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TRANSPORT ANALYSIS OF A SMALL STELLARATOR

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By

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#### TRANSPORT ANALYSIS OF A SMALL STELLARATOR

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#### ABSTRACT

A Monte Carlo method of evaluating typical particle and energy transport. coefficients is given for the case in which the particle drift orbits are a significant fraction of the plasma radius. The method is applied to a preliminary design for a helical axis (heliac) stellarator experiment.

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#### I. INTRODUCTION

Recently, there has been renewed interest in stellarators in the magnetic fusion program. The issues of equilibrium, stability, and transport can be studied in devices of modest size, about 1.5 m major radius and 25 cm minor radius. However, the small size implies that in the low collisionality regimes of reactor relevance, the ion energy confinement time  $\tau_{iE}$  may not be very large compared to the ion collision frequency  $v_i$ . In practice, one often has  $\tau_{iE}v_i \sim 10$ . The finiteness of  $\tau_{iE}v_i$ , implies that the ion drift orbits have finite radial extent and that the ions have a significant deviation from a Maxwellian distribution. In this paper, we discuss a Monte Carlo method for studying transport under these conditions. The method is then used to study transport in a small heliac stellarator.

In the heliac stellarator,<sup>1</sup> Fig. 1, the magnetic axis follows a helical path around the torus instead of a nearly circular path as in most other stellarators. Theoretically, the helical displacement of the axis permits high equilibrium and stability  $\beta$  limits, where  $\beta$  is the ratio of plasma to magnetic pressure. The coils required for a heliac can be simple planar circles.

During 1982, a preliminary design was made at Princeton for a heliac experiment. The design, Fig. 2, was consistent with the PDX experimental area and the toroidal field coils could be those of the old ATC device. The basic design parameters were 1.5 m major radius, 25 cm average minor radius, 1 T magnetic field, three periods, and a rotational transform of about 1.5 at the plasma center and 1.7 at the edge, Fig. 3.

Analytic study of transport in a heliac is difficult not only due to the finiteness of  $\tau_{iE}v_i$  but also due to the similar rates and amplitude of change of the magnetic field strength along the field lines from helical and toroidal

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effects. At present only Monte Carlo studies of transport in a heliac have been made.

#### 11. BASIC MONTE CARLO METHOD

The basic elements of the Monte Carlo code have been previously explained.<sup>2</sup> Particle drift orbits are integrated in magnetic coordinates using a drift Hamiltonian. The magnetic coordinates of the Monte Carlo code  $\psi$ ,  $\theta_0$ ,  $\chi$  are defined so that for a curl-free field

$$\dot{\mathbf{s}} = \bar{\mathbf{v}}_{\psi} \times \bar{\mathbf{v}}_{\mathbf{c}_{x}} = \bar{\mathbf{v}}_{\mathbf{x}} \,, \tag{1}$$

The flux enclosed by a magnetic surface, the toroidal flux, is  $2\pi\psi$ . The coordinate  $\theta_{c}$  is related to a poloidal angle  $\theta$  and a toroidal angle  $\phi$  by

$$\theta_{n} = \theta - \iota(\psi)\phi \tag{2}$$

with the rotational transform. The potential  $\chi$  is related to the angle  $\phi$  by

$$\chi = \mathbf{g}\phi \tag{3}$$

with 2g/c the total poloidal current outside the plasma,<sup>3</sup> The canonical coordinates of the drift Hamiltonian are  $\psi$ ,  $\Theta_0$ ,  $\chi$ , and  $\rho_0$  with  $\rho_0 = V_0/(eB/mc)$ , the parallel particle velocity divided by the cyclotron frequency. The drift Hamiltonian is

$$H = \frac{1}{2} \frac{eB^2}{mc} \rho_{\parallel}^2 + \frac{c}{e} \mu B + c\phi$$
(4)

with  $\mu$  the magnetic moment,  $\varphi$  the electric potential, c the speed of light, m the particle mass, and e its charge. Hamilton's equations for the motion are

$$\frac{d\theta_{o}}{dt} = \frac{\partial H}{\partial \psi} , \qquad \frac{d\psi}{dt} = -\frac{\partial H}{\partial \theta_{o}} , \qquad (5)$$

$$\frac{d\chi}{dt} = \frac{\partial H}{\partial \rho_{g}} , \qquad \frac{d\rho_{g}}{dt} = -\frac{\partial H}{\partial \chi} .$$

In the computer runs reported in this paper the electric potential was chosen to be of the form

$$\phi = \phi_{O} \left(1 - \frac{\psi}{\psi_{a}}\right)^{2}$$
(6)

with  $\psi_a$  being the plasma edge and  $\phi_0$  the potential at the axis. The magnetic field strength B( $\psi,\theta,\phi)$  was evaluated in Fourier decomposed form<sup>4,5</sup>

$$B(\psi,\theta,\phi) = \sum A_{nm}(\psi)\cos(n\phi - m\theta)$$
(7)

using the field produced by circular wire filaments to represent the heliac vacuum field. The Fourier coefficients, the  $A_{nm}$ , are given in Fig. 4.

After each time step using Hamilton's equations, Monte Carlo operators are evaluated to represent the pitch angle and energy scattering over that time step. The pitch  $\lambda = v_{\parallel}/v$ , the ratio of the parallel to the total velocity, is changed from the old value  $\lambda_0$  to the new value  $\lambda_n$  using

$$\lambda_{n} = \lambda_{0} (1 - v_{d}\tau) \pm [(1 - \lambda_{0}^{2})v_{d}\tau]^{\frac{1}{2}}$$
(8)

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with  $v_{\rm d}$  the angular collision frequency,  $\tau$  the length of the time step, and  $\pm$ a random sign. The kinetic energy is changed from its old value  $E_0$  to its new value  $E_n$  by

$$\mathbf{E}_{n} = \mathbf{E}_{o} - \left(\mathbf{v}_{\mathbf{E}}\tau\right)\left[\mathbf{E}_{o} - \left(\frac{3}{2} + \frac{\mathbf{E}_{o}}{\mathbf{v}_{\mathbf{E}}}\right)\frac{d\mathbf{v}_{\mathbf{E}}}{d\mathbf{E}_{o}}\right] \pm 2\left[\mathbf{T}\mathbf{E}_{o}\mathbf{v}_{\mathbf{E}}\tau\right]^{\frac{1}{2}}$$
(9)

with  $v_E$  the energy collision frequency and T the background plasma temperature. Both collision frequencies are assumed to have the simplified energy dependence

$$v = \frac{v_0}{1 + (E/T)^{3/2}}$$
 (10)

The following particle and collision simulation routines constitute the basic Monte Carlo code. The method of evaluating transport coefficients with this code is discussed in the next section.

#### III. EVALUATION OF DIPFUSION COEFFICIENT

There is a fundamental problem in the definition of diffusion coefficients when the confinement time is comparable to the collision time. A local diffusion coefficient is only rigorously defined in the limit of infinitesimal radial excursion during a collision time, which is equivalent to the collision time being infinitesimal compared to the confinement time. The radial locality of a diffusive process converges as the square root of the collision time divided by the confinement time. This follows from the diffusion coefficient being a typical radial step squared times a collision frequency. In the proposed heliac experiment, the two times differ by roughly an order of magnitude; so although one can usefully define some typical diffusion coefficient, one cannot assume a local definition of diffusion.

Although there are other methods of evaluating a typical diffusion coefficient when a radial local coefficient does not exist,<sup>6</sup> we used the following procedure. A group of 64 particles was launched halfway out in toroidal flux and the time  $t_{1/2}$  it took half the particles to leave the plasma was recorded. We chose 64 particles as the code was vectorized on the MFE CRAY computer with respect to the number of particles and for reasonable statistics. A particle was defined as leaving the plasma when its  $\psi$  position became larger than  $\psi_{a}$ . The values of  $t_{1/2}$  for ions in heliac are illustrated in Fig. 5. The points for  $\eta = 0$ , where  $\phi_{0} = \eta T$ , indicate the basic confinement without an ambipolar potential. Making the  $\eta = -3.9$  improves the confinement time by an order of magnitude. A negative  $\eta$  implies the potential pulls ions inward. These confinement times are comparable to those obtained by making heliac into a symmetric torus with

$$B = A_{oo} + A_{eq} \cos\left(\theta_{o} + \frac{1\chi}{q}\right)$$

where

$$A_{eq} = \left[\sum_{n,m} \left(\frac{m^2}{n-m} A_{nm}\right)^2\right]^{1/2}$$

and  $A_{nm}$  are the regular Fourier coefficients. The equivalent coefficient derived in this way gives the same Pfirsch-Schlüter transport as a symmetric torus.<sup>3</sup>

Figure 6 plots  $t_{1/2}$  against the potential  $\eta$  at a constant density. In general it is impractical to measure electron escape times because of their better confinement. We did, however, obtain one point at zero potential. As

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expected the electrons are an order of magnitude better confined. A potential of  $\sim -4$  T is required for the ion and electron confinement times to become comparable. The time  $t_{1/2}$  is inversely proportional to an average diffusion coefficient and the constant of proportionality can be determined by solving a diffusion equation. Since we were trying to find a typical diffusion coefficient for a heliac plasma of given density and temperature, we assumed the background plasma had a uniform density and temperature.

To understand the relation between  $t_{1/2}$  and an average diffusion coefficient, consider the cylindrical diffusion equation for charged test particles

$$\frac{\partial n}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left[ r D_{\rm g} \left( \frac{\partial n}{\partial r} + \frac{n e}{T} \frac{d \phi}{d r} \right) \right] \quad . \tag{11}$$

The relation between the density derivative and the electric potential derivative follows from the Einstein relation. That is, charged particles are in thermodynamic equilibrium for

$$n \alpha \exp\left(-\frac{e\phi}{T}\right)$$
(12)

in which case there is no net diffusion. The equivalent diffusion equation can be derived in flux coordinates (see appendix) but the final answer follows plausibly by noting that  $\psi$  is approximately proportional to  $r^2$ . The equivalent equation is

$$\frac{\partial n}{\partial t} = \frac{\partial}{\partial \psi} \left[ 2\psi D \left( \frac{\partial n}{\partial \psi} + \frac{ne}{T} \frac{d\phi}{d\psi} \right) \right] \quad . \tag{13}$$

One could let  $D_{ij} = 2\psi D$ , but a constant  $D_{ij}$  would imply a diffusion coefficient

in ordinary space which goes like  $1/r^2$  near the magnetic axis. However, our earlier Monte Carlo paper<sup>2</sup> gave the evaluation of  $D_{\psi^*}$ . The relation between the ordinary spatial diffusion coefficient  $D_g$  with units of cm<sup>2</sup>/sec and D is

$$D_{s} = \frac{2\psi}{\langle \langle \bar{\psi} \psi \rangle^{2} \rangle} D$$
(14)

with  $\langle (\sqrt[3]{\psi})^2 \rangle$  implying a flux surface average. For  $\psi = Br^2/2$  one has  $D_s = BD$ . If one assumes a form for  $D(\psi)$  (we used a constant), then the constant of proportionality between D and  $1/t \frac{1}{2}$  can be evaluated by a simple Monte Carlo solution of Eq. (13). A Monte Carlo equivalent operator to Eq. (13) is

$$\psi_{n} = \psi_{o} + \left[\frac{d}{d\psi_{o}}\left(2\psi_{o}D\right) - 2\psi_{o}D\frac{e}{T}\frac{d\phi}{d\psi_{o}}\right]\tau \pm \left(\psi_{o}D\tau\right)^{1/2}$$
(15)

In other words one starts a group of particles on the same flux surface as in the particle following Monte Carlo code. One chooses a value for  $D_T$ , which is small compared to one, with  $\tau$  the time step of the Monte Carlo operator, Eq. (15). Note that only  $D_T$  appears in Eq. (15) and not D and  $\tau$  separately. The flux  $\psi$  is stepped along for each particle until half the particles leave the plasma. If  $N_{1/2}$  is the number of applications of the Monte Carlo operator, Eq. (15), required to lose half the particles, then

$$Dt_{1/2} = (D_T)N_{1/2} = C {.} (16)$$

This defines the constant of proportionality C between D and  $1/t_{1/2}^{}$  as shown in Fig. 8.

Using this relationship we have converted the data of Figs. 5-7 into

diffusion coefficients against density, Figs. 9, 10, and 11.

Figure 12 shows diffusion coefficients for electrons obtained from measuring particle excursions in  $\psi$  over times of the order of one collision time as described in Ref. 2. We show also the only datum from a  $t_{1/2}$  measurement. The agreement between these two methods amounts to better than 10%, which is surprisingly good.

#### IV. OTHER TRANSPORT COEFFICIENTS

In general, it is the energy, not the particle, confinement time which is of primary interest. In this section an approximate method for evaluating the thermal conductivity  $\chi$  will be given.

To derive an approximation for  $\chi(n,T)$  let us first assume that most of the radial transport of heat and particles is due to superthermal particles, E >> T. Under this assumption, the scattering is independent of the temperature and is controlled by the particle energy E. The particle flux can then be written

$$\Gamma = -\int_{O}^{E} D_{*}(n,E) \left(\frac{\partial f_{M}}{\partial \psi}\right)_{E} d^{3} v \qquad (17)$$

The function  $D_{\mu}(n, E)$  is the diffusion rate of particles of energy E scattering on a background plasma of density n. On a relatively long time scale compared to the scattering that leads to radial transport the test particles form a local Maxwellian  $f_{\mu}(\psi, E)$ . For generality we have included a cutoff energy  $E_{\mu}$ . The radial derivative of the Maxwellian is

$$\frac{\partial^2 \mathbf{H}}{\partial \psi} = \left[\frac{1}{n} \frac{\mathrm{d}n}{\mathrm{d}\psi} + \frac{\mathrm{e}}{\mathrm{T}} \frac{\mathrm{d}\phi}{\mathrm{d}\psi} + \left(\frac{1}{2} \frac{\mathrm{mv}^2}{\mathrm{T}} - \frac{3}{2}\right)\frac{1}{\mathrm{T}} \frac{\mathrm{d}T}{\mathrm{d}\psi}\right] \mathbf{f}_{\mathrm{M}} \quad ; \tag{18}$$

so the particle flux is

$$\Gamma = -D(n,T)\left(\frac{dn}{d\psi} + \frac{ne}{T}\frac{d\phi}{d\psi} - \frac{3}{2}\frac{n}{T}\frac{dT}{d\psi}\right) - \left[\int_{0}^{E} n \frac{1}{2}\frac{mV^{2}}{T}D_{\star}f_{M}d^{3}V\right]\frac{\ln(T)}{d\psi}$$
(19)

with

$$D(n,T) = \frac{1}{n} \int_{O}^{E} m D_{\star} f_{H} d^{3} V , \qquad (20)$$

Using the fact that a Maxwellian is of the form

$$f_{M} \alpha \frac{n}{T^{3/2}} \exp \left(-\frac{E-e\phi}{T}\right)$$
(21)

one can write the integral in Eq. (19) in terms of partial derivatives of D(n,T) with respect to temperature and one finds

$$\Gamma = -D\left[\frac{dn}{d\psi} + \frac{ne}{T} \frac{d\phi}{d\psi} + \frac{\partial ln(D)}{\partial ln(T)} \frac{n}{T} \frac{dT}{d\psi}\right]$$
(22)

The total radial flux of energy can be written

$$q_{t} = -\int_{O}^{E} m \left(\frac{1}{2} m v^{2} + e\phi\right) D_{\star}(n, E) \frac{\partial f_{M}}{\partial \psi} d^{3}v$$
(23)

which can be written as

$$q_{t} = \left[ \left( \frac{3}{2} + \frac{3\ln(D)}{3\ln(T)} \right) T + e\phi \right] T - nD \left( \frac{3}{2} + \frac{3\ln(D)}{3\ln(T)} + \frac{3}{3\ln(D)} + \frac{3\ln(D)}{3\ln(T)} \right) \frac{dT}{d\psi} \quad .$$
(24)

In other words, the energy flux can be written as the sum of two terms. The first term says that the typical particle which leaves carries

 $T[3/2 + \partial ln(D)/\partial ln(T)] + e\phi$  units of energy with it. This energy can be measured when particles leave the volume of the plasma in the Monte Carlo simulation, Fig. 13. Let  $k_{g}$  be the average kinetic energy carried out by an exiting particle divided by the average energy of particles in a Maxwellian, 3T/2, then

$$k_{l} = 1 + \frac{2}{3} \frac{\partial \ln(D)}{\partial \ln(m)}$$
 (25)

The second term in the energy flux, Eq. (24), represents the diffusive transport of energy which persists even in the absence of a particle flux. It is natural co call this term the heat flux q. If we ignore the generally small term,  $2^{2}\ln(D)/2\ln(T)^{2}$ , then

$$q = -\frac{3}{2} n k_g D \frac{dT}{d\psi}$$
(26)

and the total energy flux is

$$q_{t} \simeq \left(\frac{3}{2}k_{g}T + e\phi\right)\Gamma + q \quad .$$
 (27)

The diffusion coefficient for heat can then be defined as

$$\chi(\mathbf{n},\mathbf{T}) = \mathbf{k}_{g} \mathbf{D}(\mathbf{n},\mathbf{T}) \quad . \tag{28}$$

Equation (28) allows one to define a typical heat diffusive coefficient and therefore make a reasonable estimate of the energy confinement time.

Up until now we have based our approximation procedure on high energy particles dominating the transport, which is equivalent to  $k_{j} >> 1$ . Suppose

low energy particles dominate the transport. Under this circumstance the scattering rate of particles is determined by the thermal scattering rate  $\nu \propto n/T^{3/2}$  and instead of Eq. (17) we have

$$\Gamma = -\int_{0}^{E_{m}} D_{\star}(v, E) \left(\frac{\partial f_{M}}{\partial \psi}\right)_{E} d^{3}V \qquad (29)$$

An analysis similar to that in the earlier part of the section gives the same results but with  $[\partial \ln(D)/\partial \ln(T)]_{D}$  replaced by  $[\partial \ln(D)/\partial \ln(T)]_{D}$ . Now

$$\left(\frac{\partial \ln(D)}{\partial \ln(T)}\right)_{v} = \left(\frac{\partial \ln(D)}{\partial \ln(T)}\right)_{n} + \frac{3}{2} \left(\frac{\partial \ln(D)}{\partial \ln(n)}\right)_{T} \quad . \tag{30}$$

The primary case when  $\{\partial \ln(D)/\partial \ln(T)\}_n$  is small is in classical transport scaling with  $D = n/T^{1/2}$ . In this case  $[\partial \ln(D)/\partial \ln(T)]_v = 1$ . Consequently we expect  $k_g \ge 1$ , but for  $k_g \approx 1$  we do not expect a simple dependence between the temperature dependence of D and  $k_g$ . However, the relation between the heat flux, Eq. (28), and the average kinetic energy of heat particles,  $3k_gT/2$ , should remain valid.

## . V. CONFINEMENT TIME

We are now in a position to estimate the energy confinement time of a heliac with and without an electric field. For simplicity we assume a flat density profile and that

$$\chi(\mathbf{T}) \alpha \mathbf{T}^{\alpha}$$
 (31)

The power deposition per unit volume is assumed to be p(r) = 1/r. Letting a be the plasma radius, one has equations for the heat flux q(r), the

temperature T(r), and the total power P.

$$q(r) = \frac{1}{r} \int_0^r p(r) r dr$$
, (32)

$$\int_{0}^{T} \chi(T) dT = \frac{2}{3} \int_{r}^{a} q(r) dr , \qquad (33)$$

$$P = \int_0^a p(r) r \, dr \quad . \tag{34}$$

The energy confinement time  $\tau_{\rm E}$  is defined by

$$\tau_{\rm E} = \frac{3}{2} \frac{\int_0^{\rm a} T(\mathbf{r}) d\mathbf{r}}{\rm P}$$
(35)

or

$$\tau_{\rm E} = \frac{(1+\alpha)^3}{(2+\alpha)(3+2\alpha)} \frac{a^2}{\chi_{\rm D}}$$
(36)

with  $\chi_0$  the value of  $\chi$  at r = 0. Note that for  $\alpha = 0$  one obtains the wellknown estimate  $\tau_E = a^2/(6\chi_0)$ . Using the estimate of  $\alpha = 3/2$  from Fig. 11 one finds the heliac ion energy confinement time is about 4.6 msec with the central temperature and potential at 1 kev. A larger potential could give a factor of two or three improvement. The 4.6 msec is approximately the ion energy confinement time required to reach the goals of the proposed HX1 experiment.

## APPENDIX - DIFFUSION EQUATION

To derive a diffusion equation in general magnetic geometry, we assume the density n remains a function of  $\psi$  and t alone, or

$$\frac{\partial n(\psi,t)}{\partial t} = \frac{\int J \vec{\nabla} \cdot \left[ D_{g} \left[ \vec{\nabla} n(\psi,t) + en/T \vec{\nabla} \phi(\psi) \right] \right] d\theta d\phi}{\int J d\theta d\phi}$$
(A1)

with J the magnetic coordinate Jacobian

$$\frac{1}{J} = \left( \vec{\nabla} \psi \times \vec{\nabla} \theta \right) \cdot \vec{\nabla} \phi \quad (A2)$$

Using the properties of the divergence operator in general coordinates one then has

$$\frac{\partial n}{\partial t} = \frac{1}{\overline{J}} \frac{\partial}{\partial \psi} \left[ 2 \overline{J} \psi \Omega \left( \frac{\partial n}{\partial \psi} + \frac{en}{T} \frac{\partial \phi}{\partial \psi} \right) \right]$$
(A3)

with

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$$\overline{J} = \frac{1}{(2\pi)^2} \int J d \theta d \phi$$
(A4)

$$D \equiv \frac{1}{2\psi} \frac{\left( \overline{\mathcal{D}}_{s} \left( \overline{\psi} \psi \right)^{2} d \theta d \phi}{\int J d \theta d \phi}$$
(A5)

In magnetic coordinates,  $\overline{J} = g/B_0^2$  for a curl-free field and  $B_0^2(\psi)$  measures the magnetic well depth. Since the magnetic well in practice is shallow,  $\overline{J}$  is to a good approximation a constant. Consequently, Eqs. (A3) and (A5) imply Eqs. (13) and (14).

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#### FIGURE CAPTIONS

- FIG. 1. Basic design of the heliac snowing the bean-shaped flux surfaces and the helical spiral of the magnetic axis.
- FIG. 2. An engineer's view of heliac.
- FIG. 3. Variation of rotational transform 1 as a function of toroidal magnetic flux  $\psi$ .
- FIG. 4. Variations of the Fourier amplitudes for the magnetic field,  $A_{nm},$  as a function of toroidal magnetic flux  $\psi \cdot$
- FIG. 5. Time for half the initial number of ions to escape confinement,  $t_{1/2}$ , as a function of density n.
- FIG. 6. Time for half the initial number of ions to escape confinement,  $t_{1/2}$ , as a function of n, where nT is the potential on axis and T the temperature of ions. An analogous point for electrons is also shown.
- FIG. 7. Time for half the initial number of ions to escape confinement,  $t_{1/2}$ , as a function of ion temperature T at constant density  $n = 10^{14}/cc$ .
- FIG. 8 The constant of proportionality,  $C = Dt_{1/2}$ , for ions as a function of  $\eta$ , where  $\eta T$  = the ambipolar potential, D is the diffusion coefficient as defined by Eq. (13). A typical standard deviation is shown for one point.

- PIG. 9. The diffusion coefficient D for ions as a function of density n. These data have been derived from those of Fig. 5 using the constant of Fig. 8.
- FIG. 10. The diffusion coefficient D for ions as a function of  $\eta$  where  $\eta T$  is the ambipolar potential. One point of reference for electrons is also shown. The ion density  $n = 10^{14}/cc$ .
- FIG. 11. The diffusion coefficent D for ions as a function of ion temperature T at constant density  $n = 10^{14}/cc$ . The electric potential was 1 kev.
- FIG. 12. The diffusion coefficient D for electrons measured by the method of Ref. 2 as a function of density n.
- FIG. 13. Average energy of escaping ions in units of ion temperature T as a function of density n with several potentials and configurations. A typical standard deviation is shown on one point.

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Fig. 2



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Fig. 4



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Fig. 6

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Fig. 10



Fig. 11

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Fig. 13

#### EXTERNAL DISTRIBUTION IN ADDITION TO TIC UC-20

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