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TWO-DIMENSIONAL CALCULATION OF
FINITE-BETA MODIFICATIONS OF DRIFT
AND TRAPPED-ELECTRON MODES

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An electrostatic two-dimensional calculation of drift and trapped-electron modes was described in detail in Ref. 1. Here, the extension of that calculation to the case of nonzero $\beta \equiv 8\pi n(T_e + T_i) / B_0^2$, where B_0 is the magnetic field strength at the magnetic axis, is described. The calculation still assumes $\beta \ll 1$, so that $A_\perp$, the component of the perturbed magnetic vector potential perpendicular to the equilibrium magnetic field B , can be omitted and only $A_\parallel$, the component parallel to the equilibrium magnetic field, need be retained. Also, the equilibrium magnetic surfaces are assumed to be circular and concentric, and this will not remain true for β approaching unity.²

Thus, a solution is required for $A_\parallel$ as well as for $\tilde{\phi}$, the perturbed electrostatic potential. In Ref. 1, the equation solved for $\tilde{\phi}$ was just the quasi-neutrality condition,

$$\sum_j e_j n_j^{(1)} = \sum_j e_j \int d^3v f_j^{(1)} = 0. \quad (1)$$

An additional equation is now required, and it is just the component of Ampere's law along the equilibrium magnetic field

$$\nabla^2 A_\parallel = -\frac{4\pi}{c} \sum_j e_j n v_{\parallel j}^{(1)} = -\frac{4\pi}{c} \sum_j e_j \int d^3v v_\parallel f_j^{(1)}. \quad (2)$$

Here $j=e, i$ for electrons and ions respectively, $e_e = -|e|$, $n_j^{(1)}$ is the perturbed density response, $f_j^{(1)}$ is the perturbed distribution function, n is the equilibrium density, and $v_{\parallel j}^{(1)}$ is the perturbed parallel velocity response.

The perturbed distribution function $f_j^{(1)}$ is obtained by solving the gyrokinetic equation as in Refs. 1 and 3, except that $\tilde{\phi}$ is replaced by^{4,5}
 $\tilde{\phi} - v_\parallel A_\parallel / c$ in the non-adiabatic terms, i.e., for electrons

$$\left(\frac{\partial}{\partial t} + (\mathbf{v}_{||} + \mathbf{v}_B) \cdot \nabla \right) \left(f_e^{(1)} - \frac{|e|\tilde{\phi}}{T_e} f_{me} \right) = \frac{c}{B^2} \nabla(\tilde{\phi} - v_{||} A_{||}/c) \times \mathbf{B} \cdot \nabla f_{me} - \frac{|e|}{T_e} f_{me} \frac{\partial}{\partial t} (\tilde{\phi} - v_{||} A_{||}/c) - v_f \left(f_e^{(1)} - \frac{|e|\tilde{\phi}}{T_e} f_{me} \right) + v_f^h \frac{f_{me}}{n} \left(n_e^{(1)} - \frac{|e|\tilde{\phi}}{T_e} n \right). \quad (3)$$

The notation here is the same as Ref. 1. Specifically, $\mathbf{v}_B$ is the magnetic (gradient and curvature) drift velocity, v_f is the effective electron collision frequency given in Ref. 1, and v_f^h is the high collision frequency limiting form of v_f . Here the lowest order equilibrium distribution function is taken to be an unshifted Maxwellian f_{me} ; the equilibrium circulating electron flow velocity considered in Ref. 1 is taken here to be zero. For ions the replacement of $\tilde{\phi}$ by $\tilde{\phi} - v_{||} A_{||}/c$ is to be made in the gyrokinetic equation,⁶ which accounts for finite Larmor radius effects, as is discussed in Ref. 1.

Equation (3) may be solved as in Ref. 1 for $f_e^{(1)}$, and the appropriate moments taken to obtain $n_e^{(1)} \equiv \tilde{n}_e(\theta, S) \exp(-i\omega t + i\ell\zeta - im^0\theta)$ and $v_{||e}^{(1)} \equiv \tilde{v}_{||e}(\theta, S) \times \exp(-i\omega t + i\ell\zeta - im^0\theta)$. Here ζ and θ are the toroidal and poloidal angles, respectively, ℓ is the toroidal mode number, m^0 is the poloidal mode number corresponding to a "reference" mode-rational surface in the radial region of interest, and $S(r) \equiv \ell q(r) - m^0$ is a new radial variable. Also

$$\tilde{n}_e = \frac{|e|n}{T_e} \left\{ \tilde{\phi} - \left(\frac{1}{n} \int d^3v f_{me}(\omega - \omega_{*e}^T) \sum_p \frac{\tilde{\phi}^{(p)} \exp[ip\omega_{b,t} \hat{t}(\theta)]}{\omega + iv_f - \omega_{de}^{(0)} - [p + SH(\Lambda_c - \Lambda)]\omega_{b,t}} \right) \times \left[1 + \frac{iv_f^h}{k_{||}v_e} Z \left(\frac{\omega + iv_f^h}{k_{||}v_e} \right) \right]^{-1} \right\}, \quad (4)$$

and

$$\tilde{v}_{||} = - \frac{|e|}{T_e n} \int d^3v v_{||} f_{me}(\omega - \omega_{*e}^T) \sum_p \frac{\tilde{\phi}^{(p)} \exp[ip\omega_{b,t} \hat{t}(\theta)]}{\omega + iv_f - \omega_{de}^{(0)} - [p + SH(\Lambda_c - \Lambda)]\omega_{b,t}}$$

$$\times \left[1 + \frac{1}{k_{||}} \frac{v}{v_e} \frac{h}{f} Z \left(\frac{\omega + i \frac{v}{v_e} \frac{h}{f}}{k_{||} v_e} \right) \right]^{-1} \quad (5)$$

Here $\tilde{\phi} = \tilde{\phi}(\theta, S) \exp(-i\omega t + i\ell\tau - im^0\theta)$, $A_{||} = \tilde{A}_{||} \exp(-i\omega t + i\ell\tau - im^0\theta)$, $\omega_{*e}^T = \omega_{*e} [1 + \eta_e (\epsilon/T_e - 3/2)]$, ω_{*e} is the usual diamagnetic drift frequency, $\eta_j \equiv (d \ln T_j / dr) / (d \ln n / dr)$, $\omega_{b,t}$ refers to ω_b , the bounce frequency, for trapped particles and ω_t , the transit frequency, for circulating particles. $\hat{t}(\theta)$ is a function expressing time along a single-particle orbit as a function of θ , $\omega_{de}^{(0)}$ is the time-average magnetic drift frequency, H is the Heaviside step function, $k_{||} = 1/qR_0$, R_0 is the major radius of the magnetic axis, and Z is the usual plasma dispersion function. Also $\Lambda = \mu B_0 / \epsilon$ is the dimensionless pitch angle variable, with $0 \leq \Lambda < \Lambda_c = 1 - \epsilon_0$ for circulating particles, and $\Lambda_c < \Lambda \leq 1 + \epsilon_0$ for trapped particles, with $\epsilon_0 \equiv r/R_0$, μ being the magnetic moment and ϵ the kinetic energy. These quantities are given more explicitly in Ref. 1. Also

$$\begin{aligned} \tilde{\phi}^{(p)} = \frac{1}{\tau_{b,t}} \int dt' \exp \left\{ iS(\theta' - \theta) - i\omega_t S[\hat{t}(\theta') - \hat{t}(\theta)] \right. \\ \left. \times H(\Lambda_c - \Lambda) \right\} \exp[-ip\omega_{b,t} \hat{t}(\theta)] [\tilde{\phi}(\theta', S) - \frac{v_{||}(\theta')}{c} \tilde{A}_{||}(\theta', S)] \end{aligned} \quad (6)$$

is the coefficient function for the p -th bounce or transit harmonic, with $\theta' \equiv \theta(t')$.

For the ions the derivation of $\tilde{n}_1$ and $\tilde{v}_{||1}$ follows that in Ref. 1. The differences are, first, that $\tilde{\phi}$ is to be replaced by $\tilde{\phi} - v_{||} A_{||} / c$ in the non-adiabatic terms in $\tilde{f}_1$, and, second, that terms in $\tilde{f}_1$ which are odd in $v_{||}$, and thus did not contribute to

$\tilde{n}_i$ in the electrostatic case have to be retained because they contribute now to $\tilde{n}_i$ and $\tilde{v}_{||i}$. The ion response is in three parts. The most important is the so-called "non-resonant" ion response $\tilde{n}_i^{nr}$ and $\tilde{v}_{||i}^{nr}$, which actually includes the ion magnetic drift resonances. The others are the ion-ion collisional response $\tilde{n}_i^c$ and $\tilde{v}_{||i}^c$, and the circulating ion transit resonance response, $\tilde{n}_i^r$ and $\tilde{v}_{||i}^r$. That is

$$\tilde{n}_i^{(1)} = \tilde{n}_i \exp(-i\omega t + i l \zeta - i m^0 \theta) = (\tilde{n}_i^{nr} + \tilde{n}_i^c + \tilde{n}_i^r) \exp(-i\omega t + i l \zeta - i m^0 \theta) \text{ and}$$

$$\tilde{v}_{||i}^{(1)} = \tilde{v}_{||i} \exp(-i\omega t + i l \zeta - i m^0 \theta) = (\tilde{v}_{||i}^{nr} + \tilde{v}_{||i}^c + \tilde{v}_{||i}^r) \exp(-i\omega t + i l \zeta - i m^0 \theta). \text{ Here}$$

$$\begin{aligned} \tilde{n}_i^{nr} = & - \frac{j e |n|}{T_i} \left(\hat{\phi}(\theta, S) - \frac{1}{n} \int d^3 v f_{mi}(\omega - \omega_{*i}) \right. \\ & \times \left\{ \frac{1}{\omega - \omega_{di}(1)} - i \frac{\hat{v}_{||} \bar{\omega}_{ti}}{\omega - \omega_{di}(1)} \left(\frac{\partial}{\partial \theta} + i S \right) \frac{1}{\omega - \omega_{di}(1)} \right. \\ & - \frac{\hat{v}_{||}^2 \bar{\omega}_{ti}^2}{\omega - \omega_{di}(1)} \left(\frac{\partial}{\partial \theta} + i S \right) \frac{1}{\omega - \omega_{di}(1)} \left(\frac{\partial}{\partial \theta} + i S \right) \frac{1}{\omega - \omega_{di}(1)} \\ & + i \left(\frac{\hat{v}_i^2 q \rho_i \bar{\omega}_{ti}}{2 \Delta r_s} \right) \frac{\sin \theta}{[\omega - \omega_{di}(1)]^2} \frac{\partial}{\partial S} - \left(\frac{\hat{v}_i^2 q \rho_i \bar{\omega}_{ti}}{2 \Delta r_s} \right)^2 \frac{\sin^2 \theta}{[\omega - \omega_{di}(1)]^2} \frac{\partial^2}{\partial S^2} \\ & \left. \left. + \left(\frac{\hat{v}_{||}^2 \bar{\omega}_{ti}^2}{2 \Delta r_s} \right) \frac{1}{[\omega - \omega_{di}(1)]^2} \left[\left(\frac{\partial}{\partial \theta} + i S \right) \left(\sin \theta \frac{\partial}{\partial S} \right) \right] \right\} \right) \end{aligned}$$

$$\begin{aligned}
 & + \left(\sin \theta \frac{\partial}{\partial S} \right) \left(\frac{\partial}{\partial \theta} + iS \right) \frac{1}{\omega - \omega_{di}(1)} \left\} (2\pi)^{-1/2} \int_{-\infty}^{\infty} dk_r \right. \\
 & \times \exp[ik_r(r-r_0)] J_0^2[\rho_1 \hat{v}_\perp (k_\theta^2 + k_r^2)^{1/2}] \int_{-\infty}^{\infty} d(r-r_0) \exp[-ik_r(r-r_0)] \\
 & \left. [\tilde{\phi}(\theta, S) - \frac{v_{||}}{c} \tilde{A}_{||}(\theta, S)] \right\} , \quad (7)
 \end{aligned}$$

and

$$\begin{aligned}
 \tilde{v}_{||1}^{nr} &= \frac{|e|}{nT_i} \int d^3v_{||} f_{m1}(\omega - \omega_{*i}) \{ \dots \} (2\pi)^{-1/2} \int_{-\infty}^{\infty} dk_r \\
 & \exp[ik_r(r-r_0)] J_0^2[\rho_1 \hat{v}_\perp (k_\theta^2 + k_r^2)^{1/2}] \int_{-\infty}^{\infty} d(r-r_0) \exp[-ik_r(r-r_0)] \\
 & [\tilde{\phi}(\theta, S) - \frac{v_{||}}{c} \tilde{A}_{||}(\theta, S)] , \quad (8)
 \end{aligned}$$

with the expression in curly brackets being the same as in Eq. (7). In these equations $\hat{v}_{||} \equiv v_{||}/v_1$, $\hat{v}_\perp \equiv v_\perp/v_1$, $v_1 \equiv (2T_i/m_i)^{1/2}$, $\omega_{di}(1) \equiv \omega_{di}(\hat{v}_\perp^2 + 2\hat{v}_{||}^2) \cos \theta$, $\bar{\omega}_{di} \equiv (r_n/R_0)\omega_{*i}$, $r_n \equiv -[d \ln n / dr]^{-1}$, $\omega_{ti} \equiv v_1/qR_0$, $\Delta r_s = [\ell dq/dr]^{-1}$ is the separation of mode-rational surfaces, J_0 is the usual Bessel junction, $\rho_1 = v_1/\Omega_i$, and $\Omega_i = |e|B_0/m_i c$. The ion-ion collisional response term, which is a very small term for normal tokamak parameters and is approximate to begin with, is left the same as in the electrostatic case, with $\tilde{n}_1^c$ as given in Ref. 1 and $\tilde{v}_{||1}^c = 0$. The ion transit resonance response term, which is also normally a small term, is as given in Ref. 1 for $\tilde{n}_1^r$ with $\tilde{\phi}(\theta', S')$ re-

placed by $[\tilde{\phi}(\theta', S') - v_{||}(\theta') \tilde{A}_{||}(\theta', S')/c]$, and $\tilde{v}_{||1}^r$ is the same thing with the extra weight factor $v_{||}(\theta)/n$ in the velocity space integration. Here $v_{||}(\theta) = \sigma_{||} v [1 - \Lambda/h(\theta)]$, with $h(\theta) = 1 + \epsilon_0 \cos \theta$ and $\sigma_{||} = \text{sgn}(v_{||})$.

The quasi-neutrality equation (1) and the parallel current equation (2) may be converted into matrix equations by expanding $\tilde{\phi}$ and $\tilde{A}_{||}$ in sets of poloidal and radial basis functions,

$$\tilde{\phi}(\theta, S) = \sum_{n=0}^{\infty} \sum_{j=-\infty}^{\infty} g_j(\theta) h_n(S) \tilde{\phi}_{jn} \quad , \quad (9)$$

and

$$\tilde{A}_{||}(\theta, S) = \sum_{n=0}^{\infty} \sum_{j=-\infty}^{\infty} g_j(\theta) h_n(S) \tilde{A}_{||jn} \quad , \quad (10)$$

where $g_j(\theta) = \exp(ij\theta) (2\pi)^{-1/2}$ and $h_n(S) = H_n(\sigma^{1/2} S) \exp(-\sigma S^2/2) M_n^{-1/2}$, H_n denotes a Hermite polynomial, and $M_n \equiv (\pi/\sigma)^{1/2} 2^n n!$. Here σ is a parameter [with $\text{Re}(\sigma) > 0$] which can be adjusted to minimize the number of radial basis functions which need to be retained. The boundary conditions used here are that $\tilde{\phi}$ and $\tilde{A}_{||}$ must both be evanescent as $S \rightarrow \pm\infty$. Using Eqs. (9) and (10), and operating on Eqs. (1) and (2) with

$$\int_{-\pi}^{\pi} d\theta \int_{-\infty}^{\infty} dS \quad g_j^*(\theta) h_n(S) \quad , \quad (11)$$

the resulting equations can be combined into a single matrix equation which can be written schematically as

$$\left(\begin{array}{c|c} L_{jj',nn'}^{n\phi} & L_{jj',nn'}^{nA} \\ \hline L_{jj',nn'}^{V\phi} & L_{jj',nn'}^{VA} \end{array} \right) \left(\begin{array}{c} \tilde{\phi}_{jn} \\ \vdots \\ \tilde{A}_{||jn} \\ \vdots \end{array} \right) = \left(\begin{array}{c} 0 \\ \vdots \end{array} \right) \quad . \quad (12)$$

The matrix elements of the submatrix $L_{jj',nn'}^{n\phi}$, are just those which were worked out in detail in Ref. 1. The matrix elements of the $L_{jj',nn'}^{nA}$, $L_{jj',nn'}^{V\phi}$, and $L_{jj',nn'}^{VA}$ submatrices can be worked out in exactly the same way, and just differ by the appropriate extra factors of $-v_{||}(\theta')/c$, $v_{||}(\theta)/v_e$, and $-v_{||}(\theta')v_{||}(\theta)/cv_e$,

respectively. In particular, the $\nabla^2 A_{||} = -(k_r^2 + k_\theta^2) A_{||}$ term in Eq. (2) contributes to the $L_{jj',nn'}^{VA}$ matrix elements and, with the normalization used here, its contribution is

$$L_{jj',nn'}^\beta = \frac{T_e}{T_i} \frac{v_i^2}{v_e c} \frac{(1 + T_e/T_i)}{\beta} \delta_{jj'} \frac{(-1)^{n-n'}}{\sigma} \int_{-\infty}^{\infty} dk_s h_n(k_s/\sigma) h_{n'}(k_s/\sigma) b_1, \quad (13)$$

where $k_s \equiv k_r \Delta r_s$, $b_1 \equiv \frac{1}{2} \rho_1^2 (k_\theta^2 + k_r^2)$, and $k_\theta \equiv -m^0/r$. Also the adiabatic terms occur only in the $L_{jj',nn'}^{n\phi}$ submatrix.

This matrix equation is solved just as in the electrostatic case for the eigenvalue ω and the eigenfunctions $\tilde{\phi}(\theta, S)$ and $\tilde{A}_{||}(\theta, S)$.

The calculation that has been described is rather general. It can treat shear-Alfvén modes as well as finite- β modified drift modes, though the evanescent boundary conditions as $S \rightarrow \pm\infty$ used here are not the most general that might be considered. The procedure for finding eigenvalues of ω requires an initial guess for ω , and for finite- β modified drift modes, this is conveniently available from the electrostatic calculations. This is not the case for shear-Alfvén modes, and the study of these will therefore be deferred to a future publication.

It is possible to study eigenfunctions with various symmetries in θ and S for $\tilde{\phi}(\theta, S)$ and $\tilde{A}_{||}(\theta, S)$. Here we will concentrate on the finite- β modifications of the lowest mode in the electrostatic limit, i.e., the one with the least number of radial and poloidal modes. This is typically the eigenfunction with the highest linear growth rate. This mode, studied in Ref. 1, has an eigenfunction for $\tilde{\phi}(\theta, S)$ which was even in θ at $S=0$, was even in S at $\theta=0$, was even in $(S\theta)$ everywhere, and ballooned strongly to the outside of the torus, around $\theta=0$.

For the cases studied here, we use $\sigma \approx 0.7 + 0.15i$, with 9 poloidal

harmonics, 9 radial basis functions, and 11 bounce and transit harmonics. The truncation to 9 poloidal harmonics artificially limits the width of the eigenfunctions, but does not appreciably affect the eigenvalues.

The parameters for the first case to be considered here are:

$\epsilon_0 = 0.1$, $T_e/T_i = 2$, $q = 2.5$, $q'r/q = 1$, $n_i = n_e = 0$, $r_n/r = 1$, $m_i/m_e = 3672$, $v_e^* \equiv v_{e1}/(\epsilon_0^{3/2} \bar{\omega}_{te}) = 0.03$, and $b_{i0} = 0.2$ (or $k_0 \rho_i = 0.63$), as in Ref. 1. In addition T_e is specified to be 1 keV and β is varied from 10^{-5} to 3×10^{-2} .

The corresponding eigenvalues for $\omega = \omega_r + i\gamma$ in units of ω_{*e} are shown in Fig. 1. Note that only the parameter β in the coefficient of the $V^2 A_1$ term given by Eq. (13) is varied in this figure; the parameters v_e^* and b_{i0} , (which depend on n, T , and B_0) are held fixed. The amount of coupling between the drift branch and the shear-Alfvén branch increases as β increases, but the parameters which determine the electrostatic growth rate are held fixed. This is a somewhat artificial case, but is still of interest. For very low $\beta (= 10^{-5})$ the eigenvalue agrees with the electrostatic ($\beta = 0$) value. Then as β increases to 3×10^{-5} the growth rate decreases very slightly. This stabilizing effect was derived in several previous perturbative calculations in slab⁷ and toroidal⁴ geometry. For β increasing from 3×10^{-5} to 3×10^{-3} , the effect is destabilizing. This is apparently a specifically toroidal effect which has been obtained also from approximate analytic calculations.⁸ For β greater than 3×10^{-3} , the effect of increasing β is again stabilizing. This seems to be an indirect effect due to the decrease in ω_r which is monotonic in β . The last two points in Fig. 1, for $\beta = 10^{-2}$ and $\beta = 3 \times 10^{-2}$, should be regarded as somewhat unreliable, since the assumptions made earlier, that coupling to A_1 is negligible and that the flux surfaces remain circular and concentric, start to break down for these higher β values. Also, computationally, it becomes more difficult to follow a particular eigenfunction as the β value

changes, for these higher β values. Notice that all of these fractional changes in the growth rate are small, and finite- β effects are far too weak in this case to stabilize the instability.

The eigenfunctions for the real and imaginary parts of $\tilde{\phi}(\theta, S)$ and $\tilde{A}_{||}(\theta, S)$ are shown in Fig. 2 for the case $\beta = 10^{-4}$. The vertical scales are the same in the four plots. The maximum magnitude of $\tilde{A}_{||}$ is only slightly smaller than that of $\tilde{\phi}$ for this small β value. This is not surprising when we recall that $\tilde{A}_{||}$ enters the dispersion equation in most terms multiplied by quantities like $v_{||}/c$, which are at most of order $v_e/c \ll 1$, with respect to $\tilde{\phi}$. Thus a substantial $\tilde{A}_{||}$ only has a small effect on the eigenvalue.

Next we consider a more realistic set of parameters, appropriate for neutral-beam heated PLT.⁹ The case considered has $T_i(r=0) = 5.9$ keV, and the quantities needed are evaluated at $r = 22$ cm., where the limiter radius $a = 40$ cm., from profiles calculated with a one-dimensional radial transport code.¹⁰ The parameters are: $\epsilon_0 = 0.17$, $T_e/T_i = 0.81$, $q = 1.37$, $q'r/q = 1.64$, $n_i = 3.70$, $n_e = 2.30$, $r_n/r = 0.88$, $m_i/m_e = 1836$, $v_e^* = 0.053$, $b_{i0} = 2.64$, $T_e = 1.05$ keV, and $\beta = 0.0022$. The electrostatic result in this case is $\omega/\omega_{*e} = -0.4616 + 0.2012i$, while the electromagnetic result is $\omega/\omega_{*e} = -0.4627 + 0.2014i$. Thus the electromagnetic corrections to the eigenvalue are very small, $|\delta\omega_{EM}|/|\omega| \approx 0.2\%$, and in this case are mostly to ω_r . Therefore, purely electrostatic calculations for drift modes would be quite accurate for these sorts of parameters.

The corresponding eigenfunctions for the real and imaginary parts of $\tilde{\phi}(\theta, S)$ and $\tilde{A}_{||}(\theta, S)$ are shown in Fig 3. Here the maximum magnitude of $\tilde{A}_{||}$ is larger, by a factor of two to three, than the maximum magnitude of $\tilde{\phi}$, for the reason given earlier.

In the future, we hope to employ the "ballooning formalism" to do the

calculation of electromagnetic trapped-particle modes in a more computationally efficient manner.¹¹

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FIGURE CAPTIONS

Fig. 1. Real frequency and growth rate in units of ω_{*e} versus the β parameter in the coefficient of the $\nabla^2 A_{||}$ term. The electrostatic ($\beta = 0$) values are also shown.

Fig. 2. Real and imaginary parts of $\tilde{\phi}(\theta, S)$ and $\tilde{A}_{||}(\theta, S)$. Here θ is the poloidal angle with $\theta = 0$ at the outside of the torus, and $S = (r - r_o)/\Delta r_g$, where r_o is the minor radius of the "reference" mode-rational surface, and Δr_g is the separation of mode rational surfaces. The parameters are: $\epsilon_o = 0.1$, $T_e/T_i = 2$, $q = 2.5$, $q'r/q = 1$, $\eta_i = \eta_e = 0$, $r_n/r = 1$, $m_i/m_e = 3672$, $v_e^* = 0.03$, $b_{i0} = 0.2$, $T_e = 1$ keV, and $\beta = 10^{-4}$.

Fig. 3. Real and imaginary parts of $\tilde{\phi}(\theta, S)$ and $\tilde{A}_{||}(\theta, S)$ for parameters typical of the neutral-beam-heated PLT. The parameters are: $\epsilon_o = 0.17$, $T_e/T_i = 0.81$, $q = 1.37$, $q'r/q = 1.64$, $\eta_i = 3.70$, $\eta_e = 2.30$, $r_n/r = 0.88$, $m_i/m_e = 1836$, $v_e^* = 0.053$, $b_{i0} = 2.64$, $T_e = 1.05$ keV, and $\beta = 0.0022$.

Fig. 1. (PFPL-802062)

Fig. 2. (PPPL-792153)

Fig. 3. (PPPL-802072)