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NONLINEAR INTERACTION OF COLLIDING BEAMS IN PARTICLE STORAGE RINGS

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NONLINEAR INTERACTION OF COLLIDING BEAMS IN PARTICLE STORAGE RINGS*

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I. INTRODUCTION

When two beams of high energy particles moving in opposite directions are brought into collision, a large amount of energy is available for the production of new particles. However to obtain a sufficiently high event rate for rare processes, such as the production of the intermediate vector boson (Z_0 and $W^\pm$), large beam currents are also required. Under this circumstance, the high charge density of one beam results in a classical electromagnetic interaction on the particles in the other beam. This very nonlinear space charge force, called the beam-beam force, limits the total circulating charge and, thereby, the ultimate performance of the colliding ring system. In the following section, we shall discuss the basic nature of the beam-beam force, indicating how it is quite different in the case of continuous beams, which cross each other at an angle as compared to the case of bunched beams which collide head-on. Some experimental observations on the beam-beam interaction in proton-proton and electron-positron beams are then reviewed and interpreted. An important aspect of the beam-beam problem in storage rings is to determine at what point in the analysis of the particle dynamics is it relevant to bring in the concepts of stochasticity, slow diffusion, and resonance overlap. These ideas will be briefly discussed.

II. THE BEAM-BEAM FORCE

When two beams of particles pass through each other, the particles of each beam will be acted on by the electromagnetic fields arising from the other beam. Clearly, this classical interaction will perturb the

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the motion of the charges circulating in the storage ring. Whether the beams are uniform or bunched, and whether they collide at an angle or head-on, will determine the exact influence on an individual particle. It is the nature of this "beam-beam force" that we wish to outline in this section.

In Fig. 1 we show a particle trajectory crossing a continuous beam at an angle α_o . Assuming that the transverse (x,y) distribution of current (I) in the beam is Gaussian, that is,

$$\rho(x,y) = \frac{I}{2\pi\sigma_V\sigma_H} \exp \left[-\frac{x^2}{2\sigma_H^2} - \frac{y^2}{2\sigma_V^2} \right], \quad (1)$$

one can calculate the net change in betatron tune for a central trajectory (y = 0). The results¹ for proton beams are

$$\Delta\nu_V = \frac{\beta_V^* r_p I}{\sqrt{2\pi} e c \gamma \beta^2 \sigma_V^* \tan(\alpha_o/2)}, \quad (2)$$

and

$$\Delta\nu_H = 0, \quad (3)$$

where we have used the conventional notation:

α_o = the horizontal crossing angle (rad),

σ_V^* = Gaussian rms (vertical) size at crossing (m),

β_V^* = vertical betatron function at crossing (m),

r_p = classical radius of proton (1.535×10^{-18} m),

(γ, β) = relativistic particle factors of colliding protons,

I = average current per beam (A),

($\Delta\nu$)_{V,H} = vertical or horizontal tune shift per crossing,

e = charge of proton (1.602×10^{-19} C), and

c = velocity of light (2.9979×10^8 ms⁻¹).

From the viewpoint of the particle dynamics, it is important to know the net angular deflection undergone by a particle going through the collision region. Because of the opposite effect experienced by the particle in passing from one side of the beam to the other, the net horizontal angular deflection is equal to zero, while the vertical one is

$$\Delta \left(\frac{dy}{ds} \right) = \frac{2\pi r_p I}{ec\gamma\beta^2 \tan(\alpha_0/2)} \Phi(y/\sqrt{2}\sigma_V^*), \quad (4)$$

where Φ is the error function defined by

$$\Phi(x) = \frac{2}{\sqrt{\pi}} \int_0^x dt e^{-t^2}. \quad (5)$$

In Fig. 2. we have sketched the variation of this vertical deflection as a function of the displacement of the particle path from the horizontal symmetry plane of the colliding beams. It can be seen that the force is very nonlinear, approaching a limiting value at the outer fringes of the beam. At this point we should emphasize that for this type of crossing, the net deflection occurs only in the vertical plane, and, therefore, the dynamics of the motion is one-dimensional.

For the case of the head on collision of bunches the situation is quite different. Thus in the example of an electron traversing a bunch of positrons, such as takes place in e^+e^- storage rings, the horizontal and vertical motions are coupled by the beam-beam interaction. As was the case for crossing uniform beams, the strength of the beam beam interaction is measured by the changes in linear tunes. These we quote from Ref. 1.:

$$\Delta\nu_V = \frac{\beta_V^* r_e N_B}{2\pi\gamma\beta^2 \sigma_V^* (\sigma_H^* + \sigma_V^*)}, \quad (6)$$

and

$$\Delta v_H = \frac{\beta_H^* r_e N_B}{2\pi\gamma\beta^2 \sigma_H^* (\sigma_H^* + \sigma_V^*)} \quad (7)$$

Here r_e is the classical radius of the electron (2.818×10^{-15} m) and N_B is the number of particles per bunch. The coupling of the net angular deflections is clearly illustrated if we consider the collision of cylindrically symmetric Gaussian bunches. Referring again to Ref. 1, we can write for these net deflections

$$\Delta\left(\frac{dx}{ds}\right) = \frac{-2N_B r_e X}{\gamma\beta^2 r^2} (1 - e^{-r^2/2\sigma^2}) , \quad (8)$$

and

$$\Delta\left(\frac{dy}{ds}\right) = \frac{-2N_B r_e y}{\gamma\beta^2 r^2} (1 - e^{-r^2/2\sigma^2}) . \quad (9)$$

When we consider these relationships, we see that for the head-on collisions of bunches, the deflection decreases to zero for particles passing through the outer fringes of the bunch ($r \gg \sigma$). Though the strength parameters, Eqs. (6) and (7), apply to unsymmetrical Gaussian bunches, the net deflections are only applicable to cylindrically symmetric beams. The more general case of a beam of arbitrary aspect ratio, which is actually the important one since e^+e^- rings operate with a ratio of horizontal to vertical size at the collision point of about 20 to 1, has been treated in detail in Ref. 1.

III. MODEL FOR A COLLIDING BEAM MACHINE

In Fig. 3. we give a pictorial representation of a particle trajectory passing on each revolution through a linear machine followed by a nonlinear deflection at a point. Though greatly simplified, this model includes the essential aspects arising from the beam-beam interaction in such machines as the ISR at CERN and the future storage accelerator

ISABELLE at Brookhaven. Between each pair of interaction regions, whose lengths are small compared with their spacing along the central trajectory, the particle coordinates are transformed by a linear transformation characteristic of the magnetic lattice of the ring. In describing this transformation, it is convenient to use the normalized coordinates², $[\eta(\psi), \dot{\eta}(\psi)]$, which depend on the betatron phase ψ along the central trajectory. If we introduce the complex variable, $Z = \eta(\psi) + i\dot{\eta}(\psi)$, we can write the variable Z_2 (see Fig. 3) after the collision point in terms of its initial value Z_0 and the increment $\Delta\dot{\eta}$ experienced at the collision point. Thus, we have

$$Z_2 = Z_0 e^{-i\psi_1} + i \Delta\dot{\eta}, \quad (10)$$

or, using Eq. (4)

$$Z_2 = Z_0 e^{-i\psi_1} + i\Delta\dot{\eta} \frac{(2\pi)^{3/2} \sigma_V^* |\Delta v_V|}{\beta_V^{*1/2}} \Phi\left(\frac{\beta_V^{*1/2} \eta_1}{\sqrt{2} \sigma_V^*}\right). \quad (11)$$

Here σ_V^* is the rms size of the strong beam at the interaction point and β_V^* is the lattice betatron function at the same point for the linear machine. In so far as we are interested in the excursions of the intersecting particle trajectory on successive revolution with respect to the fixed strong beam, it is desirable to normalize the phase space variable Z of the particle to the square root of the linear invariant W_s of the strong beam. On choosing W_s such that the associated boundary curve contains 95% of the particles of the strong beam, that is,

$$W_s = \frac{6\sigma_V^{*2}}{\beta_V^*} \quad (12)$$

we obtain the simplified form of Eq. (11) which expresses the particle coordinate transformation for one turn, i.e.

$$\hat{z}_2 = \hat{z}_0 e^{-i\psi_1} + i4\pi|\Delta\nu_V| \Theta(\hat{\eta}_1). \quad (13)$$

Specifically, we have introduced the $\hat{z}$ variable according to

$$z \rightarrow \hat{z} = \frac{\eta + i\dot{\eta}}{\sqrt{w_s}} = \hat{\eta} + i\dot{\hat{\eta}}, \quad (14)$$

and

$$\Theta(\hat{\eta}) = \frac{\sqrt{\pi}}{2\sqrt{3}} \Phi(\sqrt{3}\hat{\eta}_1). \quad (15)$$

It is clear from Eq. (13) that the motion of the particle over many revolutions is represented by a mapping, consisting of the rotation of the representative phase space vector through an angle equal to the betatron phase followed by a nonlinear increment in $\hat{\eta}$ with a magnitude proportional to the beam-beam strength parameter $\xi_V = |\Delta\nu_V|$.

IV. PROTON-PROTON EXPERIMENTS AT THE ISR

The model we have just developed is directly applicable to the proton-proton colliding beam system such as the ISR at CERN. In this ideal configuration the important feature of the mapping is that only one degree of freedom is involved — the force for the crossing beam geometry is one dimensional. As we have shown, when the scale of oscillation magnitude is taken to be the size of the fixed strong beam, there is only one parameter determining the force, the beam-beam strength, or linear tune shift ξ_V . We can therefore study the one dimensional mapping in terms of the betatron tune ($\nu = \psi_1/2\pi$) and ξ_V . This problem has essentially been solved. For $\xi_V < 0.3$, one finds³ regular bounded motion; while for $\xi_V > 0.3$, some irregular motion appears in the particle coordinate phase space. Some examples of phase space trajectories are shown in Fig. 4.

In the ISR, or as planned for ISABELLE, the strength ξ_V is about

two orders of magnitude smaller than 0.03. Thus, the onset of irregular or stochastic motion due to the beam beam interaction would be anomalous. However, some beam-beam effects are observable in the ISR at levels of $\xi_V \sim 10^{-3}$. An example is shown in Fig. 5. This is a scan of the vertical oscillation amplitude obtained by observing Schottky noise, i.e. very high frequency components of the vertical beam displacement signals. The horizontal axis is proportional to the vertical betatron frequency, ν_V . In this example, the effects of high order resonances, the 8th and 11th can be seen. Even these subtle effects are explained by classical resonance theory. The main excitation parameters turns out to be the vertical misalignments at the crossing point of one beam with respect to the other. The greater their relative displacements, the larger the amplitude growth. The time scale, which in Fig. 5. is one the order of 1-2 hours, is determined by the rate of vertical tune fluctuations caused by the multiple Coulomb scattering of the particles in each high current beam. If the beam is allowed to rest on lower order resonances, such as the 5th, the amplitude growth is much more pronounced. The beam loss rate in parts per million per minute (ppm) can also be measured. When the beam is situated as in Fig. 5, i.e. on the 8th or 11th order resonances, a small loss rate of about 1 ppm is observed. On the 5th order resonance, this is increased by two orders of magnitude to ~ 100 ppm, an unacceptable rate as far as machine radiation background is concerned.

In ISABELLE, a new type of proton-proton collision configuration may be tried. If the current is limited, one could gain in luminosity by bunching the beam current using radio-frequency power. This, however, introduces an energy-phase oscillation, the phase representing the longitudinal distance of a particle from the bunch center. The charge density

along the bunch is not uniform, as in the unbunched case, but has a distribution which after some time becomes essentially Gaussian over a few rms bunch lengths. If we now consider the crossing geometry, we note that a particle at the head of a bunch, when it crosses the collision point sees almost zero charge - no interaction. As its position changes, moving toward the bunch center, the beam-beam strength at the collision point reaches a maximum — and so on. In this way there is a slow modulation of the beam-beam force at a frequency equal to twice the synchrotron frequency. In proton rings, this frequency is very slow, $\nu_s \sim 10^{-4}$ to 10^{-5} in ISABELLE, compared to a betatron frequency of $\nu \sim 20$.

Although there is some indication that this slow modulation of the beam-beam force produces irregular behavior in the phase space motion at values of ξ_v well below the stochastic limit for uniform beams, this question is open. Does a slow diffusion set in? Are new resonances introduced causing phase space overlap and thereby inducing chaotic motion? Can the motion be described by adiabatic theory?

V. THE BEAM-BEAM LIMIT IN ELECTRON POSITRON COLLISIONS

Although proton (heavy particle) beams have phase space areas that are essentially determined at the production source, and so are governed by Liouville's theorem [the invariant emittance is constrained, i.e. $\epsilon \geq \epsilon(\text{source})$], the dynamics of electron beams circulating in a storage ring are controlled by the synchrotron radiation emitted in the bending fields combined with the radio frequency power absorbed to replace the lost energy. The net result of the equilibrium between radiation damping and quantum fluctuations (due to the discrete nature of the photon emission) is a beam which in SPEAR has the transverse cross section (determined by observing the emitted synchrotron light) shown in Fig. 6.

The electron and positron beams are at low current, $< 1\text{mA}$, and no beam-beam effects are discernible here. As the current in each beam is increased, a new phenomenon becomes apparent: the equilibrium cross section exhibits a substantial enlargement when the beams are brought into collision, and it reduces to the natural single beam dimensions when the beams are separated. An example of an enlarged beam at 4mA current in SPEAR is shown in Fig. 6b.

Since the luminosity, L , is proportional to the square of the current, I , and inversely proportional to the beam cross section, A , i.e. $L \propto I^2/A$, we expect that, for no beam-beam effect, $L \propto I^2$. We can, hence, study the growth in the beam cross sectional area by observing L as a function of I . We can look at a variety of parametric dependencies and, in particular, we can consider the effect of the vertical betatron tune. An example of a SPEAR analysis⁴ performed two years ago is depicted in Fig. 7. The deviation from the $L \propto I^2$ law shows clearly the onset of a beam-beam effect. The saturation and beam-beam limit is evident along with a strong dependence on the betatron frequency. At the ACO storage ring (Frascati), a study⁵ of allowable tunes was also carried out. The results are shown in Fig. 8. The criterion for acceptability was a beam lifetime greater than 1 hour. As the current is increased from 10mA to 21mA to 33mA , the region of tunes useable shrinks to almost zero. These and other studies of a similar nature are strong indications that the mechanisms underlying the beam blow up is connected with nonlinear resonances excited by the beam-beam space charge force.

Other parametric dependencies have been studied -- for example, maximum luminosity as a function of β_V^* (the betatron focusing function at the collision point), as a function of the number of collision points

points per revolution, and as a function of the radiation damping rate and the synchrotron frequency. There have also been quantitation studies of the limiting beam-beam strength parameter, ξ . The results of all these studies have not led to a consistent physical mechanism. No clear picture has emerged. Even the limiting value of ξ is different in different rings, at different beam energies, and with a different number of collision point, its value covering the range: $0.01 \leq \xi \leq 0.06$.

Thus, although there has been a great deal of experimentation, the basic physical mechanism consistent with all the beam beam observations has not been discovered. Nonlinear resonances are clearly involved, but whether these act as isolated resonances amenable to perturbation analysis or whether a more complicated resonant interaction mediates the beam-beam behavior has not yet been determined. Near the beam-beam limit the two transverse motions are almost fully coupled, and there is the potential for a pre-stochastic slow diffusion-like process. There are also questions relating to the radiation damping and the bunched beam structure. How important is the radiation damping in determining the equilibrium size at high current? Since it appears that the transverse tails of the beams are greatly enlarged, is there an influence from the quantum noise due to the discrete photon emission? What role is played by the force modulation caused by the head to tail motion of particles executing synchrotron oscillations? These theoretical aspects remain unsolved for e^+e^- collisions.

VI. IMPLICATIONS FOR PROTON-ANTIPROTON COLLIDING SYSTEMS

Recently a method for accumulating antiproton beams has led to proposals for developing and constructing proton-antiproton colliding beam facilities. CERN has in fact begun construction and expects to be

operating such a facility by late 1981 or early 1982. Fermilab is also making plans to build such a facility, but at higher energy, and they hope to be operating by 1983 or 1984.

If such a facility is to be productive, the luminosity should reach about $10^{30} \text{ cm}^{-2} \text{ sec}^{-1}$. To do this, it seems to be necessary to achieve beam-beam tune shifts on the order of 0.005 to 0.01. The question as to whether such magnitudes of beam-beam strength can be reached in proton-antiproton system is therefore not only of intellectual interest but it is also of practical importance to high energy physics. On formulating the $p\bar{p}$ beam beam problem, one becomes immediately aware of basic differences between this system and the two existing colliding beam systems, e^+e^- and pp . Since $p\bar{p}$ systems are one ring systems, the collision geometry is head-on and the two transverse motions are coupled. This makes the $p\bar{p}$ system unlike the pp system, but similar to the e^+e^- system. However, protons and antiprotons are heavy particles and, unlike electrons and positrons, have negligible radiation at the energies involved. Thus, we are faced in the $p\bar{p}$ system with a nonlinear problem with two degrees of freedom, opening the possibility of slow diffusion-like behavior. In this system however, there is no radiation damping and so no equilibrium size. It appears that a direct extrapolation from existing systems to $p\bar{p}$ behavior is not a reasonable expectation. However, one attempt has been made at SPEAR. Using beam wigglers in the straight section, the radiation damping time can be significantly affected. By increasing it to very long times it is hoped to simulate a system with no radiation damping. This is an interesting idea, but fraught with conceptual and experimental pitfalls. No results on these experiments have yet been published.

The requirements in the $p\bar{p}$ system are many: long lifetime, ~ 24

hours and large tune shift, 0.01. The question of whether it is possible to achieve this is more uncertain as a result of the fact that no radiation damping is present to inhibit growth. On the other hand, the lack of radiation effects simplifies somewhat the theoretical analysis, and some theoretical studies should indeed be forthcoming. In addition to the basic nonlinear beam-beam effects, we might also ask others more detailed questions: What is the effect of slow force or frequency modulation superimposed on the coupled beam beam system? What is the dependence of the limiting ξ value on energy, synchrotron frequency, and the collision region β -function? What is the influence of multi-bunch collisions per revolution? And what is the influence of relative orbit errors?

VII. CONCLUSIONS

High energy physics facilities for colliding beams are expected to expand rapidly in the 1980's and 1990's. We display this trend in a plot of center-of-mass energy made available at the facilities as ordinate, and the time of onset of facility operation as abscissa, Fig. 9.

As implied previously, the beam energy is a measure of ring size, and, for electrons, rf power needed. It is, therefore, a good measure of facility costs. The 80's and 90's could well see very large electron-positron rings, of the order of 30 km in circumference, and larger proton rings with very high field superconducting magnets. The 80's may also be the decade when proton-antiproton colliding beams are first developed. Although the production of high energy beams may be costly, their creation seems to be within the means of modern technology. It is not so clear that sufficient event rates for rare processes can be achieved. In particular, it may be that the beam-beam interaction will be a significant limitation to both

luminosity and beam lifetime. If for no other reason, the practical need to understand beam-beam phenomena and perhaps be able to influence funding or design directions is important enough.

VIII. REFERENCES

1. J. C. Herrera, 1979. Symposium on Nonlinear Dynamics and the Beam-Beam Interaction, M. Month and J. C. Herrera. Eds. American Institute of Physics, New York, N.Y. (to be published).
2. E. D. Courant, and H. S. Snyder, 1958, Ann. Phys. 3, 1-48.
3. F. M. Izrailev, S. I. Misnev, G. M. Tumaiken, 1977. Institute of Nuclear Physics, Report INF 77-43, Novosibirsk, USSR.
4. SPEAR-Group, 1973. IEEE Trans. Nucl. Sci, NS-20, No. 3: 838-842.
5. M. Bergher, J. Buon, A. Jejcic, J. LeDuff, P. Marin, M. Sommer, and H. Zynger, 1974, Proc. IX International Conference on High Energy Accelerators, Stanford: 410-413.

Figure Captions

Fig. 1. Particle crossing a continuous beam.

Fig. 2. Angular deflection for a particle crossing an unbunched beam at an angle.

Fig. 3. Pictorial representation of a particle trajectory in a linear machine with a nonlinear point interaction.

Fig. 4. Computer simulation: Phase space structure of crossing beams.

Fig. 5. Observation of amplitude growth in the ISR beam: 8th and 11th order resonances.

Fig. 6. Photometric measurements of beam cross section for electron-positron colliding beams.

Fig. 7. Luminosity as a function of beam current at SPEAR.

Fig. 8. Beam lifetime at ACO as a function of horizontal and vertical betatron tune for various beam current.

Fig. 9. Center-of-mass energy for high energy accelerators and storage rings over recent decades.

FIG. 1.

FIG. 2.

FIG. 3.

$$\xi = 0.2$$

REGULAR, BOUNDED
MOTION
RICH RESONANCE
STRUCTURE

$$\xi = 0.32$$

SOME IRREGULAR
MOTION DEVELOPS.
WITHIN BEAM ($\pm 2\sigma$)
MOTION STILL IS
REGULAR

FIG. 4.

FIG. 5.

(a) SPEAR : 1mA (e^+) x 1mA (e^-)
 LOW CURRENT- NO BLOWUP
 WHEN BEAMS IN COLLISION

(b) SPEAR : 4mA (e^+) x 4mA (e^-)
 BLOWUP WHEN BEAMS COLLIDING

FIG. 7.

BOUNDED REGION: LIFETIME < 1 HOUR
SOME BEAM ENLARGEMENT

FIG. 8.

FIG. 9.