SCATTERING CROSS-SECTIONS OF

4/1 PROLATE SPHEROIDS

APPROVED:

L. J. [Signature]
Major Professor

[Signature]
Minor Professor

[Signature]
Minor Professor

[Signature]
Director of the Department of Physics

[Signature]
Dean of the Graduate School
SCATTERING CROSS-SECTIONS OF
4/1 PROLATE SPHEROIDS

THESIS

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By

Keith W. Tompkins, B. A.

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CHAPTER I

INTRODUCTION

This is a report of the second in a projected series of experiments at North Texas State College designed for obtaining information about the microwave scattering properties of various shaped objects. The background leading up to and the justification for this work have been thoroughly outlined by Rhoads (2).

The electromagnetic scattering properties of an object are customarily described in terms of the scattering (or radar) cross-section defined from the radar range equation (1):

\[ P_r = \sigma \cdot P_t \frac{G^2 \lambda^2}{(4\pi)^3 R^4} \]

where

- \( P_r \) = signal power received by reflection
- \( P_t \) = signal power transmitted
- \( G \) = gain of the antenna
- \( \lambda \) = wavelength
- \( R \) = distance to target
- \( \sigma \) = radar or scattering cross-section of the target.

The radar cross-section, \( \sigma \), may be interpreted as the fictitious area intercepting that amount of power which, when
scattered isotropically, produces an echo at the source of radiation equal to that observed from the target. This factor, $O'$, having the units of area, is the most important parameter characterizing the scattering from an object. It should be noted that quite generally $O'$ is a function of the geometrical shape of the object, its orientation in space, the polarization of the incident radiation, the material of the model (if not near perfectly conducting), and the wavelength.

In principle, at least, $O'$ for simple shapes can be computed beginning with Maxwell's Equations (1). However, for even the simplest shape, the sphere, this computation is very tedious and to date has only been carried out for a limited range of parameters. The exact computation of $O'$ for a perfectly conducting sphere is summarized graphically in Figure 1 (1). Less than two dozen points for a similar graph for 10/1 prolate spheroids have been computed, but one can hardly draw a meaningful curve through these points (3).

There are several approximate methods available for solving scattering problems. Rayleigh's scattering law applies when the wavelength is large compared with the dimensions of the scatterer (4). The Rayleigh region is indicated to the left in Figure 1. Application of this law is limited practically to meteorological studies and the scattering of light. As the wavelength of the transmitted signal increases in length and approaches the dimensions of the scatterer, a resonance effect is set up due to the conducting currents on
the surface of the scatterer. (See Figure 1.) In this region it is possible to have the cross-section varying between quite large and small values, depending on whether the conducting currents are of a constructive or destructive nature. To date there are no general approximation techniques applicable to the resonance region. As the wavelength of the transmitted signal becomes small compared to the dimensions of the scatterer, the cross-section of the scatterer approaches some constant value (1). It is in this region that the laws of geometrical optics apply. Any perfectly conducting scatterer, if held fixed in one position and if irradiated with a signal whose wavelength can be varied from much smaller than the dimensions of the scatterer to much larger than those of the scatterer, should give a plot similar to that shown in Figure 1. The length of the models used in this experiment varied from about one wavelength of the transmitted signal to seven or eight wavelengths. The cross-section of each should fall in the resonance region.
BIBLIOGRAPHY FOR CHAPTER I


CHAPTER II

THE EXPERIMENTAL PROBLEM

To date there has been no serious effort put forth to collect large amounts of systematic data on the scattering properties of any simple three-dimensional object other than the sphere. Not only is this so, but it remains a fact that at the present state of theoretical development, the experimental approach is the most accurate available. Hence, a large amount of experimental scattering data for simple shapes would have a twofold usefulness; first, it would serve as a gauge on the usefulness of presently employed approximate mathematical techniques (geometrical optics), especially in defining more precisely their range of validity; second, a large mass of such experimental evidence would no doubt be useful as a starting point for precise, even though yet approximate, mathematical methods. This of course, is all in addition to the immediate engineering applications that such data would have.

For several reasons (geometrical simplicity, military interest and the already evident interest of theoreticians) the prolate spheroid has been chosen as the geometry for special emphasis here at North Texas State College. Since practically no systematic data have been heretofore available, a variety of possibilities presented themselves for initial study.
Rather arbitrarily it was decided to study the scattering properties of different sized prolate spheroids all having a common eccentricity. An a/b ratio (semi-major axis/semi-minor axis) of four to one was chosen for these initial measurements. Eight models were actually constructed, the shortest being two inches in length with each successive model in the set being one inch longer, and the largest model nine inches. From a purely theoretical standpoint considerably smaller models would also have been useful. However, such models would have been difficult to construct and practically impossible to "see" with the present apparatus.

The experimental problem involved setting up the models to be studied in the field of the transmitted signal and measuring the back-scattered field as the model was rotated through ninety degrees. A relative indication of how the cross-section varied with orientation was thus obtained. A standard spherical reflector was used to provide absolute calibration.

There were limitations to the experimental procedure which had to be understood before any experimental observations could be made. It was quite certain that the reflected power would be very small for certain orientations of the models. It was necessary, therefore, to position the model sufficiently near the antenna to be able to obtain a reflected signal that was greater than the noise level of the receiving system. However, placing the models very near the
antenna produces a variation in both phase and amplitude of the incident field on the model. Either of these effects gives a cross-section which is a function of range. The usual criterion (1) that \( R \geq 2d^2/\lambda \) was not strictly adhered to. However, experiment indicated that likely range was an unimportant factor for all except the largest model.

A block diagram of the instrumentation is shown in Figure 2. With a few exceptions this is the same as that used by Rhoads (2). A Varian model X-13 reflex klystron was flange-mounted in a small wooden box and insulated with a quantity of pyrex glass wool. The signal from the klystron was coupled through standard wave guide fittings to a slide screw tuner and ferrite load isolator, providing greater than 15\( \text{db} \) isolation between klystron and load. This heavy effective padding plus the fact that the klystron was run at only six watts input gave sufficient stability for our purposes without automatic compensating circuits. A flap attenuator followed the load isolator to provide for adjustments in the transmitted power level. Next was placed a ferrite modulator to modulate the transmitted signal at 1,000 cycles. Following the modulator was a 20\( \text{db} \) directional coupler which provided a one per cent sample of the transmitted signal for power monitoring and frequency checks.

The transmitted signal was fed into the H-arm of a hybrid tee arranged as a standard duplexer (1), with the
receiver connected to the E-arm. The antenna and slide screw tuner were connected to the same side of the tee, rather than placing the tuner in the matching load arm, as is customary. This gave greater mechanical stability to the tee in the particular set-up. With the transmitted power at approximately five milliwatts, 73 db isolation was obtained in the tee between transmitter and receiver. Reflected signals appearing in the receiver arm of the tee were detected in a tuned crystal mount, the 1,000-cycle modulation being removed in the detecting process and fed into a sharp-tuned VSWR amplifier.

The experimental set-up atop the Science Building is shown in Figures 3 and 4.
BIBLIOGRAPHY FOR CHAPTER II


CHAPTER III

CONSTRUCTIONAL DETAILS

The models used in this experiment were constructed from hardwood and accurately shaped on a metal lathe. They were given several coats of shellac, spray painted with Du Pont # 4548 conducting paint, and finally copper plated. The Du Pont paint was very satisfactory as a conducting coating, but offered practically no resistance to scratching. Hence, the copper plate was added to give a hard surface. A photograph of the models appears in Figure 5.

The tower and turntable used to support and rotate the models during measurements are shown in Figure 6. This is a modification of an arrangement constructed and described by Rhoads (1) the principal modification being in replacing the original styrofoam legs by the guy wires shown in the figure. This modification was to produce greater stability, though at the expense of an increased reflectivity of the tower. This increased reflectivity was no handicap, however (as explained in Chapter IV) since automatic rotation and recording were not employed.

The transmitting and receiving horn was that constructed by Rhoads (1), an optimum horn forty wavelengths long. At a
range of 200 centimeters, where the measurements were made, the signal was down 3 db at ± 21 centimeters off-axis.
BIBLIOGRAPHY FOR CHAPTER III

CHAPTER IV

MEASUREMENT PROCEDURES

To assure that thermal equilibrium had been reached in the klystron, the equipment was turned on several hours before any measurements were made. The beam supply voltage was maintained at 300 volts, which gave a cathode current of twenty milliamperes. The reflector voltage was then tuned for the mode which produced maximum frequency stability. The slide screw tuner following the klystron was then adjusted for maximum power into the line.

With the styrofoam tower in place, but no model inserted, the hybrid tee was balanced by adjusting the turnable load to give minimum cross-coupled signal into the receiver arm. This procedure was a combination process cancelling out the small reflections from the throat and mouth of the horn and from the tower. It was possible by careful tuning to keep the signal in the receiver arm within the noise level of the VSWR amplifier (about minus 71 dbm). The tower had been previously aligned by sighting through the wave guide and horn. A standard sphere, two inches in diameter was then placed in the tower and the receiver indication noted on the db scale of the VSWR amplifier. On removing the sphere, the model to be tested
was then placed in the tower and the receiver indication noted and recorded. The model was then removed and the tower manually rotated two degrees. Since the reflection from the tower was not uniform in all positions, the tee was rebalanced each time the position of the tower was changed. The model was again placed in the tower and the receiver indication at this position noted. These steps were repeated until the model had been viewed through ninety degrees. This same procedure was followed for each of the models. As was done before testing a model, the reflected power of the standard sphere was determined at the end of each test.

The transmitted wavelength was periodically checked to be sure that it did not change during a test.

Since the receiver VSWR meter was calibrated for an accurate square law detector, it was necessary to run a check of the scale readings with the particular crystal in use. Comparison was made with a calibrated attenuator which showed an error of only 2 db over a range of 40 db. Hence, no corrections on the scale readings were made as it was felt that errors greater than this were introduced by other uncontrollable factors.
CHAPTER V

ANALYSIS OF DATA

Plots of the data collected in this experiment are shown in Figures 7 and 8. The data for the nine-inch model were taken at a distance of ten feet and five inches from the mouth of the horn antenna to the model; whereas the data for the rest of the models were taken at a distance of six feet and six inches. This increase in distance was necessary with the nine-inch model in order to get sufficiently uniform illumination over the model.

The level of the calibrating sphere is indicated on each plot by the small dash on the vertical axis. The radius of the sphere was one inch and, for the wavelength used in this experiment ($\lambda = 3.26$ centimeters), the cross-section of the sphere was 4.08 square inches as read from Figure 1.

The largest model measured was seven wavelengths long and, thus, it should not be anticipated that the approximations of geometrical optics would be strictly applicable to any of the models. Nonetheless, one of the first questions of interest is just that: how well do the measured cross-sections compare with the predictions of geometrical optics? Two cases are considered—broadside and nose-on.
The broadside cross-section approximation for the prolate spheroid is \( \sigma = \pi a^2 \) (1). Table I gives the theoretical and experimental values of the models tested. The experimental cross-sections for every model were found to be low. This is shown in another way in Figure 9. The behavior of the sphere (see Figure 1) suggests that these data should be scattered around \( \sigma/\pi a^2 = 1 \). The fact that every measured point falls below this line could be due to a quite fortuitous choice of model sizes, but most likely some systematic error was involved. Non-uniform illumination, for example, would be expected to produce lower cross-sections than uniform illumination. This particular problem needs further investigation because no positive information is yet available on the variation of \( \sigma \) with range for even simple models. Most "intuitive" criteria are based on the known variations of antenna gain with range.

Nose-on cross-section data for all the models are summarized in Table II and in Figure 10. One would expect the points in Figure 10 to be scattered around \( \sigma/\pi \frac{b^2}{a^2} = 1 \), that is, even in the resonance region the cross-section is expected to be oscillatory about the geometrical optics value. It appears that the average of the measured values here is high. However, this may truly be only apparent and not real for at least two reasons. First of all, the nose-on cross-sections in all cases are quite small, in some
cases right at the receiver noise level, and in one case not measurable at all. Consequently, these small measurements are the most uncertain of any made, and the measurements in the presence of extraneous signals would tend to give higher than normal receiver outputs. Then secondly, the proximity of the receiving horn to the models was such as to produce an averaging of the data over two degree intervals, lopping off high peaks and filling in the nulls. Any sharp null at nose-on would then most likely be averaged out.

On the other hand, there is a very real reason why one might expect the nose-on measurements to be high for the larger models. The calibrating sphere was always placed at the position of the geometric center of the models. However, the nose of each model was somewhat closer to the horn than this in the nose-on position. At very large ranges this difference is insignificant—for our measurements it was not. Moreover, it is believed (1) that the principal scattering in this nose-on aspect takes place right at the tip, or at the nose. Conclusive work remains to be done in determining just where to place the calibrating sphere in such instances.

In order to transform the data of Figures 9 and 10 into a curve comparable to Figure 1 would require perhaps ten times the data presented. A complete understanding of spheroid scattering, however, awaits just such an undertaking.
BIBLIOGRAPHY FOR CHAPTER V

FIG. 1.--BACKSCATTERING FROM A PERFECTLY CONDUCTING SPHERE OF RADIUS R
INSTRUMENTATION BLOCK DIAGRAM

FIG. 2--INSTRUMENTATION BLOCK DIAGRAM
Fig. 3—Instrumentation
Fig. 4--Instrumentation
Fig. 6--Tower and Turntable
FIG. 7 -- PLOT OF DATA
FIG. 8--PLOT OF DATA
FIG. 9--MEASURED BROADSIDE CROSS-SECTIONS FOR 4/1 PROlate SPHERoIDS
FIG. 10—MEASURED NOSE-ON CROSS-SECTIONS FOR 4/1 PROLATE SPHEROIDS
TABLE I

SUMMARY OF DATA
(BROADSIDE)

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<th>Dimensions of Model in Inches (a x b)</th>
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TABLE II

SUMMARY OF DATA (NOSE ON)

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