ATMOSPHERIC GUSTS AND THEIR EFFECT
ON AIRCRAFT

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ATMOSPHERIC GUSTS AND THEIR EFFECT ON AIRCRAFT

THESIS

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CHAPTER I

INTRODUCTION

The fact that all aircraft fly in rough air at some time poses a number of problems relative to safe flight. One of the most important of these problems is that of designing the aircraft structure to withstand the loads imposed by gusts. Gust load factors result from a gust of wind striking the airplane in such a manner as to change the flight path suddenly. An airplane encountering a gust is similar to an automobile striking a bump in the road. Since the gust is not controllable by the pilot, the load factors cannot be limited for various types of airplanes as they are limited during maneuvers. However, certain departments of the government connected with defense specify gust velocities for which the airplanes have to be designed (2, p. 8). The gust loads on an aircraft are important from the standpoint of both static strength and fatigue. Records of aircraft accelerations in flight are obtained from counting accelerometers which record the number of times various acceleration levels are reached. The accelerometer is mounted in the fuselage and records the response to a gust. The accelerometer counts are directly applicable to the particular aircraft and operating conditions under which they are recorded. In order to give the results more general application, it is necessary to estimate the appropriate atmospheric conditions. For this purpose the altitude and speed of the aircraft are required. These are recorded photographically, together with the counts, at a specified time interval (1, p. 3).

The three principal phases of the gust load problem are: (1) the determination of the gust structure (the size, shape, intensity, and frequency of occurrence), (2) the reaction of any aircraft to gusts of known structure, and (3) the determination of aircraft operating conditions. No order of importance can be given to the three phases of the problem since the final loads are a function of the gust, the aircraft, and how the aircraft is flown.

The characteristics of gusts and where they occur are of fundamental importance because the gust is the source of the problem. Available information of the structure of atmospheric gusts has shown the probable size, the distance from entry to exit, of the gust is twenty-five chords, and the probable gust-gradient distance, the distance to its maximum velocity, for the standard gust is twelve and a half chords. For a sequence of gusts, the gusts may be in either direction or both with an upward gust being positive and a downward gust being negative in sign. If the sequence is of like sign, the sequence immediately following will be of like sign but opposite in direction.

Example:

(1) Two sequences of like sign but opposite in direction follow: +↑ +↑ +↑ +↑   −↓ −↓ −↓ −↓

(2) A sequence of unlike sign will be as follows: ↓↑ ↓↑ ↓↑

Current data indicate that for a downward-acting gust the magnitude is the same as an upward-acting gust. Hence, there is an equal number of upward and downward gusts with the same magnitude.

Analytical and experimental work on what happens to an aircraft when it strikes a known gust is of importance since a knowledge of aircraft reactions permit the load calculation for any aircraft. Operating conditions are of importance in setting the level of loading for operating aircraft since they define the gusts encountered and the speeds at which the gusts are encountered.

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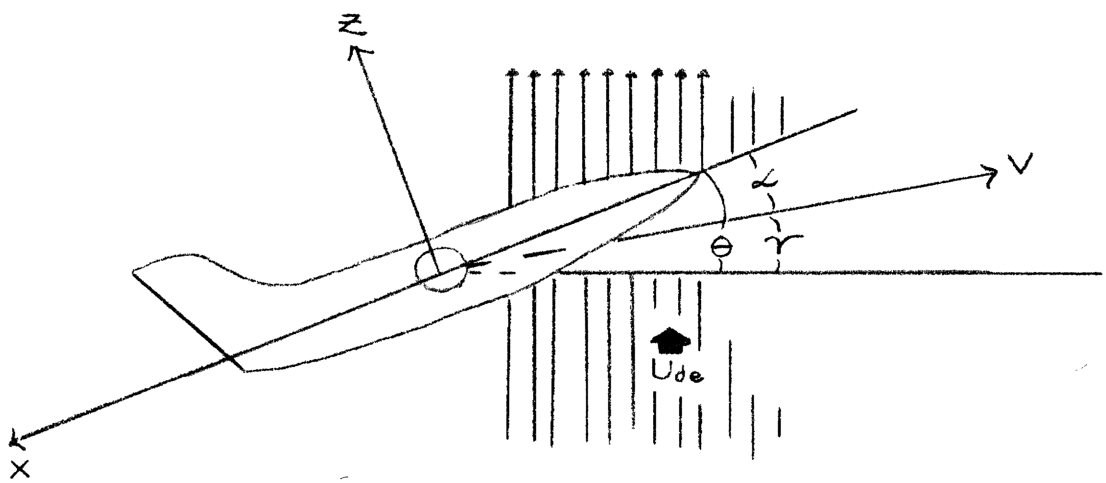
CHAPTER II

DERIVATION OF RIGID AIRPLANE FORMULA

Consider an aircraft which moves forward in level steady flight and suddenly encounters a uniform field of vertical velocity (sharp-edged gust). The boundary of the field is oriented perpendicular to the horizontal and extends on either side beyond the widest span of the airplane. As the airplane penetrates the gust front, loading develops on that portion of its surface influenced by the gust. In the figure is shown an aircraft silhouette at an instant when only a



portion of the aircraft has penetrated the gust front. An x, y, z coordinate system is placed in the aircraft, the origin being fixed at the center of gravity. The instantaneous direction of the path of the center of gravity relative to still air is indicated by the direction of the flight velocity vector V; it is assumed that the magnitude of V remains constant throughout the motion. The positive branch of the x axis is rearward and alined with the chord line of the wing. The y axis is perpendicular to the vertical plane of symmetry and coincides with the lateral axis running through the center of gravity; the z axis lies in the vertical plane of symmetry, perpendicular to the xy plane. The angle of attack α is defined to be the angle between the xy plane and the velocity vector; the angle of pitch θ is the angle between the xy plane and the horizontal plane. The flight path angle γ is the angle between the horizontal and the velocity vector. The gust velocity is indicated by U_{de}, and is measured relative to still air. All quantities are indicated in their positive direction (6, p. 6).

The term, load factor, may be defined as follows:

$$n = \text{load factor} = \frac{\text{total airload on airplane}}{\text{gross weight of airplane}}$$

Thus, a load factor of 8 indicates a total load of 8 times the gross weight of the airplane or 8 times the gravitational pull. The actual maximum load factor experienced in landing or flight is noted as limit or applied load factor. The structure is so designed that it will not yield or have permanent set when subjected to loadings from limit load factor. A safety factor of 1.5 is used above the limit loading for the ultimate strength of the airplane. Limit loads are developed during a maneuver or gust condition. The incremental load factor or normal acceleration increment is defined to be $\Delta n = n-1$ which is the load factor minus the one "g" level flight condition.

Definition of Symbols

A	aspect ratio, b^2/S
b	wing span, feet
$C_{Lg}(s)$	indicial lift response to penetration of sharp-edged gust
$C_{L\alpha}(s)$	indicial lift response to instantaneous change in angle of attack
c	mean geometric wing chord, $\frac{\text{wing area}}{\text{wing span}}$, feet
g	acceleration due to gravity, feet/second^2
K_g	gust alleviation factor
m	slope of lift curve per radian, $\frac{6A}{A+2}$
M	airplane mass, slugs
n	normal acceleration, g's
Δn	nondimensional vertical or normal acceleration increment, $\frac{d^2z}{dt^2}/g$
Δn_s	reference nondimensional vertical or normal acceleration increment, $\frac{m\rho SVU}{2W}$
S	wing area, feet^2
s	distance of penetration into a gust, chords
s_1	dummy variable, chords
t	time, $\frac{cs}{V_e}$, seconds

t_1	dummy variable, seconds
U_{de}	derived gust velocity, feet/second
u	gust velocity at any penetration distance, feet/sec.
V_e	equivalent airspeed, feet/second
W	airplane weight, pounds
z	airplane vertical displacement, feet
μ_g	airplane mass ratio, $\frac{2W}{m\rho cgS}$
ρ	air density at sea level, .002378, slugs/cubic feet

The gust load formula to be derived is obtained from solutions of an equation of airplane vertical motion in an isolated gust. The use of the formula to transfer accelerations from one airplane to another for continuous rough air implies the assumption that relative loads for single isolated gusts are a measure of the relative loads in a sequence of gusts.

Basic Assumptions and Equation of Motion

(1) The airplane is a rigid body.

(2) The airplane forward speed is constant.

(3) The airplane is in steady level flight prior to entry into the gust.

(4) The airplane can rise but not pitch.

(5) The lift increments of the fuselage and horizontal tail are negligible in comparison with the wing lift increment.

(6) The gust velocity is uniform across the wing span and is parallel to the vertical axis of the airplane at any instant.

Disregarding the forces associated with steady level flight, a summation of vertical or normal forces on the airplane in a gust yields the motion of a rigid body disturbed from its equilibrium position which according to Newton's second law of motion is as follows:

$$\frac{Md^2z}{dt^2} = \sum \text{Forces} \qquad (5, p. 5). \qquad 1$$

The forces to be summed are the aerodynamic forces due to the gust velocity, u, and the motion of the wing from its position of equilibrium. In the above equation M is the airplane mass and $\frac{d^2z}{dt^2}$ is the vertical acceleration at the center of gravity of the aircraft.

Given the indicial lift response, the lift due to an arbitrary change in angle of attack caused by the vertical velocity of the wing is determined from the following equation:

$$F_\alpha(t) = -\frac{\rho V^2 Sm}{2} \int_0^t C_{L\alpha}(t-t_1) \frac{d^2z}{dt_1^2} \frac{1}{V} dt_1. \qquad 2$$

Also given the indicial lift response, the lift to an arbitrary gust is determined as follows:

$$F_g(t) = \frac{\rho V^2 SmU}{2V} \int_0^t C_{Lg}(t-t_1) \frac{d\left[\frac{u(t_1)}{U}\right]}{dt_1} dt_1. \qquad 3$$

Summing all the forces acting, the equation of motion due to the gust is obtained. This is

$$M \frac{d^2z}{dt^2} + \frac{\rho V^2 Sm}{2} \int_0^t C_{L\alpha}(t-t_1) \frac{d^2z}{dt_1^2} \frac{1}{V} dt_1$$

$$= \frac{\rho V^2 SmU}{2V} \int_0^t C_{Lg}(t-t_1) \frac{d\left[\frac{u(t_1)}{U}\right]}{dt_1} dt_1 \quad (3,\ p.\ 6). \qquad 4$$

However, the equation of motion is usually developed with respect to the variable, s, instead of with respect to time, t. The relationship is

$$t = \frac{cs}{V} \qquad \text{and} \qquad \frac{d^2z}{dt^2} = \Delta n\ g. \qquad 5$$

Equation (4) can be written in nondimensional form as follows:

$$\frac{W}{g} \Delta n(t)\ g + \frac{1}{\mu g} \frac{V^2 W}{cg} \int_0^t C_{L\alpha}(t-t_1) \Delta n(t_1) g \frac{1}{V} dt_1$$

$$= \Delta n_s\ W \int_0^t C_{Lg}(t-t_1) \frac{d\left[\frac{u(t_1)}{U}\right]}{dt_1} dt_1 \qquad 6$$

where $\Delta n_s = \frac{\rho mSVU}{2W}$ and $\frac{\frac{W}{gc}}{\frac{1}{2}\rho mS} = \frac{2W}{m\rho cgS} = \mu g.$

The basic parameter relating the inertia and aerodynamic forces on an aircraft is μg, the mass ratio.

Now (6) becomes

$$\frac{\Delta n(t)}{\Delta n_s} + \frac{1}{\mu g}\frac{V^2}{cg}\int_0^t C_{L\alpha}(t-t_1)\frac{\Delta n(t_1)g}{\Delta n_s}\frac{1}{V}\,dt_1 = \int_0^t C_{Lg}(t-t_1)\frac{d\left[\frac{u(t_1)}{U}\right]}{dt_1}\,dt_1 \qquad 7$$

by dividing by $W \cdot \Delta n_s$. Now with the change of variable (7) becomes

$$\frac{\Delta n(s)}{\Delta n_s} + \frac{1}{\mu g}\frac{V}{c}\int_0^s C_{L\alpha}(s-s_1)\,\Delta n(s_1)\,\frac{c}{V}\,ds_1 = \int_0^s C_{Lg}(s-s_1)\frac{d\left[\frac{u(s_1)}{U}\right]}{ds_1}\,ds_1. \qquad 8$$

The equation which obtains the effect of an arbitrary gust on an aircraft follows:

$$\frac{\Delta n(s)}{\Delta n_s} = \int_0^s C_{Lg}(s-s_1)\frac{d\left[\frac{u(s_1)}{U}\right]}{ds_1}\,ds_1 - \frac{1}{\mu g}\int_0^s C_{L\alpha}(s-s_1)\frac{\Delta n(s_1)}{\Delta n_s}\,ds_1. \qquad 9$$

The arbitrary gust shape may be a triangular shape, one-minus-cosine shape, or some other desired shape (1, p. 7).

A sharp-edged gust shape is chosen here. For a sharp-edged gust $u(s_1)$ is constant at U and (9) becomes

$$\frac{\Delta n(s)}{\Delta n_s} = C_{Lg}(s) - \frac{1}{\mu g}\int_0^s C_{L\alpha}(s-s_1)\frac{\Delta n(s_1)}{\Delta n_{s1}}\,ds_1. \qquad 10$$

Now let

$$\frac{\Delta n(s)}{\Delta n_s} = f(s) \qquad \text{and} \qquad \frac{\Delta n(s_1)}{\Delta n_{s1}} = f(s_1); \qquad 11$$

so

$$f(s) = C_{Lg}(s) - \frac{1}{\mu g}\int_0^s C_{L\alpha}(s-s_1)\; f(s_1)\; ds_1, \qquad 12$$

which is known as Volterra's equation (4, p. 13). By successively substituting for $f(s_1)$ its value as given by equation (12) where $s = s_1$ at $s_1 = s_2$, the following equation is obtained:

$$f(s) = C_{Lg}(s) - \frac{1}{\mu g}\int_0^s C_{L\alpha}(s-s_1)\Big[C_{Lg}(s_1) - \frac{1}{\mu g}\int_0^{s_1} C_{L\alpha}(s_1-s_2)f(s_2)\,ds_2\Big]ds_1 \qquad 13$$

which is the same as

$$f(s) = C_{Lg}(s) - \frac{1}{\mu g}\int_0^s C_{L\alpha}(s-s_1)C_{Lg}(s_1)\,ds_1 + \frac{1}{\mu g^2}\int_0^s C_{L\alpha}(s-s_1)\int_0^{s_1} C_L\,(s_1-s_2)\,f(s_2)\,ds_2\,ds_1. \qquad 14$$

Having substituted successively, the following equation is created:

$$f(s) = C_{Lg}(s) + \frac{1}{(-\mu g)}\int_0^s C_{L\alpha}(s-s_1)C_{Lg}(s_1)\,ds_1 + \left(\frac{1}{-\mu g}\right)^2\int_0^s C_{L\alpha}(s-s_1)\int_0^{s_1} C_{L\alpha}(s_1-s_2)C_{Lg}(s_2)\,ds_2\,ds_1 + \dots + \left(\frac{1}{-\mu g}\right)^n\int_0^s C_{L\alpha}(s-s_1)\int_0^{s_1} C_{L\alpha}(s_1-s_2)\dots\int_0^{s_{n-1}} C_{L\alpha}(s_{n-1}-s_n)C_{Lg}(s_n)\,ds_n\dots ds_2\,ds_1 + R_{n+1}(s_1) \qquad 15$$

where

$$R_{n+1}(s) = \frac{1}{(-\mu g)^{n+1}}\int_0^s C_{L\alpha}(s-s_1)\int_0^{s_1} C_{L\alpha}(s_1-s_2)\dots\int_0^{s_n} C_{L\alpha}(s_n-s_{n+1})f(s_{n+1})\,ds_{n+1}\dots ds_2\,ds_1.$$

Next consider the infinite series (4, p. 14)

$$f(s) = C_{Lg}(s) + \frac{1}{(-\mu g)}1 \int_0^s C_{L\alpha}(s-s_1)C_{Lg}(s_1)ds_1$$

$$+ \frac{1}{(-\mu g)}2 \int_0^s C_{L\alpha}(s-s_1) \int_0^{s_1} C_{L\alpha}(s_1-s_2)C_{Lg}(s_2)ds_2 \; ds_1 + \ldots$$

$$+ \frac{1}{(-\mu g)}n \int_0^s C_{L\alpha}(s-s_1) \int_0^{s_1} C_{L\alpha}(s_1-s_2) \ldots$$

$$\int_0^{s_{n-1}} C_{L\alpha}(s_{n-1}-s_n)C_{Lg}(s_n)ds_n \ldots ds_2 \; ds_1 + \ldots \qquad 16$$

Since the greatest values of $C_{L\alpha}$ and C_{Lg} are $C_{L\alpha}(\infty)$ and $C_{Lg}(\infty)$, the general term of the series does not exceed

$$\left|\frac{1}{(\mu g)}n\right| \left[C_{L\alpha}(\infty)\right]^n C_{Lg}(\infty) \int_0^s ds_1 \int_0^{s_1} ds_2 \int_0^{s_2} ds_3 \ldots \int_0^{s_{n-1}} ds_n$$

that is, does not exceed

$$\left|\frac{1}{(\mu g)}n\right| \left[C_{L\alpha}(\infty)\right]^n C_{Lg}(\infty) \quad \frac{1}{n!} \quad s^n \qquad 17$$

and, the series for which (17) is the general expression for the n^{th} term is uniformly convergent (4, pp. 14-15;5, p. 11).

Solution of the Equation of Motion

Now the equation

$$\frac{\Delta n(s)}{\Delta n_s} = C_{Lg}(s) - \frac{1}{\mu g}\int_0^s C_{L\alpha}(s-s_1)\,\frac{\Delta n(s_1)}{\Delta n_{s_1}}\,ds_1, \qquad 18$$

which is known as Volterra's equation, is solved. Equation (18) is solved for the acceleration ratio $\frac{\Delta n(s)}{\Delta n_s}$ on the basis of the following indicial lift functions.

$$C_{L\alpha}(s) = 1 - .165e^{-.09s} - .335e^{-.60s} \qquad 19$$

$$C_{Lg}(s) = 1 - .236e^{-.116s} - .513e^{-.728s} - .171e^{-4.84s} \qquad 20$$

(2, pp. 410-424; 7, pp. 17-35; 8, pp. 32-33)

Considering only the first four terms of the infinite series, the following equation is obtained:

$$
\begin{aligned}
\frac{\Delta n(s)}{\Delta n_s} = {} & 1 - .236e^{-.116s} - .513e^{-.728s} - .171e^{-4.84s} \\
& - \frac{1}{\mu g}\int_0^s \left[1 - .165e^{-.09(s-s_1)} - .335e^{-.60(s-s_1)}\right] \\
& \qquad \left[1 - .236e^{-.116s_1} - .513e^{-.728s_1} - .171e^{-4.84s_1}\right] ds_1 \\
& + \left(\frac{1}{\mu g}\right)^2 \int_0^s \left[1 - .165e^{-.09(s-s_1)} - .335e^{-.60(s-s_1)}\right] \\
& \qquad \int_0^{s_1} \left[1 - .165e^{-.09(s_1-s_2)} - .335e^{-.60(s_1-s_2)}\right] \\
& \qquad \left[1 - .236e^{-.116s_2} - .513e^{-.728s_2} - .171e^{-4.84s_2}\right] ds_2\, ds_1 \\
& - \left(\frac{1}{\mu g}\right)^3 \int_0^s \left[1 - .165e^{-.09(s-s_1)} - .335e^{-.60(s-s_1)}\right] \\
& \qquad \int_0^{s_1} \left[1 - .165e^{-.09(s_1-s_2)} - .335e^{-.60(s_1-s_2)}\right] \\
& \qquad \int_0^{s_2} \left[1 - .165e^{-.09(s_2-s_3)} - .335e^{-.60(s_2-s_3)}\right] \\
& \qquad \left[1 - .236e^{-.116s_3} - .513e^{-.728s_3} - .171e^{-4.84s_3}\right] ds_3\, ds_2\, ds_1.
\end{aligned}
\tag{21}
$$

Hence,

$$\frac{\Delta n(s)}{\Delta n_s} = 1 - .236e^{-.116s} - .513e^{-.728s} - .171e^{-4.84s}$$

$$-\frac{1}{\mu g}\Big[s - 5.165 + .699e^{-.116s} - .771e^{-.728s}$$

$$+ .015e^{-4.84s} + 3.47e^{-.09s} + 1.75e^{-.60s}\Big]$$

$$+\frac{1}{\mu g^2}\Big[2.989 + .5(s)^2 - 5.165(s) - .573e^{-.09s}(s)$$

$$-.587e^{-.60s}(s) - 2.07e^{-.116s} - 1.158e^{-.728s}$$

$$-.0015e^{-4.84s} + 1.632e^{-.09s} - 1.388e^{-.60s}\Big]$$

22

$$-\frac{1}{\mu g^3}\Big[-370.799 + 35.695(s) + 3.777(s)^2 + .167(s)^3$$

$$+6.743e^{-.09s}(s) + 1.443e^{-.60s}(s) + .098e^{-.60s}(s)^2$$

$$+.047e^{-.09s}(s)^2 - 29.549e^{-.116s} - 4.921e^{-.728s}$$

$$-.00012e^{-4.84s} + 401.928e^{-.09s} + 14.559e^{-.60s}\Big]$$

is obtained containing only the mass parameter μg and the distance s.

Since the maximum value of $\frac{\Delta n}{\Delta n_s}$ defines the maximum acceleration experienced by the airplane, it is of primary concern in design (5, p. 8). This maximum value is designated as the gust alleviation factor and is labeled K_g, that is

$$\left(\frac{\Delta n}{\Delta n_s}\right)_{max} = K_g . \qquad 23$$

The variation of this gust alleviation factor with mass ratio is shown in Figure 1.

The gust load formula follows directly from equation (23), that is

$$\Delta n_{max} = \Delta n_s K_g$$

$$= \frac{m\rho SVU}{2W} K_g . \qquad 24$$

In terms of equivalent speeds this equation becomes

$$\Delta n_{max} = \frac{m\rho_o V_e U_{de} S}{2W} K_g \qquad 25$$

where the subscript e is used to denote that both the airspeed and gust velocity are equivalent speeds. The subscript d has been added also to the gust velocity to denote that, when the formula is used to evaluate gust velocity from measured accelerations, the gust velocity obtained is **a derived not a measured value.** For application in design, as in this case, U_{de} may be a stipulated value.

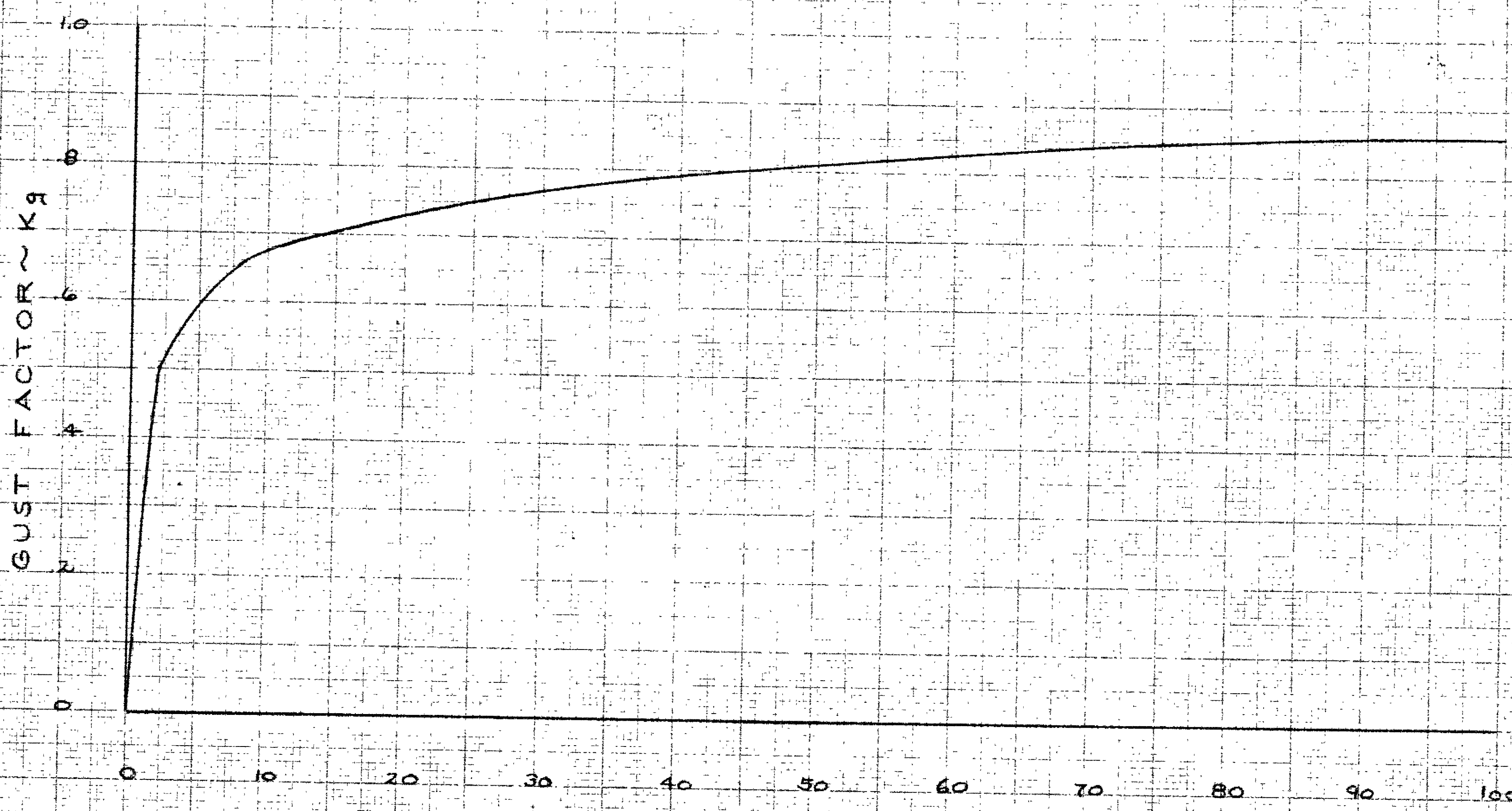


Fig. 1 -- AIRPLANE MASS RATIO ~ μ_g

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CHAPTER III

GUST EXPECTANCY AND ANALYSIS

The purpose of this chapter can be expressed in two parts, the first being to present for each of several altitude levels cumulative frequencies of gusts per mile exceeding any given gust velocity and the second part which presents a method of evaluating the cumulative frequency of gust velocities encountered during aircraft life (as defined by a specified mission) and/or the cumulative frequency of normal accelerations due to the encountered gusts (1, 2, 3, 4, 5).

Gust Frequencies

The cumulative frequency of gusts per mile with respect to altitude variation is presented in figures two through seven. The shaded band in figures two through five and figure seven is formed by data from references 1, 4, and 5. The single curves in figures two through five are derived from references 2 and 3. The upper limit on the shaded bands, formed by data from reference 4 and checked with reference 1, contains moderate storm frequency. The shaded band of figure six is obtained from data contained in references 1, 3, and 5 while the upper curve, containing moderate storm frequency, is again obtained from reference 4 and checked

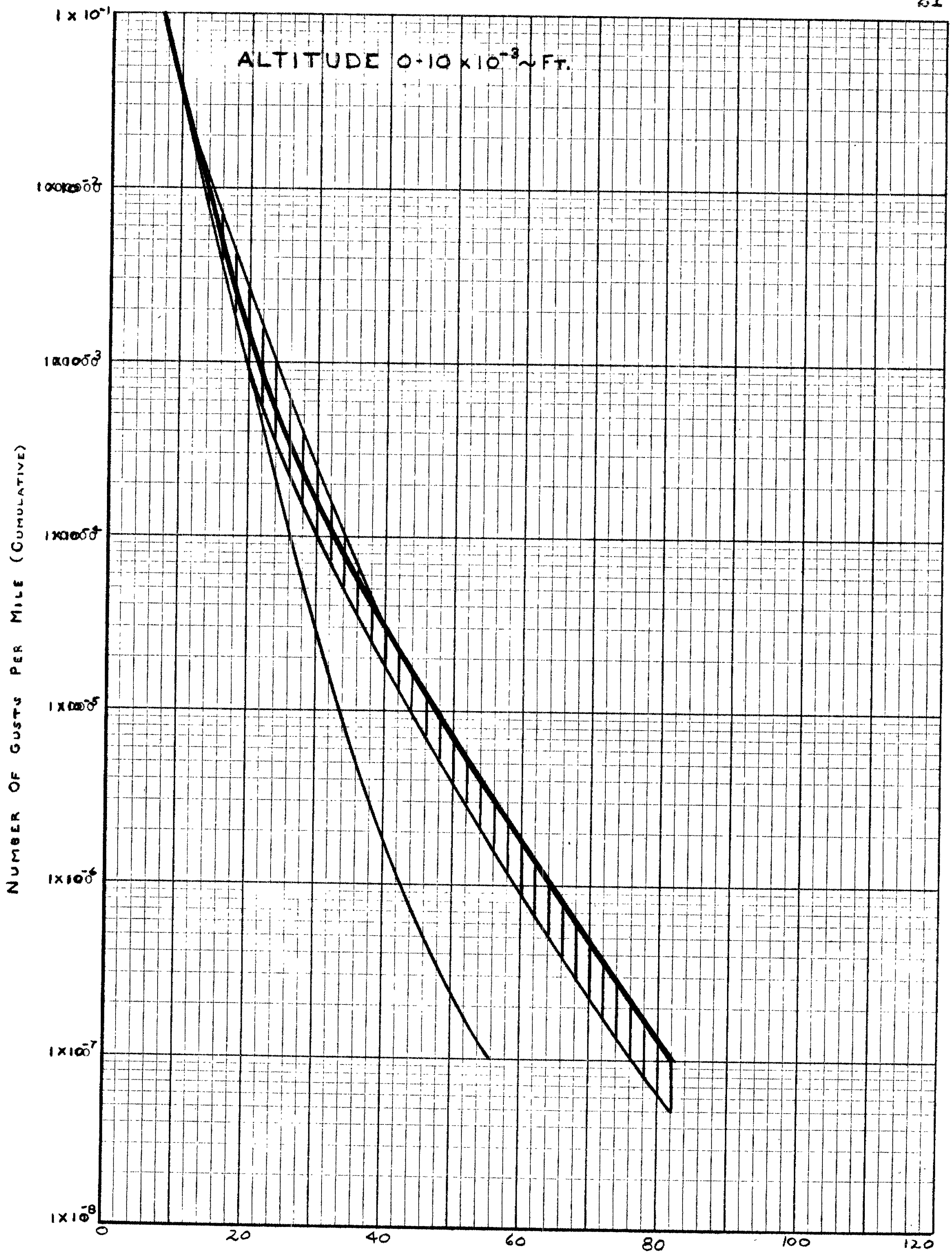

Fig. 2 --GUST VELOCITY ~ Ft./Sec.

AIRCRAFT GUST EXPECTANCY

MODEL

DATE

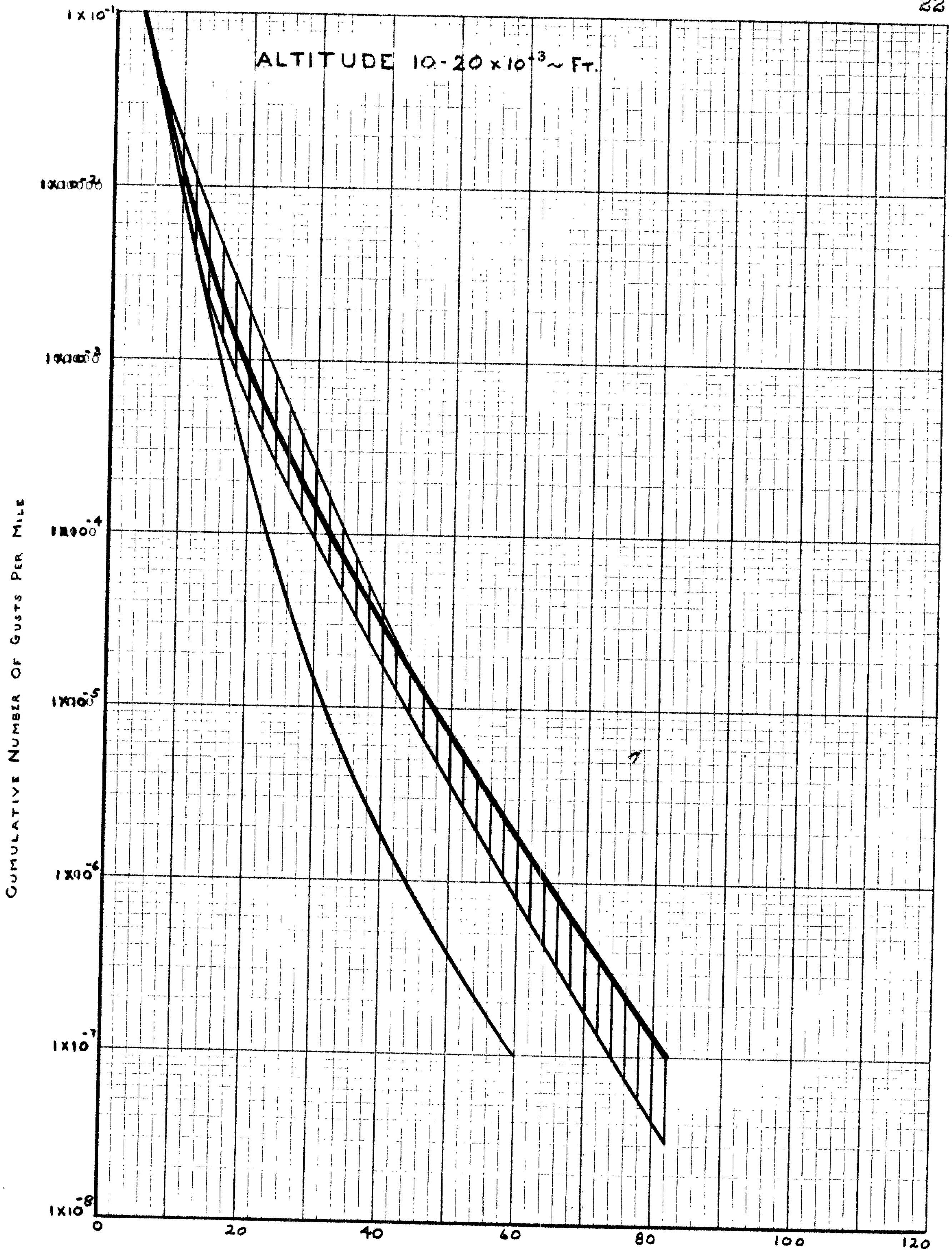

Fig. 3 --GUST VELOCITY ~ FT./SEC.

7 CYC. 5 X 60 DIVISIONS

AIRCRAFT GUST EXPECTANCY

MODEL

DATE

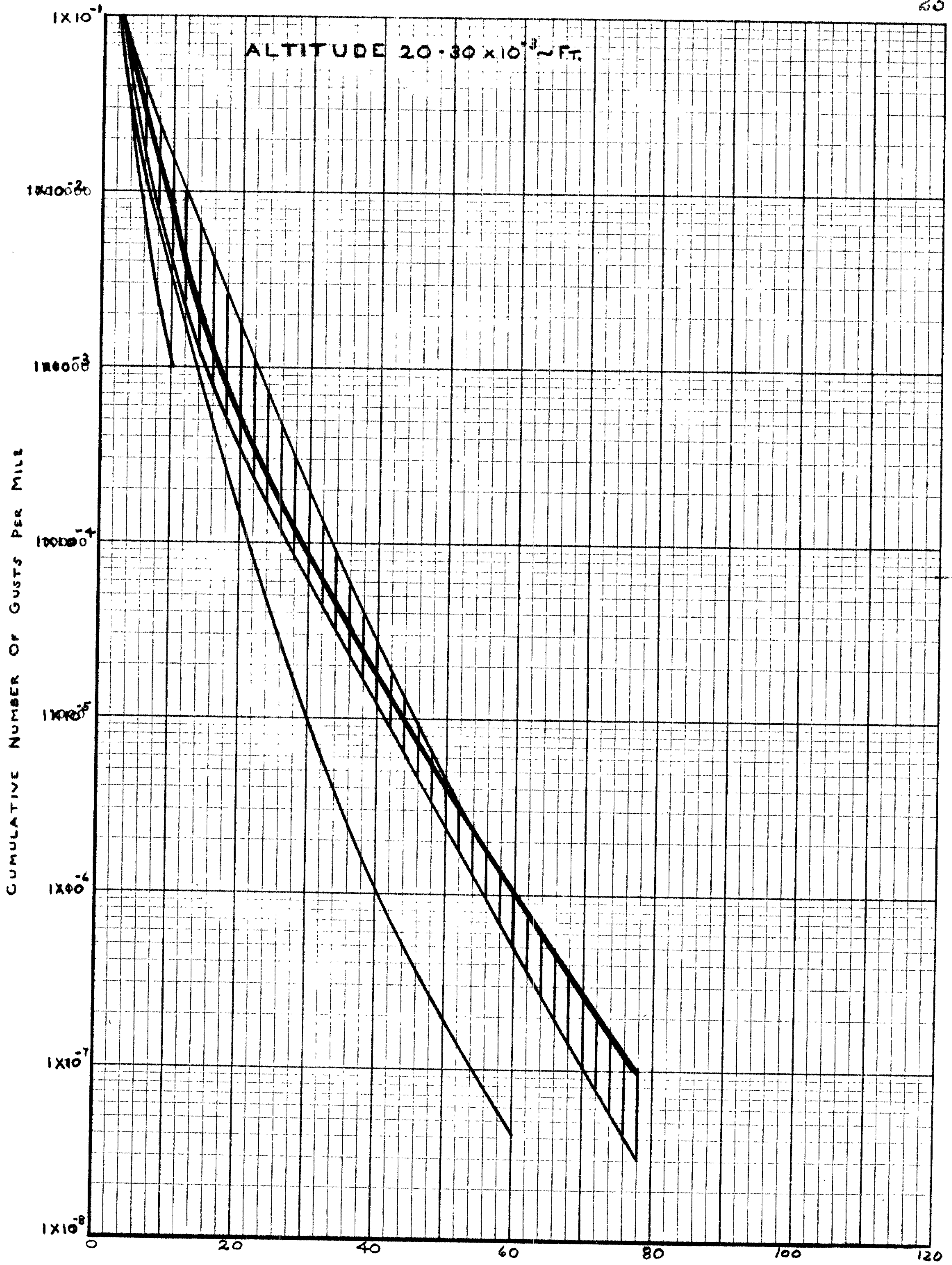

Fig. 4 —GUST VELOCITY

AIRCRAFT GUST EXPECTANCY

MODEL

DATE

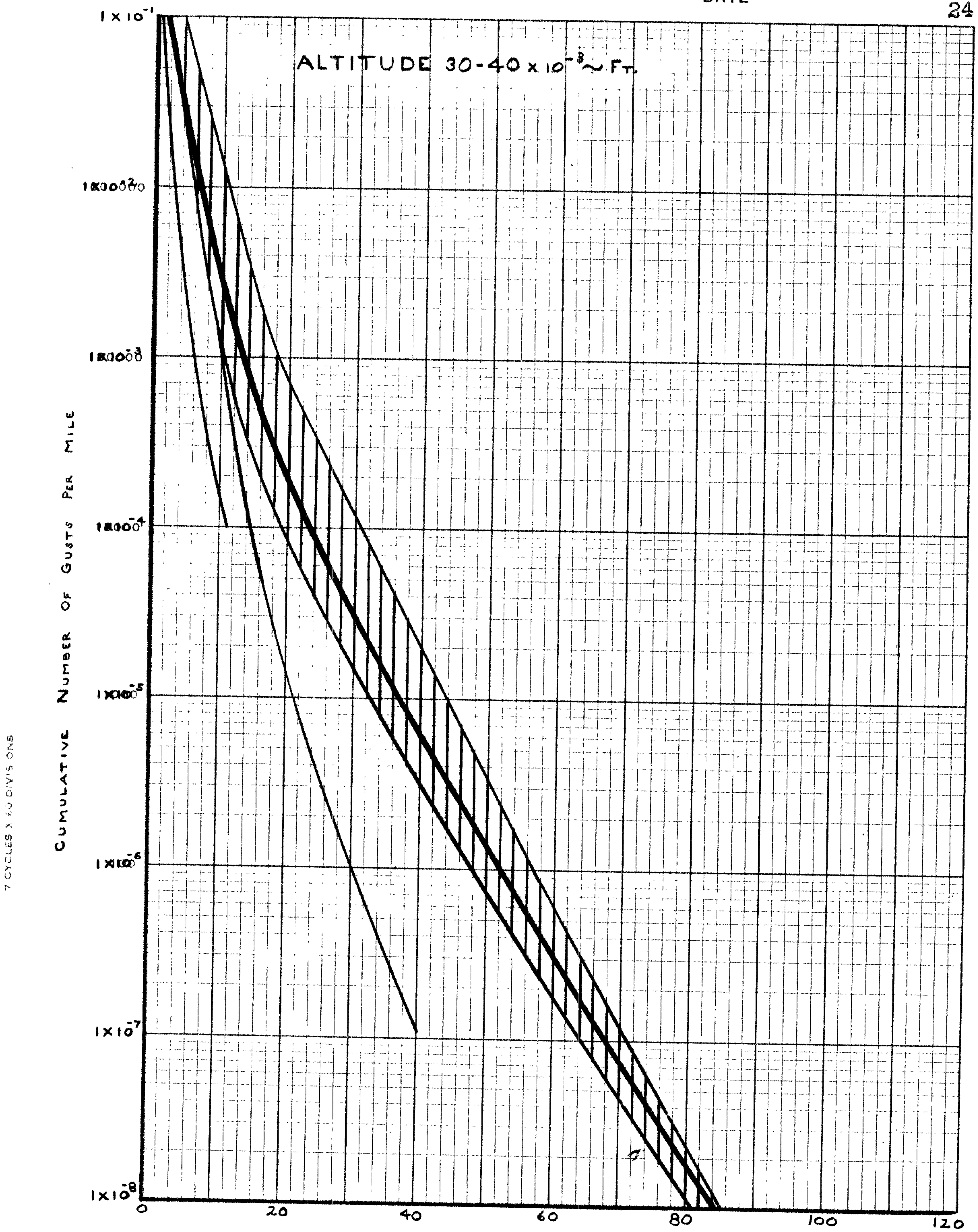

Fig. 5 --GUST VELOCITY

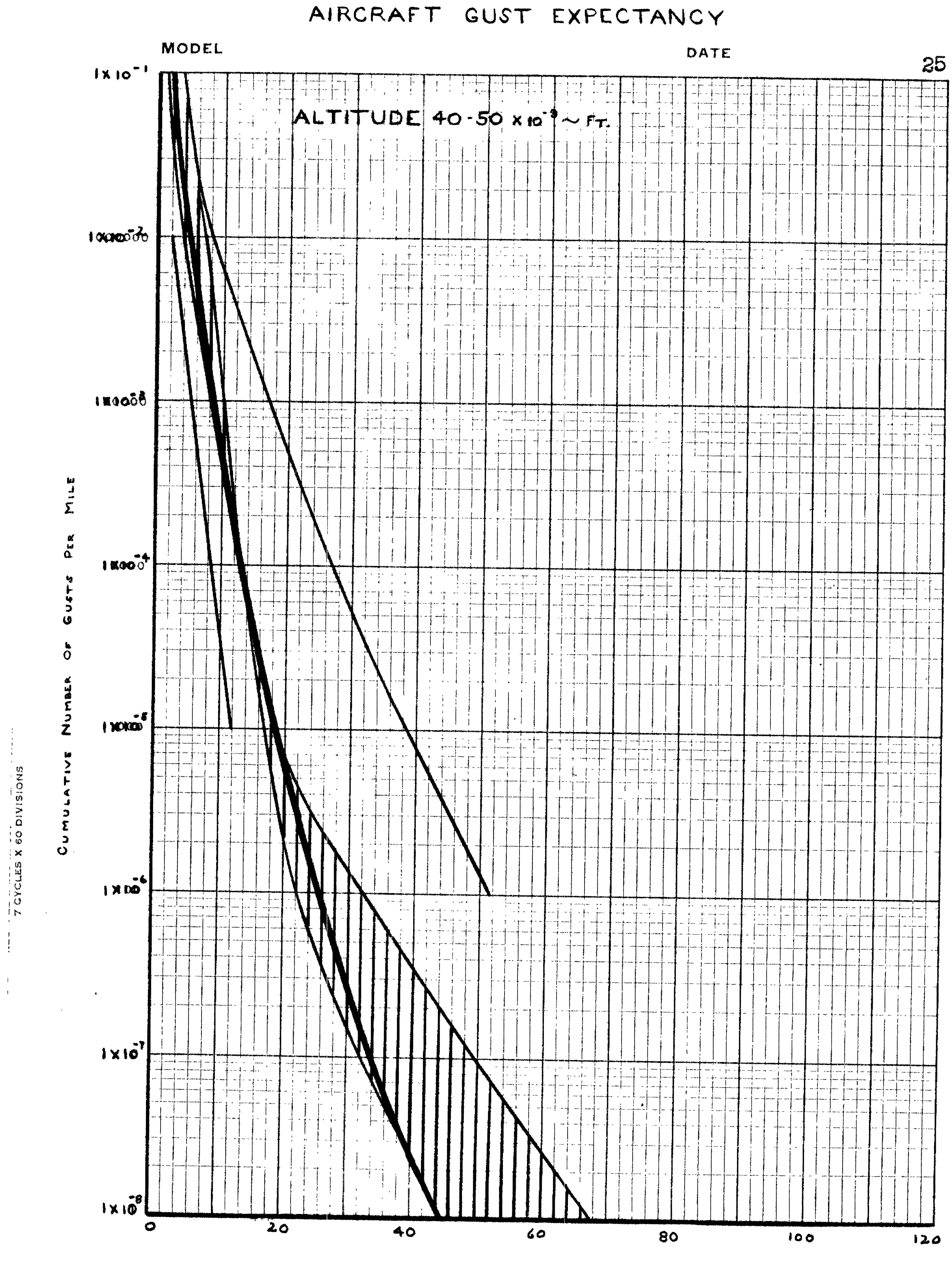

Fig. 6 —GUST VELOCITY ~ FT / SEC

AIRCRAFT GUST EXPECTANCY

MODEL

DATE

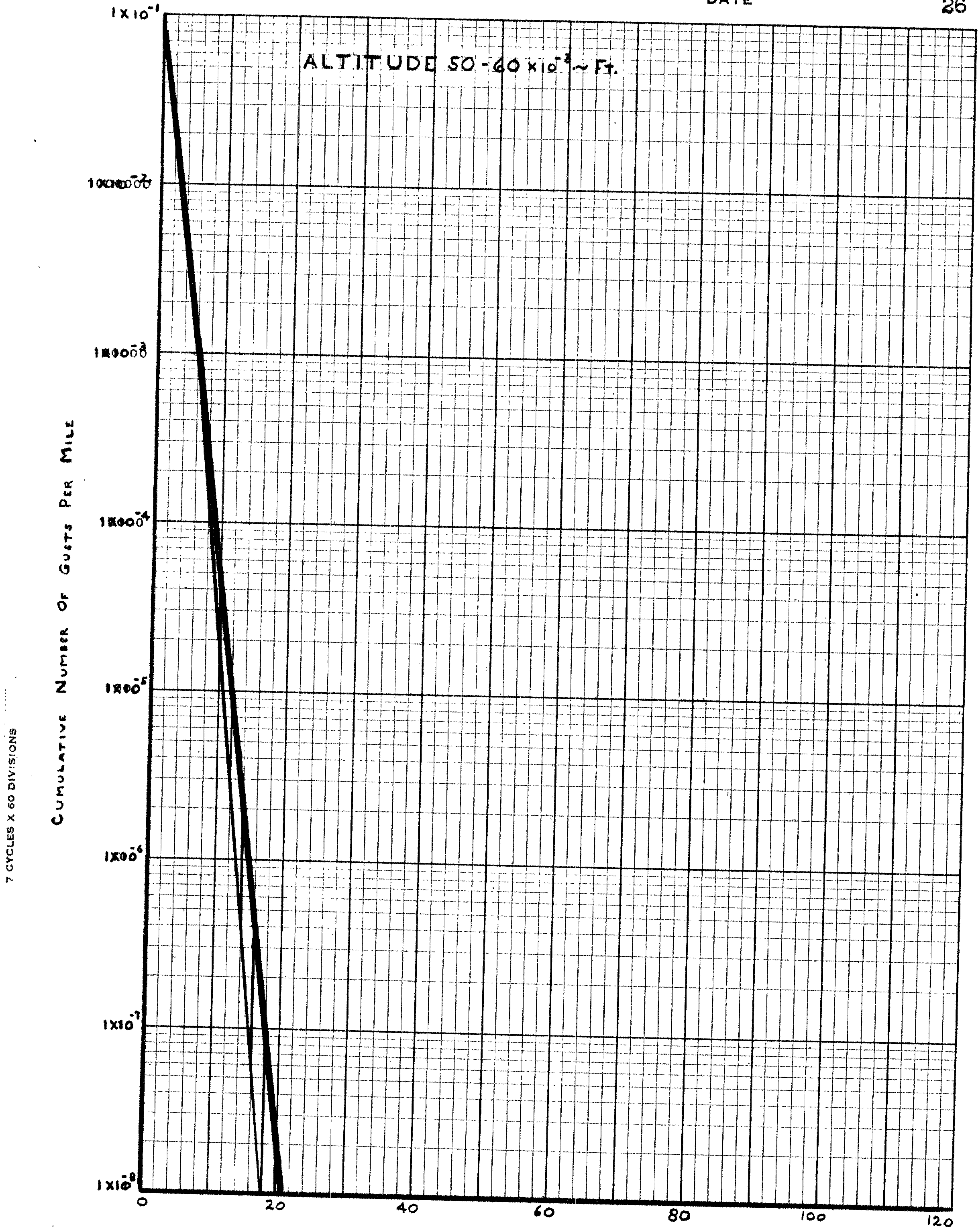

Fig. 7 —GUST VELOCITY ~ FT./SEC.

with reference 1. The lower curve of figure six is found in reference 2. It should be noted that the occurrence of a gust implies the occurrence of a gust cycle which is a positive gust immediately followed by a negative gust of like velocity.

The heavy curve of figure two through seven is chosen for use in the gust analysis (5). On a comparative basis, these data are representative and are used to evaluate a gust expectancy table which is presented as Table I.

Gust Analysis

The following assumptions are made for a gust analysis of a specified aircraft.

1. It has been assumed that gust behavior is independent of airplane weight or internal fuel distribution. This would be a signficant factor in the case of a wing having tip fuel pods or other forms of outboard fuel storage. In such a case, the fuel loads would subtract from the wing loads, making the structure more susceptible to gust damage in the light-weight than in the heavy-weight conditions. The mission would thus have to be divided into portions having significantly different fuel-weight configurations.
2. The ratio of wing loading to lift curve slope has been assumed to be a constant, independent of altitude and Mach number.

TABLE I

THE NUMBER OF GUSTS PER MILE FOR GIVEN RANGES OF ALTITUDE AND GUST VELOCITY

Altitude $\times 10^{-3}$ feet / Gust Velocity feet per second	0-5	5-10	10-15	15-20	20-25	25-30
0-15	9.90×10^{-1}	9.96×10^{-1}	9.96×10^{-1}	4.98×10^{-1}	4.98×10^{-1}	3.99×10^{-1}
15-25	9.60×10^{-3}	4.10×10^{-3}	3.20×10^{-3}	1.72×10^{-3}	1.20×10^{-3}	1.12×10^{-3}
25-35	3.43×10^{-4}	3.20×10^{-4}	3.20×10^{-4}	2.23×10^{-4}	1.60×10^{-4}	1.42×10^{-4}
35-45	4.30×10^{-5}	6.20×10^{-5}	6.20×10^{-5}	4.30×10^{-5}	3.10×10^{-5}	3.00×10^{-5}
45-55	1.08×10^{-5}	1.38×10^{-5}	1.38×10^{-5}	1.08×10^{-5}	6.70×10^{-6}	6.00×10^{-6}
55-65	2.42×10^{-6}	3.00×10^{-6}	3.00×10^{-6}	2.42×10^{-6}	1.72×10^{-6}	1.50×10^{-6}
65-75	5.80×10^{-7}	9.20×10^{-7}	9.20×10^{-7}	5.80×10^{-7}	4.20×10^{-7}	3.60×10^{-7}

TABLE I --Continued

30-35	35-40	40-45	45-50	50-55	55-60
3.99×10^{-1}	9.97×10^{-2}	6.92×10^{-3}	2.99×10^{-3}	9.99×10^{-4}	9.10×10^{-6}
7.50×10^{-4}	2.60×10^{-4}	7.52×10^{-5}	7.86×10^{-6}	1.20×10^{-6}	0
1.22×10^{-4}	2.95×10^{-5}	4.35×10^{-6}	1.36×10^{-7}	0	0
2.20×10^{-5}	4.60×10^{-6}	4.00×10^{-7}	4.20×10^{-9}	0	0
4.40×10^{-6}	7.40×10^{-7}	4.35×10^{-8}	0	0	0
1.22×10^{-7}	1.30×10^{-8}	5.60×10^{-9}	0	0	0
2.80×10^{-7}	2.45×10^{-8}	7.50×10^{-10}	0	0	0

The evaluation of the distribution of gust velocities encountered during the life of a given aircraft is now presented. Given an airplane configuration, mission profile, and expected life, divide the profile into phases by defining the altitude, airspeed, and distance. For each mission phase, determine the frequency of occurrence of gust velocity per flight mile for each of a discrete set of values of gust velocities which are selected from the range ten through sixty feet per second. The gust expectancy table has been evaluated for this purpose which expresses for each flight mile the frequency of occurrence of gusts of a given velocity as a function of altitude. For each gust velocity, sum the frequency of occurrence over altitude, airspeed, and phase, thus obtaining the total number of gusts per mission and/or life.

The evaluation of the distribution of normal accelerations due to gusts for the life of a given aircraft is given. As presented in the above paragraph, determine the total number of gusts per mission and/or per life for each gust velocity and airspeed combination. The incremental accelerations due to gusts are evaluated as functions of gust velocity and equivalent airspeed assuming a rigid aircraft. Thus for each pair of values of velocity and airspeed which occur jointly in the various mission phases, a corresponding acceleration may be calculated. Grouping the data by choosing various intervals of incremental accelerations and summing the frequencies of occurrence of gust velocity within the interval, a total number of incremental accelerations per life and/or per mission is obtained (6).

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CHAPTER IV

EXAMPLE PROBLEM

An example is presented illustrating the use of the gust analysis with the following rigid airplane formula and data for an assumed aircraft.

$$U_{de} = \left[\frac{2(W/S)}{\rho\,(m)}\right]\left[\frac{1}{K_g}\right]\left[\frac{1}{V_e}\right]\left[\Delta n\right]$$

or

$$\Delta n = \frac{m\rho S V_e U_{de}}{2W} K_g ,$$

where

W = 20,000 pounds

S = 300 square feet

b = 40 feet

c = 8 feet

K_g = .81.

Hence,

$$A = \frac{b^2}{S} = 5.33, \quad m = \frac{6A}{A+2} = 4.36/\text{rad.}, \quad \mu g = \frac{2W}{m\rho cgS} = 50.$$

The mission profile, Figure 8, is that of a long-range mission. An airplane life of 10,000 hours or 296 missions has been assumed. Table II presents a summary of the speeds, distances, and times for the individual segments of the mission.

The gust spectrum for each mission is established by applying the gust expectancy table, Table I, to the actual distances of the mission segment for each discrete set of values of gust velocity; thus obtaining the number of cycles per mission in that flight segment. Using the same discrete set of values of gust velocity, the corresponding values of incremental accelerations (above and below 1.0 g) are calculated from the rigid airplane formula. The results of these calculations are shown in Table III.

By summing these frequencies for all phases over each gust velocity interval, the frequency per interval of U_{de} and cumulative frequency per mission are obtained. Table IV presents the interval frequencies and cumulative frequencies per life. The number of gust cycles per life equalling or exceeding a given gust velocity is shown in Figure 9.

Grouping the data in Table III by choosing various intervals of incremental acceleration and summing the frequency of occurrence within the interval, the total number of incremental accelerations per mission is obtained. The summary of frequencies and cumulative frequencies per life in each

MISSION PROFILE

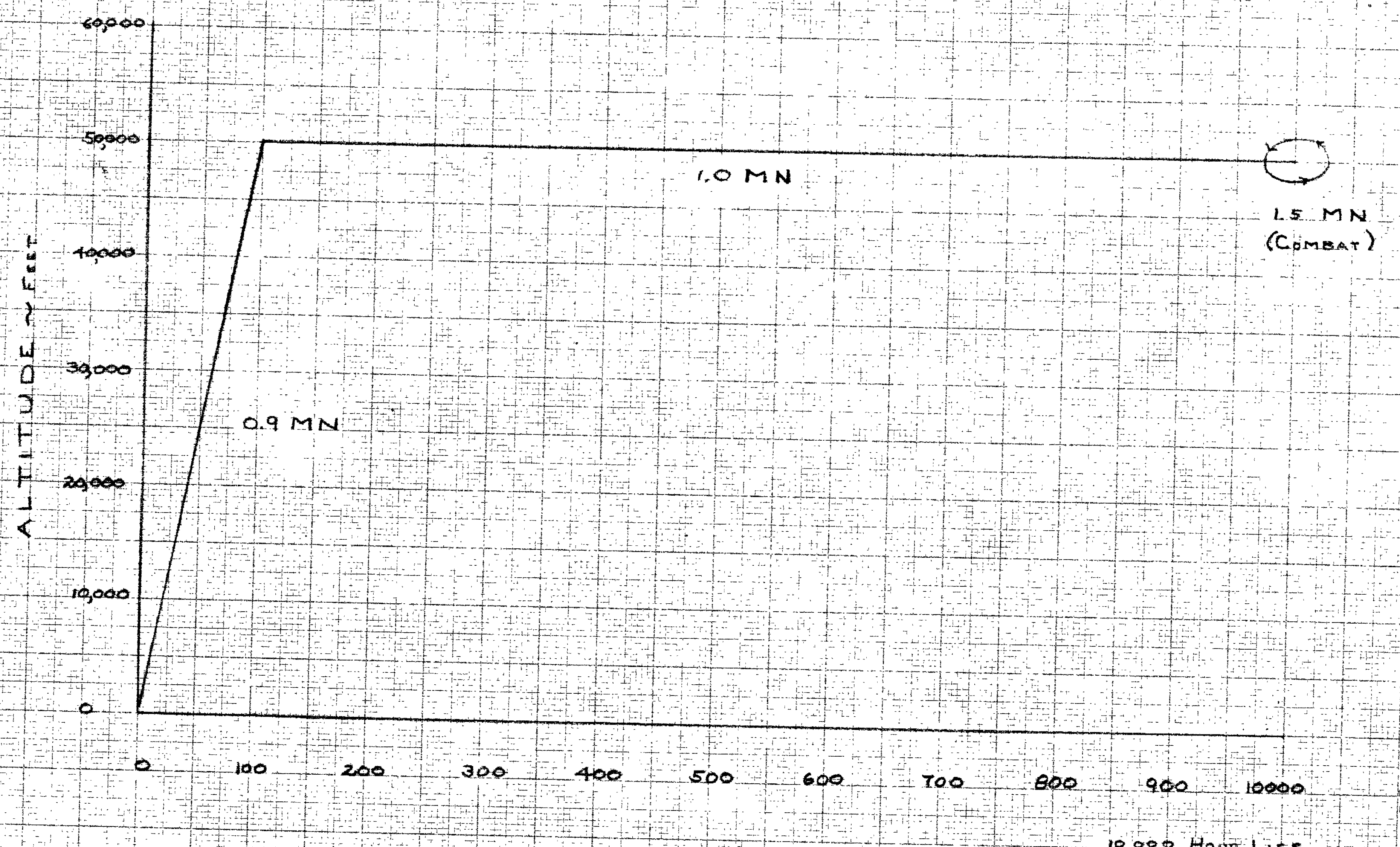

Fig. 8 --DISTANCE ~ STATUE MILES

TABLE II

SUMMARY OF SPEEDS, DISTANCES, AND TIMES FOR A TYPICAL LONG-RANGE INTERCEPTOR MISSION

Phase	Altitude Interval 1000 ft.	Mean Altitude 1000 ft.	MN	Ve kts.	Ve mph.	Ve ft./sec.	Miles per Mission	Hours per Mission
Climb and Descent	0-10	5	.9	584	673	987	40	.059
	10-20	15	.9	563	649	951	40	.061
	20-30	25	.9	541	623	914	40	.064
	30-40	35	.9	518	597	875	40	.067
	40-50	45	.9	517	596	874	40	.067
Cruise-Out and Back	50	50	1.0	575	662	971	1800	2.719
Combat or Inter-cept	50	50	1.5	862	993	1457	331	.333
Total							2331	3.370

TABLE III

CALCULATION OF GUST CYCLE OCCURRENCES PER MISSION IN EACH FLIGHT PHASE, ALTITUDE INTERVAL, AND AIRSPEED INTERVAL

Ude Interval feet/second	Phase	Altitude Interval 1000feet	Ve kts.	Miles per Mission (Statute Miles)
0-15	Climb and Descent	0-10	584	40
		10-20	563	40
		20-30	541	40
		30-40	518	40
		40-50	517	40
	Cruise	50	575	1800
	Combat	50	862	331
15-25	Climb and Descent	0-10	584	40
		10-20	563	40
		20-30	541	40
		30-40	518	40
		40-50	517	40
	Cruise	50	575	1800
	Combat	50	862	331
25-35	Climb and Descent	0-10	584	40
		10-20	563	40
		20-30	541	40
		30-40	518	40
		40-50	517	40
	Cruise	50	575	1800
	Combat	50	862	331

TABLE III --Continued

Δn for Mean Ude	Number Gusts per Mile for Mean Ude	Occurrences per Mission which = Mean Ude
.62	9.93×10^{-1}	3.97×10^{1}
.60	9.97×10^{-1}	3.99×10^{1}
.58	4.48×10^{-1}	1.79×10^{1}
.55	2.49×10^{-1}	9.96×10^{0}
.55	4.95×10^{-3}	1.98×10^{-1}
.61	1.99×10^{-3}	3.58×10^{0}
.92	1.99×10^{-3}	6.59×10^{-1}
		$\Sigma = 111.919 \times 10^{0}$
1.24	6.85×10^{-3}	2.74×10^{-1}
1.20	2.46×10^{-3}	9.84×10^{-2}
1.15	1.16×10^{-3}	4.64×10^{-2}
1.10	5.05×10^{-4}	2.02×10^{-2}
1.10	4.15×10^{-5}	1.66×10^{-3}
1.22	4.53×10^{-6}	8.15×10^{-3}
1.84	4.53×10^{-6}	1.50×10^{-3}
		$\Sigma = 4.503 \times 10^{-1}$
1.86	4.81×10^{-4}	1.92×10^{-2}
1.80	2.71×10^{-4}	1.08×10^{-2}
1.73	1.51×10^{-4}	6.04×10^{-3}
1.65	7.58×10^{-5}	3.03×10^{-3}
1.65	2.24×10^{-6}	8.96×10^{-5}
....	....	....
....	....	....
		$\Sigma = 3.916 \times 10^{-2}$

TABLE III --Continued

Ude Interval feet/second	Phase	Altitude Interval 1000feet	Ve kts.	Miles per Mission (Statute Miles)
35-45	Climb and Descent	0-10	584	40
		10-20	563	40
		20-30	541	40
		30-40	518	40
		40-50	517	40
	Cruise	50	575	1800
	Combat	50	862	331
45-55	Climb and Descent	0-10	584	40
		10-20	563	40
		20-30	541	40
		30-40	518	40
		40-50	517	40
	Cruise	50	575	1800
	Combat	50	862	331
55-65	Climb and Descent	0-10	584	40
		10-20	563	40
		20-30	541	40
		30-40	518	40
		40-50	517	40
	Cruise	50	575	1800
	Combat	50	862	331

TABLE III --Continued

Δn for Mean Ude	Number Gusts per Mile for Mean Ude	Occurrences per Mission which $\geq$ Mean Ude
2.49	5.25×10^{-5}	2.10×10^{-3}
2.40	5.25×10^{-5}	2.10×10^{-3}
2.30	3.05×10^{-5}	1.22×10^{-3}
2.20	1.33×10^{-5}	5.32×10^{-4}
....	....	....
....	....	....
....	....	....
		$\Sigma = 5.952 \times 10^{-3}$
3.11	1.23×10^{-5}	4.92×10^{-4}
2.99	1.23×10^{-5}	4.92×10^{-4}
2.88	6.40×10^{-6}	2.56×10^{-4}
2.76	2.54×10^{-6}	1.03×10^{-4}
....	....	....
....	....	....
....	....	....
		$\Sigma = 1.343 \times 10^{-3}$
3.73	2.71×10^{-6}	1.08×10^{-4}
3.59	2.71×10^{-6}	1.08×10^{-4}
3.45	1.61×10^{-6}	6.44×10^{-5}
....	....	....
....	....	....
....	....	....
....	....	....
		$\Sigma = 2.804 \times 10^{-4}$

TABLE IV

SUMMARY OF NET GUST AND CUMULATIVE CYCLES
PER LIFE IN EACH GUST
VELOCITY INTERVAL

Ude Interval	Net Cycles Occurring in Ude Interval	Total Cycles which = or exceed Lower Ude
0-15	33128.024	33275.150
15-25	133.292	147.126
25-35	11.591	13.834
35-45	1.762	2.243
45-55	.398	.481
55-65	.083	.083

TABLE V

SUMMARY OF NET GUST AND CUMULATIVE CYCLES
PER LIFE IN EACH LOAD FACTOR INTERVAL

Δn Interval	Net Cycles Occurring in Δn Interval	Total Cycles which = or exceed Lower Δn
.50- .75	32932.96	33275.02
.75-1.00	195.06	342.06
1.00-1.25	132.73	147.00
1.25-1.50	......	14.27
1.50-1.75	2.71	14.27
1.75-2.00	9.32	11.56
2.00-2.25	.16	2.24
2.25-2.50	1.60	2.08
2.50-2.75	....	.48
2.75-3.00	.25	.48
3.00-3.25	.15	.23
3.25-3.50	.02	.08
3.50-3.75	.06	.06

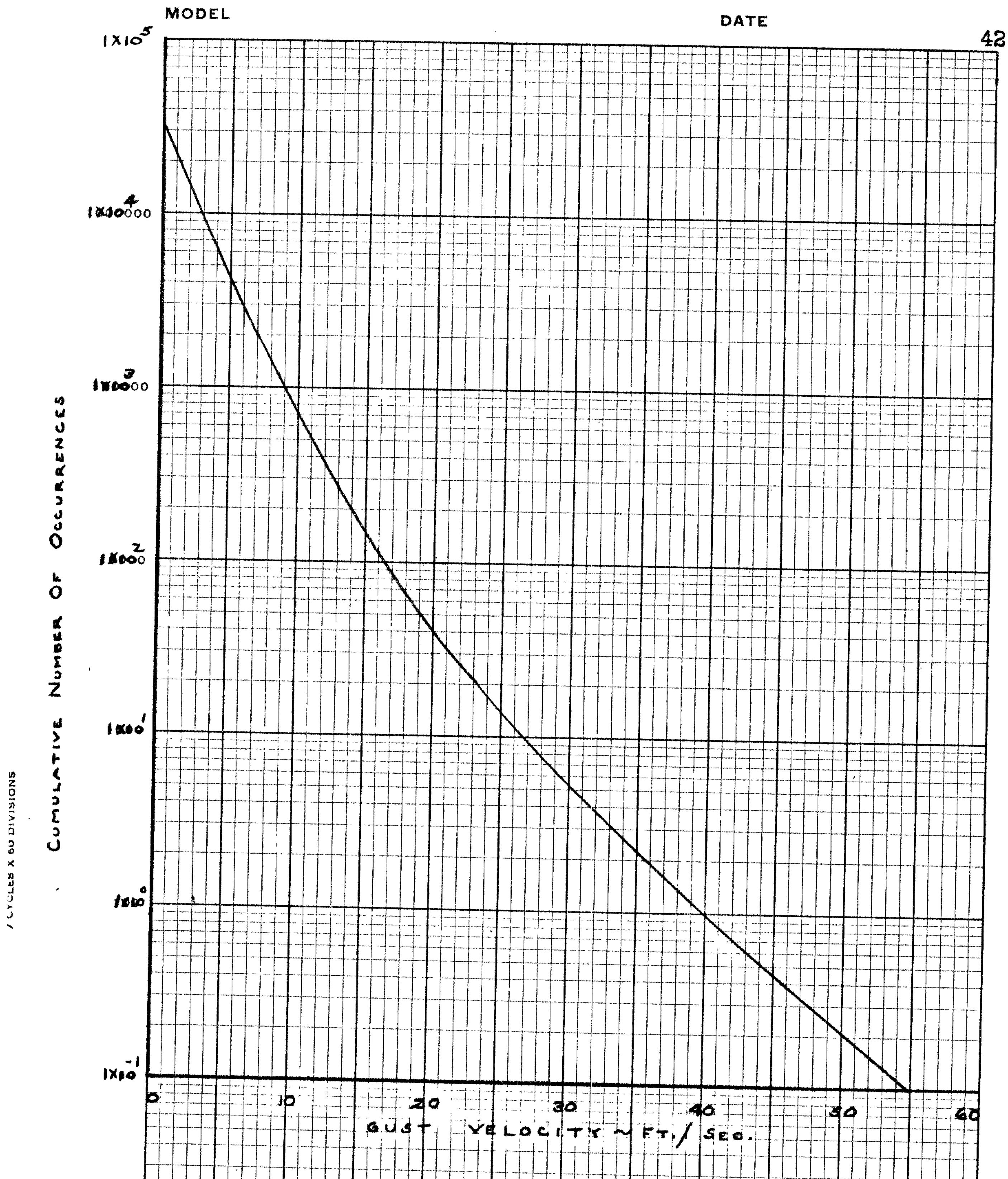

Fig. 9--NUMBER OF OCCURRENCES EQUALLING OR EXCEEDING A GIVEN VALUE OF GUST VELOCITY

MODEL DATE

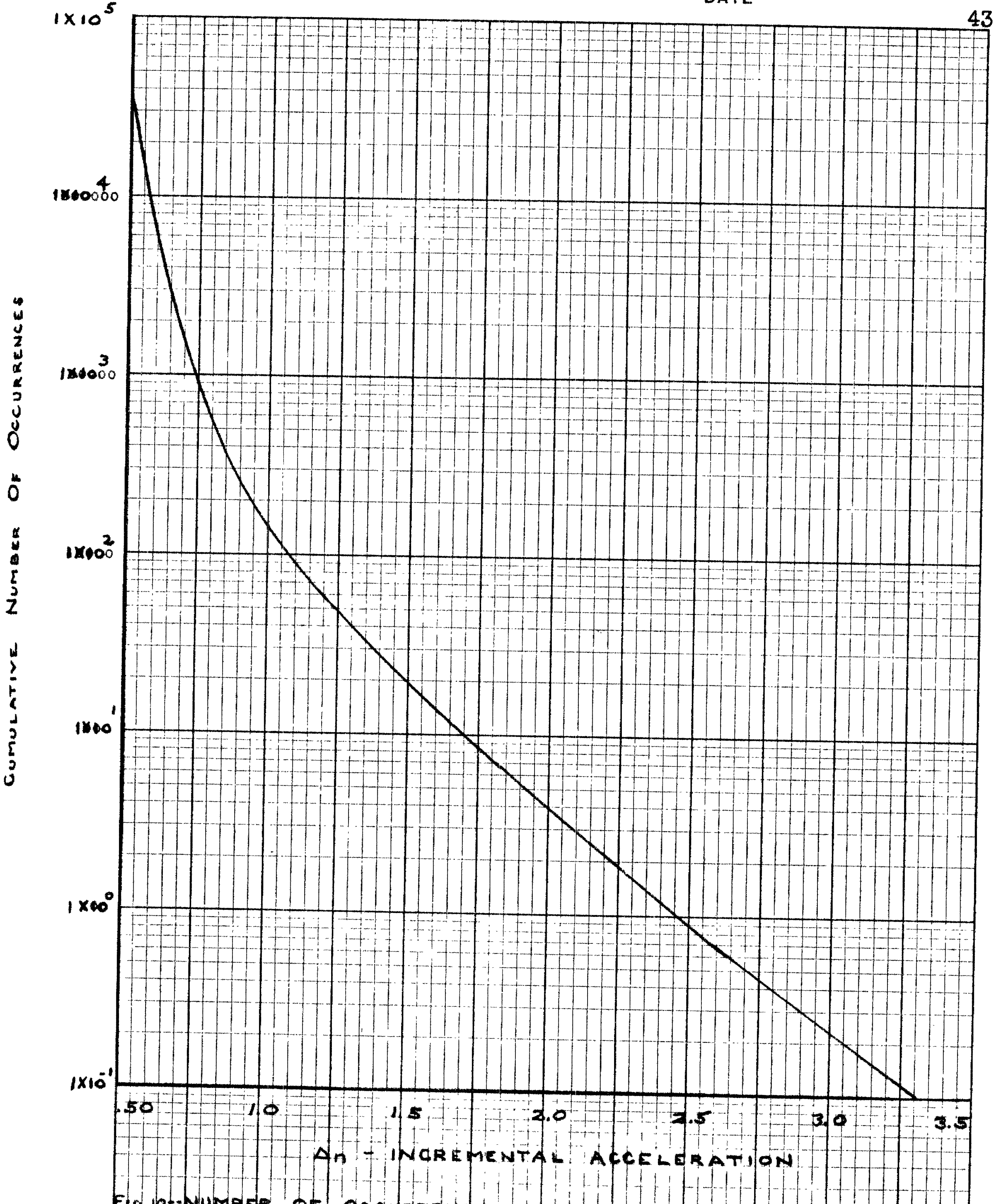

Fig. 10--NUMBER OF OCCURRENCES EQUALLING OR EXCEEDING A GIVEN VALUE OF INCREMENTAL ACCELERATION

incremental acceleration interval is presented in Table V. The number of normal accelerations per life, due to gusts, equalling or exceeding a given "g" level is presented as Figure 10 (1).

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