EFFECTS OF DIPOLE MAGNET INHOMOGENEITIES ON THE BEAM ELLIPSOID*

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JUL 1 1 1986ABSTRACT

The RAYTRACE computer code [1] has been modified [2] to accept magnetic fields measured in the median plane of a dipole magnet. This modification allows one to study the effects of a non-ideal dipole magnet on the beam ellipsoid (as defined by the TRANSPORT code manual [3]).

The effects on the beam ellipsoid are due to:

1. Field inhomogeneities in the interior region of the dipole, and
2. Discrepancies from design conditions of the magnetic field values in the fringe field region.

The results of the RAYTRACE code calculations based on experimentally measured fields will be compared with the results derived using both an ideal (no inhomogeneities) dipole with sharp boundaries and an ideal dipole with perfect (according to design) fringe fields.

INTRODUCTION

The TRANSPORT computer code [4] is often made use of to study the transport of a charged particle beam and to optimize the focusing of the beam. The code assumes that each magnetic element is ideal, having constant

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magnetic multipole strength along the effective length of the element. The fringe fields are also treated in terms of matrix elements in such a way that only the integral effects of the elements act on the beam. Irregularities in either the interior or fringe field regions can only be treated by actual raytracing of the beam particles.

Accurate magnetic field measurements show that real magnetic elements depart from ideal and show imperfections in the field regions. For instance, well inside the interior region of a dipole magnet (two or more gaps from the effective field boundaries (EFB)) the magnetic field is not uniform but is subject to inhomogeneities due to eddy currents [5], material imperfections, and non-parallelism of the pole faces. In the fringe field region the magnetic fields may deviate from the original design, in that the equal magnetic field lines (IsoB-lines) are not parallel to the entrance or exit EFB of the dipole.

In this paper we examine the effects of a specific dipole design (Fig. 1) on the beam ellipsoid, when the dipole is characterized by imperfections as mentioned above, namely, a non-uniform interior region and discrepancies in the IsoB-lines from the design conditions.

DESCRIPTION OF THE PROBLEM

An erect beam ellipsoid enters a dipole magnet at point A, Fig. 1, and appears at point D at the exit. We want to determine the shape of the beam ellipsoid after the beam passes through the dipole, which is characterized by the magnetic fields in the various regions described in Table 1. The shape of the beam ellipsoid at the exit point D will be a measure of the effect of the dipole magnet on the beam ellipsoid.

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BEAM ELLIPSOID FOR VARIOUS CHARACTERIZATIONS OF THE MAGNETIC FIELD REGIONS OF THE DIPOLE

In this section we study the beam ellipsoid at point D at the exit of the dipole for the various cases corresponding to the field characterization of the regions of the dipole, as described in Table 1.

The beam ellipsoid is described in terms of the r-matrix as defined in the TRANSPORT code manual [3] and summarized in Appendix A.

Table 2 presents the r-matrix elements of the ellipsoids at points A and D for the cases A through E discussed below.

CASE A

The dipole is characterized by a uniform field in the interior region and zero fringe fields in the entrance and exit regions (SCOFF).

In case A the TRANSPORT code was used to determine the shape of the beam ellipsoid at point D. In this configuration of the dipole magnet and entrance beam ellipsoid, the second order effects are too small to be shown in the output of the code.

The r-matrix which describes the beam ellipsoid at point D for this case is shown in Table 2.

CASE B

The field in the interior region of the dipole is uniform, and the IsoB-lines in the fringe field regions are parallel to the EFB.

The RAYTRACE code is used to derive results from which the beam ellipsoid at point D is calculated. The r-matrix corresponding to the beam ellipsoid is shown in Table 2. Appendix B discusses how to compute the r-matrix of the beam ellipsoid at point D from RAYTRACE code results. The r-matrix results of cases A and B are almost identical, verifying the fact that higher order aberrations are not significant.

CASE C

The actual magnetic fields in both the interior region and the fringe field regions are known from measurements on grid points of the rectangular plane grids shown in Fig. 2. Appendix C discusses how the magnetic field measurements were performed.

Since the original RAYTRACE code did not have the flexibility to accept experimentally measured fields, the code has been modified to accept them. Appendix C discusses in more detail how the median plane fields are introduced into the code. The RAYTRACE results show that higher order aberrations are significant when the experimentally measured fields are used. Although the beam profile at point D is not an ellipsoid any more - due to higher order aberrations - we characterize it by the enveloping ellipsoid whose maximum dimensions are the maximum values of each particular variable (x or θ) obtained from the RAYTRACE code. With such an assumption the r -matrix corresponding to the envelope ellipsoid is shown in Table 2. Comparing the r -matrix of case C with the r -matrices of the ideal cases A and B above we conclude that using the measured fields introduce second order effects that increase the beam divergence from 60.0 μ rad to 118.0 μ rad and cause rotation of the ellipsoid in the (x, θ) plane, as can be seen from the comparison of r_{12} matrix elements.

CASE D

The magnetic fields in the interior region are measured as in case C, but the fields in the fringe regions are ideal as in case B. We use this case to obtain a measure of the effect of the interior region inhomogeneities on the beam ellipsoid. The RAYTRACE code results show that second order effects are present but not significant (the higher order aberrations introduced, affect the x , or θ beam coordinates by less than 1%). By comparing the various

r-matrices in Table 2 we conclude that the inhomogeneities of the interior region do not have a significant effect on the beam ellipsoid. In Fig. 3 the equal B-field contour lines give an idea of the size of the inhomogeneities in the interior region.

CASE E

The magnetic field of the interior region is uniform but the fringe field is taken to be the measured one and is introduced into the RAYTRACE code as described in Appendix C. The RAYTRACE results show higher order aberrations to be present as in case C. The calculated r-matrix for this case is shown in Table 2 and corresponds to the ellipsoid which envelops the actual beam profile at point D. By comparing the r-matrices of cases E and C we conclude that at point D the deviation of the beam profile from the profile produced by an ideal magnet is due mainly to the discrepancies of the fringe fields from the ideal conditions. Indeed by looking at Fig. 4, where the IsoB-lines of the entrance fringe field are plotted, we note that the lines are not parallel to the EFB of the magnet as is called for in the design.

CONCLUSIONS

By comparing the r-matrices of the beam ellipsoids (Table 2) at the exit point D, (Fig. 1) we conclude that:

- a) The B-field inhomogeneities of the interior region of the dipole shown in Figure 3 do not effect the angular divergence of the beam ellipsoid by more than 0.5% and the transverse size of the beam by more than 0.1%.
(Compare case B with case D.)
- b) The deviation of the equal B-field lines (IsoB-lines) shown in Figure 4, from the design conditions, (nonparallel to the effective field boundary of either entrance or exit of the magnet) introduces appreciable second order aberrations and increases the angular beam divergence at the exit

point D, from 60.0 μ rad to 118.0 μ rad, and the transverse size of the beam by 0.5%. (Compare case B with cases C and E.)

- c) The combined effect of both B-field inhomogeneities and deviation of IsoB-lines from the design conditions on the (y, ϕ, δ) ellipsoid is less than 0.5% in the y or ϕ coordinates based on studies of the beam ellipsoid corresponding to the (y, ϕ, δ) coordinate system [3]. The higher order effects that a magnetic element normally introduces couple the x, θ coordinates to the y, ϕ coordinates. The study shows that the x, θ coordinates are affected by less than 0.2% when coupled to the y, ϕ coordinates through the higher order effects introduced when the measured fields are used.

FIGURE 1. Schematic of a 30° Bend Dipole Magnet With the Field Regions Shown.

FIGURE 1A. Projection of the Beam Ellipsoid on the (i,j) Plane. The Physical Meaning of the r-Matrix Elements is Shown on the Figure.

FIGURE 2. Schematic Diagram of the Median Plane Grids Over Which the Magnetic Field Measurements Were Performed.

FIGURE 3. Contours on Median Plane of the Equal B-Field Lines (IsoB-Lines) of the Interior Region of the Dipole as Calculated From the B-Field Measurements Described in Appendix C. The B-Field of the Inner Most Contour Line is 0.9565 Tesla and the Relative Difference Between the Contour Lines is $\frac{\Delta B}{B_0} = 2.5 \times 10^{-5}$.

FIGURE 4. Equal B-Field Lines (IsoB-Lines) as Calculated From the Measurements of the Entrance Fringe Region (Appendix C). The First Column Corresponds to the B-Field Magnitude of the IsoB-lines and the Second Column to the Angular Deviation (in Degrees) From the Ideal Direction (Parallel to the EFB of the Magnet).

FIGURE 5. Experimental Points Along the Beam Direction at the Entrance Fringe Field of the Dipole From Measurement of B-Field as Described in Appendix C: The Solid Line is a Fit to the Experimental Points as Described in Appendix D.

TABLE 1

Characterizes the Fields in the Regions of the Dipole Magnet for the Various Cases Described in the Text. The Last Column Shows Whether Higher First Order Aberrations are Present.

CASE	DIPOLE REGION		HIGHER ORDERS
	INTERIOR	FRINGE	
A	IDEAL	NONE	NO
B	IDEAL	IDEAL	NO
C	MEASURED	MEASURED	YES
D	MEASURED	IDEAL	YES
E	IDEAL	MEASURED	YES

TABLE 2

r-Matrix Description of the Beam Ellipsoid at the Entrance of the Dipole
(Point A in Figure 1) and Exit (Point D) for the Various Cases A Through E
Discussed in the Text

<u>r-Matrix</u>			<u>r-Matrix at Entrance</u>		
r ₁₁			1.237 (cm)		
r ₁₂	r ₂₂		0.0	27.0 (μrad)	
r ₁₆	r ₂₆	r ₆₆	0.0	0.0	0.0 (%)

<u>r-Matrix at Exit</u>					
<u>Case A</u>			<u>Case B</u>		
1.237 (cm)			1.237 (cm)		
0.0	60. (μrad)		0.0	59.87 (μrad)	
0.0	0.893	0.01%	0.004	0.893	0.01%

<u>Case C</u>			<u>Case D</u>		
1.244 (cm)			1.238 (cm)		
0.860	117.79 (μrad)		0.126	60.32 (μrad)	
0.004	0.455	0.01%	0.004	0.885	0.01%

<u>Case E</u>		
1.244 (cm)		
0.842	111.35 (μrad)	
0.004	0.481	0.01%

APPENDIX A

Physical Interpretation of the r-Matrix

The r-matrix as defined in the TRANSPORT code manual for first order aberration calculations, is symmetric $r_{ij} = r_{ji}$ and the diagonal elements have the following physical meaning.

$$r_{11} = X_{\max}, r_{22} = \theta_{\max}, r_{33} = Y_{\max}, r_{44} = \phi_{\max}, r_{55} = \ell, r_{66} = \delta_{\max}$$

Figure 1A shows the projection of the r-matrix on the (i,j) plane. The physical meaning of the off-diagonal elements r_{ij} is also shown.

APPENDIX B

Computation of the r-Matrix Element of an Ellipsoid at the Exit Point D (Fig. 1) Using Results from RAYTRACE Computer Code

To compute the r-matrix elements of a beam ellipsoid from the results of the RAYTRACE computer code, we make use of the formulas (8A) and (9A) below.

These formulas can be derived by the following procedures:

- a) The σ -matrix at point D along the beam (Fig. 1) is related to the σ -matrix at point A through the formula:

$$\sigma_{ij}(D) = \sum_k R_{ik} \sigma_{kk}(A) R_{jl} \quad (1A)$$

of the first order transport theory[3].

- b) Assuming that the ellipsoid at point A is erect $\sigma_{ij}(A) = 0$, $i \neq j$ the relation (1A) gives:

$$\sigma_{ii}(D) = \sum_k R_{ik}^2 \sigma_{kk}(A) \quad (2A)$$

$$\sigma_{ij}(D) = \sum_k R_{jk} R_{ik} \sigma_{kk}(A) \quad (3A)$$

- c) Any given beam coordinate X_i , at a point D along the beam, is related to the beam coordinates at a point A, by:

$$X_i(D) = \sum_j R_{ij} X_j(A) \quad (4A)$$

$$\text{where } X_i = \sqrt{\sigma_{ii}} \quad (5A)$$

Making use of the relations (4A) and (5A), (2A) and (4A) become:

$$\sigma_{ii}(D) = \sum_k R_{ik}^2 (X_k(A))^2 \quad (6A)$$

$$\sigma_{ij}(D) = \sum_k R_{ik} R_{jk} (X_k(A))^2 \quad (7A)$$

- d) Making use of the RAYTRACE code and sending into the magnet the following rays, initiating from point A,

$$\begin{array}{cccc} & 0.0 & 1.237 \text{ cm} & 0.0 & 0.0 \\ \text{Ray0} = & 0.0, & \text{Ray1} = 0.0, & \text{Ray2} = 27.0 \text{ } \mu\text{rad}, & \text{Ray3} = 0.0 \\ & 0.0 & 0.0 & 0.0 & 0.01\% \end{array}$$

The notation for the ray coordinates being Ray = $\begin{matrix} X \\ \theta \\ \delta \end{matrix}$

we obtain from the output of the RAYTRACE code the coordinates of each ray at point D. The z-axis of the coordinate system is along the direction of the central ray.

The relations (6A) and (7A) can be written as:

$$\sigma_{ii}(D) = \sum_k X_i(D) \text{ Rayk}^2 \quad (8A)$$

$$\sigma_{ij}(D) = \sum_k X_i(D) \text{ Rayk} \cdot X_j(D) \text{ Rayk} \quad i \neq j \quad (9A)$$

Formulas (8A) and (9A) express the σ -matrix elements of a beam ellipsoid at point D, in terms of the beam coordinates $X_i(D)$, calculated from the RAYTRACE code at the same point D.

Thus knowing the σ -matrix elements of the beam ellipsoid at the point D of the exit of the dipole we can easily calculate the r-matrix elements at the same point using the relationship:

$$r_{ij} = \frac{\sigma_{ij}}{\sqrt{\sigma_{ii}} \sqrt{\sigma_{jj}}}$$

APPENDIX C

Magnetic Field Measurements

The magnetic field measurements were performed on grid points of rectangular plane grids at the median plane of the dipole. Figure 2 shows the three plane grids over which the measurements were performed.

The separation of the grid points on the grid of the interior region are 5 cm and 2 cm along and normal to the beam direction respectively. The corresponding grid point separation on the grids of the fringe field regions are 4 cm and 2 cm. (Measurements performed by DANFYSIK A/S.)

APPENDIX D

Implementation of the Median Plane Magnetic Field Data Into the Raytrace Code

A. Fringe Field Regions

Perform a least squares fit of the function:

$$v(x,z) = \frac{B_0}{1 + \exp \left(\sum_{i=0}^3 \sum_{j=0}^{15} C_{ij} x^i z^j \right)}$$

to the experimentally measured magnetic field values of the median plane in the fringe field regions of the magnet. (Appendix C). The coefficients C_{ij} are computed by the least squares fit programs and entered as data into the RAYTRACE code. The magnetic field at any given point (x,z) on the median plane is calculated by the code using the fit formula above. Figure 5 shows the quality of the fit to the experimental magnetic field values taken along the beam direction.

B. Interior Region

The measured magnetic field values at the grid points are entered into the code in the form of an array.

The magnetic field at any given point (x,z) on the median plane is calculated by linear interpolation of the magnetic field values of the four adjacent grid points.

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Fig. 1

Fig. 2

Fig. 3

$B_0 = 0.9565 \text{ T}$

IsoB-Field $\delta\alpha$ Deviat
Tesla DEG.

0.954 +1.5

0.90 -1.0

0.80 0.0

0.70 +0.4

0.60 +0.5

0.50 0.0

0.40 -0.8

0.30 -1.6

0.20 0.0

0.10 +2.5

INSIDE MAGNET

EFB OF MAGNET

Entrance Fringe Field

5.0 cm

8.0 cm

Fig. 4

Fig. 5

APPENDIX A

Fig. 1A PROJECTION OF BEAM ELLIPSOID
ON (I,J) PLANE