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NON-LINEAR VARIATION OF THE BETA FUNCTION WITH MOMENTUM

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Abstract

A theory is presented for computing the non-linear dependence of the β -functions on momentum. Results are found for the quadratic term. The results of the theory are compared with computed results. A procedure is proposed for computing the strengths of the sextupole correctors to correct the dependence of the β -function on momentum.

1. Introduction

The dependence of the β -function on particle momentum, $\Delta p/p$, may depart appreciably from being linear in $\Delta p/p$ for some lattices. This paper finds results for the quadratic term in the β -function dependence on $\Delta p/p$.

For the case where the operating ν -value is close to a half-integer value, and the momentum dependence of the β -function is dominated by the corresponding half-integer stop band, it has been found that the maximum variation in the β -function with $\Delta p/p$ can be written as

$$\frac{\Delta\beta}{\beta} = \left(\frac{\Delta\beta}{\beta}\right)^{(1)} + \left(\frac{\Delta\beta}{\beta}\right)^{(2)} \quad (1.1a)$$

$\left(\frac{\Delta\beta}{\beta}\right)^{(1)}$ is the term which is linear in $\Delta p/p$,

$$\left(\frac{\Delta\beta}{\beta}\right)^{(1)} = \frac{2}{n-2\nu} |\Delta\nu_2|, \quad (1.1b)$$

where n is the integer closest to 2ν , and $\Delta\nu_2$ is the half-width of the half-integer stop band at $\nu = n/2$. $\left(\frac{\Delta\beta}{\beta}\right)^{(2)}$ is the term which is quadratic in $\Delta p/p$,

$$\left(\frac{\Delta\beta}{\beta}\right)^{(2)} = \frac{3}{(n-2\nu)^2} C \frac{\Delta p}{p} |\Delta\nu_2| + \frac{1}{(n-2\nu)^2} |\Delta\nu_2|^2, \quad (1.1c)$$

where C is the chromaticity, $C = p \, d\nu/dp$. The above results are derived in Section 2.

The above results of the theory were compared with computed results using the SYNCH program for a lattice with a rapid dependence of the β -function on momentum. Using the above theoretical results, a least squares procedure is

proposed for choosing the sextupole strengths that will minimize the variation of the β -function with momentum.

2. Theory of Non-Linear $\beta_x(p)$, $\beta_y(p)$

Courant and Snyder¹ presented the linear theory for the change in the β -function due to a change in the gradient function. They showed that the change in β -function $\Delta\beta$ caused by a change in the gradient function ΔK , obeys the equation

$$\frac{d^2}{ds^2} \left(\frac{\Delta\beta}{\beta} \right) + 4\nu^2 \left(\frac{\Delta\beta}{\beta} \right) = -\nu^2 \beta^2 \Delta K, \quad (2.1)$$

$$\Delta K = \Delta G/B\rho,$$

$$ds = \nu\beta.$$

Equation (2.1) can be used to compute the linear variation of β with momentum by putting

$$\Delta K = (-G \Delta p/p + 2B_2 X_p \Delta p/p)/B\rho \quad (2.2)$$

where $B_2(s)x^2$ is the sextupole field and $X_p(s)$ is the horizontal closed orbit dispersion function.

To obtain an equation for $\Delta\beta$ which includes higher order terms in ΔK , we start from the equation for $b = \beta^{1/2}$,

$$\frac{d^2}{ds^2} b + Kb - \frac{1}{b^3} = 0. \quad (2.3)$$

Letting $K \rightarrow K + \Delta K$, and $b \rightarrow b + \Delta b$, and keeping up to second order terms in Δb and ΔK , one finds the equation for Δb

$$\frac{d^2}{ds^2} \Delta b + (K + \frac{3}{4}) \Delta b = -b\Delta K - \Delta K \Delta b + \frac{6}{b^3} (\frac{\Delta b}{b})^2 \quad (2.4)$$

Now we carry out a transformation similar to the usual orbit theory transformation

$\eta = x/\beta^{1/2}$. We put

$$\zeta = \Delta b/\beta^{1/2} = \Delta b/b \quad (2.5)$$

Let ψ , the betatron phase function, obey $d\psi/ds = 1/\beta = 1/b^2$, then

$$\frac{d}{ds} \Delta b = \frac{db}{ds} \zeta + b \frac{d\zeta}{ds} \quad (2.6)$$

$$= \frac{db}{ds} \zeta + \frac{1}{b} \frac{d\zeta}{d\psi}$$

$$\frac{d^2}{ds^2} \Delta b = \frac{d^2 b}{ds^2} \zeta + \frac{db}{ds} \frac{d\zeta}{d\psi} \frac{1}{b^2} + \frac{d^2 \zeta}{d\psi^2} \frac{1}{b^3} - \frac{d\zeta}{d\psi} \frac{1}{b^2} \frac{db}{ds}$$

$$= (-Kb + \frac{1}{b^3}) \zeta + \frac{d^2 \zeta}{d\psi^2} \frac{1}{b^3} .$$

Putting this result into Eq. (2.4) for Δb , one finds the equation for $\zeta = \Delta b/b$

$$\frac{1}{b^3} \frac{d^2 \zeta}{d\psi^2} + (-Kb + \frac{1}{b^3}) \zeta + (K + \frac{3}{4}) b\zeta = -b\Delta K - b\Delta K \zeta + \frac{6}{b^3} \zeta^2 ,$$

or

$$\frac{d^2 \zeta}{d\psi^2} + 4v^2 \zeta = -v^2 b^4 \Delta K - v^2 \Delta K b^4 \zeta + 6v^2 \zeta^2 \quad (2.7)$$

Eq. (2.7) is correct to second order in ΔK . It may be interesting to

note that one can also obtain the exact equation by replacing $6\zeta^2$ by $1/(1 + \zeta)^3$

$-1 + 3\zeta$, which gives

$$\frac{d^2 \zeta}{d\theta^2} + 4 v^2 \zeta = -v^2 b^4 \Delta K - v^2 \Delta K b^4 \zeta + v^2 \left\{ \frac{1}{(1+\zeta)^3} - 1 + 3\zeta \right\} \quad (2.8)$$

Eq. (2.7) reduces to the linear Courant-Snyder equation, Eq. (2.1), if one drops the second order terms on the right hand side.

Eq. (2.7) can be solved by successive approximations to find $\Delta\beta/\beta$ up to second order terms. The linear solution $\zeta^{(1)}$ can be found by neglecting the second order terms. This gives the solution found by Courant and Snyder.

$$\zeta^{(1)} = -\frac{1}{4\sin(2\pi\nu)} \int ds_0 \cos(2\pi\nu - |\psi(s) - \psi(s_0)|) \beta(s_0) \Delta K(s_0), \quad (2.9)$$

$$\Delta K = (-G \Delta p/p + 2 B_2 X_p \Delta p/p) / B\rho.$$

Note that $\zeta^{(1)} = \Delta\beta/2\beta$ to first order.

An alternative solution for $\zeta^{(1)}$ is

$$\zeta^{(1)} = \sum_m \zeta_m^{(1)} e^{im\theta}, \quad (2.10)$$

$$\zeta_m^{(1)} = \frac{(v^2 b^4 \Delta K)_m}{m^2 - 4\nu^2},$$

$$(v^2 b^4 \Delta K)_m = 2\nu \frac{1}{4\pi} \int ds \beta \Delta K e^{-im\theta},$$

$$\Delta K = (-G \Delta p/p + 2B_2 X_p \Delta p/p) / B\rho.$$

If 2ν is close to the integer n , then the dominant term is

$$\zeta_n^{(1)} = \frac{(v^2 b^4 \Delta K)_n}{n^2 - 4v^2}, \quad (2.11)$$

$$\zeta_n^{(1)} = \frac{2v}{n^2 - 4v^2} \Delta v_2,$$

$$\Delta v_2 = \frac{1}{4\pi} \int ds \, B \, \Delta K e^{-in\theta}.$$

Δv_2 is the half-width of the stopband at $v = n/2$, driven by the off momentum gradient $\Delta K = (-G \Delta p/p + 2B_2 X_p \Delta p/p)/B\rho$.

The second order term in ζ may be found by putting $\zeta = \zeta^{(1)} + \zeta^{(2)}$, and solving the following equation for $\zeta^{(2)}$.

$$\frac{d^2}{d\theta^2} \zeta^{(2)} + (4v^2 + v^2 \Delta K b^4) \zeta^{(2)} = 6v^2 \zeta^{(1)2} \quad (2.12)$$

The $v^2 b^4 \Delta K$ term will shift the v value by Δv given by

$$\begin{aligned} 8v\Delta v &= (v^2 b^4 \Delta K)_0 \\ &= 2v \, C \, \Delta p/p, \end{aligned} \quad (2.13)$$

or

$$\Delta v = \frac{1}{4} C \, \Delta p/p,$$

$$C = \frac{1}{4\pi} \int ds \, B \, \Delta K,$$

where C is the chromaticity $C = p \, dv/dp$.

We will write Eq. (2.12) as

$$\frac{d^2}{d\theta^2} \zeta^{(2)} + 4v^2 \zeta^{(2)} = 6v^2 \zeta^{(1)2}, \quad (2.14)$$

with the understanding that ν is shifted by the amount given by Eq. (2.13). This amount is often small and can be neglected. For example, if $C = 2$, and $\Delta p/p = .01$, $\Delta \nu = .005$ which is not appreciable unless 2ν is very close to integer,

Eq. (2.14) for $\zeta^{(2)}$ can be solved by writing

$$\zeta^{(2)} = \sum_m \zeta_m^{(2)} e^{i \theta} \quad (2.15)$$

$$\begin{aligned} \zeta_m^{(2)} &= \frac{6\nu^2 (\zeta^{(1)2})_m}{4\nu^2 - m^2} \\ &= \frac{6\nu^2}{4\nu^2 - m^2} \sum_{n_1 + n_2 = m} \zeta_{n_1}^{(1)} \zeta_{n_2}^{(1)} \end{aligned}$$

An approximate more usable result can be found in the case where 2ν is close to an integer n . In this case $\zeta_n^{(1)}$ given by Eq. (2.11) dominates, and

$$\zeta_n^{(2)} = \frac{6\nu^2}{4\nu^2 - n^2} 2 \zeta_n^{(1)} \zeta_0^{(1)}, \quad (2.16)$$

$$\zeta_0^{(1)} = \frac{1}{4\nu^2} (-\nu^2 b^4 \Delta K)_0,$$

$$= -\frac{1}{2\nu} C \Delta p/p.$$

C is the chromaticity $C = p \, d\nu/dp$ and $C = \int ds \, \beta \Delta K / 4\pi$. Terms like $\zeta_{-n}^{(1)} \zeta_{2n}^{(1)}$ have been dropped. Thus one finds for $\zeta_n^{(2)}$,

$$\zeta_n^{(2)} = \frac{12v^2}{4v^2 - n^2} \frac{2v\Delta v_2}{n^2 - 4v^2} \left(-\frac{1}{2v} c \Delta p/p \right) , \quad (2.17)$$

$$\zeta_n^{(2)} = \frac{12v^2}{(4v^2 - n^2)^2} c \frac{\Delta p}{p} \Delta v_2 ,$$

$$\zeta_n^{(2)} = \frac{3}{4} \frac{1}{(2v-n)^2} c \frac{\Delta p}{p} \Delta v_2 . \quad (2.18a)$$

Note that $\zeta_n^{(1)}$ is given by

$$\zeta_n^{(1)} = \frac{1}{2} \frac{1}{(n-2v)} \Delta v_2 \quad (2.18b)$$

It may be noted that $\zeta_n^{(2)}$ is quadratic in $\Delta p/p$ as Δv_2 is linear in $\Delta p/p$. Eq. (2.17) shows that the second order term varies inversely as the square of $(2v-n)$, the distance from the resonance line.

The variation of β with momentum can be found from the exact relationship

$$\frac{\Delta\beta}{\beta} = 2 \frac{\Delta b}{b} + \left(\frac{\Delta b}{b} \right)^2 , \quad (2.19)$$

$$\frac{\Delta\beta}{\beta} = 2 \zeta + \zeta^2 .$$

ζ is given by

$$\zeta = 2 \left| \zeta_n^{(1)} + \zeta_n^{(2)} \right| \cos (n\theta + \delta_2) ,$$

since $\zeta_n^{(1)}$ and $\zeta_n^{(2)}$ have the same phase, the phase of Δv_2 which is denoted by δ_2 . Thus $\Delta\beta/\beta$ is given by

$$\frac{\Delta\beta}{\beta} = 4|\zeta_n^{(1)} + \zeta_n^{(2)}| \cos(n\theta + \delta_2) + 4|\zeta_n^{(1)}|^2 \cos^2(n\theta + \delta_2) . \quad (2.20)$$

$$\zeta_n^{(1)} = \frac{1}{2} \frac{1}{(n-2\nu)} \Delta\nu_2 ,$$

$$\zeta_n^{(2)} = \frac{3}{4} \frac{1}{(n-2\nu)^2} c \frac{\Delta p}{p} \Delta\nu_2 .$$

The maximum variation of β with $\Delta p/p$ can then be written as

$$\left(\frac{\Delta\beta}{\beta}\right)_{\max} = \left(\frac{\Delta\beta}{\beta}\right)_{\max}^{(1)} + \left(\frac{\Delta\beta}{\beta}\right)_{\max}^{(2)} \quad (2.21)$$

$$\left(\frac{\Delta\beta}{\beta}\right)_{\max}^{(1)} = 4 |\zeta_n^{(1)}| = \frac{2}{|n-2\nu|} |\Delta\nu_2|$$

$$\begin{aligned} \left(\frac{\Delta\beta}{\beta}\right)_{\max}^{(2)} &= 4 |\zeta_n^{(2)}| + 4 |\zeta_n^{(1)}|^2 \\ &= \frac{3}{(n-2\nu)^2} c \frac{\Delta p}{p} |\Delta\nu_2| + \frac{1}{(n-2\nu)^2} |\Delta\nu_2|^2 \end{aligned}$$

3. Comparison of Theory with Computed Results

During the design study of the CBA, a lattice was considered which has a strong half-integer stop-band at $\nu = 22.5 = 45/2$. The operating ν -value is $\nu = 22.65$. This lattice exhibited a strong dependence of the β -function on $\Delta p/p$. β_x varied from $\beta_x = 67$ at $\Delta p/p = 0$ to $\beta_x = 150$ at $\Delta p/p = .01$. The dependence of β_x on $\Delta p/p$ and on azimuth was computed using the SYNCH program², and the results compared with the theoretical results of the previous section.

For this lattice, the stop band half-width at $\nu = 22.5$ was computed to be $\Delta\nu_2 = .13$. In order to compute $\Delta\beta/\beta$ from Eq. (2.20), one needs the chromaticity

C. A difficulty arises here, because accelerators tend to operate with low value of C . The lattice has a $C_x = -40$ before the sextupoles are excited which is then reduced to $C_x \sqrt{2}$ using the sextupoles.

In the SYNCH run, the sextupoles were adjusted to give $C_x = 1.2$. If one computes C_x from Eq. (2.13) using the same sextupoles as used by SYNCH, one finds $C_x = -5.7$. These two results for C_x give considerably different results for $\zeta_n^{(2)}$ in Eq. (2.20) which is proportional to C_x . In what follows, the value $C_x = 1.2$ found by the SYNCH program was used. One then finds from Eq. (2.20)

$$\zeta_n^{(1)} = -.22$$

$$\zeta_n^{(2)} = .013$$

The $\zeta_n^{(1)}$ term gives a linear variation in β of $4 \zeta_n^{(1)}$ or $\Delta\beta = 59$ for $\Delta p/p = .01$, and a quadratic variation in $\Delta\beta/\beta$ of $4|\zeta_n^{(1)}|^2$, or $\Delta\beta = 13$. The $\zeta_n^{(2)}$ term gives a quadratic variation in $\Delta\beta/\beta$ of $4\zeta_n^{(2)}$ or $\Delta\beta = 3.5$.

The linear result predicts that $\Delta\beta/\beta$ will vary azimuthally like $\cos 2\psi$ where ψ is the betatron phase. In the lattice considered here, $\Delta\psi$ for one cell is $\pi/2$ and the variation of $\Delta\beta/\beta$ with θ has a period of 2 cells. The non-linear term in Eq. (2.2), $4|\zeta_n^{(1)}|^2 \cos^2(n\theta + \delta_2)$ changes this θ -dependence of $\Delta\beta/\beta$ adding a term that goes like $\cos^2 2\psi$. The following table shows the θ variation of β_x as given by the theory and by the SYNCH program.

ψ	0	$\pi/2$	π
β_x -Theory	143	18	143
β_x -SYNCH	147	30	147

The agreement between theory and SYNCH at $\psi = \pi/2$ is only fair, but β_x is small there and large errors may be expected. There appears to be some evidence for the existence of the $\cos^2 2\psi$ term.

The linear result predicts that $\Delta\beta/\beta$ will vary linearly with $\Delta p/p$. The SYNCH results indicate an appreciable $(\Delta p/p)^2$ term. This is illustrated by the following table.

$\Delta p/p$	.01	0	-.01
β_x -Theory	143	67	25
β_x -SYNCH	147	67	30

4. Correction of $\beta(p)$ in Proton Storage Rings

According to linear theory the variation of β with momentum given by (see Eq. (2.a)).

$$\frac{\Delta\beta}{\beta} = - \frac{1}{2\sin 2\pi\nu} \int ds_0 \cos(2\pi\nu - |\psi(s) - \psi(s_0)|) \beta(s_0) \Delta K(s_0) . \quad (4.1)$$

For $\Delta\beta_x/\beta_x$,

$$\beta \rightarrow \beta_x$$

$$\Delta K_x = (-C\Delta p/p + 2B_{2x} \Delta p/p)/B\rho ,$$

and for $\Delta\beta_x/\beta_y$, $\beta \rightarrow \beta_y$ and $\Delta K_y = -\Delta K_x$.

The sextupoles must be chosen not only to minimize the β -variation at each magnet location but also to give the proper chromaticity, C_x , C_y which is given by

$$C = \frac{1}{4\pi} \int ds \beta \Delta K . \quad (4.2)$$

ΔK_x and ΔK_y are given above.

One needs two families of sextupoles to determine C_x and C_y . The problem of using linear theory to minimize $\Delta\beta/\beta$ and get the correct chromaticity has been treated by many people, including those listed in Reference (3). If one is fortunate in the choice of the location of the sextupoles, it may be possible to

minimize $\Delta\beta_x/\beta_x$ and $\Delta\beta_y/\beta_y$ and correct C_x and C_y with 4 families of sextupoles. It is more likely that 6 or more families of sextupoles would be required.

For the lattice described in Section 3, the choice of the location of the sextupole families was determined by the location of the sextupole coils already in the accelerator for the purpose of correcting the chromaticity. These sextupole coils were arranged in four families to correct $\beta_x(p)$, and not $\beta_y(p)$, and to correct both horizontal and vertical chromaticities, C_x and C_y . Two groups of the horizontal sextupole coils were observed to be located at the peaks and valleys of the azimuthal variation of β_x at $\Delta p/p = .01$, and these two families were used to correct $\beta_x(p)$. The remaining horizontal sextupole coils was a third family, and the vertical sextupole coils was a fourth family.

Having chosen the location of the sextupoles and the number of sextupole families, one could find the strength of the sextupoles by computing $\Delta\beta/\beta$ at various suitable observation points using the linear result Eq. (4.1), and then minimizing

$$I = \frac{1}{k_{\max}} \sum_k \left(\frac{\Delta\beta}{\beta} \right)_k^2, \quad (4.3a)$$

subject to the chromaticity conditions

$$\begin{aligned} C_x &= C_{x,o}, \\ C_y &= C_{y,o}, \end{aligned} \quad (4.3b)$$

where C_x , C_y are computed using Eq. (4.2), and $C_{x,o}$, $C_{y,o}$ are the desired chromaticities. The index k enumerates the places where $\Delta\beta/\beta$ is observed; $k_{\max}$ is the total number of observation points.

The obvious defect in the above procedure is that $\Delta\beta/\beta$ may have an appreciable non-linear part, as given by Eq. (2.20), which is not included in

the above formulation. Nevertheless, minimizing just the linear part of $\Delta\beta/\beta$ may in some cases give reasonable results. If the $\Delta\beta/\beta$ variation is dominated by a nearby stopband of half-width $\Delta\nu_2$, then minimizing the linear part of $\Delta\beta/\beta$ may result in a selection of the sextupole strengths that also reduces $\Delta\nu_2$, thus reducing the non-linear part of $\Delta\beta/\beta$. However computer studies carried out for the lattice discussed in Section 3 shows that Eq. (4.3a) sometimes leads to solutions that do not reduce $\Delta\nu_2$ and which actually increase the non-linear part of $\Delta\beta/\beta$.

A procedure we have used to avoid the above difficulty is to minimize

$$I = \frac{1}{K_{\max}} \sum_k \left(\frac{\Delta\beta^{(1)}}{\beta} \right)^2 + \lambda^2 |\Delta\nu_2|^2, \quad (4.4a)$$

where $\Delta\beta^{(1)}/\beta$ is the linear part of $\Delta\beta/\beta$ given by Eq. (4.1); $\Delta\nu_2$ is the half-width of the nearby half-integer stopband and is given by

$$\Delta\nu_2 = \frac{1}{4\pi} \int ds \beta_x \Delta K_x e^{-in\theta_x}. \quad (4.4b)$$

λ has to be chosen so as to make the $\lambda^2 |\Delta\nu_2|^2$ term in Eq. (4.4a) approximately equal to the non-linear contribution to $\Delta\beta/\beta$; that is $\Delta\beta^{(2)}/\beta \approx \lambda |\Delta\nu_2|$. Using Eq. (2.20), a rough result for λ is

$$\lambda = \left| \frac{3}{(n-2\nu_x)^2} c_x \Delta p/p + \frac{1}{2(n-2\nu_x)^2} |\Delta\nu_2^{(0)}| \right| \quad (4.4c)$$

$\Delta\nu_2^{(0)}$ is the stopband width before exciting the sextupoles.

In the above formulation, we are only correcting $\Delta\beta_x/\beta_x$ and not $\Delta\beta_y/\beta_y$ and have assumed only one stop-band dominates. If more than one stopband plays a role, or one wants to correct $\Delta\beta_y/\beta_y$ as well, then one would need to add the ap-

proprate terms to Eq. (4.4) giving the contributions of $\Delta\beta_y/\beta_y$ and of the other relevant stop-bands. The choice of λ can be adjusted empirically by computing $\Delta\beta^{(2)}/\beta$ using Eq. (2.21) and comparing it with $\Delta\beta^{(1)}/\beta$.

The formulation given by Eq. (4.4) was tested by computing the sextupole strengths required to correct the $\beta(p)$ variation in the lattice described in Section 3. This lattice has a large β -variation as β_x changes from $\beta_x = 67$ at $\Delta p/p = 0$ to $\beta_x = 147$ as $\Delta p/p = .01$, a relative variation of $\Delta\beta/\beta \approx 100\%$. Four families of sextupoles were used and results were computed using the SYNCH program, and using Eq. (4.4). A comparison of the sextuple strengths found to correct the $\beta_x(p)$ variation is given in the following table.

	Theory	SYNCH
$b_2(1)$	3.949	3.319
$b_2(2)$	1.902	1.869
$b_2(3)$	2.912	2.594
$b_2(4)$	-2.048	-1.911

b_2 is the sextupole strength, $b_2 = B''/2B$, and is given in units of 10^{-4} cm^{-2} .

Using the SYNCH program, the β_x variation was reduced from $67 \rightarrow 147$ before correction to $67 \rightarrow 75$ after correction. Using Eq. 4.4, the β_x variation after correction was found to be $67 \rightarrow 74$.

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