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**CENTROID FINDING METHOD FOR
POSITION-SENSITIVE DETECTORS***

V. Radaka and R. A. Boie

**Brookhaven National Laboratory
Upton, New York 11973**

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V. Radeka and R. A. Boie

Brookhaven National Laboratory
Upton, New York 11973

ABSTRACT

A new centroid finding method for all detectors where the signal charge is collected or induced on strips or wires, or on subdivided resistive electrodes, is presented. The centroid of charge is determined by convolution of the sequentially switched outputs from these subdivisions or from the strips with a linear centroid finding filter. The position line width is inversely proportional to $N^{3/2}$, where N is the number of subdivisions.

1. Introduction

The problem of accurate centroid finding of the charge collected or induced on discrete or continuous resistive electrodes appears in most applications of all ionization and electron multiplying position-sensitive detectors. The centroid is usually determined by a global continuous readout employing delay lines or charge division, Fig. 1(a). Alternatively, it could be determined by computation from the outputs of discrete strips or wires, Fig. 1(b). The global continuous readouts have the advantage of relative simplicity. Their main disadvantage stems from the fact that both the position line width and the position error (i.e., absolute integral nonlinearity) increase with the size of the detector, since the line width is determined as

$$\text{FWHM} \propto \frac{C_D^a \cdot L}{Q_S} \quad (1)$$

where $a = \frac{1}{2}$ for position sensing on resistive electrodes, $a = 1$ for position sensing with delay lines;

C_D = total readout electrode capacitance;

L = total length of the readout electrode;

Q_S = charge induced or collected on the readout cathode.

Thus, with the increase in the detector area, one reaches a limit to the accuracy and to the position resolution for a given amount of charge. The necessity to keep the required signal charge at a low level appears in all detectors for various reasons, in order to avoid space charge effects at high rates, to maintain amplitude (energy linearity), to reduce deposit formation in gas detectors, etc.

Centroid computation, usually referred to as the "center of gravity method",¹ from the outputs of discrete strips or wires, as performed so far, requires a considerable amount of electronics and it is inherently slow. If performed digitally, it requires a large dynamic range in the analog-to-digital converters, and it results in differential nonlinearity due to the quantization errors.

This paper describes a centroid finding method which substantially reduces these problems. The basic problem is how to subdivide the detector to avoid the limitations imposed by its size and capacitance, and yet, to have a continuous readout without dead regions or discontinuities and nonlinearity. The first step in this direction was made by Alberi,^{2,3} and applied in several cases (Ref. 4, for example). This solution is illustrated in Fig. 2. It is essentially the charge division within each section L/N . The output from each preamplifier is weighted linearly according to its position. The centroid is determined as $\sum Q_k Q_k / \sum Q_k$. The position line width is inversely proportional to N^2 , where N is the number of subdivisions. The line width improves slowly with N , because the noise from all outputs is included in the centroid determination. The centroid computation is global, i.e., with respect to the whole length of the detector. Thus the position error due to the ratio computation, quantization, nonlinearity, etc., is the same as for the case without subdivision.

The solution lies, obviously, in (1) subdivided resistive chain, (2) in using only those outputs for centroid determination where the signal is present, and (3) in using output nodes as fiducial points. Thus the noise from only these outputs (3 to 4) is included, resulting in much improved position resolution. The absolute position error is determined by the error in centroid finding within one subdivision, and it is independent of the number of subdivisions, i.e., the size of the detector. Linear relation of the centroid with the event position for the signal charge straddling an output node, as in Fig. 1(c), can easily be proved by calculation of the first moment.

Our method to accomplish these functions is illustrated in Fig. 3. The solution is a unique one, taking advantage of the best properties of both analog and digital techniques. The centroid of (induced) charge is determined by convolution of the sequentially switched subdivision output signals with a linear centroid filter. The zero crossing time of the filter $w(t)$, shown in Fig. 3, is given by

$$t_{fo} = T_M/2 + \frac{\int t f(t) dt}{\int f(t) dt} \quad (2)$$

The function $f(t)$ consists, in this case, of sequentially switched samples of charge from the subdivision outputs. According to Eq. (2), the filter provides a linear measure of the centroid by its zero crossing time. The width of the filter weighting function is determined by the number of samples required to determine the centroid (three to four, in most cases). The filter has the required property of including signal and noise only from outputs containing centroid information, and thereby minimizes the line width broadening due to the electronic noise. In Fig. 3, each charge amplifier is followed by pulse shaping and a sample and hold circuit. The signal on all outputs is sampled simultaneously. Then, the switching sequence and the digital timer for zero crossing are started synchronously. The zero crossing from the centroid filter

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stops the timer.

The position resolution of this method is determined as,

$$\text{FWHM} \approx 3 a_F \frac{(kTC_p)^{1/2}}{Q_s} \frac{1}{N^{3/2}} \quad (3)$$

where: a_F = constant containing input filter and centroid filter form factors (usually $2 < a_F < 3$)

N = number of subdivisions

k = Boltzmann constant

T = absolute temperature.

The position resolution improves strongly with the number of subdivisions. The number of subdivisions is determined by the detector size, capacitance, required resolution and the signal charge. It can be any number between the following two extremes. The upper limit is one output per strip as in Fig. 1(b), where the resistor chain is not needed. This method provides a simple and fast solution to centroid determination in the "center of gravity method".¹ The optimization of electrode strip widths with respect to the detector geometry, noise, and nonlinearity, is treated in Ref. 6. At the other end of the scale, $N = 1$, the centroid filter can be used instead of the ratio computation in global charge division.

The method provides a continuous linear position readout over the whole length of the detector without any discontinuities. The nodes on the resistive chain where the amplifiers are connected, and the synchronization of the sample multiplexer with the digital position readout provide an inherent position calibration, resulting in very high position accuracy and linearity. The time scales for input pulse shaping and for centroid finding are independent and each can be optimized separately. The time required to determine the centroid by these methods is limited principally by the need of the time digitization.

This method is based on transformation of the spatial coordinate into time by the sequential switching of subdivision outputs into the centroid filter. An alternative way of accomplishing this transformation is by simultaneous switching of subdivision outputs into a delay line with equally spaced taps. The induced charge samples propagate sequentially in the delay line and then into the centroid finding filter. This is accomplished at high signal level and thus the noise from the delay line readout is negligible. The principal problem of this approach is the nonuniformity of delay time intervals between the taps if a high position accuracy is required.

In this paper, the principle of position centroid finding by convolution is described and the questions concerning position accuracy, resolution and speed of operation are considered. Some applications are suggested. We have developed a position readout system, based on this method, for two-dimensional position-sensitive detectors for thermal neutrons. For this development, we have built a device for simulation of the charge injection and distribution in charge collecting detectors, by which the position readouts and the effects of electrode geometry and induced charge distribution can be studied. These developments are described in a companion paper.⁷ The measurement

results used in this paper were obtained in the course of this development work, and are given in more detail in Ref. 7.

2. Centroid Finding by Convolution

The method is based on the centroid finding property of a class of filters with bipolar impulse response (weighting function). The concept of such a filter was introduced in a previous work.³ The impulse response of an ideal centroid finding filter is given by

$$h(t) = \frac{4}{T_M} \left(\frac{T_M}{2} - t \right) [U(t) - U(t - T_M)] \quad (4)$$

where T_M is the width of the filter impulse response as shown in Fig. 4, and $U(t)$ is the unit step function. The response of the filter to an input function $f(t)$ is given by the convolution integral,

$$v(t) = \frac{4}{T_M} \int_{-\infty}^{\infty} \left(\frac{T_M}{2} - t + \tau \right) [U(t - \tau) - U(t - \tau - T_M)] f(\tau) d\tau \quad (5)$$

We determine now the time t_{fo} of the zero crossing of the output response $v(t)$. By setting $v(t_{fo}) = 0$, and by determining the integration limits to cover the range of nonzero values of the weighting function, we obtain,

$$\int_{t_{fo} - T_M}^{t_{fo}} \left(\frac{T_M}{2} - t_{fo} + \tau \right) f(\tau) d\tau = 0 \quad (6)$$

The time of zero crossing is then,

$$t_{fo} = \frac{T_M}{2} + \frac{\int_{t_{fo} - T_M}^{t_{fo}} \tau f(\tau) d\tau}{\int_{t_{fo} - T_M}^{t_{fo}} f(\tau) d\tau} \quad (7)$$

$$t_{fo} = \frac{T_M}{2} + t_c \quad (8)$$

The detection of zero crossing thus provides a measurement of the centroid t_c of the function $f(t)$, provided that $f(t)$ is zero before $t_c - T_M/2$ and after $t_c + T_M/2$.

The relation of various functions and time parameters in the centroid finding is illustrated in Fig. 4. In our case, the function $f(t)$ is formed by the sequentially switched charge samples as shown in Fig. 3. If the induced charge distribution is narrower than the length $1/N$ of one section of the detector (between two adjacent amplifier connections to the resistor chain), there will be at most three consecutive samples with nonzero values of signal charge for each detected event, as is evident from Fig. 1(c). The magnitude of the charge samples is determined by simple linear division of charge. Each part of the induced charge distribution propagates along its corresponding section of the resistor chain k , $k+1$ and $k+1$, $k+2$. One can show by calculation of the first moment that the centroid in time of the sequence of these charge samples corresponds

to the centroid in position of the induced charge distribution. As shown in Fig. 4, the filter performs linear weighting of the function $f(x)$ with respect to the centroid x_c . The minimum width of the filter weighting function is then determined from the requirement that each element of the input function $f(x)$ must be weighted for any position of the centroid. Figure 4 shows the extrema position $x/(L/N) \approx 1/4$ in which Q_0 is still not negligible for the induced charge distribution shown in Fig. 1(c). This results in a weighting function with a width

$$T_M = 3\frac{1}{2} T_s \quad (9)$$

where T_s is the width of one sample. The output response shows that the zero crossing is delayed with respect to the centroid by $T_M/2$ in agreement with Eqs. (7) and (8). The displacement of the zero crossing by one sample width T_s corresponds to the displacement in position by one section L/N . Thus the "conversion sensitivity" position-to-time of the readout is

$$G_c = \frac{T_s}{L/N} \quad (10)$$

The zero crossing slope of $v(t)$ is $4Q_0/T_M$, where Q_0 is the area of $f(x)$. This slope is independent of the shape of $f(x)$ since every element $f(x)dx = dQ_0$ of the exciting pulse gives a contribution $(4/T_M) dQ_0$ to the total slope. Because of the linear weighting function, this contribution is independent of the position of the element dQ_0 .

Realization of such a centroid filter is fairly simple. A filter whose impulse response approaches closely the ideal function, Eq. (4) and Fig. 4, is shown in Fig. 5. Realization of a symmetrical impulse response of finite width by a time-invariant filter requires a delay line and, in this particular case, two integrators. Switched transconductance amplifiers G_1 and G_2 in feedback of the integrators serve as gated base line restorers. While abrupt jumps and sharp corners do not exist in physical systems, the ideal response can be approached in its essential features, i.e., in linearity over a large fraction of the function width. The impulse response of this filter is shown in Fig. 6(a). T_{fo} represents the delay between the centroid of the input function and the zero crossing. Figures 6(b), (c), (d), and (e) represent the output response for the positions of the particle (i.e., avalanche and induced charge) $x/(L/N) = 0; 0.3; 0.5; \text{ and } 0.7$, respectively, observed in an actual system according to Fig. 3 and described in Ref. 7.

We note that while the shapes of the input and output of the filter change with position, the slope at zero crossing remains constant as stated above. It is interesting to see how the filter selects two or three samples, as the case may be, and produces a measure of the centroid. Three samples appear when the signal charge straddles a connection to the resistor chain, and there are two samples when the charge is all within one section L/N .

One can, of course, increase the filter width to handle more samples. This might be necessary in the case where there is no resistor chain and the charge is measured on each electrode strip (4 to 5 samples may be required to achieve a linear response⁶).

The width of the impulse response of the centroid finding filter should be as small as possible, in order

to reduce the noise contribution from other samples where there is no signal charge (see next section), and to reduce the resolving time of the filter.

The resolving time of the filter, after which the next event (sampled simultaneously at the input) could be measured, is equal to the width T_M of the impulse response on the basis of the criterion that the zero crossing corresponding to either event should not be disturbed. This corresponds to the minimum centroid separation of the two events by 3 to 4 sample widths, T_s .

The time scale for centroid finding and thus the conversion sensitivity according to Eq. (10) is arbitrary, since it is completely defined by the sample width T_s . In practical implementation of various functions outlined in Fig. 3 (charge sampling, sequential switching, filter response, sensing of zero crossing, and time digitization), the limitation one encounters first is the speed of time digitization. If we assume, for example, 100 position resolution elements in one section L/N of the detector, and a sample width T_s of 100 nsec, the required clock frequency would be 1 GHz. All other functions are easily implemented on this time scale. Even with much more moderate speed of time digitization, a short overall processing time for finding the centroid is achieved. For example, in the case of the detector with 10^3 position resolution elements and a clock frequency of 200 MHz, the total processing time is 5 μ sec. In this sense, the processing time of this method with simple sequential switching is equal to that of the delay line method.

The processing time could be significantly reduced, if instead of sequential switching a more elaborate switching scheme is developed. In such a scheme, a fast search by sequential switching could be used to determine the coarse position of the event by finding a group of charge samples. This group of charge samples is then switched sequentially at lower speed into the centroid finding filter. This "fast-slow" approach would reduce the processing time from NT_s to about $5 T_s$. This is of interest in cases where the number of subdivisions N is large, and a high counting rate is required. In the example considered above with $T_s = 100$ nsec, the centroid would be determined in 0.5 μ sec. Another advantage of this approach results from the fact that only those samples where the signal is present are switched into the filter. The shape of the filter outside of the linear region is then of lesser importance because there are no other samples, except those containing the signal, contributing noise to the measurement.

3. Noise and Position Resolution

A principal purpose of this method for centroid finding is to reduce the contribution of the electronic noise to the position line width. The position resolution of a detector is, of course, limited also by other dispersive effects arising from the interaction of particles with matter in the detection process and from the mechanisms of signal development in the detector. The position line broadening due to the electronic noise is always inversely proportional to the signal charge. Most of the difficult problems in the detector design arise from the requirement for a large signal charge, which frequently results in increased dispersion due to the space charge and propagation of avalanche, for example. In our method, the noise contribution is reduced at the expense of the number of sections N which require signal processing circuits. Obviously, it is of interest to find the relation between the position resolution and the signal

charge, detector capacitance and size, and the number of sections.

An accurate determination of the noise contribution should include all the noise sources and an exact description of the input circuit (e.g., thermal noise in the resistive chain represented as a diffusive line, and the amplifier noise). The fluctuations in the zero crossing time of the centroid finding filter can best be determined by graphical methods. A detailed treatment will be published in a separate report.⁹

We derive here, analytically, an approximate relation for the position resolution by a simplified model which provides the main results and an insight into the resolution properties of this position finding method. A simplified model for the noise calculation is shown in Fig. 7(a). An example of signal and noise as it appears at the input of the centroid finding filter is shown in Fig. 7(b). We assume that the amplifier noise is negligible, and that the thermal noise of each resistor in the chain can be represented by a single noise current generator. The noise, due to such a source, appears (anti)correlated at the outputs of the two adjacent amplifiers. The measurement of the centroid is affected only by the noise of those samples in the vicinity of the centroid which are within the filter weighting function placed with its zero crossing at the centroid, as in Fig. 4. This corresponds to 3 to 4 samples (3.5 samples in the example in the previous section).

Calculation of the position resolution is simpler in the case where the centroid is determined by computation from the samples of charge (Q_{k+1} , Q_{k+2} , Q_{k+3}). Since centroid computation is equivalent to centroid finding by convolution, we shall use it here to determine the position resolution. According to Fig. 7(a), the measured centroid x_m with reference to the output $k+2$, assuming that 3 samples are used to determine the centroid, is given by,

$$\frac{x_m}{L/N} = \frac{-Q_1 + Q_3}{Q_1 + Q_2 + Q_3} \quad (11)$$

Q 's represent the samples of the signal plus instantaneous values of the noise charge; numbers 0 to 4 are used here in place of k to $k+4$ for brevity. The numerator represents the position coordinate x normalized to L/N , plus instantaneous values of noise at points 1 and 3,

$$-Q_1 + Q_3 = Q_s \frac{x}{L/N} - q_{01} + q_{12} + q_{23} - q_{34} \quad (12)$$

where Q_s is the total signal charge. Noting that in the sum $Q_1 + Q_2 + Q_3$, the anticorrelated noise outputs cancel, and Eq. (11) becomes,

$$\frac{x_m}{L/N} = \frac{Q_s \frac{x}{L/N} - q_{01} + q_{12} + q_{23} - q_{34}}{Q_s + q_{01} - q_{34}} \quad (13)$$

since $q_{1j} \ll Q_s$, it follows,

$$\frac{x_m}{L/N} \approx \left[\frac{x}{L/N} - \frac{1}{Q_s} (q_{01} - q_{12} - q_{23} + q_{34}) \right] \left[1 - \frac{1}{Q_s} (q_{01} - q_{34}) \right] \quad (14)$$

After multiplication and neglecting the second order terms, the position error becomes,

$$\frac{\Delta x}{L/N} = \frac{x_m - x}{L/N} \approx \frac{1}{Q_s} \left[-q_{01} \left(1 + \frac{x}{L/N} \right) + q_{34} \left(\frac{x}{L/N} - 1 \right) + q_{12} + q_{23} \right] \quad (15)$$

We now determine the mean square value by squaring this equation and taking the average value. The mean square value of all noise generators is the same, as determined by the resistors R_D , $q_{ij}^2 = q_R^2$. All products q_{ij}

$q_{(i+1)(j+1)} = 0$, since these sources are independent. Thus, we obtain

$$\frac{\sigma_x^2}{(L/N)^2} = \frac{\overline{\Delta x^2}}{(L/N)^2} \approx \frac{q_R^2}{Q_s^2} 4 \left[1 + \frac{1}{2} \left(\frac{x}{L/N} \right)^2 \right] \quad (16)$$

The position variance is a function of the position. The range of x with respect to any one output used as a reference (fiducial) point is $\pm 1/2 L/N$. Thus, the maximum value of standard deviation is

$$\frac{\sigma_{xm}}{L/N} \approx \frac{(q_R^2)^{1/2}}{Q_s} 2.12 \quad (17)$$

We can write, more generally,

$$\frac{\sigma_{xm}}{L/N} \approx \frac{(q_R^2)^{1/2}}{Q_s} a_{cf} \quad (18)$$

We define a_{cf} as the form factor of the centroid finding filter, which takes into account, in a general case, the number of samples considered and the shape of the filter's weighting function.

The mean square charge noise q_R^2 is determined from consideration of the charge division. It is simply the equivalent noise charge due to the thermal noise in the resistance R_{DS} (Refs. 3 and 10),

$$q_R^2 = \overline{ENC_P^2} = 2kT \frac{1}{R_{DS}} a_{F2} \tau_F \quad (19)$$

where, R_D = resistance of one section, τ_F = the width of the weighting function of the input filter for unipolar functions, or the width of the first lobe for bipolar functions, a_{F2} = the form factor of the input filter weighting function for the parallel white noise.¹⁰ As usual, in the charge division we define

$$\tau_{DS} = \frac{C_D}{N} R_{DS} \quad \text{or} \quad \tau_{DS} = C_{DS} R_{DS} \quad (20)$$

where τ_{DS} is the product of distributed detector capacitance and resistance for one section, and C_D is the total capacitance of the detector readout electrode. Linear charge division requires that $\tau_F \approx 0.8 \tau_{DS}$ (for a trapezoidal filter with a flat portion of the weighting function equal to $0.6 \tau_F$). In this case, the noise due to the diffusive nature of this RC line is larger only by a small amount than the noise from resistor R_{DS} . Substituting R_{DS} from Eq. (20) into (19),

$$\frac{1}{q_R^2} = \frac{ENC_p^2}{2kT} = \frac{C_D}{L/N} a_{F2} \frac{\tau_F}{\tau_{DS}} \quad (21)$$

From Eqs. (18) and (21) and $FWHM_m = 2.35 C_{xm}$, we obtain the position resolution,

$$FWHM_m \approx 3.32 (a_{F2} a_{cf})^{\frac{1}{2}} \frac{(kTC_D)^{\frac{1}{2}}}{Q_s} \cdot \frac{1}{N^{\frac{3}{2}}} \left(\frac{\tau_F}{\tau_{DS}} \right)^{\frac{1}{2}} \quad (22)$$

In the case $\tau_F = 0.8 \tau_{DS}$,

$$FWHM_m \approx 3 (a_{F2} a_{cf})^{\frac{1}{2}} \frac{(kTC_D)^{\frac{1}{2}}}{Q_s} \frac{1}{N^{\frac{3}{2}}} \quad (23)$$

The form factor a_{F2} of the input filter is between 0.733 for unipolar trapezoidal function and 1.47 for bipolar trapezoidal function. These functions can be approximated by single and double delay line clipping, respectively. Another filter of interest is single delay line clipping with a base line restorer, resulting in a bipolar weighting function with a positive trapezoidal lobe and a smaller and longer negative lobe. Such a function realized by switched (time-variant) filters, is described in Ref. 7, and its form factor is $a_{F2} \approx 1$.

The factor $a_{cf}^{\frac{1}{2}}$ is the ratio of the position line width for the subdivided readout considered here, and for the charge division on one section alone with no connections to the rest of the resistor chain. This increase in the line width is due to the noise contribution by several sections, Fig. 7(a).

With $a_{F2} \approx 1$, and $a_{cf}^{\frac{1}{2}} = 2.12$,

$$FWHM_m \approx 6.3 \frac{(kTC_D)^{\frac{1}{2}}}{Q_s} \frac{1}{N^{\frac{3}{2}}} \quad \text{for absolute resolution, and} \quad (24a)$$

$$\frac{FWHM_m}{L/N} \approx 6.3 \frac{(kTC_{DS})^{\frac{1}{2}}}{Q_s} \quad \text{relative to} \quad (24b)$$

one section.

The position line width is inversely proportional to $N^{\frac{3}{2}}$, where N is the number of subdivisions. Factor N arises because the position (centroid) is determined with respect to one section (subdivision). Factor $N^{\frac{1}{2}}$ is due to the increased resistance and lower noise in one section according to Eqs. (19) and (20) for a given τ_{DS} .

The results of position resolution measurements⁷ by this method are presented in Fig. 8. The calculated curve shown can be used as a universal one since it scales with detector capacitance as $(C_{DS}[pF]/30)^{\frac{1}{2}}$. The measured noise is in good agreement with Eq. (24). This shows that Eqs. (22), (23), and (24) are accurate enough for design purposes in all cases where amplifier noise is negligible, e.g., to determine the number of subdivisions for a given detector capacitance, C_D , and signal charge Q_s .

An accurate picture of noise and resolution is more complicated than presented here, as is usually the case, and it will be discussed elsewhere. One question of interest is the variation of position line width with the position. The line width varies with position in most interpolating centroid finding methods. In this method, maximum variation is between 6 and 30%, depending on the weighting function width and slope. Another question is the resolution in cases where amplifier noise is not negligible. This occurs with resistance chain at a short measurement time where the amplifier, e.g., series noise resistance, is no longer negligible compared to resistance of the section R_{DS} . Finally, the case with one output per electrode strip and no resistor chain is of special interest, since it has the lowest electronic noise and the best position resolution.

4. Position Accuracy

In this method for centroid finding, the absolute position accuracy achieved in one section is independent of the number of sections and thus of the length of the detector. Amplifier connections to the resistor chain may serve as fiducial points. In any subdivided readout the problem arises in position encoding in matching the coarse position information (amplifier connections) to the fine position information (interpolation within one section). This results in discontinuities and a large differential nonlinearity. One method of avoiding this is to use continuous readout (global position encoding) for the entire length of the detector, but then the advantage of higher absolute accuracy is lost. This problem has been encountered and considered by other authors working on subdivided readouts (see, for example, Refs. 11, 12, and 13).

In centroid finding by convolution, the position information is converted into time and this allows a solution to this problem. The time scale of the signal in Figs. 3 and 4, consisting of sequentially switched samples, is determined by the sequential switch. The sequential switch thus determines the coarse position and it also provides the time scale for the centroid, i.e., for the fine scale. The solution is to synchronize the sequential switch with the clock used for fine scale time digitization. This is easily done by deriving the switch clock from the time digitization clock by frequency division. This is illustrated in Fig. 9. In the case of the nonintegral ratio of the clock frequencies, there is a time channel width error causing discontinuity at the boundaries between sections. One can conclude more generally that, if the coarse and fine position are determined by independent means, discontinuity results. The measured position error (integral nonlinearity referred to one section) is shown in Fig. 10. Integral nonlinearity of $\pm 0.5\%$ corresponds to the performance achieved with global position readouts (referred to the whole length of the detector). Our results were obtained on the centroid finding system⁷ for a detector with $L \approx 20$ cm, so that the maximum error of $\sim \pm 80 \mu m$ corresponds to an integral nonlinearity of $\sim 4 \times 10^{-4}$. We suspect that the main cause of the remaining error is the offset of the sample and hold circuits.

Differential nonlinearity, i.e., the spectrum for a uniform illumination or for a continuous scan with a point source, is shown in Fig. 11. Differential nonlinearity is derivative dx_m/dx of the function $x_m = f(x)$, (i.e., position calibration curve), where x_m is the measured and x the true position. This is a very sensitive measure of discontinuities in the position calibration curve. In both Fig. 10 and Fig. 11, there is no indication of discontinuities at section boundaries.

The effect of finite cathode strip width can be studied by measuring differential nonlinearity. In Fig. 11(a), for a ratio of strip width to the anode-cathode spacing $s/d = 0.76$, differential nonlinearity is $\approx \pm 4\%$ and there is no periodic structure. In Fig. 11(b) for $s/d = 1.1$, periodic structure corresponding to 4 strips per section is apparent. While the differential nonlinearity is already significant at $\pm 8\%$, the integral nonlinearity and absolute position error are still smaller than the errors in Fig. 10 due to other sources. (This is because of the short period of the nonlinearity.)

5. Discussion of Results and Applications

The method described in this paper performs direct conversion of position into time and thereby it replaces the computation of the first moment by a simple measurement of the zero crossing time. The principal properties of the method are:

- (1) The input shaping prior to sampling is independent of the centroid finding filter. The input shaping can be optimized with respect to charge collection in the detector and to the amplifier noise.
- (2) The time scale for centroid finding and thus the conversion sensitivity position-to-time is determined only by the sample width T_s , Eq. (10), i.e., by the system clock, and it is independent of the detector and of other readout system parameters. (This is not the case with readouts using RC position encoding and delay line position encoding, where RC or LC parameters of the position sensing medium determine the conversion sensitivity.)
- (3) Centroid finding by convolution avoids dynamic range restrictions and quantization errors associated with analog operations on signal amplitudes or with digital operations after analog-to-digital conversion.
- (4) The position resolution for a given signal charge is better by a factor $\approx N^{3/2}/2.1$ than by any other global method with resistive or RC position encoding. (N is the number of subdivisions.) An improvement in the product of the position line width and the signal charge ($FWHM \cdot Q_s$) between one and two orders of magnitude is easily achieved. This improvement can be used to operate the detectors at lower levels of signal charge. Another remarkable improvement is in that the integral nonlinearity and absolute position calibration errors are reduced by a factor N .

We are pursuing a number of applications of this method. Some of these are:

- (1) High precision position readout for large area neutron detectors for use in biological structure determination by neutron scattering.
- (2) High resolution x-ray detectors at high photon counting rates for use in molecular and crystal structure determination by diffraction of synchrotron radiation.
- (3) Microchannel plate detectors for use in high energy physics experiments. Resistive

anodes are contemplated with subdivisions at a much smaller scale than in gas detectors ($1/N \approx 1$ mm) and a resolution and accuracy $\sigma \approx 10$ μ m. The method can also be used with cross grid anodes.

- (4) Silicon detectors for use in high energy physics. These detectors would have a resistive layer contact with fine subdivisions of $1/N \approx 1$ mm, and an expected resolution of $\sigma \approx 10$ -20 μ m.

The method can also be used in gas proportional, ionization, and time projection chambers where the centroid of the charge induced or collected on strip or pad electrodes is to be determined.

Some interesting variations and further improvements of the method are possible. First is the development of "fast-slow" methods for switching of samples as outlined in Section 2 in order to reduce further the time required for centroid finding. Second is an investigation of the possibility of using a time-variant (switched) centroid finding filter. Such a filter could be realized in hybrid or monolithic technology without components such as a delay line, which is of interest in large scale detector systems.

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Fig. 1. Centroid finding

- (a) Charge division and delay line position sensing (two outputs for the whole length of the detector).
- (b) "Amplifier per strip" or "center of gravity" method.^{1,5}
- (c) Subdivided charge division. Charge outputs Q_k are determined by the position, shape and width of the induced charge distribution. Sample amplitudes are shown qualitatively.

All amplifiers shown are "charge sensitive" with input impedance small compared to the resistance between adjacent amplifiers.

Fig. 2. Subdivided charge division with global centroid computation.^{2,3} $\bar{x} = \sum x_k Q_k / \sum Q_k$. Noise from all preamplifier outputs enters the result.

Fig. 3. Centroid finding by convolution.

Preamplifier outputs from subdivided resistive chain (or a continuous resistive sheet) are switched sequentially into the centroid finding filter. Noise from three outputs only enters the result. Zero crossing time t_{fo} is a continuous linear function of the charge (centroid) position x over the whole detector.

Fig. 4. Centroid finding by convolution. Relation of the signal width to the width of the weighting function (impulse response) of the centroid finding filter. The minimum width for the case shown is $T_M \approx 3.5 T_s$. The relation of the charge samples to the induced charge distribution is shown in Fig. 1(c). The conversion sensitivity position-to-time is uniquely determined only by the sample width T_s as $T_s/(L/N)$, (see Eq. (10) in the text).

Fig. 5. Functional diagram of the centroid finding filter. A_1 and A_2 are operational amplifiers. G_1 and G_2 are operational transconductance amplifiers comprising gated base line restorers. The response of the filter in Laplace domain is,

$$h(s) = \frac{2}{s} [1 - e^{-sT_M}] - \frac{2}{T_M} \frac{1}{s} (1 - e^{-sT_M})].$$

Fig. 6. Response of the centroid finding filter as a function of the position of the induced charge

(a) Impulse response

(b) $x = 0$

(c) $\frac{x}{(L/N)} = 0.3$

(d) $\frac{x}{(L/N)} = 0.5$

(e) $\frac{x}{(L/N)} = 0.7$

$\frac{x}{(L/N)} = 1$ corresponds to the single section

between adjacent amplifier connections (fiducial points) to the resistive chain. Note that the slope at zero crossing is independent of position x . In this example, $T_S \approx 250$ nsec and $T_M \approx 750$ nsec.

Fig. 7. A simplified model for noise calculation in the centroid finding method. It is assumed that the noise from the resistor chain is dominant.

- (a) Equivalent circuit. Q_s = signal charge
 q_{1j} = noise current generators.
- (b) Sequence of samples with a signal and noise.

Fig. 8. Position resolution versus signal charge calculated (solid line) and measured on the centroid finding system described in Ref. 7. Detector capacitance per section is $C_{DS} = \frac{C_D}{N} \approx 30$ pF. FWHM scales as $C_D^{1/2}$ with capacitance. Section length is $l/N = 15.875$ mm. Relative resolution is determined only by the capacitance.

Fig. 9. The relation of the time digitizer clock and the sequential switch clock.

- (a) The ratio of clock frequencies is not an integral number. An error in the channel width at the boundary of two sections results in differential nonlinearity.
- (b) The sequential switch clock is derived from the time digitizer clock by frequency division.

Fig. 10. Measured position error (integral non-linearity). Position error is independent of the detector length and it is given by the section length l/N . Numbers on horizontal axis indicate section boundaries.

(a)

(b)

Fig. 11. Measured differential nonlinearity.

- (a) for cathode strip/anode-cathode gap ratio $s/d = 0.76$
- (b) for cathode strip/anode-cathode gap ratio $s/d = 1.1$

Numbers indicate section boundaries. There are four strips per section L/N .