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IMAGE COLLAPSING CONCENTRATORS

Interim Scientific Report No. 2

By
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Solar Energy

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DESIGN AND COMPARISON OF STEPPED PRISM LENSES AND IMAGE COLLAPSING SUBREFLECTORS FOR SOLAR ENERGY APPLICATIONS

ABSTRACT

A cylindrical Fresnel-type lens has been designed and ray traced that focuses approximately to lines over a 60° elevation angle interval. Forty stepped prism facets are configured on the inside surface of the lens, and the lens outer surface can be well approximated by a smooth circular arc. Especially contoured cylindrical subreflectors are described for concentrating the broad image fields of the lens onto a narrow fixed shelf. These image collapsing subreflectors also compensate for focal shortening with azimuth angle incidence variations over nearly a $\pm 60^\circ$ sector. Ray tracing on the lens and subreflector provide approximate energy distributions on the absorber shelf over the large acceptance angle intervals. The expected performance characteristics of a solar concentrator using the curved lens with facets on the inside surface are compared with a concentrator employing a straight stepped prism lens with facets cut on the outer lens surface.

DESIGN AND COMPARISON OF STEPPED PRISM LENSES AND IMAGE COLLAPSING SUBREFLECTORS FOR SOLAR ENERGY APPLICATIONS

by

F. S. Holt and C. J. Sletten

Introduction

Thin wide-angle Fresnel lenses provide attractive components for solar collectors. In earlier reports^{1,2,3} a novel method for synthesis and construction of a two-point corrected cylindrical Fresnel lens together with a new technique for contracting the broad image fields of the lens onto a narrow strip were described. In this paper a curved stepped prism lens (essentially a Fresnel type lens) with facets on the inside lens surface is compared with a nearly straight stepped prism lens with prism facets on the outside lens surface. Image collapsing subreflectors are synthesized to concentrate the large focal fields of each lens onto a narrow "shelf" for acceptance angles of 60° in the elevation plane by 120° in the azimuth plane. Performance parameters including optical efficiencies, concentration ratios, and collector sizes are compared for concentrators using the two lenses.

General Design Considerations

An ideal non-tracking solar collector for heating and air-conditioning of buildings would concentrate all solar energy incident on its aperture onto a very small absorber to provide high output temperatures with very low optical and thermal losses. Ideal collector panels should, besides assuring trouble-free

heat extraction and long lasting maintenance-free installations, be constructed at low cost per unit collector aperture area. The wide angle Fresnel lenses with image collapsing subreflectors reported here can be configured to approach some of these goals. Recently, too, other workers^{4,5,6} have addressed problems in lens design for solar energy applications.

In order to keep dirt and snow from accumulating on the steps of the Fresnel lens it is better to have the prism facets on the inside surface of the lens rather than on the outer lens surface. FIGS. 1A and 1B show cross sections through the two lenses, one designed with prism facets on the inside lens surface (curved prism lens) and the other with prism facets on the outside lens surface (straight prism lens). In general, the concentration ratios are greater for the straight prism lens than for the curved prism lens. For both lenses the smooth lens surface could be well approximated by circles. These approximations to the flat prism surfaces on the opposite sides of the lenses to the facets result in slight focal broadening which is evident by a small central gap in the ray bundles traced on the lens focal surfaces. Actual ray traces presented here for the curved lens were done with flat outer facets on each prism rather than with the circular arc approximations.

Note in FIGS. 1A, 1B that the concentrator depth T measured from the outer surface of the lens to the vertex P_0 of the subreflector is larger for the curved lens than for the straight lens. (The straight lens was made unusually thick because it was constructed by a milling process. It could be as thin as

the curved lens if it were cast or extruded.) When the aperture heights H are made equal for the two cylindrical lenses then their aperture areas are also equal and structural and optical comparisons are facilitated. A design objective is usually to make T/H small which is tantamount to making F/D (focal length over lens aperture diameter) as small as possible.

Because the optical properties and all dimensions of the concentrator scale with the lens height H , one can build larger or smaller concentrators for arraying in collector panels by choosing a convenient value of H . Scaling of lens, subreflector and absorber dimensions to different sizes or units is done easily by changing H . For actual concentrator construction we chose a value $H = 12$ units and in lens synthesis and ray tracing $H = 20$ units was used.

The Fresnel lens itself should be made as thin as is practical structurally to reduce material costs and optical transmission losses. The lens functions both to concentrate solar energy and to provide weather protection as a cover plate in front of the absorber. The primary lens losses are due to lens surface reflections which are dependent on the angle of solar incidence relative to the normals to the lens' surfaces. Referring to the chart on page 46 of reference 7 it is clear that the optical power reflected from a single lens exterior surface will exceed 50% for incidence angles greater than 75° from the lens normal. (An index of refraction of 1.6 was assumed.) Therefore, for azimuth angle ϕ incident at greater than about $\phi = \pm 60^\circ$, corresponding to early morning and late

evening operations with a cylindrical non-tracking collector oriented along an East-West line, optical losses become excessive for any glass or plastic lens or cover plate. These reflection losses can be reduced to some extent by coating lenses with quarterwave plates or other non-reflecting surface layers.⁸

Fresnel lenses are also subject to optical losses due to shadowing by the prism facets at each return segment or step. For a lens with $\theta = \pm 30^\circ$ elevation angle coverage, these return steps cannot be made entirely invisible. This shadow loss is due to reflection and refraction from the return steps of the prisms shown as short horizontal lines in FIGS. 1A and 1B. For $\theta = 0^\circ$ shadow losses are small for both lenses being about 5.97% (-0.26 dB) for the curved lens and 9.0% (-0.42dB) for the straight lens. Shadowing losses increase with elevation angle incidence and are calculated to be approximately 11.9% (-0.55 dB) on the curved lens and up to 20.4% (-0.99 dB) on the straight lens at $\theta = \pm 25^\circ$. In reference 9 shadowing and other lens losses for this straight lens are reported. Some of the energy (rays) striking the return prism steps actually reaches the absorber shelf due to the wide acceptance angles of the sack-like shaped subreflector. Many of the rays internal to the lenses are totally reflected by these (horizontal) return prism steps. In general increasing the lens F/D reduces shadow losses while increasing the number of prisms in the lens aperture tends to increase shadow losses.

The number of prism elements in the lens aperture effectively determines the sharpness of the line focusing. The ray traces shown are for rays through the centers of each prism element. Because the prism surfaces are flat, not curved, each prism element produces a broadened image. Of course the finite angular width of the sun, the lens aberrations and construction errors also broaden the focal lines. However, unless very high intrinsic concentration ratios are required of the lens, prism step size and construction tolerance can be fairly large. The straight lens which we actually constructed of acrylic (index 1.49) had 40 prism steps/^{each} about $3/4$ cms high with $H = 30\frac{1}{2}$ cms. The axial (x-axis) focal length was about $33\frac{1}{2}$ cms and the image width was about 1 cm. For sharper focusing the number of prisms can be increased and each prism surface will be curved.

There are fundamental limitations in solar concentrator design with lenses and subreflectors. In general, a lens with an F/D of 1 or greater is required to achieve wide-angle performance and low step shadowing. Also, cylindrical lenses exhibit considerable shortening of the line focal length as azimuth angle ϕ varies. The image collapsing subreflector provides a second optical element which compensates for this focal shortening effect. Another fundamental aspect of non-tracking concentrators is the theoretical dependence of concentration ratio on the acceptance angle interval. This means that for the collector geometries shown in FIGS. 1A and 1B the absorber shelf must have an area about $1/3$ the lens aperture area, assuming θ coverage of 60° and ϕ coverage of 120° with "100%" optical efficiency (all rays incident on the lens within

this acceptance zone strike the absorber shelf). Higher concentration ratios can be attained by restricting the angular coverage to smaller absorber elements by segmenting the sub-reflectors.^{1,2} Circulants for removing heat can be switched to circulate mainly through solar illuminated absorbers and segments. Likewise, circulants can be pumped through portions of the shelf with highest concentration of solar rays to increase effective concentration ratio and operating temperatures. Detailed energy distributions on the shelves of the two principal concentrators compared in this report are provided to aid in implementing this option.

These cylindrical Fresnel lenses can be used also in tracking applications. For example, daily and yearly tracking of the sun can be accomplished by azimuthal rotation of the lens. Such scanning avoids the focal shortening effects, reduces reflections from the lens surfaces, and reduces step shadowing losses (assuming θ and ϕ are always small). When the lens has a sufficient number of prisms very sharp line focusing over the elevation plane focal surface results. A very small diameter absorber tube can be located along this focal line producing a collector with concentration ratio of 50 or more. The techniques described here for designing wide-angle lenses can be employed for smaller or larger θ correction angles and, in particular, excellent focusing can be obtained in more limited elevation angle zones of 5° to 20° . Also, lenses of revolution focusing approximately to points rather than to lines can be designed simply by rotating the x-y plane profile of the lens about the

x-axis. It is expected that for such lenses having a spherical (or nearly flat) outer surface the tangential plane focusing will be good but the sagittal plane performance will be poorer resulting in a lens with astigmatism. Other collector configurations with absorbers shaped, positioned or moved in the lens focal field or with circulants switched to illuminated portions of the focal surfaces are easy to imagine. However, this paper reports on analytical and experimental results for non-tracking applications using especially contoured image cylindrical subreflectors focused on simple absorber tubes or strips.

Design of Fresnel Lens with Prism Facets on Inside Surface of Lens

In this section the design details of the two-point corrected stepped prism lens shown in FIG. 1A are described. The elevation plane focusing characteristics are shown in FIG. 3 for this curved Fresnel lens. Design details for the straight lens and subreflector shown in FIG. 1B are given in References 1 and 2 with ray tracing diagrams depicting its focusing characteristics. The curved lens parameters are: Index of refraction $N = 1.49$, aperture height $H = 20$ (units), number of prism steps 40, focal points $x = -15$, $y = \pm 8.69$, step height 0.5, correction angles $\theta_0 = \pm 25^\circ$ in the elevation plane. The outer lens surface can be approximated by the circle

$$(x + 11.711)^2 + y^2 = (15.598)^2$$

The coordinate system used in the lens design is shown in FIG. 2(a).

The two-point corrected prism lens with stepped inner face is composed of a collection of modified symmetric prism pairs.

All prism pairs are initially designed such that incoming rays making an angle θ_0 with the system axis (x-axis) and incident at the prism vertices, all pass through the same off-axis focus point F (see FIG. 2(B)). It follows from the symmetry of each prism pair that incoming rays making an angle of $-\theta_0$ relative to the system axis and incident at the prism vertices will all pass through a focus point F' symmetrical to F relative to the system axis. It is important to note that if the design rays are traced starting at F then, as the outer faces of the prisms are moved away from the focus but parallel to themselves, the exit rays still make an angle θ_0 with the system axis. By moving the outer faces and truncating in a symmetric manner, the prism pairs are modified so that the prisms have constant height, their outer faces join to form a broken line, a prescribed overlap is maintained from one prism to the next, and the design rays pass through the centers of the inner faces. This creates essentially a smooth outer face, stepped inner face prism lens.

The overall design is an iterative procedure in that for a fixed incoming ray angle θ_0 and a given lens height H, the focus point must be adjusted to produce an optimum lens design. Optimum in this case means that there is minimum jump in prism angles for the central (nearest axis) prism pair. Ideally, the prisms of the central prism pair should be nearly plane parallel with faces approximately perpendicular to the system axis (see FIG. 1(A)).

The angles that prescribe the prisms in each prism pair are determined from the roots of a very complicated transcendental

expression. Newton's method was quite satisfactory for finding the roots for the prism lens with stepped outer face but for the stepped inner face design difficulties in convergence were encountered as the optimum design condition was approached. These difficulties were removed by modifying Newton's method. This modification produced roots in five iterations in cases that previously had not converged in over one hundred iterations.

The ray tracing diagrams shown in FIG. 3 verify the wide-angle focusing capabilities for rays passing through the centers of the 40 prisms inclined at 5° intervals in the elevation plane from $\theta = 0^\circ$ to $\theta = 30^\circ$ incidence ($\phi = 0$). Identical ray traces are obtained in the angular interval $\theta = 0^\circ$ to $\theta = -30^\circ$. Nearly perfect focusing is attained for correction angles $\theta_o = \pm 25^\circ$ and at $\theta = 0^\circ$. Many ray tracing diagrams were made with azimuth incidence ϕ varied also. These diagrams showed similar sharp focusing characteristics with focal shortening as ϕ increased and a shift in the sharpest focus from $\theta_o = \pm 25^\circ$ to smaller θ values. For example, for oblique ray incidence in which the azimuth angle ϕ is fixed at $\phi = 20^\circ$ the sharpest focal point on the elevation plane focal arc is at approximately $\theta = 15^\circ$. To illustrate the effects of focal shortening with change in azimuth angle incidence the composite ray traces shown in FIG. 4 are included in which θ is fixed at zero degrees and ϕ is varied. Note that on-axis focusing remains sharp over large variations in azimuth incidence ϕ . Representative skewed or oblique angle ray tracing diagrams are shown in FIG. 5. Although corresponding ray tracing diagrams for the straight lens are not shown here

they are very similar to those of the curved lens. Both lenses are two-point corrected at $\theta_0 = \pm 25^\circ$ and have aperture heights of 12 units. The straight lens has on-axis focal length of 12.60 units compared to the curved lens' on-axis focal length of 13.77 units.

Design of Image Collapsing Subreflectors

The procedure used to synthesize a subreflector shape to reflect all the rays refracted by the lens over the acceptance interval onto an absorber shelf makes use of the ray tracing information given in FIGS. 3 and 6. The focal arc produced by the lens for θ varied and $\phi = 0$ can be represented analytically using measured data points. When the subreflecting mirror surface is contoured such that all rays over the $\theta = \pm 30^\circ$ elevation plane sector are reflected on to an axial shelf of given width S for $\phi = 0^\circ$, the same subreflector also reflects the rays for $\phi \leq |\pm 60^\circ|$ and $\theta \leq |\pm 30^\circ|$ onto this shelf.

Computer algorithms employed for subreflector synthesis can be based either on the leading ray parameters of each bundle of rays focused onto the lens focal arc or on an envelope method which uses a family of ellipses as generating functions. The leading ray method is an incremental technique for subreflector contour synthesis. It is a powerful surface shaping method which allows rapid investigation of concentrator performance parameters. Many subreflectors were synthesized for the two lenses shown in FIGS. 1A and 1B and then ray tracing using lens rays was carried out on the shaped subreflector contours to find

impact points in the absorber shelf region. Although the absorber structures themselves can be flat, tubular or have special shapes, the rays reflected from the image collapsing subreflectors were traced onto a flat shelf extending from the vertex of the subreflector along the lens axis.

The Leading Ray Subreflector Synthesis Method

The 3-ray diagrams traced in FIG. 6 for θ between 0° and 30° and $\phi = 0$ were used for finding the ^{curved} lens focal curve. Because rays are not focused exactly to points on the focal arc we can define an analytic curve characterizing an average focal curve by locating the centers of triangles formed by the crossings of the 2 edge rays through the lens (extreme rays on the focused ray bundle for a given θ) and the central ray. Because we are primarily interested in reflecting the leading or extreme ray of these focused bundles to an axial point F_0 , the crossings of these two edge rays provide another set of data points for defining the focal curve. Although many different mathematical functions can be passed through points connecting the centers of the triangles or the edge ray crossings, an accurate, smooth focal curve f_c is obtained simply by assuming the lens focal arc is an ellipse of the form:

$$(1) \quad f_c(x_1, y_1) = x_1^2 + M y_1^2 + N x_1 + P = 0$$

where x_1 and y_1 are points on the focal curve f_c and the constants M , N , and P are determined by the simultaneous solution of equation (1) using 3 point pairs measured from FIG. 6. Because of

the even symmetry of (1) about the x-axis shown in FIG. 6, equation (1) passes exactly through 6 data points.

When the curved lens of FIG. 1A has an aperture height of 12 units then the three points on the focal arc taken from FIG. 6 for $\theta = 5^\circ$, 15° and 25° are measured to be approximately:

θ	Edge Ray Crossings		Triangle Center Locations	
5°	$x_{11}=13.557$	$y_{11}=1.271(\text{units})$	$x_{11}=13.557$	$y_{11}=1.333(\text{units})$
15°	$x_{12}=11.906$	$y_{12}=3.606$	$x_{12}=11.906$	$y_{12}=3.702$
25°	$x_{13}=8.886$	$y_{13}=5.192$	$x_{13}=8.886$	$y_{13}=5.192$

Substituting these points into equation (1) gives for the edge ray crossings the constants:

$$M = 2.03918, \quad N = -11.3799, \quad P = -32.8093$$

For the centers of triangles the constants in equation (1) are:

$$M = 1.64671, \quad N = -13.5661, \quad P = -2.80314$$

For the straight lens of FIG. 1(B) the constants based on leading ray crossings are:

$$M = 2.3300, \quad N = -10.1613, \quad P = -30.207$$

In scaling equation (1) to other concentrator sizes (with different H values) or units for a scaling factor k multiply N by k and P by k^2 .

To synthesize the subreflector contour $f_s(x,y)$ that will focus the leading ray of each focused bundle to a point F_0 on the x-axis we assume that all lens rays are focused on the focal

curve $f_c(x_1, y_1)$, eq. (1), as derived above from the leading ray crossings (or the triangle centers points). Referring to FIG. 6 the leading ray (one with greatest slope ψ referred to the x-axis) of each focused ray bundle is seen to pass through the edge of the lens at a point $y_h = -5.835$ (units scaled to $H = 6$) and $x_h = -0.078$. The angles ψ for the leading rays of each bundle focused on $f_c(x_1, y_1)$ are given by:

$$(2) \quad \tan \psi = \frac{y_1 + 5.835}{x_1 + 0.078}$$

With equations (1) and (2) we can define a discrete set of rays, approximately equally spaced in ψ which rays populate the region where $f_s(x, y)$ will be located. Only two numbers are needed to define each leading ray in this discrete set: the angle ψ (with the x-axis direction) and the x-axis crossing points $(f, 0)$. By placing these number pairs $(\tan \psi_i, f_i)$ in computer memory we can generate the subreflector contour $f_s(x, y)$ with an incremental routine which interpolates between the given discrete rays and requires that incremental surfaces on f_s reflect the leading rays by Snell's law of reflection to F_0 . All other rays in the focused bundles intercept the x-axis on the shelf between the focus F_0 and the vertex point P_0 of the subreflector surface f_s . Also, the leading (edge of lens) rays from ray bundles which are focused inside of f_c when $\theta \neq 0^\circ$ also strike the point F_0 . In general all rays in the acceptance zone were found by ray tracing to intersect the x-axis on the absorber shelf.

Synthesis of the subreflector curve f_s begins by selecting a starting point near the end of curve f_c (for $\theta = 30^\circ$). The end point of f_s should be back of f_c to avoid shadowing of converging rays although the computer routines work for f_s nearer to the lens than f_c . For the lenses shown in FIGS. 1A and 1B the incremental shaping method is very accurate for calculating f_s when Δx on the surface is the independent variable and Δy is computed using Euler, Taylor or Adams predictors.^{10,11} Near the vertex P_0 of the subreflector, the slopes $\Delta y/\Delta x$ of f_s become large and it is useful, when extreme surface accuracy is required in the region near P_0 , to make Δy the independent variable and compute the subreflector contours twice, overlapping the curves in the range of y between 1 and 4 units (for the concentrators of FIGS. 1A and 1B).

When the end points of the mirror f_s are fixed near the ends of f_c , only the focal point F_0 for leading or edge rays and the collector efficiency are at the designer's disposal. Varying F_0 changes shelf width S and hence collector concentration ratio such that the larger F_0 the smaller the concentration ratio. In this study we define optical efficiency to be "100%" when all the rays emerging from the lens intercept the shelf. Losses due to step shadowing, surface reflections, lens transmission absorption and mirror reflections are ignored. We can reduce the subreflector optical efficiency by choosing the leading rays of each bundle to be, not the edge of the lens ray, but another ray set which enter the lens aperture between the edge of the

lens and the lens center. Reducing optical efficiency reduces shelf width S somewhat. Because a practical definition of collector concentration ratio is given by lens aperture area divided by the absorber area it is desirable to make the shelf width S small.

A subreflector (leading ray) focus F_0 near the on-axis lens focus F_A is a reasonable compromise and allows many rays to strike the absorber shelf without reflections from the subreflector surface (when θ is near zero and the absorber is tubular or thick). Increasing F_0 beyond F_A slowly reduces the shelf width S but results in a deeper collector (dimension T) and increases the number of multiple reflections off the subreflector surface for θ incidences greater than 25° . Although multiple reflected rays also strike the absorber shelf each reflection results in additional energy loss. Table I shows the effect of varying F_0 and optical efficiency on shelf width S and collector thickness (depth) T .

TABLE I

CURVED LENS
aperture height $H = 12.0$ units

F_o Subreflector Focus	% of Lens' Rays on S	S Shelf Width	T Collector Thickness
14.0 (FIG.1A)	100	3.63	19.66
14.0	50	1.80	18.13
13.5	100	3.66	20.00
13.5	92.5	3.35	19.18
15.0	100	3.50	20.83
12.0	100	4.00	18.33

STRAIGHT LENS
aperture height $H = 12.0$ units

F_o Subreflector Focus	% of Lens' Rays on S	S Shelf Width	T Collector Thickness
13.0 (FIG.1B)	100	3.15	17.00
11.0	100	3.45	15.30
9.0	100	4.00	13.85
7.0	100	4.84	12.68

Concentration Ratios $\approx H/S$

Energy Distributions on Absorber Shelf

Once the subreflector surfaces have been synthesized ray tracing can be done to approximate the radiant energy distributions on the absorber shelves. Bearing in mind that ray optics is inaccurate for determining energy density when rays cross or in caustic regions, one can, however, use the distance between ray intersections on the shelf to find relative hot and cold portions of the absorber. Assuming that each angular segment, $\Delta\psi$, of the ray bundles carries approximately the same quantity of solar energy then the density of ray intersections (per constant $\Delta\psi$) on the shelf is proportional to energy density. This study was done to make sure all rays for all θ , ϕ values in the acceptance zones do intersect the shelf and to identify hot regions where shaped absorbers could be located at different seasons of the year.

The charts in FIGS. 7A, B, C, D, E show shelf energy distributions for both the curved and straight lenses when these lenses are provided with image collapsing subreflectors that direct nearly all the lens rays in the acceptance zone onto the shelf areas. For the straight lens and its subreflector the shelf width extends along the x-axis from 13.0 units to 16.125 units ($S = 7.74$ cms), and for the curved lens and subreflector the shelf extends from 14.0 to 17.63 units. Ray tracing to find the lens rays reflected from the curved subreflectors to intercept points on the shelves was accomplished using a computer algorithm which operates on a set of discrete points chosen approximately

at equal intervals along the sack-like curves $f_s(x,y)$. Because of the very large changes in slope along the contours care must be taken in determining the first and second derivatives in different regions of f_s . In this iterative procedure two sets of points on f_s are needed to compute the second derivative. This means that when the computer search begins at one end of f_s to find the impact point of a given ray on the subreflector cylinder it is difficult to get accurate data on intercept points corresponding to $\theta \approx \pm 30^\circ$. The ray tracing program requires that rays incident on the subreflector be designated by their slopes $\tan \psi$ (measured with respect with the x-axis) and any point on the incident ray. This ray information can be obtained from lens ray tracing diagrams similar to those shown in FIGS. 3, 4, 5, and 6.

In FIG. 7A a typical ray tracing diagram for lens rays focused to point (x_1, y_1) when $\theta = 15^\circ$ and $\phi = 0^\circ$ is presented. The density of ray intercept points on the x-axis shelf between F_0 and P_0 indicates the solar energy distributions for equal angular increments $\Delta\psi$ within the ray bundle focused at x_1, y_1 . For ray bundles focused on or inside f_c for $\theta = 20^\circ$ the reflected rays tend to cross and those in the central portion of the focused ray bundle may strike the subreflector surface more than once before intersecting the absorber shelf. The ray tracing algorithm allows for multiple reflections on f_s . It is difficult however to assign numerical values to shelf energy density (spacing of x-axis intercept points) when there are multiple

reflections and reflected ray crossings. Clearly the shelf energy density becomes more intense in the vertex region near P_0 when θ incidence is greater than 20° . In all cases measured, however, the leading (or lens edge) ray strikes very near to F_0 for any θ or ϕ in the acceptance zone. This means that for large θ s some energy is always distributed over the entire shelf as well as at the maximum near P_0 .

FIG. 7B presents the approximate shelf energy distributions for the curved lens when $\phi = 0$ (noon time) and θ is varied from 0° to 25° . Although all rays are concentrated near F_0 and F_A when $\theta = 0$ and $\phi = 0$ for small θ incidence values ($\theta = 5^\circ$), the shelf will be illuminated by a mixture of direct and reflected rays over most of its length. FIG. 7E shows that all the sharply focused rays for $\theta = 0$ and ϕ varied (focal shortening) shown in FIG. 4 are actually focused near to F_0 upon reflection from the subreflector. FIG. 7D illustrates that skewed or oblique angle rays produce about the same shelf energy distributions as the $\phi = 0$ rays. The ray intercepts for skewed rays are shifted laterally (along the z-axis) on the shelf but this effect results in energy loss only at the ends of cylindrical collectors deployed along East-West lines. To minimize morning and evening losses at the ends of collectors it is well to make the lens height H small with respect to the East-West length of the collector cylinders. In FIG. 7E some representative shelf energy distributions for the straight lens of FIG. 1B are presented. It is seen that the shelf energy distributions and, therefore, the focusing characteristics of the two lenses with their respective subreflectors are nearly identical.

In FIG. 8 is shown a ray tracing diagram for a subreflector contour (for the curved lens) calculated to reflect central rays through the lens onto a focal point F_0 at 14 units. Now only 50% of the rays will strike the shelf between F_0 and P_0 ($S = 1.8$ units). Of course a tubular absorber, for example, could be located with its center at F_0 and many rays intercepting the x-axis between F_0 and the inner surface of the lens would be received by such an absorber. It is difficult, however, to bound a region on the x-axis which will receive all rays. In this design much of the pile-up of rays near the subreflector vertex is avoided for larger θ values. Because the collector thickness T is less and multiple reflections are less image collapsing subreflectors as shown in FIG. 8 which focus central or other than lens edge rays can have advantages.

Subreflectors shown in FIGS. 1A, 1B and 8 were computed using the incremental or Δx method which is a numerical procedure. Designers may prefer a closed-form solution for reflector surface synthesis and this can be obtained using an envelope theory. Again the lens focusing properties are characterized by f_c of equation (1) and a family of ellipses having one focus at F_0 and another on the curve f_c is defined. To calculate subreflector contours which are equivalent to the contours gotten by the incremental method it is necessary to find a characteristic function $C_2(y_2)$ which function expresses the C_2 path length for each generating ellipse whose envelopes define f_s . C_2 is the sum of 2 distances, a constant for a given ellipse, measured from the two foci of the ellipse which are located at F_0 and a point (x_2, y_2)

on f_c out to any point on the generating ellipse. References 1 and 2 give the functions relating f_c , the slope of f_c , F_0 and the points x_2, y_2 which, when programmed to include the characteristic function $C_2(y_2)$, result in approximately the same subreflector contours as obtained by the Δx incremental calculations. In FIG. 9 two characteristic curves $C_2(y_2)$ are plotted according to the polynomial given in the figure. In FIG. 8 the dots on the curve indicate values of f_s obtained using the envelope synthesis method in comparison with subreflector contours shown by the solid curve computed by the Δx method. The coefficients of the polynomial expressing $C_2(y_2)$ are:

$a_0 = 7.45007455$, $a_2 = -0.2074766104$, $a_4 = 0.0290528663$,
 $a_6 = 0.0013625025$ and $a_8 = 0.0000224352$. The subreflector contour shown in FIG. 1A can be closely approximated using the envelope method when $C_2(y_2)$ is given by polynomial shown in FIG. 8 with coefficients $a_0 = 3.850$, $a_2 = 0.017386922$,
 $a_4 = 0.0137640843$, $a_6 = -0.007944174$ and $a_8 = 0.0000165312$.

Although the envelope method provides an alternative and perhaps superior conceptual approach to reflector surface synthesis, the Δx (or Δy) leading ray incremental method is simpler and more flexible to use.

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FIGURE TITLES

- FIG. 1A Image Collapsing Concentrator with Prism Facets on Inside Lens Surface
- FIG. 1B Image Collapsing Concentrator with Prism Facets on Outside Lens Surface
- FIG. 2A Coordinate System for Lens Design
- FIG. 2B Ray Traces Through Vertices of Symmetric Prism Pair for Incident Rays at $\pm \theta^\circ$
- FIG. 3 Elevation (θ) Plane Ray Tracing on 2-Point Corrected Cylindrical Fresnel Lens
- FIG. 4 Focal Shortening with Azimuth Angle (ϕ) Variation
- FIG. 5 Skewed Ray Scan
- FIG. 6 Extreme and Central Rays for Defining Lens Focal Curve
- FIG. 7A Typical Distribution of Ray Intercepts on Shelf
- FIG. 7B Shelf Energy Distributions for Curved Lens ($\theta = 0$, ϕ Varied)

TABLE I

- FIG. 7C Shelf Energy Distributions for Curved Lens ($\theta = 0$, ϕ Varied)
- FIG. 7D Shelf Energy Distributions for Curved Lens Skewed Ray Incidence
- FIG. 7E Shelf Energy Distributions for Straight Lens
 $\theta = 20^\circ$
 $\phi = 0^\circ, 10^\circ, 20^\circ, 30^\circ, 40^\circ$
- FIG. 8 Subreflector Contours for Focusing Central Rays of Curved Lens to $F_0 = 14.0$
- FIG. 9 Characteristic Functions for Generating Ellipse Using Envelope Method of Subreflector Contour Synthesis

FIG.1A IMAGE COLLAPSING CONCENTRATOR WITH PRISM FACETS
ON INSIDE LENS SURFACE

FIG.1B IMAGE COLLAPSING CONCENTRATOR WITH
PRISM FACES ON OUTSIDE LENS SURFACE

FIG.2A COORDINATE SYSTEM FOR LENS DESIGN

FIG. 2B RAY PASSES THROUGH VERTICES OF SYMMETRIC PRISM PAIR FOR INCIDENT RAYS AT $\pm \theta_0$

FIG.3 ELEVATION (θ) PLANE RAY
TRACING ON 2-POINT CORRECTED
CYLINDRICAL FRESNEL LENS

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FIG. 4 SCAN PARALLEL TO CYLINDRICAL LENS

FIG. 5 SKEWED RAY SCAN

FIG.6 EXTREME AND CENTRAL RAYS FOR DEFINING LENS FOCAL CURVE

$$\phi = 0^\circ$$

FIG.7A TYPICAL DISTRIBUTION OF RAY INTERCEPTS ON SHELF

FIG.7B SHELF ENERGY DISTRIBUTION FOR CURVED LENS

FIG.7C SHELF ENERGY DISTRIBUTIONS FOR CURVED LENS

FIG.7D SHELF ENERGY DISTRIBUTIONS FOR CURVED LENS
(SKEWED RAY INCIDENCE)

FIG.7E SHELF ENERGY DISTRIBUTIONS FOR STRAIGHT LENS

FIG.8 SUBREFLECTOR CONTOURS FOR FOCUSING
CENTRAL RAYS OF CURVED LENS TO $F=14.0$

$$C_2 f(y_1) = a_0 + a_2 y_1^2 + a_4 y_1^4 + a_6 y_1^6 + a_8 y_1^8$$

FIG.9 CHARACTERISTIC FUNCTIONS FOR GENERATING ELLIPSE USING ENVELOPE METHOD OF SUBREFLECTOR CONTOUR SYNTHESIS