A Technique for Measuring Parity Nonconservation in Hydrogenic Atoms

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ABSTRACT

A scheme for precise measurement of the parity mixing of the $2s_{1/2}$ and $2p_{1/2}$ states in hydrogen and deuterium atoms is discussed.

MASTER

I. INTRODUCTION

The parity nonconserving (PNC) interactions between electrons and nucleons provide a detailed probe of the neutral current weak interaction. One may, in lowest order, distinguish 4 separate PNC couplings: $g_A^e g_V^p$, $g_V^e g_A^p$, $g_A^e g_V^n$, and $g_V^e g_A^n$ whose strengths (following ref. 1) we denote by C_1^p , C_2^p , C_1^n and C_2^n . Measurements of the parity mixing between specific hyperfine components of the $2s_{1/2}$ and $2p_{1/2}$ levels of hydrogen and deuterium atoms can, in principle, yield all four of the PNC coupling constants with essentially no uncertainty from atomic theory. Note that for deuterium we have

$$C_1^d = C_1^p + C_1^n \quad \text{and} \quad C_2^d = C_2^p + C_2^n$$

If experiments in H and D achieve sufficient accuracy one can resolve some very fundamental issues. In the Weinberg-Salam (WS) model the lowest order expressions for the coupling constants are:

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$$C_1^P = \frac{1}{2} \left(1 - 4 \sin^2 \theta_W \right) \quad C_1^N = -\frac{1}{2}$$

$$C_2^P = \frac{1}{2} g_A \left(1 - 4 \sin^2 \theta_W \right) \quad C_2^N = -\frac{1}{2} g_A \left(1 - 4 \sin^2 \theta_W \right)$$

It would be clearly interesting to measure all four of these constants and check whether they agree with the WS predictions. Existing experiments in heavy atoms^{2,3,4,5} and high-energy electron scattering⁶ determine only certain linear combinations of the constants and significant uncertainties occur in the theoretical interpretations—especially in the heavy atoms. Perhaps even more interesting than checking the lowest order WS predictions are the higher order effects. In the WS model the coefficient C_2^d vanishes identically in lowest order. However, the radiative corrections do not vanish—in fact the second order terms are expected to contribute to C_2^d at the 0.01-0.05 level.⁷ The experimental determination of C_2^d thus directly measures the weak radiative corrections in any model in which the axial weak neutral current is purely isovector! There may also be a contribution to C_2^d from induced isoscalar axial currents due to virtual $s\bar{s}$ quark excitations in the nucleon.⁸

In this article we describe a new technique for measuring the PNC mixing with a sensitivity much greater than that of a method we have described previously.⁹ The relevant $n=2$ levels of hydrogen are shown in Fig. 1 to establish our notation. The PNC interaction can mix the following levels: $\alpha_0 e_0$, $\alpha_0 f_0$, $\alpha_+ e_+$, $\beta_0 e_0$, $\beta_0 f_0$, and $\beta_- f_-$. Due to the $1/r$ Coulomb potential the $\beta_0 e_0$ mixing in hydrogen is sensitive only to C_2^P and in deuterium to C_2^d .^{1,10} Hence measurements of the $\beta_0 e_0$ mixings

in hydrogen and deuterium provide a fundamental test of electroweak interaction theory.

II. GENERAL PRINCIPLES OF THE EXPERIMENT

Several groups^{9,10,11} have proposed microwave schemes for measuring β_0 - e_0 and β -f PNC mixing at level crossings using 500 eV beams of metastable hydrogen. Our new scheme, in common with those proposed in refs. 9 and 10 detects the PNC β_0 - e_0 mixing by driving $\alpha_0 \rightarrow \beta_0$ electric-dipole transitions in a uniform magnetic field of ~ 570 Gauss near the β -e crossings. The transition proceeds predominantly via intermediate e states and we detect the interference between weak interaction and Stark effect β_0 - e_0 mixing. However, in our new scheme the PNC amplitude $A_{\text{PNC}} \begin{pmatrix} R_z & \beta_0 \\ \alpha_0 & e_0 \end{pmatrix} V_{\text{PNC}}^0$ produced in one RF cavity interferes with a parity conserving amplitude $A_{\text{PC}} \begin{pmatrix} R_z & E_z \\ \alpha_0 & \beta_0 \end{pmatrix}$ produced in another cavity. Our signal is the change in the $\alpha_0 \rightarrow \beta_0$ transition rate when the relative phase of A_{PC} and A_{PNC} is changed by 180° . This can be achieved by changing the relative phase of the RF in the two cavities by 180° , by reversing the direction of $\vec{E}$, and by reversing $\vec{B}$. In our notation $R = \langle e | \frac{e\vec{R} \cdot \vec{r}}{2\hbar} | \alpha \rangle$ is the matrix element of a microwave electric field ($\vec{R}$) oscillating at $\omega_{\text{RF}} = 2\pi \cdot 1608 \cdot 10^6 \text{ sec}^{-1}$, $V_{\text{PNC}} = i\tilde{V}_{\text{PNC}} = \langle \beta_0 | \frac{H_{\text{PNC}}}{\hbar} | e_0 \rangle = -0.16iC_2^{\text{P}} \text{ sec}^{-1}$ is the weak matrix element, $E = \langle e | \frac{e\vec{E} \cdot \vec{r}}{\hbar} | \beta \rangle$ is the matrix element of a static electric field ($\vec{E}$), and $\hat{z}$ is the magnetic field axis. (In this paper all frequencies are angular frequencies measured in radians/sec). At our RF frequency the unperturbed $\alpha_0 \rightarrow \beta_0$ resonance occurs at 569.4 gauss. The transition rate is $R_{\alpha_0 \rightarrow \beta_0} \propto |A_{\text{PC}} + A_{\text{PNC}}|^2 \approx |A_{\text{PC}}|^2 + 2\text{Re}|A_{\text{PC}} A_{\text{PNC}}^*|$. Our signal obtained by reversing the relative sign of A_{PC} and A_{PNC} , is $\Delta R = 4\text{Re}|A_{\text{PC}} A_{\text{PNC}}^*|$.

The experimental geometry is shown in Fig. 2. 500 eV protons from a duoplasmatron, converted by charge exchange in Cs vapor into a beam of $H(2s)$, enter a 570 gauss solenoid. The beam passes through a transverse, static $\vec{E}$ field to quench the β levels. The resulting beam of atoms in the α_+ and α_0 states then enters two successive cylindrical RF cavities oscillating coherently in the TM_{010} mode. In each cavity there are RF and static $\vec{E}$ fields along z . Since the RF $\vec{B}$ field vanishes on the cavity axes we shall temporarily neglect M1 transitions between the $2s$ states. The first, shorter, cavity drives A_{PC} . It contains weak RF and strong static electric fields $\vec{R}_z^I$ and $\vec{E}_z^I$ with matrix elements R_z^I and E_z^I , respectively. Therefore the RF quenching of the α states is small and the PNC mixing can be neglected in comparison to the Stark mixing induced by the static $\vec{E}$ field. The second, longer, cavity drives A_{PNC} . The RF in this cavity is strong ($R_z^{II} = \sqrt{\gamma/t_2}$) and is shifted in phase with respect to that in the first cavity. The weak static $\vec{E}$ field along z (with matrix element E_z^{II}) drives a probe transition, $A_P \begin{pmatrix} \alpha_0 & \xrightarrow{R_z} & e_0 \\ & \xrightarrow{E_z} & \beta_0 \end{pmatrix}$. The purpose of A_P will be discussed below. The beam then passes through a cavity containing a perpendicular RF $\vec{E}$ field oscillating at $2\pi \cdot 2143 \cdot 10^6 \text{ sec}^{-1}$ which depopulates the α_+ and α_0 levels by α -f mixing. The remaining β states are detected by passing the beam through a static perpendicular $\vec{E}$ field. We determine the $\alpha_0 \rightarrow \beta_0$ transition rate by detecting the Lyman radiation from this β -e mixing.

The main features of our scheme can be understood by treating E_z^I , E_z^{II} , R_z^I , R_z^{II} , and V_{PNC} as perturbations and neglecting intermediate f states. We assume $R_z^{II} \ll \gamma$, $E_z^I \ll \gamma$, $E_z^{II} \ll R_z^{II}$, and $R_z^I \ll E_z^I$, where γ is the

$2p \rightarrow 1s$ decay rate. Then at the $\alpha_0 \beta_0$ resonance, an atom initially in the α_0 state which spends times t_1 and t_2 in cavities 1 and 2, respectively, has a probability to be in the β_0 state given by:

$$|\beta|^2 = \left| -\frac{0.11E_z^I R_z^I}{\frac{\gamma}{2} - i\omega_{\beta_0 e_0}} \left(\frac{1 - e^{-\Gamma_1 t_1}}{\Gamma_1} \right) - \frac{\left(i\tilde{V}_{PNC} + 0.11E_z^{II} R_z^{II} e^{i\phi} e^{-\Gamma_2 t_1} \right) \left(\frac{1 - e^{-\Gamma_2 t_2}}{\Gamma_2} \right)}{\frac{\gamma}{2} - i\omega_{\beta_0 e_0}} \right|^2$$

where $\hbar\omega_{\beta_0 e_0} = E_{\alpha_0} - E_{e_0} - \hbar\omega_{RF}$, $\hbar\omega_{e\beta} = E_{e_0} - E_{\beta_0}$, $\hbar\delta = E_{\alpha_0} - E_{\beta_0} - \hbar\omega_{RF}$,

$$\Gamma_1 = \frac{\left(E_z^I\right)^2}{\frac{\gamma}{2} + i\omega_{e_0 \beta_0}} + i\delta, \quad \text{and} \quad \Gamma_2 = \frac{\left(R_z^{II}\right)^2}{\frac{\gamma}{2} - i\omega_{\alpha_0 e_0}} + i\delta$$

and we have ignored any drift space between the two cavities. The relative phase of the oscillations in the two cavities has been included by defining R_z^I to be real and setting $R_z^{II} = \tilde{R}_z^{II} e^{i\phi}$ where ϕ is the relative phase.

The phase ϕ is set to make the interference of A_{PNC} and A_{PC} as large as possible (A_{PNC} and A_{PC} relatively real). This can be achieved by requiring the interference of A_{PC} with A_p to vanish (since A_p and A_{PNC} are 90° out of phase). The angle ϕ is controlled as follows. The weak static $\vec{E}$ field in the second cavity is modulated at frequency ω_m and lock-in techniques are used to detect any component of the β state signal which

occurs at a frequency ω_m . This component arises from A_{PC} , A_P interference. It vanishes if ϕ has been set correctly and is used to control an electronic phase-shifter in a feedback loop.

V_{PNC} can be measured by detecting the change in the counting rate of β_0 states upon the independent reversal of the solenoid field ($R_z^I \rightarrow -R_z^I$, $R_z^{II} \rightarrow -R_z^{II}$, $E_z^I \rightarrow -E_z^I$), the Stark mixing field ($E_z^I \rightarrow -E_z^I$) or the RF phase $\phi \rightarrow \phi + 180^\circ$. This change is

$$\Delta|\beta|^2 = 4\text{Re} \frac{0.11E_z^I R_z^I \left(iV_{PNC} + 0.11E_z^{II} \right) R_z^{II} e^{i\phi - \Gamma_2 t_1}}{\left(\frac{\gamma}{2} - i\omega_{\beta_0 e_0} \right) \left(\frac{\gamma}{2} + i\omega_{\beta_0 e_0} \right)} \times \frac{1 - e^{-\Gamma_1^* t_1}}{\Gamma_1^*} \frac{1 - e^{-\Gamma_2 t_2}}{\Gamma_2}$$

In Section III we shall discuss an additional subtraction involving δ .

The results of a numerical calculation modelling our experiment are displayed in Fig. 3. The lengths of the first and second cavities are 12 and 36 cm respectively. For simplicity the fields $\vec{E}_z^I$, $\vec{E}_z^{II}$, and $\vec{R}_z^I$ and $\vec{R}_z^{II}$ are assumed to turn on and off abruptly and the magnitudes are roughly optimized for the ideal case of a negligible background in the Lyman α detectors. Our present ion source produces a metastable beam intensity of ~ 1 particle μm^2 in each hyperfine state into a solid angle of 2×10^{-5} sr and we have constructed a β -state detector with a total efficiency of $1/8$. With this intensity a statistical error in C_2^P of ~ 0.16 can be reached in one day of running time.

III. SYSTEMATIC ERRORS

The various schemes^{9,10,11} which have been suggested to study PNC effects in H(2s) are roughly equivalent as regards statistical errors since for a given metastable atom flux and counting time, the signal/noise ratio is proportional to $V_{\text{PNC}} \sqrt{\frac{T}{\gamma}}$ where T is the time the beam spends on the interaction region. Our scheme is a little more favorable than that of ref. 10 since we have a larger effect (A_{PC} and A_{PNC} are relatively real) and β 's produced by V_{PNC} are not quenched by transverse $\vec{E}$ fields.

However, the different schemes have quite different sensitivities to systematic errors. These are dominated, in decreasing order of importance, by 1) motional $\vec{E}$ fields, $\vec{E}_m = \frac{\vec{v}}{c} \times \vec{B}$, 2) misalignment and non-uniformity of the applied fields, 3) stray electric fields, and 4) M1 transitions.

It is convenient to express the $\alpha_0 \rightarrow \beta_0$ amplitudes as scalar products of the external fields acting on the H(2s) atoms. In this notation

$$A_{\text{PNC}} = 0.5i\tilde{V}_{\text{PNC}} V_0 (\vec{R}^{\text{II}} \cdot \hat{B}) t_2 / D_0 ,$$

$$A_{\text{P}} = 0.055V_0^2 (\vec{R}^{\text{II}} \cdot \hat{B} \vec{E}^{\text{II}} \cdot \hat{B}) t_2 / D_0 ,$$

and

$$A_{\text{PC}} = 0.055V_0^2 (\vec{R}^{\text{I}} \cdot \hat{B} \vec{E}^{\text{I}} \cdot \hat{B}) t_1 / D_0$$

$$\text{where } D_0 = \left(\frac{\gamma}{2} - i\omega_{\beta_0} e_0 \right) .$$

One must realize that the states α_0 and β_0 can also be connected by the amplitudes

$$A_M = 0.055iU_o (\vec{M}^{II} \cdot \hat{B}) t_2 ,$$

$$A_{J_1} = -0.0275V_o^2 (\vec{R}^{II} \times \hat{B}) (\vec{E}^{II} \times \hat{B}) t_2 / D_+ ,$$

and

$$A_{J_2} = -0.0275iV_o^2 (\vec{R}^{II} \times \vec{E}^{II} \cdot \hat{B}) t_2 / D_+$$

where $\vec{M}$ is the RF magnetic field with matrix element $M = \langle \beta | \frac{\vec{\mu} \cdot \vec{M}}{2\hbar} | \alpha \rangle$, $\hat{B}$ is a unit vector along $\vec{B}$ and $D_+ = \left(\frac{\gamma}{2} - i\omega_{\beta_o} e_+ \right)$. The constants V_o and U_o have numerical values $V_o = \frac{\sqrt{3} e a_o}{\hbar} = 13.93 \times 10^6 \text{ sec}^{-1}/V(\text{cm}^{-1})$ and $U_o = \frac{\mu_o}{\hbar} = 8.79 \times 10^6 \text{ sec}^{-1}/\text{gauss}$. We concentrate on the $\alpha_o \rightarrow \beta_o$ amplitudes in the second cavity since the sum of all $\alpha_o \rightarrow \beta_o$ amplitudes in the first cavity can be combined into an overall amplitude A_I which then interferes with the various amplitudes in the second cavity.* Let us decompose all the $\vec{E}$ and $\vec{B}$ fields into applied, fringing, motional and stray components. For example in an obvious notation $\vec{E} = \vec{E}_a + \vec{E}_f + \vec{E}_m + \vec{E}_s$ where $\vec{E}_m = \frac{\vec{v}}{c} \times \vec{B}$. The matrix elements involving the fields are listed in Table I.

In our geometry the applied external fields, $\vec{E}_a$, $\vec{E}_a^I$, $\vec{R}_a^I$ and $\vec{R}_a^{II}$ are ideally parallel to $\hat{B}$ and the symmetry axis of the atomic beam also lies along $\hat{B}$. This is possible in an apparatus which is perfectly aligned but which contains fringing fields and a finite emittance of the atomic beam. If the alignment were ideal in this sense all the spurious amplitudes listed in Table I either vanish by cylindrical symmetry or are

*This is strictly correct only if the various amplitudes in the final cavity are constant over the cross section of the beam.

cancelled by the reversals of $\vec{B}$ and ϕ . However, the alignment cannot be perfect. In the following discussion we make a first order approximation—i.e., we assume the various fields are constant but not colinear. Let the angle between $\vec{R}^{II}$ and $\hat{B}$ be θ_R and between $\vec{E}^{II}$ and $\hat{B}$ be θ_E . It is easy to see that both A_{J_1} and A_{J_2} are proportional to $\theta_E \theta_R$ —i.e., second order in small quantities. In addition $A_M \propto \theta_R$ since $M \perp R$ in the TM_{010} mode and $\vec{R} \parallel \hat{B}$. Furthermore the induced motional $\vec{M}$ seen in the rest frame of the atoms $\vec{M}_m = -\frac{\vec{v}}{c} \times \vec{R}$ is also small since $\vec{v}$ lies along $\vec{R}$. As we shall see later the M1 contributions are small for other reasons as well. This favorable situation should be contrasted with those occurring in the schemes discussed in refs. 9, 10, and 11 all of which involve applied transverse fields and hence contain systematic errors which are first order in misalignment angles.

The most important systematic error involves the motional field. At 500 eV a H(2s) atom travelling at an angle of 1 milliradian to a 570 gauss B field experiences a field $E_m \approx 0.18$ V/cm, while for $C_2 = 0.05$ (the value predicted by the WS theory with $\sin^2 \theta_w = 0.23$) the weak matrix element is equivalent to a field of 5×10^{-9} V/cm. The motional field contribution to the amplitudes A_{J_1} and A_{J_2} are

$$A_{J_1}^m = -8.25 V_o^2 R \cdot \left(\frac{\vec{v}}{c} \right) \times \vec{B} \cdot \vec{t}_2 / D_+$$

and

$$A_{J_2}^m = -i 8.25 V_o^2 \left[\left(\frac{\vec{v}}{c} \cdot \hat{B} \right) (\vec{R} \cdot \vec{B}) - |B| \left(\vec{R} \cdot \frac{\vec{v}}{c} \right) \right] t_2 / D_+$$

Under reversal of the direction of $\vec{B}$

$$A_{J_1}^m \rightarrow -A_{J_1}^m \quad \text{while} \quad A_{J_2}^m \rightarrow +A_{J_2}^m$$

Therefore the $A_{J_2}^m$ term is greatly reduced by the subtraction associated with the reversal of $\vec{B}$, while $A_{J_1}^m$ is not affected.

Since $A_{J_1}^m$ is responsible for the most serious systematic errors in our scheme it is worth considering in some detail. If $\vec{R}$ were always parallel to $\hat{B}$, $A_{J_1}^m$ would vanish identically. However because of fringing fields at the ends of the cavity an atom travelling parallel to $\hat{B}$ a distance r from the axis of the cavity experiences an unavoidable $R_{\perp}$ from each end given by $\int R_{\perp} dt / \int R_z dt = \frac{r}{2L}$ where L is the length of the cavity. If the atomic beam were not quenched by RF quenching and if each atom in the beam had a constant value r , then on resonance, the effects of the two ends would cancel on an atom by atom basis. But since neither of these conditions is satisfied the cancellation is ineffective. On the other hand, if $\vec{R}$, $\hat{B}$, and the atomic beam have cylindrical symmetry around a common axis $A_{J_1}^m$ does vanish on an atom by atom basis.

To reduce the spurious amplitudes such as $A_{J_1}^m$ to a sufficiently small level one must align $\vec{R}$ and the atomic beam with $\hat{B}$ and minimize the effects of fringing fields. The average values of $R_{\perp}$ and V_m can be efficiently minimized as follows. Set $\vec{B}$ to the $\alpha_o \xrightarrow{R_z} e_o \xrightarrow{E} \beta_-$ resonance at 603.8 gauss. The amplitude for this resonance is $A_{\alpha_o \beta_-} = -0.05 V_o^2 |R_z| |E_{\perp}| t_2 / D_-$ where $D_- = \left(\frac{\gamma}{2} - i\omega_{\beta_- e_o} \right)$. Vary the direction of the atomic beam so as to

minimize $A_{\alpha_0 \beta_-}$. This provides a sensitive way to align $\vec{B}$ with the symmetry axis of the atomic beam. Then align the RF cavity with $\vec{B}$ by returning to the $\alpha_0 \rightarrow \beta_0$ resonance. Deliberately apply a strong transverse electric field $\vec{E}_1$. Then vary the alignment of the cavity with respect to $\hat{B}$ to minimize the amplitude $A_{J_2} = -0.0275 V_0^2 |R_1| |E_1| t_2 / D_+$. This procedure greatly magnifies the effects of any misalignment and allows one to measure and correct the misalignment.

The effects of the fringing fields at the cavity ends can be cancelled by running on the sides of the $\alpha_0 \rightarrow \beta_0$ resonance ($\delta \neq 0$). Off resonance the phase of the $\alpha_0 \rightarrow \beta_0$ amplitude depends upon z , the position inside cavity where the transition occurs. Therefore the phases of amplitudes generated in the beginning and end of cavity (e.g., from fringing fields) are different from the phase of an amplitude which is generated throughout the cavity (e.g., A_{PNC}). One can always choose δ in such a way that when $\delta \rightarrow -\delta$ the amplitude driven by fringing fields does not change its phase relative to some constant interfering amplitude (e.g., A_{PC}) while the amplitude driven by the applied field will change sign. For simplicity consider an ideal situation where the fringing fields have negligible width, the applied fields are small and the phase is chosen to be zero if the transition occurs in the beginning of cavity. Then the phases of the amplitudes rising from fringing fields at the beginning and end of the cavity are 0 and $e^{-i\delta t_2}$ respectively while the phase of A_{PNC} is $e^{-i\delta t_2/2}$. So if δt_2 is chosen to be $\pm\pi$ then as $\delta \rightarrow -\delta$, the amplitudes driven by fringing fields do not change their phase but A_{PNC} changes its sign. This can be used to cancel the effects of the fringing fields. The cancellation is limited by an

unavoidable variation in times t_2 due to a spread in the velocities of the atomic beam. A duoplasmatron produces a velocity spread $\frac{\Delta v}{v} \sim 10^{-2}$. This ultimately limits the cancellation to a factor of $\left(\frac{\Delta t}{t}\right)^2 = \left(\frac{\Delta v}{v}\right)^2 \sim 10^{-4}$ even if the frequency shift δ could be chosen exactly.

There is, in principle, a fifth subtraction which can be very useful in further reducing systematic errors. This subtraction is based on the imaginary nature of the weak matrix element V_{PNC} . Time reversal invariance requires that all amplitudes which contain an odd number of T-odd factors ($\vec{B}$'s and $\vec{v}$'s) must also contain a factor of i , for example A_{PNC} contains one factor of $\hat{B}$ and has a factor of i , while A_p contains two factors of $\hat{B}$ and no factor of i . Amplitude with this factor of i will change sign when initial and final states are reversed ($\alpha_o \rightarrow \beta_o$ is changed to $\beta_o \rightarrow \alpha_o$), while those without this factor of i do not. Now compare the effects of the transformations $B \rightarrow -B$ and $\alpha\beta \rightarrow \beta\alpha$. All amplitudes which do not contain an odd power of beam velocity behave similarly under the two transformations. However those amplitudes which do not contain a factor of the beam velocity will transform oppositely under $\vec{B} \rightarrow -\vec{B}$ and $\alpha\beta \rightarrow \beta\alpha$. Hence all motional field amplitudes are either cancelled by the subtraction $\vec{B} \rightarrow -\vec{B}$ or $\alpha\beta \rightarrow \beta\alpha$. This behavior is shown more clearly in Table I. The subtraction $\alpha\beta \rightarrow \beta\alpha$ can be implemented by alternating between RF and static $\vec{E}$ field quenching before the main RF cavity and the L_α detector.

It is obvious that further reduction of the systematic errors would be achieved if one could reduce the velocity v of the atomic beam. Intense thermal beams of H(2s) are being developed.¹³ A thermal beam would reduce all motional field effects by a factor of ≈ 100 . In principle it would

also increase the PNC effect by a factor of $\sqrt{T} \propto \frac{1}{\sqrt{V}} \approx 10$. However to obtain full benefit of this factor of 10 the magnetic field must be uniform and stable to better than 4 parts in 10^5 .

Numerical estimates of the various amplitudes discussed above are presented in Table I for the case of a 500 eV atomic beam without the $\alpha\beta \rightarrow \beta\alpha$ subtraction. It seems likely that a thermal atomic beam or the $\alpha\beta \rightarrow \beta\alpha$ subtraction will be necessary to reach a sensitivity in C_2 as small as that predicted by the WS theory.

IV. SUMMARY

We have described a two cavity microwave experiment for measuring the $2s_{1/2}-2p_{1/2}$ mixing in hydrogen and deuterium atoms at a level crossing. Compared to previously proposed level crossing schemes this experiment has several significant advantages. In our scheme all the applied fields are parallel to $\vec{B}$. This has the desirable feature that the most troublesome systematic errors are proportional to a product of two small misalignment angles. Any two-cavity scheme is inherently superior to the corresponding single cavity scheme because an RF phase reversal can be carried out much more precisely than the reversal of a static field since the RF reversal is not affected by stray fields, etc. In our scheme there are, in principle, 5 different subtractions [$\phi \rightarrow \phi + \pi$, $E \rightarrow -E$, $B \rightarrow -B$, $\delta \rightarrow -\delta$ and $(\alpha \rightarrow \beta) \rightarrow (\beta \rightarrow \alpha)$] which can be used to drive systematic errors to a very low level. Finally, since R_z and E_x (neglecting finite beam emittance) are not both present in the same cavity the very strong, but uninteresting, $\alpha_+ \beta_0$ and $\alpha_0 \beta_-$ resonances do not occur as they do in the schemes of refs. 9 and 10.

This is particularly advantageous for experiments in deuterium where the resonance spacing is much closer than in hydrogen.

One disadvantage of any $\alpha \rightarrow \beta$ scheme at level crossings is the stringent requirement placed on the magnetic field because $E_\alpha - E_\beta$ is such a strong function of B_z . In our case this is reflected in both the narrowness of the PNC resonance (~ 0.2 Gauss FWHM) and the rapid variation of the phase ϕ with B_z $\left\{ \frac{\partial \phi}{\partial B_z} \sim 70^\circ / 0.1 \text{ Gauss} \right\}$. However, the required constancy and uniformity of B_z is achievable with NMR stabilization and careful magnet design.

It is worth noting that the $\alpha_+ \rightarrow \beta_-$ transition has transverse Stark mixing matrix elements which are almost identical to those of $\alpha_0 \rightarrow \beta_0$ transition. However, the PNC $\alpha_+ \rightarrow \beta_-$ matrix element vanishes identically. At $\omega_{RF} = 2\pi \cdot 1608 \text{ sec}^{-1}$ the unperturbed $\alpha_+ \rightarrow \beta_-$ resonance occurs at 574.6 Gauss. Thus in principle the $\alpha_+ \rightarrow \beta_-$ transition provides a means to monitor many of our potential systematic errors (which are due to transverse Stark coupling) by making a ~ 5 Gauss change in B_z .

We gratefully acknowledge informative conversations about this experiment with a large number of colleagues. We are particularly indebted to Prof. E.A. Hinds for introducing us to attractive features of two cavity experiments and for helping to clarify some issues concerning systematic errors in measurements of this type.

References

1. R.R. Lewis and W.L. Williams, Phys. Lett. 59B, 70 (1975).
2. L.L. Lewis et al., Phys. Rev. Lett. 39, 795 (1977).
3. P. Baird et al., Phys. Rev. Lett. 39, 798 (1977).
4. L.M. Barkov and M.S. Zolotoryov, JETP Lett. 27, 379 (1978).
5. R. Conti et al., Phys. Rev. Lett. 42, 343 (1979).
6. C.Y. Prescott et al., Phys. Lett. 77B, 347 (1978).
7. W.J. Marciano and A.I. Sanda, Phys. Rev. D 17, 1313 (1978).
8. L. Wolfenstein, Phys. Rev. D 19, 3450 (1979).
9. E.G. Adelberger, T.A. Trainor, and E.N. Fortson, Bull. Am. Phys. Soc. 23, 546 (1978).
10. R.W. Dunford, R.R. Lewis, and W.L. Williams, Phys. Rev. A13, 2421 (1978).
11. E.A. Hinds and V.W. Hughes, Phys. Lett. 67B, 487 (1977), and E.A. Hinds, private communication, 1979.
12. Our code is an adaptation of one described in G.G. Ohlson and J.L. McKibben, Los Alamos report LA-3725 (1967) unpublished.
(We are indebted to W.L. Williams for informing us of an error in the Los Alamos code.)
13. R.D. Deslattes (private communication), 1979.

Figure Captions

1. $n=2$ $j=1/2$ levels of the hydrogen atom.
2. Schematic diagram of the experimental configuration.
3. Calculated signal for the case when the front and rear cavities have lengths of 12 cm and 36 cm respectively. Electric fields are assumed to turn on and off abruptly and are roughly optimum if there is a negligible background in the L_α detectors.

Lower Part: the α_β resonance yield

Upper part: the modulation of the α_β yield when the relative phases of A_{PC} and A_{PNC} are reversed. We assume $C_2 = 0.5$.

Fig. 1

Fig. 2

Fig. 3

Table I

 $\alpha_0 \beta_0$ Amplitudes Involved in the 570 Gauss Parity Mixing Measurement

Amplitude	Form ^b	Numerical Value (Units of $R_a T$)	Phase Setting	Subtractions ^a				Value ^c After B, δ and ϕ subtraction
				Cyl. Symn.	B $\downarrow$ -B	δ $\downarrow$ - δ	$\alpha \rightarrow \beta$ $\downarrow$ $\beta \rightarrow \alpha$	
A_{PV}	$i(C_2) \frac{V_{PV} V_o (\vec{R}_a \cdot \vec{B})}{D_o} T$	$3 \times 10^{-9} C_2 e^{i60^\circ}$	1.00					$3 \times 10^{-9} C_2$
A_P	$0.055 \frac{V_o^2 (\vec{R}_a \cdot \hat{B}) (\vec{E}_a \cdot \hat{B})}{D_o} T$	$2.9 \times 10^{-2} (\vec{E}_a \cdot \hat{B}) e^{-i30^\circ}$	≈ 0		X		X	≈ 0
	$0.055 \frac{V_o^2 (\vec{R}_a \cdot \hat{B}) (\vec{E}_s \cdot \hat{B})}{D_o} T$	$2.9 \times 10^{-5} e^{-i30^\circ}$	10^{-4}		X		X	2.9×10^{-13}
A_{J_1}	$-8.25 \frac{V_o^2 \vec{R}_f \cdot \left[\frac{\vec{v}}{c} \times \vec{B} \right]}{D_+} T$	$-7.9 \times 10^{-5} e^{-i50^\circ}$	-0.33	X		X	X	7.8×10^{-11}
	$-8.25 \frac{V_o^2 \vec{R}_{ma} \cdot \left[\frac{\vec{v}}{c} \times \vec{B} \right]}{D_+} T$	$-4.7 \times 10^{-7} e^{-i50^\circ}$	-0.33	X			X	1.5×10^{-9}
	$-0.027 \frac{V_o^2 (\vec{R}_f \times \hat{B}) (\vec{E}_s \cdot \hat{B})}{2D_+} T$	$-1.8 \times 10^{-7} e^{-i50^\circ}$	-0.33	X	X	X	X	1.8×10^{-17}

Table I (Continued)

Amplitude	Form ^b	Numerical Value (Units of $R_a T$)	Phase Setting	Subtraction ^a				Value ^c After B, δ and ϕ subtraction
				Cyl. Symn.	B ↓ -B	δ ↓ - δ	$\alpha \rightarrow \beta$ ↓ $\beta \rightarrow \alpha$	
A_{J_2}	$-i(8.25) \frac{V_o^2 \left(\vec{R}_f \times \left(\frac{\vec{v}}{c} \times \vec{B} \right) \right) \cdot \hat{B}}{D_+} T$	$-7.9 \times 10^{-5} e^{i40^\circ}$	0.95		X	X		-2.2×10^{-12}
	$-i(8.25) \frac{V_o^2 \left(\vec{R}_{ma} \times \left(\frac{\vec{v}}{c} \times \vec{B} \right) \right) \cdot \hat{B}}{D_+} T$	$-4.7 \times 10^{-7} e^{i40^\circ}$	0.95	X	X			-4.4×10^{-13}
	$-i(0.027) \frac{V_o^2 \left(\vec{R}_f \times \vec{E}_s \right) \cdot \hat{B}}{D_+} T$	$-1.8 \times 10^{-7} e^{i40^\circ}$	0.95	X		X		-5.1×10^{-13}
A_M	$(0.94 \times 10^{-4}) U_o \left(\vec{R}_{ma} \times r \right) \cdot \hat{B} \left(\frac{r}{h} \right) T$	1.4×10^{-8}	0.50	X				7.0×10^{-11}
	$-i(1.84 \times 10^{-4}) U_o \left(\frac{\vec{v}}{c} \times \vec{R}_f \right) \cdot \hat{B} T$	$-2.7 \times 10^{-9} e^{i90^\circ}$	0.87	X		X	X	-7.0×10^{-15}

^aAn X indicates that the corresponding quantity is reduced by the subtraction.

^bAll fields are decomposed into applied, misaligned and fringing components; for example; $\vec{R} = \vec{R}_a + \vec{R}_{ma} + \vec{R}_f$, where the component is uniform and $\parallel$ to $\vec{B}$ and the misaligned component is uniform but $\nparallel$ to $\vec{B}$.

^cReduction factors of 10^{-2} , 10^{-4} and 3×10^{-4} are assumed for the subtractions due to cylindrical symmetry, B and δ .