 \frac{\eta_n}{\mu_n} \{ \exp(-\mu_n x_0) - \exp(-\mu_n \ell_i) \}. \quad (4.6)$$

Table 4.1 presents values for some of these terms, computed at normal incidence as well as with the $\cos r$ correction for Albuquerque (latitude $35^\circ 03'$, north).

Evaluating eq.(4.6) at ℓ_i and substituting the result into eq.(4.4) gives

$$T = T_a + \frac{\tau F H_s}{k_w} - \frac{\ell_i - x_0}{k_w} (Q + R \frac{dT}{dt}).$$

Substitution of the expressions presented in eq.(4.3) leads to an equation which can be separated for average and sinusoidal components, which are

$$\bar{T} = \bar{T}_a + \frac{\tau F \bar{H}_s}{k_w} - (\frac{\ell_i - x_0}{k_w}) \bar{Q}, \quad (4.7)$$

$$[1 + i\omega R(\frac{\ell_i - x_0}{k_w})] \tilde{T} e^{-i\delta} = \tilde{T}_a e^{-i\delta_a} + \frac{\tau F \tilde{H}_s}{k_w} - (\frac{\ell_i - x_0}{k_w}) \tilde{Q} e^{-i\delta_q}.$$

TABLE 4.1 - Distribution of Sunlight as a Function of Depth

Depth (m)	Fraction of Input*		$\sum_n \frac{n_n}{\bar{\mu}_n} e^{-\bar{\mu}_n x} \text{ (m) }^{**}$	
	$\cos r=1$	$\cos r=\frac{1}{1.2}$	$\cos r=1$	$\cos r=\frac{1}{1.2}$
0.01	0.72	0.71	7.90	6.57
0.1	0.55	0.54	7.83	6.52
0.2	0.50	0.49	7.78	6.47
0.3	0.47	0.46	7.73	6.42
0.4	0.45	0.43	7.69	6.37
0.6	0.41	0.39	7.60	6.29
0.8	0.38	0.37	7.52	6.22
1.0	0.36	0.35	7.45	6.15
1.2	0.35	0.33	7.38	6.08
1.4	0.33	0.32	7.31	6.01

* Fraction of the surface insolation which is transmitted through a depth x of water.

Example: at 100cm, only 36% of the surface insolation has reached this depth; the rest was absorbed at the surface and in the gradient.

** Components of the factor F in eq.(4.6).

Solving the second equation for real and imaginary parts gives

$$\tan \delta = \frac{\bar{T}_a(\sin \delta_a + \alpha \cos \delta_a) - \frac{l_i - x_0}{k_w} \bar{Q}(\sin \delta_q + \alpha \cos \delta_q) + \frac{\alpha \tau F \bar{H}_s}{k_w}}{\bar{T}_a(\cos \delta_a - \alpha \sin \delta_a) + \frac{l_i - x_0}{k_w} \bar{Q}(\cos \delta_q - \alpha \sin \delta_q) + \frac{\tau F \bar{H}_s}{k_w}} \quad (4.8)$$

$$\bar{T} = \frac{\bar{T}_a(\cos \delta_a - \alpha \sin \delta_a) + \frac{l_i - x_0}{k_w} \bar{Q}(\cos \delta_q - \alpha \sin \delta_q)}{(\alpha^2 + 1) \cos \delta} \quad (4.9)$$

where $\alpha = (\frac{l_i - x_0}{k_w}) R \omega = (\frac{l_i - x_0}{k_w}) \rho_c C_v l_c \omega$. Equation (4.7) is of course identical to eq.(4.5) presented in the R&N paper. Equations (4.8) and (4.9) instead must be tested with the tedious eqs.(4.10),(4.11) of the R&N paper, to see how closely this simple form can predict temperature amplitude and phase lag.

Using data provided in the R&N paper, a 60m² pond in Albuquerque, supplying 25,000BTU/⁰F-day (4.75J/⁰C-day) of winter heating, would be subjected to the following parameters:

$$\begin{array}{llll} \bar{T}_a = 13.4^{\circ}\text{C} & \bar{H}_s = 254\text{W/m}^2 & \delta_a = \pi/6 & \rho_c = 1.13 \cdot 10^3 \text{Kg/m}^3 \\ \bar{T}_a = 12.2^{\circ}\text{C} & \bar{H}_s = 112\text{W/m}^2 & \delta_q = 7\pi/6 & C_v = 3.43 \cdot 10^3 \text{J/Kg}^{\circ}\text{C} \\ l_i = 0.6\text{m} & \bar{Q} = 60.6\text{W/m}^2 & \tau = 0.97 & k_w = 0.60\text{W/m}^{\circ}\text{C} \\ l_c = 2.6\text{m} & \bar{Q} = 30.3\text{W/m}^2 & & (0.63 \text{ at } 60^{\circ}\text{C}, 20\%) \end{array}$$

The average temperature of the storage layer results in both cases in 76⁰C; for the temperature amplitude instead, eq.(4.9) above gives 41⁰C, compared with 36⁰C and 45⁰C for cases with and without ground storage. The phase lag predicted by eq.(4.8) is 76⁰ while computation of the R&N equations gives 75⁰ and 79⁰ for cases again with and without ground storage. Results in similar agreement were obtained by Nielsen (1979?).

It is interesting to note that a variation of 3°C can be obtained just by raising k_w to the correct value at 60°C and 20% concentration by weight. The difference between the results computed above is therefore well within the margin of possible error. Still, these predictions are applicable only to an ideal pond and they are only to be used as guidelines for the estimation of operating ranges for an actual pond. For example, the performance of the UNM pond at steady-state, supplying the entire heating demand for one house is predicted as: $\bar{T} = 117^{\circ}\text{C}$, $\tilde{T} = 54^{\circ}\text{C}$, $\delta = 64^{\circ}$. The new parameters used for this computation are:

$$\begin{aligned} A &= 105\text{m}^2 & \bar{Q} &= 34.6\text{W/m}^2 & \alpha &= 1.32 \\ l_i &= 0.8\text{m} & \tilde{Q} &= 17.3\text{W/m}^2 \\ l_c &= 1.6\text{m} \end{aligned}$$

Although the UNM pond is not yet at steady-state, the average experimental temperature has been 65°C , with about half of the heat load only. If the heat output is then reduced to $\bar{Q} = 20.5\text{W/m}^2$, which was the actual total heat extracted in 1977-78, the temperature prediction from eq.(4.7) climbs even higher: $\bar{T} = 135^{\circ}\text{C}$, $\tilde{T} = 47^{\circ}\text{C}$, $\delta = 62^{\circ}$. This large discrepancy can only be accounted for by inclusion of the following items into the derivation:

1. Increased heat loss due to a surface convective layer.
2. Increased absorption in the gradient due to suspended matter.
3. Heat loss by conduction to the sides and bottom.

If a 10cm convective layer is present at the surface, the prediction of eq.(4.7) is reduced from 135°C to 114°C ; if 20cm are convective, the average temperature prediction goes down to 97°C . As can be seen here,

the effect of a surface convective layer alone can be quite important to the operation of the pond, and thickness of this layer should definitely be controlled.

As mentioned previously, it is difficult to obtain accurate values for the sunlight reaching the storage layer. It is however almost as difficult to compute correct ground losses, due to the limited number of sensors buried in the ground around the pond. In the period November to February, with heat extraction in progress, the temperature gradients into the bottom and sides of the pond have been negligible. Analysis of daily data for the winter of 1977-78 shows that the energy balance requires a decrease in the assumed input (see Table 4.1) so that only 27.5% of the light present at the surface actually reaches a depth of 80cm, instead of the 37% predicted for clear water. This reduction could be accounted for by an increase of the incidence angle, by changes of the solar spectrum in winter or by increased absorption in the brine, due to suspended matter. Evaluation of eq.(4.1) at winter solstice, with the appropriate $\cos r = 1/1.35$ correction, can only decrease the predicted input at 80cm from 37.4% to 35.4%. The effect of the solar spectrum variation is even smaller. The remaining 8% or so must therefore be absorbed in the gradient because of suspended matter which makes the pond more opaque than visual analysis would predict. If this were the case, the reduction of input should affect the pond year-around and could be incorporated into the calculation by reducing the surface input appropriately, for example, by decreasing the value of the transmission coefficient τ . Accurate direct data on this item are sorely needed for a sound prediction.

In addition, edge and ground losses must also be included in the derivation, as they are not negligible over the year for a pond as small as the UNM one (see Section C). As the undisturbed ground temperature around the solar pond is very close to the average ambient temperature, the ground losses from the storage layer could be included in the conductive losses through the gradient by increasing the effective conductivity of the brine. A heat balance analysis for the year 1977-78 shows that increasing the average water conductivity to $0.75\text{W/m}^\circ\text{C}$ would account for ground losses adequately.

If one includes both the input loss due to additional absorption in the brine and the increase of the water conductivity to account for ground losses, the parameters become:

$$\begin{array}{lll} \bar{T}_a = 13.4^\circ\text{C} & \bar{H} = 254\text{W/m}^2 & \alpha = 1.32 \\ \tilde{T}_a = 12.2^\circ\text{C} & \tilde{H} = 112\text{W/m}^2 & \tau^* = 0.75 \\ x_0 = 0.15\text{m} & \bar{Q} = 20.5\text{W/m}^2 & k_w^* = 0.75\text{W/m}^\circ\text{C} \\ l_i = 0.80\text{m} & \tilde{Q} = 10.3\text{W/m}^2 & \delta_q = 7\pi/6 \\ l_c = 1.60\text{m} & & \end{array}$$

Results are: $\bar{T} = 66^\circ\text{C}$, $\tilde{T} = 35^\circ\text{C}$, in close agreement with the temperature data collected so far. The phase lag predicted is 64° or 65 days, longer than what has been registered at the pond. However, the same discrepancy is present in the R&N solution.

To model the solar pond more realistically, a two-dimensional computer code is being tested at UNM. Although this project is still in progress, preliminary results give insight into the approach to steady-state as well as the temperature regime of the ground (Dhingra and Bryant, 1979?).

C. Experimental Results

The UNM pond was originally filled in November 1975 with water at 20°C. Shortly after filling, the surface of the solar pond became a 5cm thick sheet of ice which persisted for over one month; temperature scans were not taken during this period. Figure 4.2 shows several temperature scans taken between January 1976 and August 1977, depicting the temperature distribution in the solar pond and in the ground below. While the thickness of the insulation layer was maintained between 60 and 110cm throughout this time, the thickness of the storage layer was increased from 20 to 150cm by August 1976.

The daily temperature response of the solar pond on both a sunny and a cloudy day is shown in Figure 4.3. As one can see, the daily change in ambient temperature and insolation affects the surface temperature as expected, while the effect on the convective layer is minimal. In both cases, the convective layer temperature reaches a daily maximum at about 17:00, a daily minimum at about 8:00. On a reasonably sunny day (August 22, 1977), the convective layer temperature varies by 1.3°C from daily maximum to minimum, with a net gain of 0.9°C over the 24 hours. On a cloudy day (August 11, 1977), when the insolation is well below the period average, the convective layer maintains its temperature within a range of 1.1°C, with no net gain for this day. The sharp drop in surface temperature around 18:00 is due to rain, characteristic of this time of the year. Comparison of the two convective layer temperature curves indicates a very similar response and verifies that the daily rather than hourly average temperatures and insolation are quite

FIGURE 4.2 - Temperature profiles in the solar pond and the ground below (center).

FIGURE 4.3 - Daily temperature response of the convective layer and the pond surface.

satisfactory for an accurate modelling of the solar pond. Figures 4.4 and 4.5 show instead the response of the pond to long periods of low insolation and/or low average ambient temperature. Figure 4.4 portrays the behavior of the convective layer temperature in January 1977, while the pond was dropping to its yearly minimum of 31°C . As seen previously, a day's reduction in insolation below average does not entail a drop in pond temperature; in this case the duration is seven days and still no great effect is registered. Instead, at consistently low ambient temperatures (two weeks averaging below 0°C) the net temperature drop is doubled, from 0.2 to $0.4^{\circ}\text{C}/\text{day}$. At this time, of course, the surface was completely frozen and the amount of light penetrating the pond was reduced. However, similar occurrences when the surface was not frozen reflect the same behavior. Figure 4.5 depicts similar circumstances during the period March-April 1977, when the temperature of the convective layer was increasing rapidly.

For the two consecutive summers of operation without any heat extraction, the UNM pond reached the maximum yearly temperature within the same week: 71°C on August 7, 1976 and 93°C on August 6, 1977. To our knowledge this is a record high for sodium chloride ponds with large storage capacity. The minimum temperature was 31°C on January 30, 1977. The recorded lag between convective layer temperature and insolation was therefore about 46 days for the two summers and 30 days for the one winter.

Heat extraction began on November 4, 1977, when the pond was at 61°C . The minimum temperature reached was 29°C on January 10, 1978. The extraction continued through 1978 and, once the pond reached 85°C on July 7, '78,

FIGURE 4.4 - Temperature response of the convective layer in relation to ambient temperature and insolation (I).

FIGURE 4.5 - Temperature response of the convective layer in relation to ambient temperature and insolation (II).

the extraction rate was increased to maintain the temperature at this level. This procedure was continued until September 16, 1978, when the pond temperature fell below the 85°C limit. For the summer of 1978, the phase lag can only be estimated by the middle of the constant temperature period; this would again be around August 11, 1978, matching the other two previous years. The minimum, however, was shifted forward by twenty days. Although the scheduling of heat extraction during the month of January 1978 could have caused this shift (see Chapter VI for details), the same occurred in the following year, when the minimum temperature of 29.4°C was reached on January 9, 1979.

Daily storage temperature, daily average ambient temperature and daily average insolation over 24 hours are presented in Figure 4.6, for the three years of operation. Figures 4.7 through 4.9 show instead a comparison between storage temperature and ground temperatures at three locations, two below the center of the pond, at depths of 25 and 150cm, and one in the side, east of the pond, at a depth of 120cm perpendicular to the wall and at a vertical position approximately equal to the floor of the pond. The scales of these three graphs are identical, for easy superposition. The response of the pond itself is shown in Figure 4.10.

On June 10, 1978 the pond broke down in a succession of layers encompassing the entire gradient, very much like the breakdown of August 13, 1977, described in Chapter V. As no supply of salt was immediately available, correction could not take place until June 15, with the total addition of one ton of salt to reinforce the salinity gradient and raise the bottom concentration from 16 to 20% by weight. By June 25, the entire gradient was reconstructed successfully, using the new technique described in Chapter III. This period was marked by a characteristic

FIGURE 4.6 - Temperature history of the convective layer for three successive years of operation.

FIGURE 4.7 - Temperature response of the solar pond and adjacent ground, 1976.

FIGURE 4.8 - Temperature response of the solar pond and adjacent ground, 1977.

FIGURE 4.9 - Temperature response of the solar pond and adjacent ground, 1978.

FIGURE 4.10 - Comparison of convective layer temperature for the three years of operation.

sharp drop in storage temperature as the insulating effect of the gradient is sharply decreased by internal convective layers. As can be seen in Figures 4.6 and 4.9, the storage temperature dropped 15°C in this period, but regained this deficit almost as rapidly as it had lost it, as soon as the gradient was established again. This was the last occurrence of instability registered until present (April 1979) and it should not repeat itself in future operation as the stability question, discussed in Chapter V, is now well understood.

As can be seen from the above experience, the solar pond needs several years to approach steady-state. This sluggish response can be attributed mainly to the ground, which approaches steady-state very slowly, proportional to $1/\sqrt{t}$. Following Rabl and Nielsen (1975a) and considering the ground under the pond as a semi-infinite solid with skin depth of 2m, the temperature of the ground should only be about 75% of its steady-state value after two years of operation, only 80% in three years and so on. However, by letting the UNM pond warm up for two years without any heat extraction, it seems that the temperature reached matches the steady-state value with a one-house load. The yearly cycle of the UNM pond should therefore be reproduced from now on and variations should only be due to different heat extraction levels or sharp climatic changes.

CHAPTER V

STABILITY ANALYSIS

A. Summary

Double-diffusive fluid systems with linear gradients and constant properties have been studied extensively; however, neither linear gradients nor constant properties can be assumed over the entire gradient region of a solar pond as the temperature variation is quite large and the transport coefficients quite temperature sensitive.

Observation of solar pond instabilities in three years of operation indicates that the gradient tends to break into small sublayers about 3 to 10cm thick. Occasionally the non-convective layer has broken in a succession of sublayers encompassing the entire gradient; in general, though, the instability occurs only in a small section of the gradient. This suggests that only local properties and local gradients are relevant to the stability condition and thus that the linear, constant gradients solution should be adequate in predicting stability, if applied to the appropriate sublayer instead of the entire non-convective zone. The key problem, resolved in this chapter, is the determination of the appropriate sublayer and the critical wavenumber at which breakdown will occur. Because heat diffusion takes place at a much faster rate than salt diffusion, the unstable sublayer is expected to be centered on the zone with the weakest salinity gradient, but the existing stability criteria give no information as to its size. Since viscosity would stabilize an infinitesimal layer, there must be a minimum size for this

sublayer, which should depend on local fluid properties and the local shape of the gradients.

The above conclusions are supported by the analysis of a constant-property fluid system with two diffusing components and arbitrary gradients in the conductive (steady) state. The classical, linear, constant gradients results are reproduced as a special case of this analysis (Section D) and numerical solutions for the non-constant gradients case are obtained by variational principles (Section E). The results, applied to actual solar pond data (Section F), do predict localized sublayer instabilities accurately, for a variety of gradient configurations. In addition, the results confirm that the sublayer instability is the lowest (therefore most unstable) mode of the whole-layer solution, at the corresponding horizontal wavenumber a . This approach also gives the scale of the sublayer which will overturn, as well as general requirements for the optimal salinity gradient necessary to operate successfully.

With the results of this analysis in hand, easy and immediate field evaluation of solar pond stability can be performed with minimum computation by application of the following procedure, subsequently referred to as "sublayer stability analysis":

1. The sublayer to be analysed for stability can be quickly identified to a rough approximation by observation of the salt distribution; it is the one with the weakest salinity gradient.
2. The shape functions dT/dz and dS/dz are determined by differencing the measured salt concentration $S(z)$ and temperature $T(z)$ profiles. These are reduced to the dimensionless forms $\Delta T_c F(z)/h$ and $\Delta S_c G(z)/h$.

3. The function

$$\lambda(z) = \frac{(Pr + 1) \alpha}{(Pr + \sigma) \beta} \frac{\Delta T_c F(z)}{\Delta S_c G(z)}$$

is computed, using a table of properties provided in Appendix C, and a plot of $\lambda(z)$ will reveal potentially unstable sublayers as the finite regions for which $\lambda(z) > 1$.

4. When the extent of any of these sublayers reaches the minimum size d , required by the non-constant gradients solutions (Table 5.1 below), the layer is neutrally stable. The correct non-constant gradients solution has thermophysical properties evaluated at the average temperature and salinity of the sublayer, using Appendix C.

B. Governing Equations

Consider a layer of fluid of depth h and infinite horizontal dimensions; z is positive downward and the suffix zero refers to values at some reference level. This layer is subjected to non-constant gradients of temperature and salinity which are established in the layer at time $t=0$.

Proceeding in the standard form of linear theory, presented by Chandrasekhar (1961) or Turner (1973), express temperature and salinity distributions as the superposition of an initial conductive distribution (T_c, S_c) and a convective perturbation of the system ($\tilde{T}, \tilde{S}$). Hence,

$$T(x, y, z, t) = T_c(z) + \tilde{T}(x, y, z, t) \quad (5.1)$$

$$S(x, y, z, t) = S_c(z) + \tilde{S}(x, y, z, t) \quad (5.2)$$

Also express the density distribution in the layer as

$$\rho(x, y, z, t) = \rho_c(z) + \tilde{\rho}(x, y, z, t),$$

where

$$\rho_c(z) = \rho_0 \{1 - \alpha(T_c - T_0) + \beta(S_c - S_0)\}, \quad (5.3)$$

$$\tilde{\rho}(x, y, z, t) = \rho_0 \{-\alpha\tilde{T} + \beta\tilde{S}\}, \quad (5.4)$$

with $\alpha = -\frac{1}{\bar{\rho}} \frac{\partial \rho}{\partial T} \Big|_{S, p}$, $\beta = \frac{1}{\bar{\rho}} \frac{\partial \rho}{\partial S} \Big|_{T, p}$, which are taken as constant through the layer analysed and where $\bar{\rho}$ is the mean density.

The frequent restriction posed at the outset of convection studies is that the fluid must be confined to a layer small enough so that

$$\tilde{\rho}/\rho_0 \leq \Delta\rho_c/\rho_0 = \epsilon \ll 1 \quad (5.5)$$

where $\Delta\rho_c$ is the maximum variation of the conductive distribution over the whole layer (Boussinesq, 1903; Rayleigh, 1916). This assumption is quite appropriate for solar ponds as a variation of 10^0C or 1% concentration by weight of sodium chloride produces a density change of about 1%, while $\tilde{\rho}/\rho_0 = -\alpha\tilde{T} + \beta\tilde{S}$ is of order 10^{-3} . In addition, the variation of density due to compression is also negligible because the volumetric change of water for one atmosphere pressure increment is 0.005%.

Veronis's study of finite amplitude instabilities (Veronis, 1968) concludes that the most unstable modes occur for infinitesimal perturbations and therefore the velocity $\tilde{u}$ at the onset of the convective state can be assumed sufficiently small to justify neglect of its squares and products. Equivalently, the perturbation of the temperature and salinity fields must also be small and terms of second order can be dropped.

The term $\frac{1}{\rho_0} \frac{D\rho}{Dt}$ is the Lagrangian derivative of the relative density variation ($\tilde{\rho}/\rho_0$) of a parcel of fluid as it moves through the conductive stratification. Accordingly, the variation of this term can be at most of order ϵ and is therefore neglected in the governing equations, under the present approximation. For a detailed review of the wide applicability of this approximation, see Spiegel and Veronis (1960) and Gough (1969).

For any depth h through which the absolute viscosity μ , the kinematic viscosity ν , the temperature and salinity diffusivities κ_T and κ_S can be assumed constant, the equations characterizing a viscous fluid system, under the above approximation, are

1. Conservation of Mass (Continuity):

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \underline{U}) = \frac{D\rho}{Dt} + \rho \nabla \cdot \underline{U} = 0 \quad (5.6)$$

where $\underline{U}$ is the convective velocity vector in the fluid, with components (u, v, w) . As $\frac{D\rho}{Dt} \approx 0$,

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0. \quad (5.7)$$

2. Conservation of Momentum:

$$\rho \frac{D\underline{U}}{Dt} = \rho g \hat{k} - \nabla p + \mu \nabla^2 \underline{U}. \quad (5.8)$$

3. Conservation of Energy:

$$\frac{DT}{Dt} = \kappa_T \nabla^2 T. \quad (5.9)$$

4. Conservation of Salt:

$$\frac{DS}{Dt} = \kappa_S \nabla^2 S. \quad (5.10)$$

C. Linearization

To eliminate the pressure term in eq.(5.8), express

$$\vec{U} \cdot \nabla \vec{U} = \nabla \left(\frac{|\vec{U}|^2}{2} \right) - \vec{U} \times (\nabla \times \vec{U})$$

and introduce vorticity, $\xi = \nabla \times \vec{U}$

$$\frac{\partial \vec{U}}{\partial t} + \nabla \left(\frac{|\vec{U}|^2}{2} \right) - \vec{U} \times \xi = g\hat{k} - \frac{1}{\rho} \nabla p + \nu \nabla^2 \vec{U} ,$$

with $\nu = \mu/\rho$; take the curl of this expression to obtain

$$\frac{\partial \xi}{\partial t} - \nabla \times (\vec{U} \times \xi) = - \nabla \left(\frac{1}{\rho} \right) \times \nabla p + \nu \nabla^2 \xi .$$

As the convective perturbation on the density is small relative to the conductive distribution,

$$\frac{1}{\rho} = \frac{1}{\rho_c + \tilde{\rho}} \approx \frac{1}{\rho_c} \left\{ 1 - \frac{\tilde{\rho}}{\rho_c} + O\left(\frac{\tilde{\rho}^2}{\rho_c^2}\right) \right\} ,$$

where $O(\tilde{\rho}^2/\rho_c^2)$ refers to terms of order $(\tilde{\rho}/\rho_c)^2$ or higher, which are therefore negligible. Then

$$\begin{aligned} \nabla \left(\frac{1}{\rho} \right) \times \nabla p &= \nabla \left(\frac{1}{\rho} \right) \times \nabla p_c + \nabla \left(\frac{1}{\rho} \right) \times \nabla \tilde{p} \\ &= \nabla \left(\frac{1}{\rho} \right) \times \rho_c g\hat{k} + \nabla \left(\frac{1}{\rho} \right) \times \nabla \tilde{p} - \nabla \left(\tilde{\rho}/\rho_c^2 \right) \times \nabla \tilde{p} . \end{aligned}$$

The third term is negligible under the present approximation and the second is also negligible because it is about ten times smaller than the first one. This can be shown without difficulty by non-dimensioning the terms with respect to the reference values ρ_c and $p_{c,0} = \rho_c g h$. The first term becomes

$$\frac{1}{h} \frac{\nabla^* \rho_0 (\tilde{\rho}/\rho_0)}{\rho_0 (\rho_c/\rho_0)} \times g\hat{k} = \frac{g}{h} \frac{\nabla \tilde{\rho}^*}{\rho_c^*} \times \hat{k}$$

while the second becomes

$$\frac{1}{h} \frac{\nabla^* \rho_0 (\rho_c / \rho_0)}{\rho_0^2 (\rho_c / \rho_0)^2} \times \frac{\rho_c g h}{h} \{ \nabla^* (\frac{\tilde{p}}{\rho_c g h}) \} = \frac{g}{h} \rho_c^* (\nabla \rho_c)^* \times (\nabla \tilde{p})^*$$

with the asterisk referring to dimensionless quantities, ρ_c^* of order 1, $(\nabla \tilde{p})^*$ of order 0.1 and $(\nabla \tilde{p})^*$ of order 0.1 also.

The linearized form of eq.(5.8) becomes

$$(\frac{\partial}{\partial t} - \nu \nabla^2) \xi = g \frac{\nabla \tilde{\rho}}{\rho_c} \times \hat{k} . \quad (5.11)$$

To express this in terms of the velocity $\tilde{U}$, take the curl of eq.(5.11)

with $\nabla \times \xi = \nabla \times (\nabla \times \tilde{U}) = \nabla (\nabla \cdot \tilde{U}) - \nabla^2 \tilde{U}$. As $\nabla \cdot \tilde{U} = 0$ by eq.(5.7), then

$$(\frac{\partial}{\partial t} - \nu \nabla^2) (-\nabla^2 \tilde{U}) = \frac{g}{\rho_c} \nabla \times (\nabla \tilde{\rho} \times \hat{k}) .$$

The z component of this equation is

$$(\frac{\partial}{\partial t} - \nu \nabla^2) \nabla^2 w = \frac{g}{\rho_c} \nabla_2^2 \tilde{\rho} , \quad (5.12)$$

where $\nabla_2^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the Laplacian in the two horizontal dimensions.

The equation for temperature, expressed in terms of eq.(5.1), becomes

$$\frac{\partial \tilde{T}}{\partial t} + \tilde{U} \cdot \nabla T_c + \tilde{U} \cdot \nabla \tilde{T} = \kappa_T \nabla^2 \tilde{T} ,$$

with $\frac{\partial T_c}{\partial t} = 0$ and $\nabla^2 T_c = 0$ for the conductive (steady) solution. By dropping the second order terms in the perturbation, this becomes

$$\frac{\partial \tilde{T}}{\partial t} + w \frac{dT_c}{dz} = \kappa_T \nabla^2 \tilde{T} . \quad (5.13)$$

Equivalently for salt,

$$\frac{\partial \tilde{S}}{\partial t} + w \frac{dS_c}{dz} = \kappa_S \nabla^2 \tilde{S} . \quad (5.14)$$

To non-dimensionalize eqs.(5.12),(5.13),(5.14), take as scaling parameters time, $t = \frac{h^2 \tau}{\kappa_T}$; velocity, $w = \frac{\kappa_T W}{h}$; temperature, $T = \{T_c(h) - T_c(0)\} \theta = \Delta T_c \theta$; salt, $S = \{S_c(h) - S_c(0)\} \Sigma = \Delta S_c \Sigma$ and the depth is scaled by h . The tilde, specifying perturbation quantities, has been dropped. Equation (5.12) becomes

$$\left(\frac{\kappa_T}{\nu} \frac{\partial}{\partial \tau} - \nabla^2 \right) \nabla^2 W = \frac{gh^3}{\nu \kappa_T} \nabla_2^2 \left(\frac{\rho}{\rho_c} \right),$$

where ∇^2 , ∇_2^2 are dimensionless operators and

$$\nabla_2^2 \left(\frac{\rho}{\rho_c} \right) = \nabla_2^2 (-\alpha T + \beta S) = -\alpha (\Delta T_c) \nabla_2^2 \theta + \beta (\Delta S_c) \nabla_2^2 \Sigma.$$

Defining two Rayleigh numbers for temperature and salinity, respectively,

$$R = \frac{\alpha g (\Delta T_c) h^3}{\nu \kappa_T} \quad \text{and} \quad R_S = \frac{\beta g (\Delta S_c) h^3}{\nu \kappa_T},$$

eq.(5.12) can be written as

$$\left(\frac{1}{Pr} \frac{\partial}{\partial \tau} - \nabla^2 \right) \nabla^2 W = -R \nabla_2^2 \theta + R_S \nabla_2^2 \Sigma, \quad (5.15)$$

while eqs.(5.13),(5.14) become

$$\left(\frac{\partial}{\partial \tau} - \nabla^2 \right) \theta = -\frac{h}{\Delta T_c} \frac{dT_c}{dz} W = -FW, \quad (5.16)$$

$$\left(\frac{\partial}{\partial \tau} - \sigma \nabla^2 \right) \Sigma = -\frac{h}{\Delta S_c} \frac{dS_c}{dz} W = -GW, \quad (5.17)$$

where F and G are the dimensionless gradients of the conductive state,

$Pr = \frac{\nu}{\kappa_T}$ is the Prandtl number and $\sigma = \frac{\kappa_S}{\kappa_T}$ is the Schmidt number.

These equations are in agreement with Turner (1973) for the case of constant gradients, i.e., $F = G = 1$ but the direction of z reversed.

The boundary conditions for this problem are

$$W = \theta = \Sigma = 0 \quad \text{at} \quad z = 0, 1 \quad (5.18)$$

as temperature and salinity are kept constant on the boundaries and the normal component of the convective velocity must vanish. For free surfaces, an additional condition is that $\frac{\partial u}{\partial z} = \frac{\partial v}{\partial z} = 0$ as no tangential stresses can occur.

From continuity, $\nabla \cdot \mathbf{U} = 0$, hence

$$\frac{\partial^2 u}{\partial z \partial x} + \frac{\partial^2 v}{\partial z \partial y} = -\frac{\partial^2 w}{\partial z^2} = 0 \quad \text{at} \quad z = 0, 1. \quad (5.19)$$

Equations (5.18) and (5.19) provide the required eight boundary conditions for the system.

D. Normal Mode Analysis

Supposing that the perturbation is expressed as a set of periodic waves, each set associated with a horizontal wavenumber a , then

$$(W, \theta, \Sigma) = (\text{z-dependent parts}) \cdot \exp\{i(k_x x + k_y y) + pt\},$$

where $p = p_r + ip_i$ in general and $a^2 = (k_x^2 + k_y^2)$.

For these functions, $\frac{\partial}{\partial t} = p$; $\nabla_z^2 = -a^2$; $\nabla^2 = D^2 - a^2$, where $D^2 = \frac{\partial^2}{\partial z^2}$.

The governing equations become then

$$\left\{ \frac{p}{Pr} - (D^2 - a^2) \right\} (D^2 - a^2) W = Ra^2 \theta - R_S a^2 \Sigma, \quad (5.20)$$

$$\{p - (D^2 - a^2)\} \theta = -FW, \quad (5.21)$$

$$\{p - \sigma(D^2 - a^2)\} \Sigma = -GW, \quad (5.22)$$

and W, θ, Σ represent the z-dependent parts. The system will be stable with respect to arbitrary infinitesimal disturbances if p_r is negative

for all values of a ; if p_r is positive for any a , the system will be unstable. The sign of p_r will therefore determine regions in the R, R_S plane and marginal stability will be characterized by $p_r = 0$.

Let us first reproduce the solution for constant gradients to obtain the corresponding regions of stability (see for example Turner). For $F = G = 1$, solutions to eqs. (5.20), (5.21), (5.22) can be expressed as

$$(W, \theta, \Sigma) \propto \sin n\pi z,$$

and substitution into eqs. (5.20)-(5.22) leads to the matrix form

$$\begin{bmatrix} \{-\frac{p}{Pr}\ell^2 - \ell^4\} & \{-Ra^2\} & \{R_S a^2\} \\ 1 & \{p + \ell^2\} & 0 \\ 1 & 0 & \{p + \sigma\ell^2\} \end{bmatrix} \begin{bmatrix} W \\ \theta \\ \Sigma \end{bmatrix} = 0$$

where $\ell^2 = n^2\pi^2 + a^2$. For non-trivial W, θ, Σ solutions, the determinant must be equal to zero, which gives the characteristic equation for p

$$\begin{aligned} p^3 + p^2\{(\pi^2 n^2 + a^2)(Pr + \sigma + 1)\} + p\{(\pi^2 n^2 + a^2)(Pr + \sigma Pr + \sigma) + \frac{a^2 Pr}{(\pi^2 n^2 + a^2)}(R_S - R)\} \\ + \{\sigma Pr(\pi^2 n^2 + a^2)^3 + a^2 Pr(R_S - \sigma R)\} = 0 \end{aligned} \quad (5.23)$$

which is of the form: $p^3 + C_1 p^2 + C_2 p + C_3 = 0$ with $C_1 > 0$. This cubic will have negative $\text{Re}\{p\}$ if all coefficients are positive and if $C_1 C_2 > C_3$ (> 0). For particular values of Pr, σ and a , the lines $C_3 = 0$ and $C_1 C_2 = C_3$ will determine the boundaries of the stable region in the R, R_S plane.

For $C_3 = 0$, the condition is

$$R = \frac{R_S}{\sigma} + \frac{(n^2 \pi^2 + a^2)^3}{a^2}.$$

For any a , the value of R is smallest for $n = 1$, i.e., the developing convective cell will affect the entire depth. For $n = 1$, R is minimum when

$$\left. \frac{dR}{d(a^2)} \right|_{a^2_{\text{crit}}} = \frac{3(\pi^2 + a^2)^2}{a^2} - \frac{(\pi^2 + a^2)^3}{a^4} = 0 ; \text{ and } a^2_{\text{crit}} = \frac{\pi^2}{2} .$$

Thus,

$$R = \frac{R_S}{\sigma} + \frac{27 \pi^4}{4} \quad (5.24)$$

where $\frac{27 \pi^4}{4} = R_c$ is the critical Rayleigh number if salt is not present. For this solution, $p = 0$ is the growthrate, i.e., $p_i = 0$ also, and the motion is steady ("exchange of stabilities").

For $C_1 C_2 = C_3$ the condition becomes

$$R = \frac{Pr + \sigma}{Pr + 1} R_S + R_c (\sigma + 1) \left(1 + \frac{\sigma}{Pr}\right) \quad (5.25)$$

which represent the boundary of the "overstable" region, since $p_i \neq 0$ and the convective motion is oscillatory. A diagram of these regions is presented in Figure 5.1, adapted from Turner (1973). Along the line ZX, determined by eq.(5.24), one of the eigenvalues of p is zero; along XW, determined by eq.(5.25), two eigenvalues of p are purely imaginary; along XV, two complex conjugate eigenvalues coalesce on the real axis. As the system crosses the line XZ from the stability region (right) to the instability region (left), a solution which was exponentially decaying, and therefore stable, becomes time independent on XZ and then begins to grow exponentially, drawing its potential energy from the salt field. For R, R_S negative, this is referred to as the "finger regime" and is unstable well below the statically stable condition (unit slope

FIGURE 5.1 - Diagram of the convective regimes in the R , R_s plane.

with $R = R_S$). The small triangle in the second quadrant, between ZX and zero is of interest because, although both temperature and salinity are destabilizing, the Rayleigh numbers are so small and, therefore, the viscosity so high that instabilities are damped and the system is stable.

For R positive and no salt present, the point R_c is the original Rayleigh number defining the stability boundary for a fluid heated from below. The numerical value of R_c varies with the boundary conditions (Nield, 1967); for free boundaries, $R_c = \frac{27\pi^4}{4}$ (Rayleigh, 1916).

Through the line XW, the oscillatory but exponentially decaying solutions in the stable region become purely oscillatory on XW and oscillatory with increasing amplitude above XW; thus breakdown occurs as overstable modes. For conditions found in the solar pond, this is the only relevant mode as both R and R_S are positive and very large (order 10^{10} , 10^{12}). For characteristic values of a salt-heat system, $\sigma \approx 10^{-2}$, $Pr \approx 5$, $R_S \approx 10^{12}$ hence the stability locus becomes

$$R = \frac{Pr+\sigma}{Pr+1} R_S + 1.012 R_c = 0.8 R_S \{1 + O(10^{-8})\}.$$

The breakdown by overstable modes can be easily understood with the following argument, following Stern (1960). Consider a particle of fluid at a certain depth z , raised by a small disturbance to $z+\Delta z$. As heat diffuses much more rapidly than salt, this element of fluid will become colder but will maintain most of its salt. The density will increase and the restoring force will be greater than in an adiabatic parcel (overstabilizing); the particle will overshoot the position $z+\Delta z$ by, say, Δ . At $z+\Delta z+\Delta$ the element will become hotter but will not gain much salt; it will therefore be less dense and again, the restoring

force will be greater than in the adiabatic case, so that the parcel overshoots $z-(\Delta z+\Delta)$ on its next upward oscillation. In this manner a small disturbance will create an overstable oscillatory mode.

Similar results have been reproduced by Weinberger (1964) and are occasionally referred to as the Weinberger stability criterion for solar ponds.

E. Variational Approximation for Non-Constant Gradients

Considering now the previously untreated case of non-constant gradients, return to eqs.(5.15),(5.16),(5.17). The neutral stability locus is characterized by $p_r = 0$, therefore $p = ip_i$ only. Equations (5.20), (5.21),(5.22) are again applicable, if p is replaced by ip , with p being now $p = \text{Im}\{\text{growthrate}\}$.

By eliminating Θ and Σ from eq.(5.20), using eqs.(5.21) and (5.22),

$$(ip-\sigma\nabla^2)(ip-\nabla^2)\left(\frac{ip}{Pr}-\nabla^2\right)\nabla^2W = -a^2R(ip-\sigma\nabla^2)FW + a^2R_S(ip-\nabla^2)GW. \quad (5.26)$$

Equation (5.26) must be satisfied separately for real and imaginary parts as $W(z)$ must be real; then

$$\begin{aligned} \text{Re: } \{ p^2\left(\frac{\sigma}{Pr}+1+\frac{1}{Pr}\right) - \sigma\nabla^4 \} \nabla^4 W &= a^2\sigma R \nabla^2(FW) - a^2R_S \nabla^2(GW) , \\ \text{Im: } \{ -\frac{p^2}{Pr} + (\sigma+\frac{\sigma}{Pr}+1)\nabla^4 \} \nabla^2 W &= -a^2R FW + a^2R_S GW . \end{aligned}$$

To eliminate p between these two equations, differentiate the second with ∇^2 , divide the first by $(1+\sigma+Pr)$ and add one to the other. Then

$$\begin{aligned} \left(\frac{Pr\sigma+Pr+\sigma}{Pr} - \frac{\sigma}{1+\sigma+Pr}\right)\nabla^8 W &= a^2\left(\frac{\sigma}{1+\sigma+Pr} - 1\right)R\nabla^2(FW) \\ &\quad - a^2\left(\frac{1}{1+\sigma+Pr} - 1\right)R_S\nabla^2(GW) , \end{aligned}$$

which factors into

$$\nabla^2(\nabla^6 W + a^2 k_1 FW - a^2 k_2 GW) = 0, \quad (5.27)$$

$$\text{where } k_1 = \frac{Pr R}{(Pr+\sigma)(1+\sigma)} \text{ and } k_2 = \frac{Pr R_S}{(Pr+1)(1+\sigma)}. \quad (5.27A)$$

The boundary conditions at $z = 0, 1$ become

$$W = \theta = \Sigma = D^2 W = 0.$$

Then eq.(5.20), evaluated at the boundaries, is

$$\left(\frac{ip}{Pr} - \nabla^2\right)\nabla^2 W = a^2 R \theta = a^2 R_S \Sigma = 0,$$

hence $\nabla^4 W = 0$. As $\nabla^2 = (D^2 - a^2)$, this leads to $D^4 W = 0$. Also, eq.(5.21) becomes

$$(ip - \nabla^2)\theta = -FW = 0,$$

therefore $\nabla^2 \theta = 0$ and similarly $\nabla^2 \Sigma = 0$. By differentiating eq.(5.20) with ∇^2 and using these conditions on the right hand side, one obtains $\nabla^6 W = 0$.

The quantity in parentheses in eq.(5.27) vanishes on the boundaries and is therefore identically zero in the interval $z = 0, 1$; thus the problem is reduced to sixth order with

$$\nabla^6 W + a^2 k_1 FW - a^2 k_2 GW = 0 \quad (5.28)$$

and $W = \theta = \Sigma = D^2 W = D^4 W = D^6 W = 0$ at $z = 0, 1$. The functional relationship of k_1 and k_2 will again divide the R, R_S plane in regions of stability.

Equation (5.28) defines an eigenvalue problem which can be solved analytically for F, G constant, giving the results presented in eqs.(5.24) and (5.25), as well as Figure 5.1. For variable gradients, useful

approximate solutions can be obtained from variational principles.

Equation (5.28) can be expressed as

$$M(W) - \Lambda N(W) = 0 ,$$

where $M(W) = \nabla^6 W - a^2 k_2 GW$ and $N(W) = -a^2 FW$. If the problem is self-adjoint and positive (negative) definite, the Rayleigh quotient will converge from above to the correct eigenvalue

$$\Lambda = k_1 = \frac{\text{minimum over all functions } W \text{ satisfying the essential boundary conditions}}{\frac{\int_0^1 W M(W) dz}{\int_0^1 W N(W) dz}}$$

By substitution and expansion of $\nabla^6 W$

$$k_1 = \min. \frac{\int_0^1 W(D^6 W - 3a^2 D^4 W + 3a^4 D^2 W - a^6 W) - a^2 k_2 GW^2 dz}{-\int_0^1 a^2 FW^2 dz} .$$

Integrating each term of the numerator by parts and applying the boundary conditions, all terms can be reduced as shown here for the first one

$$\begin{aligned} \int_0^1 W D^6 W dz &= W D^5 W \Big|_0^1 - \int_0^1 D W D^5 W dz = -D W D^4 W \Big|_0^1 + \int_0^1 D^2 W D^4 W dz \\ &= D^2 W D^3 W \Big|_0^1 - \int_0^1 (D^3 W)^2 dz . \end{aligned}$$

The Rayleigh quotient, reduced in this manner, gives

$$k_1 = \min. \frac{-\int_0^1 (D^3 W)^2 + 3a^2 (D^2 W)^2 + 3a^4 (D W)^2 + a^6 W^2 + a^2 k_2 GW^2 dz}{-\int_0^1 a^2 FW^2 dz} \quad (5.29)$$

in which the numerator and denominator are clearly negative definite.

Continued integration by parts also shows the operators are self-adjoint.

It is also immediately apparent that increasing G will stabilize the system while increasing F will decrease the eigenvalue and make the system more unstable.

Expressing the eigenfunctions as

$$W = \sum_{n=1}^N C_n \phi_n$$

and applying the Ritz method, one obtains a set of equations for the coefficients C_n . The same result can be obtained by a slightly different method, adapted from Chandrasekhar's (1961) derivation for the case of a temperature gradient alone. Equation (5.28) can be expressed as

$$(D^2 - a^2)^2 W = H \quad (5.30)$$

$$(D^2 - a^2) H + a^2 k_1 F W - a^2 k_2 G W = 0, \quad (5.31)$$

with boundary conditions $H = W = D^2 W = 0$ at $z = 0, 1$. Then H can be expanded in a sine series as

$$H = \sum_m C_m \sin m\pi z$$

and W can therefore be determined as the exact solution to eq.(5.30), or

$$W = \sum_m C_m W_m$$

$$(D^2 - a^2)^2 W_m = \sin m\pi z,$$

which gives

$$W_m = \frac{\sin m\pi z}{L_m^2} \quad \text{and} \quad L_m^2 = (m^2 \pi^2 + a^2)^2.$$

The variational scheme in this case results in a minimization based on eq.(5.31); hence one needs to minimize

$$I = \int_0^1 H (D^2 - a^2) H dz + a^2 k_1 \int_0^1 F H W dz - a^2 k_2 \int_0^1 G H W dz \quad (5.32)$$

$$\begin{aligned} I = & \int_0^1 \sum_m C_m \sin m\pi z \left\{ - \sum_n C_n (n^2 \pi^2 + a^2) \sin n\pi z \right\} dz \\ & + a^2 k_1 \int_0^1 F \sum_m C_m \sin m\pi z \sum_n C_n \frac{\sin n\pi z}{L_n^2} dz \\ & - a^2 k_2 \int_0^1 G \sum_m C_m \sin m\pi z \sum_n C_n \frac{\sin n\pi z}{L_n^2} dz . \end{aligned}$$

Setting $(m/n) = \sin m\pi z \sin n\pi z$ one has

$$\begin{aligned} I = & \sum_m C_m \sum_n C_n \left[-(n^2 \pi^2 + a^2) \int_0^1 (m/n) dz + \frac{a^2 k_1}{L_n^2} \int_0^1 F(m/n) dz \right. \\ & \left. - \frac{a^2 k_2}{L_n^2} \int_0^1 G(m/n) dz \right] . \end{aligned} \quad (5.33)$$

Suppose that the dimensionless temperature and salinity are expressed as

$$\begin{array}{lll} \Theta = Az + Bz^2 + Yz^3 & \Theta(z=1) = 1 & \text{thus } A + B + Y = 1 \\ \Sigma = Cz + Dz^2 + Ez^3 & \Sigma(z=1) = 1 & \text{thus } C + D + E = 1 \end{array}$$

while the dimensionless gradients are

$$\begin{aligned} F &= \frac{h}{\Delta T_c} \frac{dT_c}{dz_{dm}} = \frac{d\Theta}{dz} = A + 2Bz + 3Yz^2 \\ G &= \frac{h}{\Delta S_c} \frac{dS_c}{dz_{dm}} = \frac{d\Sigma}{dz} = C + 2Dz + 3Ez^2 \end{aligned} \quad (5.34)$$

and dm refers to dimensional depth.

The integrals to be evaluated are

$$I_1 = \int_0^1 (m/n) dz = \begin{cases} 0 & \text{for } m \neq n \\ \frac{1}{2} & \text{for } m = n \end{cases}$$

$$I_2 = \int_0^1 z (m/n) dz$$

$$= \begin{cases} \frac{(-1)^{m-n} (m+n)^2 - (-1)^{m+n} (m-n)^2 - 4mn}{2(m^2-n^2)^2 \pi^2} & \text{for } m \neq n \\ \frac{1}{4} & \text{for } m = n \end{cases}$$

$$I_3 = \int_0^1 z^2 (m/n) dz$$

$$= \begin{cases} \frac{(-1)^{m-n} (m+n)^2 - (-1)^{m+n} (m-n)^2}{(m^2-n^2)^2 \pi^2} & \text{for } m \neq n \\ \frac{1}{6} - \frac{1}{4m^2 \pi^2} & \text{for } m = n \end{cases}$$

Returning to eq.(5.33), I is minimized by differentiating with respect to a C_k and setting this to zero,

$$\frac{1}{2} \frac{\partial I}{\partial C_k} = \sum_n C_n \left[-(n^2 \pi^2 + a^2) \int_0^1 (m/n) dz + \frac{a^2 k_1}{L_n^2} \int_0^1 F(m/n) dz - \frac{a^2 k_2}{L_n^2} \int_0^1 G(m/n) dz \right] = 0.$$

Hence, rearranging terms and dividing through by the large value of k_2 in order to have scaled eigenvalues (i.e., unity represents the familiar constant gradient value), this becomes

$$\sum_n C_n \left[\frac{L_n^2}{a^2 k_2} (n^2 \pi^2 + a^2) \int_0^1 (m/n) dz + \int_0^1 G(m/n) dz - \frac{k_1}{k_2} \int_0^1 F(m/n) dz \right] = 0$$

or

$$\sum_n C_n \left[\frac{(N^2 \pi^2 + a^2)^3}{a^2 k_2} I_1 + C I_1 + 2D I_2 + 3E I_3 - \frac{k_1}{k_2} (A I_1 + 2B I_2 + 3Y I_3) \right] = 0$$

which is of the form

$$\left(\underline{P} - \frac{k_1}{k_2} \underline{Q} \right) \underline{C} = \underline{0}, \quad (5.35)$$

with the terms

$$P_{nn} = \frac{1}{2} \left[\frac{(n^2 \pi^2 + a^2)^3}{a^2 k_2} + 1 - \frac{3E}{2n^2 \pi^2} \right]$$

$$P_{mn} = \begin{cases} \frac{-4mn(2D + 3E)}{(m^2 - n^2)^2 \pi^2} & (m+n) \text{ odd} \\ \frac{12mnE}{(m^2 - n^2)^2 \pi^2} & (m+n) \text{ even} \end{cases}$$

$$Q_{nn} = \frac{1}{2} \left[1 - \frac{3Y}{2n^2 \pi^2} \right]$$

$$Q_{mn} = \begin{cases} \frac{-4mn(2B + 3Y)}{(m^2 - n^2)^2 \pi^2} & (m+n) \text{ odd} \\ \frac{12mnY}{(m^2 - n^2)^2 \pi^2} & (m+n) \text{ even} \end{cases}$$

For average property values, k_2 indicates the size of the layer considered; i.e., for water with a $1^\circ\text{C}/\text{cm}$ gradient, $k_2 = 10^4$ represents a layer about 1cm thick, $k_2 = 10^8$ represents about 10cm, $k_2 = 10^{12}$ represents about 100cm. For the larger values of k_2 , the order of the approximation must increase until the contribution from the derivative terms becomes dominant in the matrix; after the n for which this occurs, coefficients in the sine series become negligible. This requires that the term $\frac{(N^2 \pi^2 + a^2)^3}{a^2 k_2}$ be order one, implying $N \propto k_2^{1/6}$. For $k_2 = 10^{12}$, which would represent the entire non-convecting layer of the solar pond, the order of the approximation required to provide converged eigenfunctions is greater than the maximum of twenty terms for which economical parameter studies could be run. Equation (5.35) was solved numerically for several values of N , a , k_2 , F , G , using data collected from the solar pond. Fully converged results for cases with $k_2 = 10^{12}$ were not routinely computed, but the qualitative behavior, specifically the localization of

the eigenfunctions, is similar to that of the cases presented, with lower values of k_2 . Details of the computer codes used can be found in Appendix D.

F. Comparison of Results to Solar Pond Data

Analysis of three years of data from the solar pond shows that the temperature profile in the gradient region, because of the relatively rapid diffusion of heat, can always be represented accurately over most of its depth by a quadratic, hence $Y = 0$. Also, the temperature is always increasing with depth therefore F can at most have one zero, and this can occur only at $z = 1$. So

$$A + B = 1$$

$$F(1) = A + 2B \quad \text{therefore}$$

$$F'(z) = 2B \leq 0$$

$$-1 \leq B \leq 0$$

$$1 \leq A \leq 2$$

Using the same kind of arguments on the salinity and requiring $G'' > 0$, so that G is a positive quadratic with a minimum in the layer, one obtains $E \geq 0$, always. As $G(z=0) \geq 0$, then $C \geq 0$ also.

Looking at one term approximation only, i.e., $N = 1$, eq.(5.35) gives

$$\frac{k_1}{k_2} = 1 - \frac{3E}{2\pi^2} + \frac{(\pi^2 + a^2)^3}{a^2 k_2}$$

Using eq.(5.27A), this can be expressed as

$$R = \frac{(Pr + \sigma)}{(Pr + 1)} \left(1 - \frac{3E}{2\pi^2}\right) R_S + (\sigma + 1) \left(1 + \frac{\sigma}{Pr}\right) R_C$$

with a minimum again at $a_{crit.}^2 = \pi^2/2$.

For linear distributions, i.e., $E = 0$, this result is identical to

eq.(5.25). For non-constant gradients, the boundary of the stability limit will always be below the linear result.

On May 23, 1978 the non-convective region of the solar pond broke down from 55 to 66cm from the surface. Temperature and salinity data from the previous day were fitted in the form of eq.(5.34) and are presented in Figure 5.2. The coefficients used are

$$\theta = 1.5z - 0.5z^2$$

$$\Sigma = 3.1455z - 4.2370z^2 + 2.0915z^3$$

Solutions to eq.(5.35) were computed for this day with values of k_2 ranging from 10^4 to 10^{12} ; this implies a set of different non-convective layers of sizes ranging from 1cm to 100cm, all having the same dimensionless gradients F and G . These represent different cases and not a succession of sublayers from the same problem.

Figures 5.3 through 5.6 show the smallest eigenvalue k_1/k_2 as a function of horizontal wavenumber a , for several values of k_2 and several orders of approximation N (N is the number of terms in the approximate solution $W = \sum_{n=1}^N C_n \phi_n$).

Analysis of the Rayleigh quotient formulation, eq.(5.29), reveals that for very small or large a , the eigenvalues tend to infinity, while the numerical computations show that the region of minimum k_1/k_2 broadens as k_2 increases. By $k_2 = 10^8$, the curves are very flat, the region of minimum eigenvalue becomes very broad and the horizontal structure has little effect. Also, the critical value of a^2 for $k_2 = 10^4$ is close to the linear result, i.e., $a_{crit}^2 = \pi^2/2$; however, as k_2 is increased, a_{crit}^2 is shifted to larger values. This fact is in agreement with results

FIGURE 5.2 - Temperature ($^{\circ}\text{C}$) and salinity (% by weight) distributions on May 22, 1978 in the upper 100cm of the solar pond. The insert presents the small convective layers established by May 23, 1978 in the region 55 to 66cm from the surface. The curve fitted region goes from 10 to 80cm.

FIGURE 5.3 - Minimum eigenvalue k_1/k_2 as a function of horizontal wavenumber a with 4 terms in the approximate solution.

FIGURE 5.4 - Minimum eigenvalue k_1/k_2 as a function of horizontal wavenumber a with 10 terms in the approximate solution.

FIGURE 5.5 - Minimum eigenvalue k_1/k_2 for 10 terms in the approximate solution and k_2 from a value of 10^8 to 10^{12} .

FIGURE 5.6 - Minimum eigenvalue k_1/k_2 for 20 terms in the approximate solution.

obtained by Baines and Gill (1969) who discuss most unstable modes over all wavenumbers for the case of constant gradients. It is interesting to note that for values of k_2 larger than 10^{10} , variations in the minimum eigenvalue are very small and that the stability condition would therefore be virtually the same for layers between about 30 and 100cm thick, all displaying the same structure in F and G.

Figures 5.7 through 5.9 represent the first eigenfunctions (i.e., the eigenfunction corresponding to minimum k_1/k_2 at critical a , for particular N and k_2) for increasingly better approximations. In Figure 5.7, the layer considered is about one centimeter, the function has converged already by $N = 5$ and the motion would involve the entire layer, the stably stratified portion being driven by the buoyant interior portion. In Figure 5.8 instead, with $k_2 = 10^8$ (equivalent to about 10cm), convergence requires twenty terms; the strong motion is localized and affects only one third of the whole layer. Figure 5.9 presents the case of a 100cm layer. As discussed previously, convergence would require $N = O(100)$, hence with only 20 terms in the solution the function has not converged but the main circulation is again localized and would affect only 12cm or so.

The numerical prediction in Figure 5.9 indicates a possible breakdown in the region from 50 to 61cm from the surface; the region which did break was from 55 to 66cm. The slight disagreement between the derived and the observed result might be attributed to the fact that the fit on the salinity distribution does not precisely follow the actual profile in the marginally stable region, as can be seen in Figure 5.2; this is investigated later in this chapter.

The same numerical analysis performed on data from April 25, 1978,

FIGURE 5.7 - First eigenfunctions for $k_2=10^4$ as a function of dimensionless layer depth z which covers approximately 1cm. The approximate solutions for different N are superposed and have converged by $N=5$.

FIGURE 5.8 - First eigenfunctions for $k_2=10^8$ or a layer 10cm thick. The solutions have converged by $N=15$.

FIGURE 5.9 - First eigenfunctions for $k_2=10^{12}$ or a layer about 100cm thick. Solutions are not converged by $N=20$ but they are localized and peak at about 56cm.

when the pond remained stable, provides similar results. Once again the data was fitted according to eq.(5.34), see Figure 5.10, with

$$\theta = 1.438z - 0.438z^2$$

$$\Sigma = 2.413z - 2.772z^2 + 1.359z^3$$

and the numerical results are presented in Figures 5.11 through 5.14. In this case, the minimum eigenvalue for $k_2 = 10^{12}$ was almost twice the previous result, but the general behavior is preserved and the eigenfunctions are very similar. The same localization is present, although the region affected is shifted by a few centimeters, as is the peak F/G for these two cases.

This procedure has been applied to several selected days and the results follow the previously described trend over a wide range of parameters. The minimum eigenvalue, through this range, shifts from 0.35 to 1.0. The eigenfunctions are consistently localized in regions of 15cm or less while the remainder of the layer shows only weak secondary flow. These results support the validity of the sublayer technique described at the beginning of this chapter. Application of this technique leads back to eqs.(5.30), (5.31), with F and G constant, but different from unity, in general. For a single term in the summation, i.e., $W = C_1 \sin \pi z$, direct substitution in these equations gives

$$R F(z) = \frac{(Pr + \sigma)}{(Pr + 1)} R_S G(z) + (\sigma + 1) \left(1 + \frac{\sigma}{Pr}\right) R_c$$

which is the linear, constant gradient result presented in eq.(5.25), with the Rayleigh numbers scaled by F and G . As values for R , R_S are of order 10^4 to 10^{12} for the centimeter to meter-scale sublayers which

FIGURE 5.10. - Temperature ($^{\circ}\text{C}$) and salinity (% by weight) distributions for April 25, 1978 in the upper 100cm of the solar pond. The curve fitted region goes from 20 to 70cm. No breakdown was registered with this configuration.

FIGURE 5.11 - Minimum eigenvalue k_1/k_2 as a function of horizontal wavenumber a , and 10 terms in the approximate solution. Values of k_2 are stepped by 10^2 and the last two curves are indistinguishable.

FIGURE 5.12 - First eigenfunctions for $k_2=10^4$ as a function of non-dimensional depth z which covers approximately 1cm. The solutions are all superposed, converging by $N=5$.

FIGURE 5.13 - First eigenfunctions for $k_2=10^8$ or a 10cm layer. The solution has converged by $N=15$.

FIGURE 5.14 - First eigenfunctions for $k_2=10^{12}$ equivalent to a layer approximately 100cm thick. No convergence by $N=20$ but solutions peak at about 52cm.

will commonly be analysed, the second term can be neglected, as it is of order 10^2 .

The local stability function

$$\lambda(z) = \frac{(Pr + 1) R F(z)}{(Pr + \sigma) R_S G(z)} = \frac{(Pr + 1) \alpha \Delta T_c F(z)}{(Pr + \sigma) \beta \Delta S_c G(z)} = \frac{k_1 F(z)}{k_2 G(z)},$$

where k_1 and k_2 are defined in eq.(5.27A), is independent of scale, although a function of position within the layer. However, $\lambda(z) = 1$ at individual points in the layer is not the marginal stability condition because viscosity would prevent motion in a zero thickness layer; it must instead be expressed as

$$\frac{k_1 F(z)}{k_2 G(z)} \geq 1 \quad \text{over a finite region } d \text{ within the layer } h. \quad (5.36)$$

This "finite region d " must either be spatially large enough to overturn or there must be enough excess energy in the region where $\lambda > 1$ to drive the stable adjoining regions. The minimum ratio d/h necessary for an instability to develop should depend both on k_2 and on the scale of the shape function F/G . A natural choice of scale for F/G would be the radius of curvature r , at the point where F/G is maximum, thus the most unstable point in the layer. The limiting case of $r \rightarrow \infty$ is the constant gradients case, where $F/G = 1$ and $d/h = 1$, for which the instability would affect the entire layer considered, regardless of size.

This dependence was analysed on the basis of data obtained at UNM, only for the range of parameters relevant to solar pond operation. Results obtained display only the dependence on k_2 , as the radius of curvature of the shape function is very small, in all cases considered.

See Table 5.1, keeping in mind that k_2 carries implicitly a layer size associated with its value, due to the way in which it is defined. Additional research should be undertaken in this area to reveal the full dependence of d/h on both r and k_2 .

TABLE 5.1 - Dependence of Minimum Breakdown Region, d , on k_2 , for Several Different Sublayers and Gradient Configurations and for Asymptotically Small r (order 10^{-2})

k_2	$(d/h)_\lambda^*$	$(d/h)_W^{**}$
10^4	0.43 - 0.49	1.0
10^6	0.20 - 0.23	0.67 - 0.70
10^8	0.10 - 0.12	0.30 - 0.35
10^{10}	~ 0.025	$< 0.15 - 0.17$
10^{12}	~ 0.025	< 0.14

* $(d/h)_\lambda$ refers to the dimensionless size of d/h for which the ratio F/G is larger than the smallest numerical eigenvalue λ ($= k_1/k_2$), obtained for each k_2 .

** $(d/h)_W$ refers to the dimensionless size of d/h between the main zeros of the first eigenfunction W , for each k_2 .

As the eigenfunctions for the cases with $k_2 = 10^{10}, 10^{12}$ were not fully converged, the results presented are only upper bounds. However, results for the cases with $k_2 = 10^6, 10^8$, converted to dimensional depths, range from 2 to 3cm, depending on the property values selected. This is in excellent agreement with observations of solar pond behavior as internal convective layers smaller than about 2cm have always disappeared spontaneously. An independent program solving the same equation

was used to check four-digit converged results at $N = 50, 100$, as part of a continuation of the present analysis. Results show that the first eigenfunctions for the $k_2 = 10^{12}$ cases remain centered on the same location but each region $(d/h)_W$ continues to narrow down. For the May 22 case, for example, the 100 terms converged solution is still centered on $z = 0.65$ (see Figure 5.9) but the width has decreased to $(d/h)_W = 0.08$.

The validity of the "sublayer stability analysis" expressed in eq.(5.36) was tested against the numerical results obtained for the cases of May 22, 1978 and April 25, 1978. Following the procedure outlined in the beginning of this chapter (Section A), the local value of F/G was computed first from the whole layer curve fits, while values of k_1/k_2 were computed according to local conditions. The accuracy of the curvefits and properties is no better than 10%; in addition, the position coordinates for the temperature and salinity profiles (recorded by two independent instruments) may differ by as much as one centimeter; this would change the shape and peak value of F/G . However, as can be seen in Figure 5.18a, in which the entire gradient region has been scanned, the sublayer from 54 to 59cm has $\lambda(z) \geq 1$, thus $d/h = 0.07$, indicating that the system is above marginal stability, for which the value is $(d/h)_\lambda = 0.025$. The numerical solution of eq.(5.35) predicts motion from 50 to 62cm in this case; motion was observed between 55 and 66cm, as reported above. Despite the fact that the numerical stability criterion proves conservative, the region of predicted motion agrees surprisingly well with observed motions.

To test the ultimate accuracy of this analysis, a sublayer 25cm thick was chosen, and data fits were applied to this region only. Visual analysis of the data (Figure 5.2) indicates that the weakest

salinity gradient occurs in the region from 55 to 70cm; the numerical technique was thus applied to the sublayer from 50 to 75cm; the coefficients used are

$$\theta = 1.341z - 0.341z^2$$

$$\Sigma = 1.2605z - 1.8230z^2 + 1.5625z^3$$

and the results are presented in Figures 5.15, 5.16, 5.17. As expected, the calculated minimum eigenvalue has risen from 0.34 to 0.52, as the region analysed was decreased from 70 to 25cm. The eigenfunctions in Figures 5.16 and 5.17 are once again localized in the region from 57 to 61cm, as compared to motions in the May 23, 1978 breakdown extending from 55 to 66cm (Figure 5.2).

Application of eq.(5.36) to local values in the sublayer leads to Figure 5.18b, in which $\lambda(z) \geq 1$ from 54 to 64cm. This is again broader than the numerically calculated marginal stability result ($\lambda \geq 1$ from 57 to 61cm) and half of the difference is not attributable to the already identified sources of error. This suggests that the model may still be too simple to be quantitatively precise, in agreement with an analysis by Rothmeyer (1979?) who finds significant magnitude of diffusivity gradient in the flux divergence term of the salt-water transport equations. Thus, once again, the numerical stability criterion is conservative but the predicted layer of motion agrees well with observations.

The break on May 23, 1978 occurred as a succession of 4 consecutive convective layers (see Figure 5.2). A possible source of this behavior may be found in the following argument. Because, for $F = G = 1$, the solution is $W_n = \sin n\pi z$, one has

$$\lambda = \frac{(n^2\pi^2 + a^2)}{a^2k_2} + 1$$

FIGURE 5.15 - Minimum eigenvalue k_1/k_2 as a function of horizontal wavenumber a with 20 terms in the approximate solution. The sublayer analysed was from 50 to 75cm.

FIGURE 5.16 - First eigenfunctions on the sublayer for $k_2=10^8$ or a 10cm sublayer. Solutions have not converged by $N=20$.

FIGURE 5.17 - First eigenfunctions for $k_2=10^{10}$ or a 25cm sublayer from 50 to 75cm below the surface. Solutions have not converged but are localized and peak at about 59cm.

(a)

(b)

FIGURE 5.18 - Stability functions. (a) Entire gradient region, May 22 and April 25, 1978. (b) Sublayer 50 to 75cm from the surface, May 22, 1978.

an expression which indicates that $\lambda_n \rightarrow 1$ as $k_2 \rightarrow \infty$ for all finite n . The lowest eigenvalues approach λ_1 , so all modes become marginally stable at nearly the same λ and thus a breakdown into more than one convective layer is possible.

Given the uncertainty of the physical properties available and the possible uncertainty of the location data, remarkable agreement was found between observation of pond breakdown and predictions of the numerical analysis.

Additional days have been analysed in the same fashion. Temperature and salinity distributions for these days are presented in Figures 5.19 through 5.22, while Table 5.2 summarizes the stability analysis results.

In Figure 5.20, data for March 22, 1977, one can see that although there is a characteristic unstable "dent" in the salinity curve from 25 to 40cm, the temperature gradient was not large enough to overturn this region.

Figure 5.19 presents data for July 29, 1978, when the storage temperature was maintained around 85°C by additional heat extraction. Comparison to Figure 5.22 provides a good indication of the importance of the salinity distribution to stability; while both temperature and salinity ranges for these two days were very similar ($\Delta T_c \sim 60^\circ\text{C}$, $\Delta S_c \sim 18\%$), the August 8, 1977 configuration in Figure 5.22 proved to be very unstable in the entire gradient region. In contrast, the distribution on July 29, 1978 would have been more than adequate to sustain a storage temperature in excess of 100°C, if the shape of the temperature curve had remained the same. This confirms once again that maximum temperatures can be obtained and maintained in a solar pond stabilized by sodium chloride.

FIGURE 5.19 - Temperature ($^{\circ}\text{C}$) and salinity (% by weight) distributions for July 29, 1978 in the upper 100cm of the solar pond. No breakdown occurred.

FIGURE 5.20 - Temperature ($^{\circ}\text{C}$) and salinity (% by weight) distributions for March 22, 1977 in the upper 100cm of the solar pond. No instabilities occurred.

FIGURE 5.21 - Temperature ($^{\circ}\text{C}$) and salinity (% by weight) distributions for June 30, 1977 in the upper 100cm of the solar pond. Instabilities occurred by July 1, 1977 in the region 61 to 69cm as shown in the insert.

FIGURE 5.22 - Temperature ($^{\circ}\text{C}$) and salinity (% by weight) distributions for August 8, 1977, in which the highest storage temperature for 1977 was achieved (93°C). Convective sublayers from 27 to 33cm appeared by August 9, 1977 while by August 13, 1977 the entire gradient broke down, as shown by the dotted curve.

TABLE 5.2 - Sublayer Stability Analysis (summary)

Date	$\frac{\Delta T}{\Delta S}$ (tot) *	$\frac{\Delta T}{\Delta S}$ (fit) **	Region of Fit (cm)	Peak $\frac{k_1 F}{k_2 G}$ †	Predicted †† Unstable Layer (cm)	Break-down?	Date of Breakdown	Region of Breakdown (cm)
April 25, '78	$\frac{52.7}{12.6} = 4.2$	$\frac{47.3}{10.8} = 4.4$	20-70	0.091 · 7.22 = 0.65	-	NO	-	-
"	"	$\frac{29.0}{4.8} = 6.0$	35-65	0.087 · 6.90 = 0.60	-	NO	-	-
May 22, '78	$\frac{47.1}{13.2} = 3.6$	$\frac{44.6}{12.6} = 3.5$	10-80	0.097 · 10.43 = 1.01	51-61	YES	May 23	[55-66 (50-61 _{num})
"	"	$\frac{34.4}{1.6} = 9.0$	50-75	0.099 · 13.41 = 1.33	54-64	YES	"	[55-66 (57-61 _{num})
July 29, '78	$\frac{58.5}{18.1} = 3.2$	$\frac{28.8}{3.4} = 3.4$	30-70	0.100 · 3.83 = 0.38	-	NO	-	-
March 22, '77	$\frac{34.9}{14.8} = 2.4$	$\frac{17.7}{4.8} = 3.7$	20-50	0.067 · 5.26 = 0.35	-	NO	-	-
June 30, '77	$\frac{51.6}{14.1} = 3.6$	$\frac{7.6}{1.1} = 6.9$	50-75	0.107 · 14.21 = 1.51	57-68	YES	July 1	61-69
August 8, '77	$\frac{67.0}{19.2} = 3.5$	$\frac{18.9}{4.0} = 4.7$	25-45	0.093 · 10.68 = 0.99	27-35	YES	[August 9 August 13	[27-33 5-85

* $\frac{\Delta T}{\Delta S}$ (tot) refers to the difference of values between the convective layer and the surface layer.

** Temperature and salinity differences at the extremes of the region selected for curve fitting (column 4).

† Peak value of $\frac{\Delta T}{\Delta S} \frac{F}{G}$ multiplied by the stability coefficient, found in Appendix C, Table C.1.

†† The 10% error margin in the data has been taken into account for these results.

TABLE 5.2 (cont.)

- I. The region to be analysed is selected by visual check of the salinity distribution; the region about the weakest salinity gradient is chosen.
- II. The region selected is fitted with two polynomials for the temperature and salinity distributions; local gradients are computed from these fits. This defines the local value of F/G .
- III. The peak value of F/G is determined, along with the position where this occurs and a value of the stability coefficient, C_s , is determined from Table C.1, at the appropriate temperature and salinity.
- IV. A plot of C_s scaling the F/G ratio will show if the gradient is stable, when (d/h) is less than the value in Table 5.1.

Note that, although May 22, 1978 has a larger overall ΔS for a smaller ΔT than April 25, 1978, the distribution is such that the first configuration is unstable, while the second is quite stable. The same comparison holds true for June 30, 1977, stressing the importance of the shape function.

Note also that the total ΔT is dependent on the hour in which the data are collected, as the surface temperature is very sensitive to ambient conditions and can vary of several degrees over a few hours' span (see Figure 4.3). The skin depth is of the order of 10cm; data fits should therefore not include this region, at least.

Finally, the density or salinity data require extreme accuracy as the results of this analysis are sensitive to small changes, due to the differentiation to obtain $G(z)$.

CHAPTER VI

HEAT EXTRACTION

The UNM solar pond was designed to supply the hot water and heating requirement of a 185m^2 house in Albuquerque; this amounts to approximately 30MWh or 10^{11} Joules per year of useful heat, of which only about half is necessary for space heating in an average year.

As no building of this approximate size was available near the facility, it was decided to model a standard 185m^2 frame-stucco, flat roof, single story house, occupied by a five-person family. This house is heated exclusively by natural gas, thus during the Summer and early Fall of 1977 the gas consumption was monitored to determine what fraction of the total energy use should be attributed to different requirements. On every morning of the heating period the gas meter was estimated to the nearest 10 cubic feet of gas and the portion used for heating was calculated. This in turn was converted directly into energy using the conversion supplied by the gas company with each bill (approximately 8.8therms/MCF) and an equivalent amount of energy was extracted from the solar pond. Inefficiencies of the floor furnaces were not taken into account as they should be roughly equivalent to heat losses in the supply pipes, if a solar pond were used for direct heating. Because of the large thermal inertia of the pond, a delay of one day between the demand of the house and the withdrawal of that energy is negligible; in fact, energy was often withdrawn on a weekly basis in

the 1978-79 heating season without any noticeable difference in pond response, except for a slightly larger temperature drop at the time of reinjection.

Heat can be extracted from the solar pond by either using the hot brine directly or by placing a heat exchanger in the storage layer and running a secondary fluid loop. Direct extraction from the hot brine was chosen not only because it is the most efficient in terms of heat transfer but also to analyse the effect of localized withdrawal of brine from the solar pond.

At first, the hot brine from the convective layer was circulated through the heat exchanger, then returned to the storage convective layer through the diffuser; several arrangements of inlet and outlet positions have been tried and little effect has been registered. Brine has been injected 30cm below the convective layer boundary to provoke perturbation by enhanced convection; despite these deliberate disturbances, no detrimental effects have been noticed. Injection tests performed periodically until present have been equally successful, even at the highest pond temperatures (thus, least stability); at this distance, the boundary has not appeared to be affected.

The heat exchanger was placed in the auxiliary pond for two reasons: the effects of salt on the outside of the heat exchanger can be monitored and the heat extracted can be used to enhance evaporation of the surface run-off, instead of being wasted to the air. While this setting represents approximately the condition of a heat exchanger at the bottom of the solar pond, it also renders the exchanger accessible for analysis of corrosion and scaling problems. As the primary interest is the

response of the pond to large, periodic and localized fluid extraction and injection, characteristics and efficiency of the heat exchanger are secondary; the relevant parameter is the amount of heat extracted, and the time scale. For reference, the exchanger is described briefly in Appendix A; pumping rates have been around 120ℓ/min and the maximum temperature drop across the exchanger has been 10°C.

A second method of heat extraction has been used more recently to increase even further the disturbance to the pond. Hot brine from the convective layer has been withdrawn and allowed to cool approximately 20°C in the auxiliary pond before returning it to the convective layer all at once and at the normal pumping rate. The effect is to cool the entire storage layer between 3 and 6°C, depending on the volume extracted; in general the mixing pond is filled with 20m³.

Cooling the storage convective layer has a stabilizing effect on the gradient as it reduces the temperature difference in the lower region, but the mixing due to such large injection of cold, salty brine is a test to the boundary. The brine injected is actually slightly saltier than it was previously as some evaporation takes place in the mixing pond, but most of the density difference is due to the large difference in temperature. When injected, the fluid sinks but exchanges enough heat in the process to maintain the storage layer isothermal. On few occasions, the saltier brine has remained at the bottom, dividing the storage layer in two regions of salinity; however, this separation only lasts a few days.

In the winter of 1977-78, only that fraction utilized by space heating was extracted from the solar pond; this amounted to $3.2 \cdot 10^{10}$ J.

The daily demand of the model house was followed fairly closely until the first two weeks of January 1978, when far more heat than necessary was extracted by mistake. To compensate for the surplus, no heat was extracted in the following two weeks. This accounts for the shift in phase lag of the pond temperature and for the minimum of 29°C which was slightly below what was expected.

From November through February, heat extraction averaged about 66% of the radiation reaching the storage layer, or equivalently, 24% of the total input at the surface. In the month of December, however, 95% of the input to storage was extracted.

Part of the heat stored in the ground below the pond and in the sides was recovered, mostly in January, but this contribution is very small for the UNM pond. The two-winter experience with this pond indicates that the assumption of retrievable ground storage in the analytical derivation by R&N is definitely not valid for small ponds and short operating times, as discussed in Chapter IV.

In April 1978, at the end of the winter heating season, additional heat was extracted from the pond to match the hot water requirement of the house, until the pond reached 85°C . At this time, additional heat was extracted to maintain the pond temperature around this value of 85°C . This temperature was chosen as it would be in the optimal range for efficient coupling of the solar pond to an absorption cooler, for example. This procedure continued until September 1978, when the pond temperature dropped below this limit and the hot water requirement alone was extracted from the pond, until house heating for the winter season of 1978-79 began. Extraction in this period amounted to $2.6 \cdot 10^{10} \text{J}$. A summary of

energy use and extraction for this period is presented in Table 6.1.

The November 1977 to November 1978 yearly average of the pond's collecting efficiency amounts to 9% of the radiation received at the surface, as measured by the pyranometer near the pond. This value will continue to increase as the ground below the pond warms up and as methods to control the surface convective layer improve. Better efficiencies would also be obtained with larger ponds as the wall losses would have a smaller effect on the energy balance.

TABLE 6.1 - Heat Extraction in 1977-78

Month	Heat Extracted from Pond * (10^9 J)	Total Energy Used (10^9 J)	Attributable to Hot Water & Space Heating** (10^9 J)	Storage Temperature ($^{\circ}$ C)
A 1977		6.22		
S		9.92		
O		9.81		
N	4.66	11.18	6.27	61.5
D	8.67	19.62	14.71	48.0
J 1978	9.16	17.09	12.18	34.5
F	6.36	17.41	12.50	32.0
M	2.66	12.76	7.85	34.0
A	<u>0.64</u>	<u>10.13</u>	<u>5.22</u>	55.0
Subtotal	32.15	114.14	58.73	
M 1978	3.97	8.76	4.90	72.0
J	1.14	8.76	"	81.5
J	6.52	7.17	"	80.5
A	8.99	5.91	"	83.5
S	5.00	6.01	"	87.0
O	<u>5.21</u>	<u>9.01</u>	<u>5.80</u>	80.5
Subtotal	30.83†	45.62	30.30	
Total	62.98	159.76	89.03	

* This energy was dispersed at the UNM facility.

** This quantity is computed from the total monthly usage by subtracting the September consumption and adding an average hot water requirement of $4.9 \cdot 10^9$ J/mo.

† If one includes the energy lost in the June breakdown, this subtotal amounts to $39.63 \cdot 10^9$ J, making the summer excess $9.3 \cdot 10^9$ J.

CHAPTER VII

MAINTENANCE REQUIREMENTS

A. Water Clarity

The surface of the solar pond is cleaned using two different procedures: the surface is flushed on a continuous basis using a low rate fresh water feed and a slow draw-off syphon which collects the run-off into the evaporation ponds for salt recycling; after severe dust storms the entire surface layer is pumped, filtered, then returned to the pond. Most materials that are blown into the solar pond either float or settle rapidly to the bottom, with the occasional exception of some leaves which remain in suspension for a longer time. If a convective layer is present at the surface, these leaves will sink to the base of this layer, marking the boundary with the gradient region below. Once at this level, the leaves appear to remain there until enough salt has crystallized on their surfaces to make them sink to the bottom; this has never been a serious problem. As there are several deciduous trees in the vicinity of the pond, a floating net, fixed to the bank, has been used to immediately collect leaves and other floating debris through the natural circulation of the surface water in times of breeze or even strong wind (at these times the surface is swept efficiently and collection of all debris at the banks is possible).

The debris that have settled to the bottom do not affect the transmission nor inhibit the absorption of the liner, as tests have indicated;

no attempt has therefore been made to remove this sediment. On occasion, leaves appear to have colored the water in the convective region; this does not affect the collecting efficiency of the pond since total absorption in this region is desired. If this effect were so strong as to absorb all the transmitted light at the top of the storage layer, it could create a problem as a new gradient would be established, inhibiting heat extraction; a temporary condition of this nature was observed in the Spring of 1976. The effect disappeared in a short time, probably due to eventual settling of the leaf pigment (see Figure 4.2, curve of June 2, 1976).

In an uncovered pond, there seems to be no reason to have a black liner; in fact, a light colored liner might reduce the tendency for localized convection since it would decrease the amount of heat absorbed in the walls. As for the bottom of the pond, it is inevitably covered with brown debris which mask the original color of the liner.

To eliminate turbidity caused by biological growths in the upper region of the pond, a copper sulfate solution is sprayed over the entire pond two or three times a year. One kilogram of copper sulfate diluted in a bucket of warm water is sufficient to remove all visible organisms and clear the pond in three days. Samples collected from the pond show very low turbidity and, if the solar angle is high, one can see details of the bottom very clearly.

B. Surface Convection and Salt Diffusion

As the surface of the UNM pond is exposed to the atmosphere, rapid night cooling induces an even stronger destabilizing temperature gradient in the upper region of the pond; wind stress on the surface also favors

mixing and the surface invariably becomes convective for a depth of about 10cm in a short time. Wind action would be minimized by an array of pipes floating on the surface of the solar pond. If, in addition, a small amount of hot brine from the convective layer were to be circulated through these pipes each evening, a higher surface temperature would be maintained during the rapid cooling period and convection could be prevented.

An obvious but costly alternative to this procedure would be to shield the solar pond with a plastic, air inflated bubble. This cover would decrease the solar input into the pond but it would also prevent the large evaporation characteristic of arid regions such as New Mexico. This would maintain a higher surface temperature, decreasing the heat loss by conduction through the gradient. A cover would also prevent dirt from blowing into the solar pond, minimizing cleaning requirements. As stated, it is a costly measure which would benefit the small residential pond, but would not be reasonable for either a low-cost pond or a very large one.

The importance of surface control for improved efficiency is stressed in Chapter IV, where it is shown that the average storage temperature prediction can be decreased by 38°C if a 20cm convective layer is present at the surface.

Until August 1978, the surface convective layer at the UNM pond was maintained around 10cm by syphoning out brine from the base of this layer and injecting fresh water on top. The volume of brine removed this way varied from about 0.2 to $1.0\text{m}^3/\text{day}$, while replenishment of fresh water to compensate both for recycling and evaporation varied from about 0.9 to $1.7\text{m}^3/\text{day}$. After this date, no further maintenance was performed to

see how this layer would develop and to measure the salt transport to the surface. Addition of fresh water to the surface was reduced to compensate only for the evaporation and to maintain the pond level constant. In August, the surface layer increased slowly from 11 to 22cm. This depth was then maintained until the middle of September, when it slowly increased again, this time to 25cm. Finally, by the middle of October, the size of this layer settled around 30cm, where it has remained until present, despite ice formation on the surface in the months of December 1978 and January 1979. It is encouraging that there appears to be a stable finite depth for this layer; however, 30cm account for too much energy loss and further analysis of development as well as prevention methods is still necessary.

Diffusion analysis through this period shows a marked decrease on the amount of salt transported to the surface in the fall and winter. Analysis in July 1978, at peak temperature, showed a change of surface salinity by an average of 0.24% concentration by weight increase per day. This amounts to a total salt transport per unit surface of about $0.36\text{kg/m}^2\text{-day}$ for the summer, far more than steady state diffusion calculations can account for. In the winter time instead, with a similar salinity distribution but a much lower temperature gradient, the salt transported averaged $0.05\text{kg/m}^2\text{-day}$. As the diffusion coefficients are functions not only of temperature and salinity but also of the respective gradients, theoretical calculations of diffusion rates are far from trivial. Analysis of this dependence and diffusion rates predictions for the solar pond should be presented shortly in a Master Thesis from UNM (Rothmeyer, 1979?). Results of this analysis are crucial for the

determination of the best salt distribution attainable (and maintainable) in solar ponds, to optimize performance.

C. Salt Reclamation

To maintain a closed salt loop in diffusing ponds, the salt transported to the surface must somehow be replaced at the bottom of the pond. Once the brine from the surface convective layer is removed, salt must be separated from the excess water by some means. At the UNM facility, open evaporation ponds are used to concentrate the brine up to the maximum salinity attainable in solution (about 26% for sodium chloride) before this concentrate is reinjected into the storage layer. Recovery of salt, if no breakdown has occurred, amounts to full recovery from April to September, and about half recovery in the winter, when low temperatures and insolation greatly reduce the evaporation, or the surface is covered with ice. Evaporation in these ponds does not occur at the optimum rate because of the variation in depth due to uneven bottoms. At times, salt crystals have been recovered from the evaporation ponds. These are mixed with the brine from the storage layer when it is pumped out for heat extraction purposes; the saltier brine is then returned to the solar pond. Salt crystals can be added either by direct mixing in the auxiliary pond, as it is done at UNM, or by dispensing small quantities of salt in the hose through which brine from the heat exchanger is being returned to the solar pond.

Occasionally, if the evaporation ponds are full and the surface salinity is too high, part of the surface layer has been wasted and replaced by additional fresh water. This procedure could be used continually if salty brines were available to flush the surface. However,

cost and possible pollution generally prevent this method from being applied.

Other alternatives could be flash evaporators, as suggested by Tabor, or reverse osmosis membranes. Both of these, however, appear to be very expensive and maintenance intensive methods which have never been tested with brines from a solar pond; as these brines do contain suspended matter, an extensive filtering operation would probably be required.

CHAPTER VIII

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH

Three years of continuous operation of a salt gradient stabilized solar pond at the University of New Mexico demonstrate that the solar pond is easy to establish, easy to maintain and a dependable source of low temperature heat. The successful operation of this pond required minimal attention; more work than strictly necessary for operation was performed, in order to simulate abnormal or extreme conditions and to acquire additional data to understand the physical processes governing solar ponds.

Experiments with fluid redistribution in the gradient region led to a new filling technique which is considerably simpler and more expedient than any previous method used as well as procedures to modify the gradient region profiles and prevent breakdown.

Realistic general predictions of solar pond operation temperatures were obtained by solution of the one-dimensional heat conduction problem with corrections for actual input and wall losses; results approximate data obtained with the UNM pond, however, an accurate two-dimensional model is still needed to incorporate all physical factors and predict the fine features of the response.

Extensive analysis of gradient stability provided a new understanding of the stability condition of double-diffusive systems, applied to the previously untreated case of arbitrary gradients, as well as new criteria to predict location and size of unstable sublayers.

Comparison of theoretical predictions to actual solar pond data showed excellent agreement and confirmed that all instabilities registered at the UNM pond were due to the purposefully weak salinity gradient, which could have been increased, by brine redistribution, to successfully stabilize the pond up to the 100°C operating temperature limit.

During one and a half years, heat was extracted from the UNM pond on a continuous basis, to match the demand of a home in Albuquerque. This experiment proved that heat can be extracted easily, without deleterious effects on the pond's performance, and that cold brine can be injected back into the convective layer without affecting the gradient boundary.

The numerous applications where a solar pond would be an economically feasible alternative to conventional fuels include domestic, commercial and industrial applications such as space heating and cooling, supply of hot water and process heating. Electricity production in remote areas has been considered, using a secondary fluid in combination with a solar pond, which would supply both hot and cold reservoirs.

The general behavior of the solar pond is understood, to the point where a pond can actually be used to supply energy under demand conditions at this time. As the storage efficiency increases with size, predictions of use go toward the concept of large ponds supplying energy to residential communities, industries or agricultural concerns. Sodium chloride has been shown to be an appropriate salt for establishment of solar ponds, and despite earlier predictions to the contrary, it has been found to stabilize ponds even at high temperatures. This is convenient, since sodium chloride is widely available and is the cheapest of salts.

However, it is not necessary to have a pure salt solution at the start of a solar pond; salty brines, especially if available at the site, would be adequate as well. Thus, some of the most appropriate locations for solar ponds are those in the vicinity of a salty brine supply; in the United States, these would be the coastal areas, the Great Salt Lake, the Salton Sea and other areas of the Southwest.

While present knowledge is sufficient to operate a solar pond successfully to meet demands posed by actual, non-experimental uses, additional research is needed to understand fully the physical behavior defining solar pond parameters. Accurate data on density distribution and light penetration as well as accurate measurements on heat losses to the ground and pond efficiency are sorely needed to refine the mathematical model, while the problems of minimizing salt transport and minimizing the effects of environmental factors must be addressed to improve operation and maintenance requirements.

As seen in Chapter V, accurate density data are crucial in monitoring the pond's behavior and in determining its approach to breakdown conditions. Drastic changes occur with very small concentration changes, as stability depends on the slope of the salt distribution curve. An accuracy of better than 0.1% concentration by weight is therefore needed. In order to obtain accurate data, density-measuring instrumentation must be developed, both to serve experimental (laboratory) conditions and to help in the operation of actual (non-laboratory) ponds. Thus, such instrumentation must be, at the same time, sophisticated enough to provide accurate measurements and rugged enough to withstand field use.

Accurate optical measurements of light penetration through the

gradient are necessary for a correct determination of the solar pond efficiency. The problem at hand calls for instrumentation capable of operating within a range of 0 to 100°C, with a rapid temperature response and designed for use under water.

The problem of salt diffusion would require a study of the time development of the salt gradient under operational conditions in order to determine the optimal initial distribution required to minimize the possibility of breakdown and salt transport to the surface; also, the design of pond walls (slanted versus vertical walls) should be analysed, to determine the role played by the walls and wall heating in the transport of salt to the surface.

The performance and cost of salts whose solubility is strongly temperature dependent should be studied as well. It is hoped that operation of ponds stabilized by such salts would proceed under saturated conditions in the bulk of the pond; in these circumstances, internal salt redistribution would take place spontaneously, without intervention of external equipment or maintenance requirements.

If salt transport to the surface cannot be economically eliminated or reduced, recycling of the salt washed from the surface should be undertaken. Different recycling methods should be analysed, with attention given to the cost effectiveness of each method.

The numerous feasible applications of a solar pond call for a wide range of sizes. Scale can change the operation and maintenance requirements, because certain environmental factors, such as winds, will be felt more acutely as sizes increase. Wind action in large ponds should be minimized in order to prevent establishment of a deep convective layer

at the surface and a subsequent reduction in collection and storage efficiency. This must be weighed against the cost of installing barriers as this could become the single most expensive item in the construction of a large pond. A good prediction of cost versus efficiency gain requires accurate predictions of the mechanisms for establishment of the surface convective layer, the development of this layer if left unattended, and whether any configuration of salt gradient would maintain this layer at a fixed size with a stable lower boundary. This in turn requires an understanding of the physical behavior of boundaries and mechanisms to maintain them stable. The position of the lower boundary between the gradient and the storage layer can be controlled by appropriate injection of the return brine from the external heat exchanger or by the location of the internal heat exchanger, if this method of extraction is chosen. This is an artificial control of boundary movement, but it will always be available in operating ponds, as long as heat is being extracted.

Prevention of excessive evaporation from the surface of the pond by film, membrane or other transparent cover must be researched with respect to the particular type of operation chosen (i.e., whether the salt used will continuously diffuse to the surface, in a manner similar to sodium chloride, or if it will redistribute itself internally, as should be the case with salts whose solubility is strongly temperature dependent), as well as environmental conditions encountered in the region. If "diffusing" salts are used, a mechanism to flush the surface must be devised that does not disturb the evaporation inhibitor chosen. In arid regions, where dust storms are common, special attention must be

given to possible entrapment of dirt on or in the evaporation inhibitor, so as to minimize opacity and not cause excessive reduction of energy input into the pond.

The areas where additional research is needed are then: laboratory testing, equipment development and mathematical modelling. The latter requires continued, long-term analysis of a solar pond such as the pond presently operating at the University of New Mexico, as well as the establishment and operation of larger ponds, in order to compare the mathematical model's predictions to observed results over a varied range of conditions.

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APPENDIX A

EQUIPMENT AND MATERIALS USED AT THE UNM SOLAR POND

Recorders: Houston Instruments OmniScribe recorders. Two independent channels or integration on the first channel. Voltage ranges are: 0.001 to 10V with built-in steps of 10 and adjustable full-scale exterior bypass, or 0.01 to 100V by steps of 10 only. The first recorder has been used to monitor the temperature gradient in the pond using two vertical scanners at different locations; the second has been used to display the insolation and record the daily total.

Conductivity Meter: Balsbaugh, Series 1200. Electrodeless conductivity monitor with automatic temperature compensation at 25°C, which measures the solution conductivity by measuring the current in a closed loop of solution coupling a pair of toroidal transformer coils. The standard DC output is proportional to mMHΩ/cm. The repeatability is only equivalent to 0.1% by weight for a sodium chloride solution. In addition, the sampling area is about 1cm, which is not a fine enough spatial resolution for the observed centimeter-scale instabilities.

Data Logger: Kaye Instruments, RAMP Scanner. Independent plug-in units provide 8 channel sets, for a maximum of 64 channels. Voltage signals registered range from 1μV to 24V, DC. The two-way ASCII communication with a computer provides computer control and selection of seven different operating modes for channel scanning.

Multimeter: Keithley 160B. Digital multimeter with analog output to monitor and calibrate other equipment.

Teletype: Model 33SAR with automatic reader and tape punch. ESC key board.

Pyranometer: Eppley 4-48, standard black and white pyranometer for total radiation measurements. Temperature range is -20 to 40°C.

Computer: Standard Engineering Corporation MIK 11 with Heathkit H9 video terminal and floppy disk storage.

Flow Meter: calibrated orifice with differential pressure gauge, on loan from the Los Alamos Scientific Laboratory.

Pond Liner: Hypalon, 45 mils, 3 plies with two nylon mesh reinforcement between the plies. This is guaranteed for twenty years against "normal" weathering and has not been tested by the factory above 77°C. Our experience is that it softens at 100°C, but appears to remain strong enough for the purpose. It is our impression that a thinner liner, i.e., 30 mils thick, would have performed equally well. Maximum operating temperature so far has been 93°C and the liner has performed satisfactorily. We have not encountered problems associated with weeds growing through the liner, as reported by other research teams using Butyl rubber liners. The Hypalon liner was manufactured into a square of 22m x 22m with all seams glued at the factory. Since the pond was circular by choice while the liner was square by necessity, the excess material from the corners was later glued in the field to line a subsidiary shallow pond. These field seams have also performed well but have never been subjected to water temperatures above 80°C.

Evaporation Pond Liner: black polyethylene, 8 mils thick, with no reinforcement. These sheets were installed directly on dirt and sand in the Summer of 1977 and have been adequate for the purpose. As small tears have occurred, two sheets are now superposed on each pond.

Pump and Filter Assembly: commercial swimming pool equipment of the Seablue Company, with a stainless steel sand filter, brass pump, 1Hp motor; maximum rated flow, 66gpm (250L/min), although such high values have never been achieved; suction and discharge ports are 1.5in (3.8cm) in diameter. No problems have been registered with the units themselves but at high operation temperature the plastic leaf strainer preceeding the pump has partially melted, the six-way valve connected to the filter has shown some deformation, while the original PVC piping from the valve to the filter developed a leak at the inlet to the filter.

Valves: gate, three-way and four-way valves, all brass; these allow use of only one pump to circulate water or brine to and from any reservoir, including circulation through the heat exchanger. Rotary, plastic, six-way valve on the filter unit.

Piping: polyethylene hose is used for all cold water lines and for hot brine syphons where the pressure drop is small. Although suggested for temperatures below 70°C, successful syphoning has been performed up to 85°C. Multipurpose steam hose is used on all other suction and discharge lines carrying hot brine. This hose is overrated for the application, but we could not find any other flexible hose which could withstand fluid temperatures around 100°C.

Diffuser Assemblies: at first, two plates of plexiglas, 30 cm in diameter, 2cm thick, with a 0.3cm gap, suspended by a tripod and attached to a cable stretched over the pond; then, two semi-circles of plexiglas of same dimensions and gap, mounted on a stainless steel support on the south bank of the pond. This entire assembly scans the depth of the pond at selected rates and it is driven on the bank by a synchronous motor and

reduction drive, which are themselves fixed at the south edge of the pond. This diffuser assembly is used for selective injection of fresh water or brine into the gradient region as well as return of the salty brine to the convective layer, after circulation through the heat exchanger. Scanning rates are: 0.5, 1, 2cm/min in the vertical direction; exit velocities from diffuser number 2 range from 1.5 to 2.6m/s, depending on which supply line is used (see Figure A.1).

Scanners: thermocouples attached to weights are suspended on fishing lines by means of a pulley arrangement. Vertical scans into the pond can be made at any selected position over the east-west or north-south diameter of the pond. The metal braided fishing line is rolled on calibrated, threaded drums driven by synchronous motors so that vertical position of the thermocouples is a known linear function of time. Motor speeds are 1 and 2 rotations per minute; resulting scanning rates are 6 and 18cm/min. Delimiting markers are attached to the lead wire and reverse the direction of motion by activating a switch. The horizontal temperature distribution is obtained by using the scanners simultaneously.

Wall Scanner: synchronous motor drive on a continuous loop to which a thermocouple is attached. The support is made of thin stainless steel tubing welded as a ladder, and rests on the bank of the pond. The drive and loop are mounted off-center to provide three distances from the wall: 5, 9 and 15cm. The scanning rate is 6cm/min and can be matched to the vertical scanners for comparison of vertical temperature distribution. Continuous submersion in the hot brine resulted in weakened or broken welded joints.

FIGURE A.1 - Diffuser drive assembly. 1. For fixed injection, the diffuser support is lowered into the pond by manually turning the position pulley until the desired depth is reached, the position pulley is then fastened in place, the pump is turned on. 2. For a distributed injection into a region of the gradient (scanning injection), the diffuser assembly is placed at the correct starting position as in 1., the direction and rate of motion are selected (motor reversing switch and gear ratio), the driving motor and the pump are turned on simultaneously.

Heat Exchanger: at present, 5cm diameter brass tubes, 3.6m long, in four parallel sections, 38cm apart. This heat exchanger has been installed in the shallow mixing pond.

Temperature Sensors: copper-constantan thermocouples; 24 gauge wire with vinyl coating was silver soldered and encapsulated in "shrink tubes". No failures have been registered yet.

Temperature Compensators: Omega Engineering, Omega-CJ cold junction compensators with reference accuracy of $\pm 0.5^{\circ}\text{C}$. All other compensators are built-in features of the equipment used.

APPENDIX B
INJECTION DATA

Although the UNM pond has not been filled in the same manner as the Miamisburg pond, injection into the gradient has been performed both to test this technique and to modify the existing gradient. The following data are provided for comparison of rates used at both facilities: at the Miamisburg pond for the filling process alone, at UNM for various tests.

	<u>Miamisburg Pond</u>	<u>UNM Pond</u>
Surface Area	2,000m ²	175m ²
Depth	3.6m	2.5m
Total Volume	~ 7,000m ³	230m ³
Diffuser Size	1.2m dia.	0.3m dia.
Diffuser Gap	3mm	3mm
Brine Exit Velocity	0.6m/s	[0.4m/s (with small pump). 1.5 to 2.6m/s
Vertical Stepping of Diffuser	5cm	[1 to 3cm or continuous scans
Duration of Injection	8 to 10hours/day (for seven consecutive days)	0.5 to 4hours/day

APPENDIX C PROPERTIES AND STABILITY COEFFICIENT

Values and ranges for properties of a sodium chloride solution between the temperatures of 0 to 100°C and concentrations of 0 to 25% by weight are presented in the following list; these properties were introduced in Chapter V. The second column gives minima and maxima while the third column gives the value of each property at 60°C and 15% concentration by weight.

<u>Property</u>	<u>Range</u>	<u>Value(60°C, 15%)</u>
ρ = density (gm/cm ³)	0.95-1.19	1.0887
C = heat capacity (J/gm°C)	3.3-4.2	3.57
k = thermal conductivity (W/cm°C)	(5.4-6.8) · 10 ⁻³	6.37 · 10 ⁻³
ν = kinematic viscosity (cm ² /s)	(2.3-25.) · 10 ⁻³	5.81 · 10 ⁻³
κ_T = thermal diffusivity (cm ² /s)	(1.3-1.8) · 10 ⁻³	1.64 · 10 ⁻³
κ_S = salt diffusivity (cm ² /s)	(0.9-5.9) · 10 ⁻⁵	3.3 · 10 ⁻⁵
α = (neg.) expansion coefficient (°C) ⁻¹	(0.3-6.9) · 10 ⁻⁴	5.3 · 10 ⁻⁴
$\beta = \frac{1}{\rho} \frac{\partial \rho}{\partial S}$ (% by weight) ⁻¹	(6.8-7.4) · 10 ⁻³	6.98 · 10 ⁻³

The first three properties were obtained from tables in Kaufmann (1960), the others from the Office of Saline Water, United States Department of the Interior (1971).

Interpolation of these tables was performed on a CDC 6600 from

the Los Alamos Scientific Laboratory, using two-dimensional bicubic spline interpolating routines. These routines calculate the coefficients of a bicubic polynomial for smooth two-dimensional interpolation (SPL2D1), interpolate for a function value or specific derivative of this function (SPL2D2), returning any derivative desired (SPL2D3).

Tables of parameters relevant to the stability analysis have been produced with the use of these routines. Values of α and β were obtained by interpolation and differencing of the density data, given to 5 digits; as differencing gives 2 or 3 digits, values of α and β may separately contain a 5% error, giving 10% errors for the ratio α/β .

The following tables give values of the stability coefficient $\frac{(Pr+1)\alpha}{(Pr+Sc)\beta}$, as well as its components α/β and $(Pr+1)/(Pr+Sc)$.

TABLE C.1 - Stability Coefficient (% by weight/⁰C)

(PR+1)/(PR+SC) * (ALPHA/BETA) * 100											
DEG.	0	5	10	15	20	25	30	35	40	45	50
%											
1	.48	.41	1.42	2.79	3.71	4.54	5.41	6.19	7.05	7.88	8.60
2	.87	.80	1.80	3.14	4.01	4.78	5.60	6.35	7.14	7.92	8.68
3	1.24	1.18	2.16	3.47	4.28	4.99	5.76	6.47	7.21	7.97	8.72
4	1.59	1.54	2.52	3.79	4.53	5.19	5.93	6.58	7.29	8.02	8.74
5	1.93	1.89	2.86	4.10	4.76	5.37	6.09	6.68	7.36	8.07	8.75
6	2.27	2.23	3.19	4.39	4.97	5.54	6.24	6.78	7.43	8.11	8.76
7	2.61	2.57	3.50	4.65	5.17	5.69	6.36	6.87	7.49	8.15	8.76
8	2.93	2.89	3.79	4.90	5.36	5.83	6.46	6.96	7.56	8.19	8.75
9	3.24	3.20	4.07	5.12	5.53	5.95	6.56	7.03	7.62	8.22	8.75
10	3.54	3.49	4.32	5.32	5.68	6.07	6.65	7.10	7.67	8.25	8.74
11	3.82	3.76	4.55	5.50	5.83	6.18	6.72	7.17	7.71	8.27	8.74
12	4.08	4.01	4.76	5.65	5.97	6.28	6.79	7.23	7.75	8.27	8.74
13	4.33	4.25	4.93	5.77	6.11	6.40	6.85	7.28	7.77	8.27	8.75
14	4.54	4.45	5.11	5.93	6.25	6.49	6.91	7.33	7.79	8.27	8.74
15	4.71	4.63	5.31	6.12	6.32	6.53	6.98	7.36	7.80	8.27	8.70
16	4.87	4.79	5.48	6.25	6.37	6.58	7.04	7.39	7.81	8.25	8.67
17	5.03	4.94	5.56	6.29	6.49	6.70	7.08	7.41	7.80	8.22	8.66
18	5.18	5.06	5.62	6.33	6.65	6.83	7.10	7.44	7.80	8.19	8.63
19	5.29	5.16	5.71	6.43	6.76	6.90	7.14	7.46	7.80	8.17	8.54
20	5.36	5.24	5.81	6.52	6.81	6.94	7.18	7.47	7.80	8.14	8.43
21	5.43	5.32	5.89	6.60	6.86	6.99	7.21	7.47	7.79	8.11	8.36
22	5.49	5.38	5.95	6.67	6.92	7.03	7.24	7.48	7.78	8.08	8.32
23	5.53	5.42	6.01	6.74	6.97	7.06	7.26	7.49	7.76	8.04	8.31
24	5.54	5.44	6.05	6.79	7.00	7.07	7.26	7.48	7.73	8.00	8.27
25	5.56	5.46	6.09	6.84	7.06	7.10	7.27	7.50	7.74	7.98	8.21

TABLE C.1 (cont.) - Stability Coefficient (% by weight/°C)

(PR+1)/(PR+SC) * (ALPHA/BETA) * 100											
%	DEG. 50	55	60	65	70	75	80	85	90	95	100
1	8.60	9.34	10.21	11.14	12.05	12.97	13.89	14.74	15.40	15.86	16.13
2	8.63	9.45	10.25	11.05	11.87	12.72	13.61	14.45	15.12	15.61	15.90
3	8.72	9.46	10.17	10.88	11.64	12.46	13.36	14.22	14.92	15.45	15.79
4	8.74	9.43	10.10	10.78	11.53	12.35	13.24	14.08	14.74	15.21	15.49
5	8.75	9.41	10.06	10.75	11.49	12.29	13.13	13.89	14.47	14.85	15.05
6	8.76	9.38	10.03	10.71	11.43	12.18	12.95	13.64	14.18	14.54	14.73
7	8.76	9.35	9.99	10.65	11.35	12.05	12.78	13.44	13.95	14.32	14.52
8	8.75	9.32	9.94	10.61	11.29	11.97	12.66	13.28	13.76	14.10	14.29
9	8.75	9.28	9.90	10.57	11.24	11.89	12.53	13.09	13.53	13.83	14.00
10	8.74	9.24	9.85	10.50	11.13	11.74	12.32	12.85	13.26	13.56	13.75
11	8.74	9.22	9.79	10.40	10.99	11.56	12.13	12.65	13.07	13.39	13.60
12	8.74	9.22	9.76	10.33	10.90	11.46	12.04	12.57	13.01	13.33	13.54
13	8.75	9.24	9.75	10.29	10.83	11.39	11.97	12.51	12.94	13.25	13.44
14	8.74	9.21	9.70	10.20	10.73	11.27	11.84	12.35	12.76	13.06	13.24
15	8.70	9.14	9.61	10.11	10.63	11.15	11.68	12.16	12.54	12.82	13.00
16	8.67	9.10	9.57	10.06	10.54	11.02	11.51	11.96	12.33	12.61	12.79
17	8.66	9.11	9.55	9.97	10.40	10.85	11.33	11.79	12.17	12.45	12.64
18	8.63	9.06	9.44	9.81	10.24	10.70	11.21	11.68	12.06	12.32	12.48
19	8.54	8.89	9.27	9.70	10.17	10.65	11.14	11.58	11.91	12.15	12.29
20	8.43	8.73	9.15	9.64	10.12	10.58	11.02	11.41	11.73	11.97	12.14
21	8.36	8.64	9.06	9.55	10.01	10.44	10.86	11.25	11.59	11.87	12.08
22	8.32	8.61	9.00	9.44	9.86	10.28	10.71	11.12	11.48	11.77	11.98
23	8.31	8.60	8.94	9.25	9.66	10.07	10.52	10.96	11.34	11.63	11.82
24	8.27	8.54	8.80	9.08	9.41	9.82	10.30	10.79	11.20	11.51	11.71
25	8.21	8.42	8.63	8.88	9.21	9.63	10.16	10.69	11.12	11.45	11.67

TABLE C.2 - Ratio α/β (% by weight/ $^{\circ}\text{C}$)

ALPHA/BETA * 100												
%	DEG.	0	10	20	30	40	50	60	70	80	90	100
1		.44	1.28	3.23	4.55	5.72	6.71	7.65	8.65	9.52	10.04	9.96
2		.81	1.62	3.50	4.72	5.81	6.80	7.71	8.57	9.39	9.93	9.91
3		1.15	1.95	3.74	4.87	5.88	6.85	7.68	8.44	9.26	9.87	9.92
4		1.48	2.28	3.97	5.02	5.96	6.89	7.66	8.39	9.23	9.81	9.80
5		1.80	2.59	4.17	5.17	6.04	6.92	7.66	8.40	9.20	9.68	9.59
6		2.12	2.89	4.37	5.30	6.11	6.95	7.66	8.39	9.11	9.54	9.44
7		2.43	3.18	4.55	5.42	6.17	6.97	7.65	8.36	9.03	9.43	9.37
8		2.74	3.44	4.72	5.52	6.24	6.98	7.64	8.35	8.99	9.35	9.26
9		3.03	3.70	4.88	5.61	6.31	7.00	7.64	8.34	8.93	9.23	9.12
10		3.31	3.93	5.02	5.70	6.37	7.02	7.62	8.29	8.82	9.09	9.00
11		3.58	4.15	5.16	5.78	6.42	7.03	7.60	8.21	8.71	9.00	8.94
12		3.83	4.34	5.29	5.85	6.47	7.05	7.60	8.18	8.68	8.99	8.93
13		4.06	4.51	5.43	5.91	6.50	7.08	7.62	8.16	8.66	8.97	8.91
14		4.27	4.68	5.56	5.98	6.54	7.10	7.61	8.11	8.60	8.89	8.81
15		4.43	4.87	5.64	6.06	6.56	7.09	7.56	8.06	8.52	8.78	8.69
16		4.58	5.03	5.70	6.13	6.59	7.09	7.56	8.03	8.43	8.67	8.59
17		4.74	5.12	5.82	6.17	6.61	7.10	7.57	7.95	8.33	8.59	8.53
18		4.88	5.18	5.97	6.21	6.63	7.10	7.51	7.87	8.29	8.55	8.47
19		4.99	5.27	6.08	6.26	6.65	7.05	7.42	7.85	8.28	8.49	8.38
20		5.07	5.37	6.15	6.31	6.67	7.00	7.35	7.85	8.23	8.41	8.33
21		5.14	5.45	6.21	6.36	6.69	6.97	7.32	7.81	8.16	8.36	8.33
22		5.20	5.53	6.28	6.41	6.71	6.97	7.30	7.73	8.09	8.33	8.32
23		5.24	5.59	6.34	6.45	6.72	6.99	7.29	7.62	8.00	8.28	8.27
24		5.26	5.64	6.39	6.48	6.72	6.99	7.22	7.47	7.89	8.24	8.26
25		5.28	5.69	6.46	6.51	6.76	6.98	7.13	7.36	7.83	8.25	8.30

TABLE C.3 - Ratio (Pr+1)/(Pr+Sc)

(PR+1)/(PR+SC)											
%	DEG. 0	10	20	30	40	50	60	70	80	90	100
1	1.08	1.11	1.15	1.19	1.23	1.28	1.33	1.39	1.46	1.53	1.62
2	1.08	1.11	1.15	1.19	1.23	1.28	1.33	1.39	1.45	1.52	1.60
3	1.07	1.11	1.14	1.18	1.23	1.27	1.32	1.38	1.44	1.51	1.59
4	1.07	1.11	1.14	1.18	1.22	1.27	1.32	1.37	1.43	1.50	1.58
5	1.07	1.11	1.14	1.18	1.22	1.26	1.31	1.37	1.43	1.49	1.57
6	1.07	1.10	1.14	1.18	1.22	1.26	1.31	1.36	1.42	1.49	1.56
7	1.07	1.10	1.14	1.17	1.21	1.26	1.30	1.35	1.41	1.48	1.55
8	1.07	1.10	1.13	1.17	1.21	1.25	1.30	1.35	1.41	1.47	1.54
9	1.07	1.10	1.13	1.17	1.21	1.25	1.30	1.35	1.40	1.47	1.54
10	1.07	1.10	1.13	1.17	1.20	1.25	1.29	1.34	1.40	1.46	1.53
11	1.07	1.10	1.13	1.16	1.20	1.24	1.29	1.34	1.39	1.45	1.52
12	1.07	1.10	1.13	1.16	1.20	1.24	1.28	1.33	1.39	1.45	1.51
13	1.07	1.09	1.12	1.16	1.20	1.24	1.28	1.33	1.38	1.44	1.51
14	1.07	1.09	1.12	1.16	1.19	1.23	1.28	1.32	1.38	1.44	1.50
15	1.06	1.09	1.12	1.15	1.19	1.23	1.27	1.32	1.37	1.43	1.50
16	1.06	1.09	1.12	1.15	1.18	1.22	1.27	1.31	1.36	1.42	1.49
17	1.06	1.09	1.12	1.15	1.18	1.22	1.26	1.31	1.36	1.42	1.48
18	1.06	1.09	1.11	1.14	1.18	1.21	1.26	1.30	1.35	1.41	1.47
19	1.06	1.08	1.11	1.14	1.17	1.21	1.25	1.30	1.35	1.40	1.47
20	1.06	1.08	1.11	1.14	1.17	1.20	1.24	1.29	1.34	1.39	1.46
21	1.06	1.08	1.10	1.13	1.16	1.20	1.24	1.28	1.33	1.39	1.45
22	1.06	1.08	1.10	1.13	1.16	1.19	1.23	1.28	1.32	1.38	1.44
23	1.05	1.08	1.10	1.13	1.16	1.19	1.23	1.27	1.31	1.37	1.43
24	1.05	1.07	1.10	1.12	1.15	1.18	1.22	1.26	1.31	1.36	1.42
25	1.05	1.07	1.09	1.12	1.15	1.18	1.21	1.25	1.30	1.35	1.41

APPENDIX D

SOFTWARE

Solutions to the matrix equation $(\underline{P} - \frac{k_1}{k_2}\underline{Q}) \underline{\xi} = \underline{Q}$ (eq.5.35) were obtained on a CDC 6600 from the Los Alamos Scientific Laboratory, using Program Library routines to compute eigenvalues and eigenvectors of a real matrix. These routines invert the matrix $\underline{Q}$ (MATINV), iteratively correct the inverted matrix if necessary (CORINV), multiply $\underline{Q}^{-1}\underline{P}$ (MATMPY) and compute all eigenvalues and corresponding eigenvectors (GREVEV); internal subroutines are part of the EISPACK collection of codes for solving the eigenvalue problem.

Minimum eigenvalues, with corresponding eigenvectors, are selected from the solutions, the eigenvectors are normalized by $\underline{Q}$ and the resulting eigenfunctions are plotted.

The plotting routines are part of the DISSPLA plotting package of the Integrated Software Systems Corporation. Plots were first displayed on a Tektronix terminal, then stored on magnetic disk to produce 35mm negatives.