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INDUCTIVE CURRENT STARTUP IN LARGE TOKAMAKS WITH EXPANDING MINOR RADIUS AND RF ASSIST*

S. K. Borowski

Fusion Engineering Design Center

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It is subject to revision or correction and therefore does not represent a
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ABSTRACT

Auxiliary rf heating of electrons before and during the current-rise phase of a large tokamak, such as the Fusion Engineering Device ($R = 4.8$ m, $a = 1.3$ m, $\sigma = 1.6$, $B_T = 3.62$ T), is examined as a means of reducing both the initiation loop voltage and resistive flux expenditure during startup. Prior to current initiation, to 2 MW of electron cyclotron resonance heating power at ~ 90 GHz is used to create a small volume of high conductivity plasma ($T_e \approx 100$ eV, $n_e \approx 10^{19}$ m $^{-3}$) near the upper hybrid resonance (UHR) region. This plasma conditioning permits a small radius ($a_0 \approx 0.2$ - 0.4 m) current channel to be established with a relatively low initial loop voltage (≤ 25 V as opposed to ~ 100 V without rf assist). During the subsequent plasma expansion and current ramp phase, a combination of rf heating (up to 5 MW) and current profile control leads to a substantial savings in volt-seconds by (1) minimizing the resistive flux consumption and (2) maintaining the internal flux at or near the flat profile limit.

To study the "preheating" phase, a near-classical particle and energy transport model is developed to estimate the electron heating efficiency in a currentless toroidal plasma. The model assumes that preferential electron heating at the UHR leads to the formation of an ambipolar sheath potential between the neutral plasma and the conducting vacuum vessel and limiter. The ambipolar electric field (E_{AMB}) enables the plasma to neutralize itself via poloidal $\vec{E}_{AMB} \times \vec{B}$ drift. This form of effective rotational transform "short circuits" the vertical charge separation and improves confinement. A comparison of theoretical estimates and experimental preheating data from the ISX-B tokamak shows reasonably good agreement.

To analyze the expansion phase of current startup, numerical algorithms have been developed and coupled to the WHIST radial tokamak transport code. These algorithms are used to examine the expansion to full size of small radius plasmas formed both on-axis and near the outboard chamber wall. The 1-D simulations of current startup indicate a significant easing of the resistive voltage and flux requirements (by more than a factor of 2) as compared to simpler, 0-D estimates. In

assessing the potential for current profile control during startup, it has been shown that linear current ramping is superior to linear size ramping. It provides hotter plasmas with broader current profiles and with reduced resistive flux consumption. Obtaining truly broad profiles necessitates the use of expansion techniques which can inhibit or counter the centralized peaking of the current density due to the thermal instability. Minor radius expansion with decreasing edge safety factor is such a technique. It utilizes a controlled skin effect to broaden the current profile.

The rf-assisted, expanding radius approach to current startup appears to be an effective method for achieving higher quality ohmic discharges with minimum voltage and flux requirements. The initial operation of the Tokamak Fusion Test Reactor and the Joint European Torus will provide a unique opportunity to test the feasibility and benefits of the various expansion options over a wide range of plasma shapes, sizes, magnetic fields, and current levels.

1. INTRODUCTION

The new generation of large tokamaks now entering into operation is expected to produce the first substantial quantities of fusion power using the deuterium-tritium (D-T) reaction. Possessing the size and current levels thought necessary to demonstrate scientific breakeven ($Q = \text{fusion power output/external input power to the plasma} = 1$), these devices will provide valuable physics information on alpha particle heating and confinement. However, because the amount of alpha power produced is small (~ 5 MW for 25 MW of external heating at $Q = 1$), it is not expected that these tokamaks will achieve a self-sustaining plasma burn, a condition known as ignition. This next step in performance will require further increases in size and current, a plasma diameter of approximately 2 meters, and a current level in excess of 5 million amperes (5 MA). By comparison, the Princeton Large Torus (PLT), one of the largest tokamaks now in operation, has a diameter of ~ 0.9 meters and a plasma current of roughly 0.5 MA.

The creation of large, multi-megampere plasmas presents some important new problems previously unencountered in smaller devices. The most troublesome and, indeed, ironic problem during current startup involves increasing tokamak size. For a fixed aspect ratio (ratio of major radius to minor radius), increasing the plasma minor radius is generally beneficial because it improves energy confinement. However, the accompanying increase in major radius leads to a decrease in the current density. Because the thermal power generated in the plasma due to current flow is proportional to the plasma resistance and the square of current density, it becomes increasingly more difficult to initiate a tokamak discharge in a large device.

To compensate for the falloff in current density, energy storage and power supply requirements are characteristically large. The cost associated with these pulsed electrical systems can be substantial, up

to 25% of the total capital cost of the plant. Steiner et al. [1] have advocated siting multiple tokamak reactor units around a central pulsed electrical plant as a means of reducing the per unit capital cost. While the multiple unit approach can lighten the economic burden of expensive energy storage and power handling equipment, it does little to remedy the basic problem of conventional current startup, namely, a low heating efficiency.

In this study, auxiliary rf heating of electrons (before and during current rise) is suggested as a means of improving the heating efficiency in large tokamaks. When coupled to an expanding radius plasma, modest amounts of rf power (<5 MW) can maintain a reasonably high plasma conductivity even with a slow current rise. This startup approach is expected to lead to substantial reductions in the applied voltage, resistive flux expenditure, and power supply requirements for tokamak reactors. A further appreciation for the potential benefits of the rf startup technique can best be obtained by first reviewing basic tokamak operations.

1.1 Tokamak: The Transformer Approach to Fusion

The tokamak confinement concept [2,3] utilizes the transformer principle for transferring energy from a central induction solenoid to a coaxial plasma. The central solenoid, also referred to as the ohmic heating (OH) coil, functions as the primary side of the transformer and the plasma functions as the secondary.

Plasma formation and current initiation in the basic tokamak (illustrated in Fig. 1.1) begins by filling the toroidal vacuum vessel with a specified amount ($\sim 10^{-5}$ to 10^{-3} torr) of neutral gas, typically hydrogen. (In future tokamak reactors the hydrogen isotopes, deuterium and tritium, will be used as the fuel gas.) A short high-voltage spike (lasting between 10 and 100 ms) is then applied to the torus by energizing the transformer primary windings with a time changing current (I_M). The resulting toroidal electric field (varying from 1 to 10 V/m) causes gas breakdown, with the ionized gas acting as the transformer secondary. As the electron density builds, a current (I_p) begins to flow in the plasma which is driven by the applied electric field.

Fig. 1.1 Schematic illustration of a tokamak system including major device components and support equipment.

Current flow is vital to the tokamak concept for two reasons: (1) it provides the poloidal magnetic field (B_p) necessary for particle confinement, and (2) it heats the plasma. The heating, referred to as ohmic heating, results from the interaction between the plasma current and the internal resistance of the plasma. The self-field of the plasma current is, in itself, incapable of adequately confining the plasma because it is subject to various pinching and kinking instabilities [4]. These magnetohydrodynamic (MHD) or configurational perturbations are stabilized by introducing a strong toroidal magnetic field (B_T) into the plasma. This field is produced by currents (I_T) running in individual toroidal field coils which wrap around the entire torus. When the pulsed poloidal magnetic field and the steady-state toroidal field are combined, helical field lines are formed which map out a set of nested magnetic surfaces. It is on these surfaces that the hot plasma particles are confined. A vertical field component (B_V) is also produced in the plasma by a separate set of external equilibrium field (EF) coils. Their purpose is to provide the toroidal plasma with horizontal and vertical stability. The OH and EF coils are usually considered to be parts of an overall poloidal field (PF) coil system.

During inductive current startup, the OH and EF power supplies function principally as energy storage devices for transferring energy in and out of the plasma. This energy appears primarily in the form of poloidal magnetic field energy plus energy dissipated in resistive losses. While being large (~ 100 MJ in a reactor plasma), the energy contained in the plasma poloidal magnetic field represents only about 10 to 20% of the total energy stored in a fully energized reactor OH coil.

During typical operation, the OH coil system utilizes a switching arrangement to transfer inductive energy from the OH primary windings to a capacitive type of energy storage and then back again. In the process, a time varying current is driven through the primary which generates the electric field necessary to induce the desired plasma current. The rate at which energy is transferred within and from the OH coil system determines the pulsed power requirements for startup.

In present-day tokamaks, such as PLT, the PF coils are located

inside the toroidal field (TF) coils and are close to the plasma. This closeness results in a relatively strong magnetic coupling between the plasma and the external coil currents. In reactors, however this coil arrangement is undesirable because it makes the tokamak extremely difficult to construct and repair. While locating the OH and EF coils outside the TF coils improves maintainability, it leads to poor coupling between the plasma and the PF system. This poor coupling, together with a need for large loop voltages for current initiation, implies a substantial current ramp rate in the OH primary and sizeable startup power requirements.

1.2 The Importance of Using Transformer Flux Efficiently

To confine the energetic alpha particles produced in a D-T tokamak power reactor, it is first necessary to drive the initial plasma current upward to levels in excess of 5 MA [5]. The ramping and subsequent maintenance of the plasma current against dissipation by Coulomb collisions necessitates a continuously increasing magnetic flux (ϕ) in the OH solenoid in accordance with Faraday's law. For a given solenoidal bore, the duration of the plasma discharge is limited by the volt-second (V-s) capability of the transformer [the weber or volt-second is used to measure magnetic flux]. Once this capability is exhausted, the plasma burn must be terminated and the transformer reset for the next operating cycle.

In future power-producing tokamaks, it is desirable to maximize the length of the burn phase for several key reasons. These include (1) improved system economics when the availability of the plant for power production is high and (2) a reduced potential for component failure due to cyclic thermal and mechanical fatigue. Increasing the longevity of the burn pulse can be accomplished in either of two ways. The most straightforward way is to increase the bore of the OH coil to gain additional volt-second capability. However, the size of the OH solenoid is important in defining the basic dimensions and cost of the tokamak. Sensitivity studies [6] indicate that modest increases in the size of the flux core can significantly increase the cost of the device. The alternative to making the tokamak bigger is

to optimize the usage of the established flux capability.

The efficient utilization of flux during startup will require an ability to tailor key plasma parameters such as the current and temperature distribution. Measurements made on PLT during current startup [7] have indicated that volt-second consumption is determined primarily by the external and internal inductive flux required to establish the current profile. A third component, the resistive flux loss on axis, was found to be small in PLT. In future large tokamaks, however, this resistive component can be substantial.

Because the external flux is a function of the plasma geometry and the integrated current (discussed in Section 9.2), it represents an irreducible lower limit on the required flux for a given tokamak design. The internal and resistive flux components, on the other hand, are influenced by a number of parameters, such as: the current profile shape, the plasma resistivity, the impurity and temperature profiles, and the mode of startup. Because each of these parameters can be modified (to varying degrees) during current rise, there is an immediate interest in seeing if the startup flux requirements can be reduced through a judicious choice of external controls. The savings in transformer flux can then be used to either increase the current level or prolong the duration of the discharge.

In this study, current startup with expanding minor radius and rf assist is used as a means of (1) achieving a quality ohmic discharge with a minimum flux expenditure and (2) simplifying the technological requirements of the OH system. The benefits of this approach are best illustrated by examining the problems of conventional startup.

1.3 Physics Problems Associated With the Inductive Startup of a Large Tokamak

The primary difficulty associated with the startup of any large tokamak is a physics limitation on the amount of current which can be driven through the plasma. As the current increases, the helical twist imparted to the toroidal magnetic field can become excessive with field lines closing upon themselves after an integral number of transits around the torus. Under these conditions, "kink-like" mag-

perturbations can be driven unstable. The plasma safety factor, q_I , is a measure of the helicity of the magnetic field lines. For a large aspect ratio circular plasma, the safety factor is given by

$$q_I(r) = rB/RB_p, \quad (1.1)$$

where r is a radial location in the plasma and R is the plasma major radius. Typically $q_I(a)$ must be greater than about 2.5 [8] to ensure configurational stability against the important surface kink modes. The $q_I(a) = 1$ case, which corresponds to a field line closing on itself after one transit around the torus, is referred to as the Kruskal-Shafranov limit [9,10].

By constraining the amount of current which can flow in a plasma, Eq. (1.1) effectively limits the achievable current density [both peak ($q_I(0) \gtrsim 1$) and average ($q_I(a) \gtrsim 2.5$)] and the amount of power which can be delivered to the plasma via ohmic heating ($P_{oh} \propto R^{-2}$ for fixed B and q_I). High current density operation with significant ohmic heating may be possible in compact, high-field tokamaks [11,12].

In sharp contrast to the declining ohmic heating in large tokamaks, the dominant energy loss mechanisms during startup, ionization and radiation, increase with tokamak size (through a volume effect). Such a situation can lead to serious discharge initiation problems. In their classic paper, Hawryluk and Schmidt [13] pointed out that radiation losses (associated with low-Z impurities) dominate the plasma energy balance early in the discharge when the electron temperature is low ($T_e \lesssim 20$ eV).

With only ohmic heating available to penetrate the low-Z impurity "radiation barrier," the discharge characteristics will depend rather sensitively on the initial filling pressure and on the applied loop voltage, V_L . Immediately after gas breakdown, T_e is low and the rate of current rise is small ($\dot{I}_p \approx 0$). The energy balance which approximately describes this situation is given by [13]

$$P_{oh} \left[\lesssim (V_L/2\pi R)^2/\eta_{SPITZER} \right] \approx P_{rad} \left[= n_e^2 \xi f(T_e) \right], \quad (1.2)$$

where $\xi (= n_I/n_e)$ is the impurity fraction, n_I and n_e are the impurity and electron densities, respectively, and η_{SPITZER} is the Spitzer plasma resistivity ($\propto Z_{\text{eff}}/T_e^{3/2}$) [14]. With too high a filling pressure ($n_e > 10^{19} \text{ m}^{-3}$ and ξ of a few percent) for a constant V_ℓ , the electron temperature can be held in check at low levels. Under these conditions, the plasma current would fail to evolve properly [13]. In the opposite limit of a low filling pressure ($n_e < 10^{18} \text{ m}^{-3}$), a small population of electrons can "run away" from the bulk plasma distribution by continuously accelerating in the toroidal electric field [15]. This mode of operation is also undesirable because it can lead to serious machine damage as these energetic electrons (with energies in excess of 10^6 eV) strike the limiters and wall.

According to Eq. (1.2), the ohmic heating power is at a maximum when V_ℓ is primarily resistive in nature. As the current and temperature rise, however, the inductive voltage becomes important because of the increased plasma conductivity. Figure 1.2 shows the interrelationship between the electron temperature (T_e), current ramp rate ($\dot{I}_p$), and plasma voltage (V_ℓ) at the beginning of a voltage flattop (specified here at 100 ms). The plot illustrates the current ramp rate possibilities for a given voltage as a function of electron temperature. (The current ramps are assumed to be linear in time with the plasma current evaluated at t_1 .) The startup plasma characteristics and geometry (a fixed-size, small radius plasma formed near the outboard wall of the torus) are typical of those envisioned for future large tokamaks [16]. The plasma is modelled as a constant inductance in series with a resistance. Further, it is assumed that the plasma is free from impurities ($Z_{\text{eff}} = 1$).

The voltage equation describing this configuration is given by (mkS units with T_e in eV)

$$V_\ell = L_p(dI_p/dt) + R_p I_p, \quad (1.3)$$

where

Fig. 1.2 A snapshot in time showing the voltage requirements necessary to support a given current ramp rate in a plasma with temperature T_e .

$$L_p = \mu_0 R \left(\ln \frac{8R}{a} + \frac{\ell_i}{2} - 2 \right) \quad (1.4)$$

is the plasma self-inductance, R and a are the major and minor radius of the plasma, and ℓ_i is the flux-related definition of the internal inductance [7]. For flat and parabolic current profiles, $\ell_i = 1.0$ and 1.5 , respectively. The parameter R_p is the plasma resistance ($= 2R\eta_{\text{SPITZER}}/a^2$).

Following breakdown, the resistance of the partially ionized gas is high and the plasma acts primarily as a resistive load. The current level remains low during this period because little, if any, inductive voltage is available to increase the plasma current. Using the constant voltage line $V_L = 200$ volts as an example, the effect of increasing electron temperature on $\dot{I}_p$ can be easily seen. At $T_e \cong 2$ eV, the maximum $\dot{I}_p$ capability at the beginning of flattop (at $t_1 = 0.1$ s) is 0.15 MA/s. With $I_p(t_1) = 15$ kA and $R_p I_p \geq 196$ volts, less than 4 volts is available to drive the current ramp. When T_e increases to ~ 12 eV (moving along path 1 to 2), the resistive voltage drops to ~ 62 volts enabling a current ramp rate of 1.5 MA/s to be sustained. Further increases in T_e , at the same loop voltage, leads to a rapid current rise (in the limit as $R_p I_p \rightarrow 0$, $\dot{I}_p \rightarrow 8$ MA/s) and a violation of the safety factor limit $q_1(a) = 2.5$. It is for this reason that driving voltages are characteristically reduced in time (e.g., for $t > t_2$) so that the current can rise gradually to its intended value. Large current ramp rates are also undesirable because they can lead to skin current distributions [17] (discussed in Section 7.1) which are prone to plasma disruptions.

1.4 Technological Problems Associated With the Inductive Startup of a Large Tokamak

Engineering considerations favor a lowering of the applied voltage and current ramp rate (path 2 to 3) in order to lessen the peak power demands during startup. The scaling trends for startup power can be determined by using the following simplified set of circuit equations,

$$V_{OH} = L_{OH} \dot{I}_{OH} \pm M_{OH,p} \dot{I}_p + R_{OH} I_{OH} \quad (1.5)$$

$$0 = \pm M_{OH,p} \dot{I}_{OH} + L_p \dot{I}_p + R_p I_p, \quad (1.6)$$

which describe an inductively coupled OH primary-plasma system. Here, V_{OH} is the primary voltage, R is the resistance, and L and M are the single-turn self and mutual inductances, respectively. The voltages of mutual inductance can be of either polarity, and generally depend on the direction of current flow selected for the primary windings [18]. Assuming a negative polarity in this discussion, a comparison of Eqs. (1.3) and (1.6) indicates that $V_\ell = M_{OH,p} \dot{I}_{OH}$. After multiplying Eqs. (1.5) and (1.6) by I_{OH} and I_p , respectively, the addition of these two equations yields the following expression for the instantaneous power [19]:

$$P_{OH} = V_{OH} I_{OH} = L_{OH} I_{OH} \dot{I}_{OH} + L_p I_p \dot{I}_p - M_{OH,p} \frac{d}{dt}(I_{OH} I_p) + R_{OH} I_{OH}^2 + R_p I_p^2, \quad (1.7)$$

where L_p has been assumed to be constant in time. Equation (1.7) indicates that power requirements during startup are associated (1) with the rate of change of the stored magnetic energy in the coupled field system and (2) with the rate at which energy is dissipated resistively in the OH-plasma circuits.

In an unassisted startup, V_ℓ scales linearly with major radius for a specified electric field requirement. Typically, $E_\ell \sim 5\text{-}10$ V/m for high-density startups ($n_e \sim 10^{19} \text{ m}^{-3}$) with a few percent impurity [13]. For this range of electric fields, V_ℓ is estimated to be $\sim 300\text{-}500$ V for tokamak fusion reactors [20]. This voltage level is normally maintained for a fraction of a second, during which time I_p is ramped to sufficiently high values so that the ohmic heating power can surmount the losses associated with ionization and radiation. Achieving such high voltages necessitates large values of $\dot{I}_{OH}$ in the primary windings. These values, in turn, imply sizeable voltages

on the primary coil and peak powers of approximately a gigawatt [20]. (A relatively weak coupling between the plasma and the OH primary also aggravates this condition.) In the Experimental Power Reactor (EPR) studies [21] conducted by Argonne National Laboratory, approximately 1000 MWe of ohmic driving power was required for a 1.0-s current startup. The major power component was associated with the rapid flux swing in the OH primary [the $L_{OH} I_{OH} \dot{I}_{OH}$ term in Eq. (1.7)]. Resistive OH losses were negligible due to the use of a superconducting primary. Typical OH coil parameters at peak power were $L_{OH} \cong 1 \mu H$, $I_{OH} \cong 20$ MA, and $\dot{I}_{OH} \cong 50$ MA/s [21].

With Eqs. (1.5) - (1.7) as a basis, a simple scaling for the startup power can be obtained. Using

$$V_{OH} \propto V_L (= M_{OH,p} \dot{I}_{OH}) \propto R, \text{ and}$$

$$I_{OH} \propto I_p/k \propto a^2 B f_\sigma / Rk$$

$$\propto R B f_\sigma / k \text{ (for fixed aspect ratio),}$$

one finds

$$P_{OH} = V_{OH} I_{OH} \propto V_L I_p / k \propto R^2 B f_\sigma / k.$$

Here, f_σ is an enhancement factor due to elongation effects (discussed in Section 8.2) and $k (= M_{OH,p} / \sqrt{L_{OH} L_p} < 1)$ is the induction coupling coefficient for the system. It is clear from the above that the OH power requirements for unassisted startup scale unfavorably with increasing tokamak size.

On the economic side, the capital outlay associated with this pulsed power system appears to be substantial. Studies have estimated the cost of the generators, switches, and energy storage and transfer systems (see Fig. 1.1) at around \$150 million for an EPR [22] and around \$250 million for a demonstration fusion reactor facility [1].

In order to reduce the loop voltage and power supply needs substantially, some form of auxiliary heating is necessary which can eliminate the resistive fraction of the loop voltage and also permit

a relatively slow current rise.

1.5 Why the Need for Electron Preheating and RF Assist?

By heating electrons before and during current rise (path 1 to 2 in Fig. 1.2), the resistive portion of the loop voltage can be greatly diminished. A reduced V_ℓ should then be sufficient to build up the plasma current to its steady-state value. With $T_e \cong 70$ eV, the plasma is primarily an inductive load. At this temperature, a loop voltage of 50 volts can support the same current ramp rate ($\dot{i}_p = 1.5$ MA/s) as $V_\ell = 200$ volts can at $T_e \cong 12$ eV. Further reductions in V_ℓ (to 25 volts at $T_e \cong 35$ eV and $\dot{i}_p = 0.5$ MA/s) are possible if electron cyclotron resonance heating (ECRH) power is used to supplement the ohmic power.

Considerable interest presently exists in the use of ECRH power to assist current startup in large tokamaks such as the Fusion Engineering Device (FED) [16]. ECRH has been used successfully on a number of small tokamaks for the purposes of breakdown and preionization prior to ohmic heating [23,24,25]. In each of these experiments, the rf power was injected into the plasma from either the vertical plane (top and bottom) or the low-field side of the device. Relatively cold plasmas ($T_e \lesssim 10$ eV) were obtained. Even with these low temperatures, the loop voltage was reduced by roughly 50% [23,24]. Such observations are consistent with the trends shown in Fig. 1.2.

Unfortunately, these tokamaks have spent their time only duplicating the voltage reduction results. They have not addressed the more important concern of heating through the radiation barriers of hydrogen and low-Z impurities which exist for electron temperatures $\lesssim 50$ eV. In a recent work by Peng, Kammash, and this author [26], the potential for electron heating prior to current initiation was examined. Their findings indicated that significant heating ($T_e \gtrsim 100$ eV) was possible following microwave irradiation from the high-field side of the torus and extraordinary wave absorption near the upper hybrid resonance (UHR). The primary motivation for studying currentless electron heating (referred to as preheating) is the potential for order-of-magnitude reductions in the loop voltage and pulsed power supplies

required for startup.

Sufficient precedent exists to warrant a study of electron preheating at the UHR. Experimental studies by Anisimov et al. [27] indicated the feasibility of microwave breakdown and wave absorption by a plasma in a purely toroidal magnetic field. In these experiments, powers up to 50 kW at 9.7 GHz were deposited from the high-field side, primarily as extraordinary waves, into a toroidal chamber with major and minor radii of 22 cm and 6 cm, respectively. Electron energies around a few hundred electron volts were observed at powers above several kilowatts. Measurements also established that the microwave absorption was localized near the values of the plasma density and magnetic field corresponding to the UHR. Recently, preionization experiments on the much larger Impurity Studies Tokamak (ISX-B) [28] have also confirmed UHR absorption using a variety of diagnostic techniques to examine the plasma evolution. Together, these experimental results suggest that significant electron heating ($T_e \approx 50 - 200$ eV) may be possible near the UHR. Actual values, however, will depend on parameter specifics such as: the polarization of the wave and its launch angle, the length and power level of the rf pulse, and the electron density. (The sensitivity of UHR preheating to these parameters will be discussed in Sections 2 through 5.)

Preheating at the UHR layer has some unique features which make it particularly well suited for assisting startup in large tokamaks. In unassisted startup, a sizeable resistive power input is required to ohmically heat through the radiation barrier in roughly the entire vacuum chamber. With an inadequate driving voltage (or with a large impurity content), the plasma current and temperature may fail to evolve successfully. Maintaining a large voltage for an extended period also consumes valuable volt-seconds. By contrast, the UHR heating process can deposit power to a localized section of the volume with high efficiency. Because only a small portion of the torus volume is "burning through" the radiation barrier at any one time, relatively low rf power levels should be adequate for starting up reactor plasmas. In this sense, the UHR preheating process should simplify the engineering design requirements of the OH system. It should also substantially

increase the pulse length capability of a large tokamak like FED (e.g., a savings of ~ 10 V-s during startup can extend the length of the burn pulse by ~ 200 s at $T_e \cong 10$ keV and $V_L \cong V_{res} \cong 0.05$ V).

High-density startups ($> 10^{19} \text{ m}^{-3}$) may also be possible with UHR preheating, provided high-power, millimeter wavelength microwave sources are available. The procurement of such tubes should be a reasonable extension of the program for developing 200-kW, 60-GHz microwave gyrotrons needed in the MFTF-B tandem mirror experiment.

According to Eq. (1.3), the essential ingredients of a low-voltage startup are a reasonably high plasma conductivity and a relatively slow current ramp rate. In a full-bore unassisted startup, these features are incompatible because a large current ramp rate is necessary for electron heating. However, with rf assist, this situation can be changed. By supplementing the ohmic heating with ECRH power, one can eliminate the need for a fast current rise and still maintain attractive electron temperatures.

In addition to this important function, ECRH may also prove useful in optimizing the evolution of the discharge during current startup. For example, ECRH could be used in conjunction with current ramping to "freeze in" (by slowing down magnetic field diffusion) current profiles which are necessary for high-beta (β) tokamak operation. (Beta, the ratio of plasma pressure to magnetic field pressure, is a measure of the effective utilization of the magnetic field and an indicator of the fusion power density.)

Because of the numerous physics and technology benefits cited above, the rf-assisted startup concept has been incorporated into a number of tokamak reactor studies. One of its first appearances was in the TNS (The Next Step) reactor study conducted by Steiner et al. [29] at Oak Ridge National Laboratory. Since then it has been adopted in the design of a future commercial reactor [30] and in studies of nearer term engineering test reactors such as FED [16].

1.6 Why the Need for an Expanding Radius Startup?

As discussed earlier, full-bore unassisted startups require sizeable current ramp rates in order to successfully initiate a plasma

discharge. In large plasmas, however, the increasing electron temperature can actually hinder the current penetration by increasing the plasma skin time $\tau_s \propto a^2 T_e^{3/2} / Z_{\text{eff}}$. The result is a pileup of the poloidal magnetic field at the outer edge of the plasma and the formation of a skin current distribution (shown in Fig. 1.3b). The $q_I(r)$ profile accompanying such a distribution exhibits a local minimum in the outer plasma radius rather than at the center of the discharge, as found during quasi-steady-state operation (Fig. 1.3a). Such a profile can drive double tearing modes [31] unstable. Subsequent small disruptions within this skin layer allow a rapid penetration of the skin current into the plasma interior. Experimental observations indicate that this "relaxation" phenomenon is accompanied by enhanced plasma-limiter interactions and the introduction of impurities into the plasma [32]. The plasma edge cooling by the partially stripped impurity ions helps current penetration by increasing the resistivity, but it aggravates the problem of resistive flux consumption. Because the skin time scales as size squared, for a given T_e , it should be possible to augment the current penetration process by initiating current startup at reduced radius.

The preheating process examined in this study leads naturally to the formation of a small radius plasma centered at the UHR layer. It is predominantly in this region of high conductivity that the plasma current should be induced once the toroidal voltage is applied. Through proper frequency selection and density control, it should be possible to position the current channel at various locations within the vacuum chamber (either on-axis or near the outboard chamber wall) with good reproducibility.

A small radius plasma should prove to be superior to a full-bore plasma in terms of the plasma characteristics produced during current initiation. Not only is a small radius plasma less susceptible to skin current formation, but it can potentially lead to cleaner discharges by ameliorating the plasma-limiter interactions accompanying tearing mode relaxation. Empirical scaling of energy confinement in ohmically heated tokamaks indicates an improvement with increasing density and size ($\tau_E \propto n_e a^2$) [33]. The coupling of ECRH power to an

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Fig. 1.3 Typical radial profiles of the plasma pressure, current density, and safety factor during the (a) quasi-steady-state and (b) early current-rise phases of a tokamak discharge.

expanding radius plasma is therefore a logical means of maintaining a positive electron energy balance during the early stages of current rise when the plasma density and minor radius are still small. Prolonged operation at reduced radius must be avoided, however, in order to prevent excessive heat loading on the startup limiters and an influx of impurities into the plasma.

By simultaneously increasing the size and current during the expansion phase, skin currents can be avoided and monotonically decreasing current profiles can be obtained. In a similar fashion, an expanding radius startup may provide a means of controlling the evolution of the current profile during the ohmic heating phase. Of particular interest are methods for obtaining broad current profiles. Such profiles are consistent with low- q_I operation and can lead to high- β burns in tokamak reactors. In this study, the requirements for creating current distributions of various shapes (from peaked to approximately flat) will also be examined. In addition to enhancing the high- β possibilities in a reactor, broad current profiles are desirable because they can be established with a lower internal flux expenditure than can more peaked current profile shapes.

Due to the breadth of the problem under investigation, it is logical to discuss key elements of the study within the context of a current-rise scenario. This study has focussed on an rf-assisted, expanding radius startup consisting of three separate phases:

1. Extraordinary mode waves ($\vec{E} \perp \vec{B}$) are first injected from the torus high-field side producing ionization and electron heating near the UHR layer before current initiation. The UHR layer, which is limited in volume, is deliberately positioned either on-axis or near the outboard side of the chamber to facilitate a small radius current startup. Away from the UHR layer, a low-temperature (a few eV) partially ionized plasma, produced by nonresonant rf breakdown of the prefill gas, is expected to fill the chamber.
2. While maintaining ECRH, a low loop voltage is applied which induces a plasma current in the preheated region. Current initiation occurs over a relatively long time scale (0.2 - 0.9 s) to avoid the formation of plasma skin currents. The

initial current channel is limited to ≤ 0.4 m in minor radius by a startup limiter which can be movable (retractable) or fixed at the outboard midplane. This size is maintained until the safety factor q_I reaches a desired value.

3. Additional supplementary heating [either ECRH or proton minority heating (ICRF)] is then introduced as the minor radius and plasma current are increased while holding q_I constant. This supplementary heating minimizes the resistive loss of volt-seconds and allows a relatively slow current ramp-up during the remainder of the current startup phase.

In the following sections, the physics and plasma engineering issues associated with each of these phases are examined. Sections 2 through 6 cover phase 1 and describe the physics of the UHR preheating model developed for this study. Specifically, Section 2 reviews the wave-plasma interactions during UHR absorption. The location of the rf power deposition and the selection of the required microwave frequency are also discussed. Formation of an ambipolar sheath potential and its effects on plasma transport are discussed in Section 3. With this as a basis, Section 4 presents global particle and energy balance equations describing the ambipolar loss of electrons and ions from a currentless toroidal plasma. A comparison between theory and experiment is found in Section 5, where model predictions are shown to provide a plausible interpretation of the recent ISX-B observations. In Section 6, the UHR heating model is used to predict the power requirements for FED under a variety of preheating conditions.

The various physics constraints which must be observed during the current initiation and ramp-up phases of tokamak operation (phases 2 and 3, respectively) are discussed in Section 7. Fixed major radius (movable limiter) and R-compression (fixed limiter location) expansion techniques are examined in Section 8 and the proposed startup scenario for FED is also outlined here. In this work, a modified version of the WHIST tokamak transport code [34] has been used to study the expansion phase in FED. Section 9 presents equations that describe particle and energy balance, poloidal magnetic field diffusion, and volt-second

consumption. Also contained in this section are descriptions of the algorithms developed for modelling the expansion of small radius plasmas, with both fixed and varying major radius. In order to assess the impact of profile effects on the overall startup process, Section 10 presents quantitative results using a "zero-dimensional" model with flat profiles. The details of the 0-D equations used for determining the electron temperature and plasma loop voltage during current rise are also presented here. This section further examines the sensitivity of both voltage and flux requirements to variations in the initial minor radius, oxygen impurity content, and electron preheating level. Radial transport (1-D) simulations of current startup, using the modified WHIST code, are presented in Section 11. Following a detailed comparison of 0-D and 1-D results, the prospects for current profile control are examined using external controls such as gas puffing, localized rf power deposition, and various expansion techniques. Completing this section is a look at the effects of magnetic axis shift (MHD effects) and neoclassical corrections to the plasma resistivity. The findings of this study are then summarized and conclusions are drawn in Section 12.

2. PHYSICS DISCUSSION OF WAVE-PLASMA INTERACTIONS

The absorption of extraordinary waves by electrons near the UHR region is an attractive heating process because the wave energy is efficiently transferred to the bulk electrons in a small volume. Sufficient experimental evidence exists to warrant a study of this heating technique. Experimental studies by Anisimov et al. [27] indicated the feasibility of UHR absorption by a plasma in a purely toroidal magnetic field, but only for short pulse lengths (<1 ms). Recently, longer pulse ($\lesssim 15$ ms) preionization experiments on the much larger ISX-B tokamak [28,35] have also confirmed UHR absorption and heating using time-resolved density measurements, temperature diagnostics and high-speed photography to examine the plasma characteristics during preheating. To provide the physics basis for the UHR preheating process examined in this study, this section briefly reviews the criterion for wave-plasma interactions at the UHR. The ability to position the UHR layer at a specific location, through proper frequency selection, is also discussed.

2.1 Extraordinary Wave Absorption at the UHR Layer

During the preheating phase, fundamental electron cyclotron resonance heating power is injected from the high-field side of the torus as extraordinary mode (EO) radiation. As the ECRH power preionizes the initial fill gas, two types of wave-plasma interactions are observed [27], depending on whether or not the electron density satisfies the condition

$$\alpha \equiv \frac{\omega_{pe}^2}{\omega_{RF}^2} \gtrless 2 \left(\frac{\omega_{ce}}{\omega_{RF}} - 1 \right) \quad (2.1)$$

Here,

$$\omega_{pe} (= \sqrt{4\pi n_e e^2 / m_e}) \cong 5.64 \times 10^4 \sqrt{n_e}$$

is the electron plasma frequency, $\omega_{RF} (= 2\pi f_{RF})$ is the applied microwave frequency, and

$$\omega_{ce} (= eB / m_e c \cong 1.76 \times 10^7 B)$$

is the electron cyclotron frequency.* The subscript r represents the boundary of strong ECR interaction. Because the cyclotron resonance is affected by both the longitudinal Doppler effect and the relativistic mass variation [36], cyclotron damping by electrons can be expected when the resonance condition

$$\omega_{RF} = n \omega_{ce}(v) + k_{\parallel} v_{\parallel} \quad (2.2)$$

is satisfied. Here, $k_{\parallel}$ is the parallel propagation vector of the microwave, n is the harmonic number ($n = 1$ corresponds to the fundamental frequency), $v_{\parallel}$ is the electron velocity parallel to the magnetic field B , and

$$\omega_{ce}(v) = \frac{eB}{m_e(v)c} = \frac{eB}{m_{e0}c} \sqrt{1 - \frac{v^2}{c^2}} \quad (2.3)$$

is the expression for the cyclotron frequency with the relativistic mass dependence explicitly indicated.

At low temperature and density, or for nearly perpendicular propagation, relativistic de-tuning [37] of the resonant electrons determines the boundary of strong cyclotron interaction. For finite temperature plasmas, the resonance boundary is determined by the Doppler broadening associated with the thermal electron velocity. In this case,

* In Sections 2 through 6, cgs units are used and temperatures are given in electron volts.

Eq. (2.1) can be written as [38]

$$\alpha \geq 2 \left(\frac{kT_e}{m_e c} \right)^{1/2} \cos \theta, \quad (2.4)$$

where θ is the angle between the propagation vector $\vec{k}$ and the direction of the toroidal magnetic field at the plasma edge.

2.1.1 Low-Density Regime

During the early stages of rf heating, when the electron density is low and Eq. (2.1) is violated, strong cyclotron absorption of the EO-wave results from a resonant energy transfer from the right-circularly polarized component of the wave to the gyrating electrons. Relativistic variation of the electron mass limits the maximum electron energy around $\omega_{RF} = \omega_{ce}(v)$ to [37]

$$\begin{aligned} W_{1R}(\text{eV}) &= 2m_e c^2 \left(\frac{e\tilde{E}_0}{m_e \omega_{RF} c} \right)^{2/3} \\ &\cong 1.54 \times 10^9 \left[\frac{\tilde{E}_0 (\text{V/cm})}{\omega_{RF}} \right]^{2/3}, \end{aligned} \quad (2.5)$$

where $\tilde{E}_0$ is the root-mean-squared (rms) vacuum wave electric field. (The value of $\tilde{E}_0$ can be approximately determined if the microwave power level and the total area of wave irradiation are known). Away from the resonance region, collision-induced absorption transforms energy gained by the electrons in the oscillating electric field into thermal energy. The magnitude of this energy is given by [27]

$$\begin{aligned} W_{1NR}(\text{eV}) &= e^2 \tilde{E}_0^2 / m_e \omega_{RF}^2 \left(\frac{\omega_{ce}}{\omega_{RF}} - 1 \right)^2 \\ &\cong 1.76 \times 10^{15} \tilde{E}_0^2 (\text{V/cm}) / \omega_{RF}^2 \left(\frac{\omega_{ce}}{\omega_{RF}} - 1 \right)^2. \end{aligned} \quad (2.6)$$

Typically, with $\tilde{E}_0 \approx 10^3$ V/cm and $\omega_{RF} \approx 2.2 \times 10^{11} \text{ s}^{-1}$ (35 GHz), one has $W_{IR} \lesssim 2$ keV while $W_{INR} \lesssim 10$ eV at $|(\omega_{ce}/\omega_{RF}) - 1| \approx 0.07$.

Characteristically, the kilovolt electrons are weakly coupled to the plasma. However, increases in tokamak size and magnetic field strength can improve coupling by ensuring that the particle confinement time exceeds the thermalization timescale for the energetic electrons.

2.1.2 High-Density Regime

As the density increases, a polarization electric field develops. This field transforms the EO-wave into a predominantly left-circularly polarized wave which can no longer accelerate electrons at the cyclotron resonance. The transition occurs when Eq. (2.4) is satisfied. In this high-density regime, the cyclotron layer becomes transparent to the EO-wave which then propagates through to the UHR region where

$$(\omega_{RF})^2 = (\omega_{UH})^2 \equiv (\omega_{pe})^2 + (\omega_{ce})^2. \quad (2.7)$$

As the almost undamped EO-wave approaches the UHR layer, a linear accumulation and amplification of the wave $\tilde{E}$ field occurs [39,40]. For small $\tilde{E}$ such that linear wave theory applies, the EO-mode is expected to go through a linear transformation into the electron Bernstein mode [41].

On the other hand, if $\tilde{E}$ near the UHR becomes larger than $\sim 10^2 - 10^3$ V/cm, a nonlinear parametric decay into ion-acoustic waves and upper hybrid (electron Bernstein) waves is likely to occur [42]. The high-frequency Bernstein waves subsequently transfer their energy to the plasma electrons via Landau damping when $\vec{k} \cdot \vec{B}$ is finite [43]. This strong damping can heat electrons to temperatures ranging from 50 - 200 eV [27,28]. Since various ECR heating experiments at low temperature [27,44,45] have resulted in bulk electron heating near the UHR, the author has assumed that this absorption mechanism is the dominant process in the high-density regime considered in this study.

The assumed interactions between the input extraordinary wave and the currentless toroidal plasma in the high- and low-density regimes are depicted in Fig. 2.1.

2.2 Region of Microwave Power Deposition

In FED, the UHR layer is positioned at $R_{UH} = 590$ cm, a location which facilitates the growth of a small radius plasma ($a_0 \cong 20$ cm) off of a fixed outboard limiter during current initiation. As the plasma density builds during the preheating phase, the electron cyclotron resonance (ECR) and UHR layers become spatially separated. The distance of separation (ΔR) is determined by dividing Eq. (2.7) through by ω_{RF}^2 . This division yields the following expression:

$$1 = \underbrace{\frac{\omega_{pe}^2}{\omega_{RF}^2}}_{\equiv \alpha} + \underbrace{\frac{\omega_{ce}^2(R_{UH})}{\omega_{RF}^2}}_{= R_{EC}^2/R_{UH}^2},$$

from which one obtains

$$R_{UH} = \frac{R_{EC}}{\sqrt{1 - \alpha}} \quad (2.8)$$

and

$$\Delta R = R_{UH} - R_{EC} = R_{EC} \left(\frac{1}{\sqrt{1 - \alpha}} - 1 \right). \quad (2.9)$$

The dimensionless parameter α can also be expressed in terms of R_{EC} as follows:

$$\alpha = 1.03 \times 10^{-5} \left(\frac{n_e(\text{cm}^{-3}) \left[R_{EC}(\text{cm}) \right]^2}{\left\{ \left[B(R_0)(\text{G}) \right] \left[R_0(\text{cm}) \right] \right\}^2} \right). \quad (2.10)$$

(a) LOW DENSITY

(b) HIGH DENSITY

Fig. 2.1 Assumed mechanisms for extraordinary mode absorption near the ECR and the UHR in a currentless tokamak plasma in the (a) low-density and (b) high-density regimes.

With $n_e \cong 10^{13} \text{ cm}^{-3}$ (at the end of preheating), $R_{UH} = 590 \text{ cm}$ and a toroidal field on-axis of $B (R_0 = 480 \text{ cm}) = 36.2 \text{ kG}$, Eqs. (2.9) and (2.10) indicate that $R_{EC} = 558 \text{ cm}$ and $\Delta R \cong 32 \text{ cm}$. The magnetic field at this value of R_{EC} implies a resonant microwave frequency requirement of $f_{RF} = 87 \text{ GHz}$ [$f_{RF}(\text{GHz}) = 28 B_{EC}(\text{T})$]. For $a_0 \cong 40 \text{ cm}$ (PLT-size plasma), $R_{UH} = 570 \text{ cm}$, and the same electron density, one obtains $R_{EC} = 541 \text{ cm}$, $\Delta R \cong 29 \text{ cm}$, and $f_{RF} = 90 \text{ GHz}$. (The expected thickness (Δ_H) of the heated UHR layer is discussed in Section 4.3).

Finally, in going to higher field tokamaks like INTOR [46] (with $\alpha \ll 1$), the ECR and UHR layers are in close proximity to each other. Under these conditions, Eqs. (2.8) and (2.9) simplify to

$$R_{UH} \cong R_{EC} \left[1 + \frac{\alpha}{2} \right] \quad (2.11)$$

and

$$\Delta R \cong R_{EC} \left(\frac{\alpha}{2} \right) \quad (2.12)$$

3. AMBIPOLAR POTENTIAL AND ITS EFFECTS ON TRANSPORT

The preferential heating of electrons during the microwave pulse is expected to lead to enhanced electron transport and the spontaneous appearance of an ambipolar sheath electric field ($\vec{E}_{AMB}$) between the neutral plasma and the conducting vacuum vessel. The process of UHR heating and plasma sheath formation is depicted in Fig. 3.1. This generalized schematic shows a currentless tokamak plasma bounded by a continuous conducting poloidal limiter and vacuum vessel. The limiter and vessel walls are assumed to be electrically connected and to have a negative potential with respect to the plasma surface. The usually expected charge separation [47] is eliminated through the use of conducting limiters and walls and the formation of an ambipolar sheath. These effects reduce the buildup of a large vertical electric field [48]. Those electrons with energies below the ambipolar potential energy ($e\phi_{AMB}$) are confined by poloidal $\vec{E}_{AMB} \times \vec{B}$ drift. This effective rotational transform prevents a buildup of negative charge near the top of the vacuum vessel and enables the plasma to neutralize itself.

3.1 Estimating the Ambipolar Potential

In a currentless toroidal plasma, the principal loss mechanisms are assumed to be toroidal magnetic drift, V_D , and parallel drift, $V_\parallel$, resulting from small, stray error fields, δB , in the poloidal direction. The particle drift velocities and their associated lifetimes are given by

$$V_{Dj} = V_R + V_{\nabla B} = \frac{cm_j}{e_j BR} \left(v_{\parallel j}^2 + \frac{v_{\perp j}^2}{2} \right),$$

Fig. 3.1 Representation of electron preheating near the UHR region prior to current initiation.

$$v_{Dj} \approx \frac{2ckT_j}{e_j BR} , \quad \tau_{Dj} = \frac{b}{v_{Dj}} \approx \frac{bRB}{2 \times 10^8 T_j} , \quad (3.1)$$

$$v_{\delta j} = v_{||j} \left(\frac{\delta B}{B} \right) , \quad \tau_{\delta j} = \frac{b}{v_{\delta j}} = \frac{b}{v_{||j}} \left(\frac{B}{\delta B} \right) , \quad (3.2)$$

where

$$v_{||j} = \left(\frac{2kT_j}{\pi m_j} \right)^{1/2} ; \quad j = e, i$$

is the average particle velocity along the direction of the magnetic field and $2b$ is the vertical height of the vacuum vessel.

In response to the predominant loss of heated electrons, an ambipolar potential develops which allows only a fraction of the drifting electrons [$\approx \exp(|e|\phi_{AMB}/kT_e)$] to escape to the limiter or vacuum vessel wall. The magnitude of this potential adjusts itself automatically so as to ensure that the electron and ion loss rates are equal. In a device like ISX-B (Fig. 3.2a), all ions are lost once they drift into the limiter shadow (see Section 3.3). Because the UHR heating model developed here is zero-dimensional (0-D) and utilizes flat density and temperature profiles, the conventional ambipolarity constraint [$\vec{\nabla} \cdot (\vec{\Gamma}_e - \vec{\Gamma}_i) = 0$] is replaced by the global "zero net current" condition

$$\sum_{k=C}^H \dot{N}_{ik} = \sum_{k=C}^H \dot{N}_{ek} (1 - \delta_k) , \quad (3.3)$$

where

$$\dot{N}_{ik} = n_{ik} \left(\frac{1}{\tau_{Di}} + \frac{1}{\tau_{\delta i}} \right) v_k , \quad (3.4)$$

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Fig. 3.2 Sketch showing (a) the ISX-B experimental configuration and (b) the various transport regimes for electrons and ions as a function of error field magnitude.

$$\dot{N}_{ek} = n_{ek} \left(\frac{1}{\tau_{De}} + \frac{1}{\tau_{\delta e}} \right)_k \exp \left\{ \frac{|e|\phi_{AMB}}{kT_{ek}} \right\} v_k . \quad (3.5)$$

The parameters δ and V are the coefficients for secondary electron emission from the limiter surface (electrons/electron) [49] and the plasma volume, respectively; C and H refer to the cold (bulk) and hot (UHR layer) components of the preheated plasma. Considering for the moment only the hot plasma component and assuming quasi-neutrality in the interior plasma ($n_{eH} = n_{iH} = n$), the ambipolar potential is given by

$$\phi_{AMB} = \frac{-kT_{eH}}{|e|} \ln \left\{ (1 - \delta_H) \left[\frac{V_{De} + V_{\delta e}}{V_{Di} + V_{\delta i}} \right] \right\} . \quad (3.6)$$

Equations (3.1) and (3.2) are used to write ϕ_{AMB} in terms of the appropriate drift velocities. As is evident in Eq. (3.6), the magnitude of ϕ_{AMB} will depend on the level of secondary electron emission and on whether transport is dominated by error field or curvature drift losses. To illustrate this sensitivity, two limiting cases are considered. Assuming negligible error fields ($\delta_B \approx 0$), the ambipolar potential reduces to

$$\phi_{AMB} = - \frac{kT_{eH}}{|e|} \ln \left\{ (1 - \delta_H) \frac{T_{eH}}{T_{iH}} \right\} ; \quad (V_D \gg V_\delta) . \quad (3.7)$$

In the opposite extreme of large error fields, one obtains

$$\phi_{AMB} = \frac{-kT_{eH}}{2|e|} \ln \left\{ (1 - \delta_H)^2 \frac{m_i T_{eH}}{m_e T_{iH}} \right\} ; \quad (V_\delta \gg V_D) . \quad (3.8)$$

This result is equivalent to that recovered when one considers plasma transport along magnetic field lines. Because of their higher thermal

velocity, error fields allow the electrons to quickly spiral out of the vacuum chamber, leading to large potential values.

As will be shown in Section 5, secondary electron effects are important in explaining the preheating results in ISX-B. Emitted by primary particle bombardment with the limiter, secondary electrons reduce the sheath potential and subsequently lessen the effectiveness of the electrostatic confinement. Following sheath acceleration and return to the interior plasma via $\vec{E}_{AMB} \times \vec{B}$ drift, these electrons can also cause additional cooling by thermalization with the hot electrons.

3.2 Energy Transport to the Limiter

During preheating a significant percentage of electrons have energies below $e\phi_{AMB}$. These confined electrons are reflected from the presheath and returned to the plasma without a change in their energy. The energetic tail electrons with energies greater than $e\phi_{AMB}$ can penetrate this potential barrier, however. These lost electrons carry part of their energy directly to the limiter ($3/2kT_e$). The remainder supplies the energy needed to sustain the sheath against losses resulting from the inward acceleration of the cooler ions ($e\phi_{AMB}$). With experimental evidence [27,28] supporting a relatively uniform density and temperature in the heating regions, the total electron and ion power transported to the limiter is estimated to be

$$\sum_{k=C}^H (P_{TR})^k = \sum_{k=C}^H \left\{ \frac{3}{2} kT_{ek} \dot{N}_{ek} + \left\{ \frac{3}{2} kT_{ik} + e\phi_{AMB} \right\} \dot{N}_{ik} \right\} . \quad (3.9)$$

By using Eq. (3.3) a simplified form of Eq. (3.9) can be written as

$$\sum_{k=C}^H (P_{TR})^k = \sum_{k=C}^H \left[\underbrace{\left\{ \frac{3}{2} + \frac{e\phi_{AMB}}{kT_{ek}} (1 - \delta_k) \right\}}_{\equiv \gamma_k} kT_{ek} \right] \dot{N}_{ek} + \frac{3}{2} kT_{ik} \dot{N}_{ik} . \quad (3.10)$$

Here, the parameter γ is defined as the electron energy coefficient, and the quantity $\gamma k T_e$ represents the average energy transported from the interior plasma by each primary electron in the presence of secondary electron emission. Finally, if only the hot plasma component is considered in the analysis, the ambipolarity criteria of Eq. (3.3) reduces to [49]

$$\dot{N}_{iH} = \dot{N}_{eH} - \dot{N}_{2e} = \dot{N}_{eH}(1 - \delta_H) , \quad (3.11)$$

and the following compact expression for total transport power is obtained:

$$P_{TR}^H = \left\{ \gamma k T_{eH} + \frac{3}{2} k T_{iH} \right\} \dot{N}_{iH} , \quad (3.12)$$

where

$$\gamma \equiv \left\{ \frac{3/2}{(1 - \delta_H)} + \frac{e\phi_{AMB}}{k T_{eH}} \right\} . \quad (3.13)$$

3.3 Effects of Ambipolar Electric Field on Particle Confinement

The ISX-B tokamak (Fig. 3.3) has upper and lower rail limiters which intercept all vertically drifting electrons and ions. The diameter of the rail limiter ($\Delta_L \gtrsim 3$ cm) is many times larger than the average toroidal drift distance of either the electrons ($\lesssim 0.1$ cm) or ions ($\lesssim 0.4$ cm) in the limiter shadow (~ 5 cm). These distances of vertical drift are, in turn, expected to be significantly larger than the thickness of the ambipolar sheath ($\Delta_s \gtrsim$ several Debye lengths). This comparison of scale lengths indicates appreciable interaction of both particle species with the ISX-B limiters. As illustrated in Fig. 3.3, the majority of ions reach the limiter while in the limiter shadow where $\vec{E}_{AMB}$ is nearly parallel to $\vec{B}$. The electrons, however, experience several possible electron-limiter interactions. Those heated electrons which just miss the limiter may undergo some poloidal

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Fig. 3.3 An illustration of the ISX-B chamber showing particle drift motion and interaction with the horizontal rail limiters and ambipolar sheath potential.

deflection. The majority, however, drift along the sheath potential in the limiter shadow until they reach the conducting vessel wall; there, they experience poloidal $\vec{E}_{AMB} \times \vec{B}$ drift which effectively increases their confinement time. Only the energetic electrons, whose energy of motion toward the limiter exceeds $e\phi_{AMB}$, can reach the limiter. Penetrating the ambipolar potential, these electrons complete an electrical circuit by closing on the conducting limiter and vessel walls.

Although the formation of a potential barrier reduces particle and energy loss, the limiters in ISX-B, in some sense, lead to a deterioration in confinement during preheating. By protruding into the plasma they provide an inescapable loss route for all ions and energetic electrons which drift into the limiter shadow. By removing these limiters [50] the motion of both electrons and ions would be affected by $\vec{E}_{AMB} \times \vec{B}$ drift, and confinement could be further improved. Ambipolar transport would then consist of a balance between barrier penetration by tail electrons and collisional diffusion by thermal ions. Stix [51] and Bulyginskii et al. [52] have examined electrostatically assisted toroidal confinement for reactor-grade plasmas.

4. PARTICLE AND ENERGY TRANSPORT PRIOR TO CURRENT INITIATION

4.1 Particle Balance

Given the effective transfer of wave energy to electrons at the UHR and the existence of an ambipolar potential, a near-classical particle and energy transport model has been developed to estimate the efficiency of electron heating in the absence of plasma current. With electron impact ionization as the primary particle source, the equation for particle balance is given by

$$\frac{dn_e}{dt} = n_e (n_o \langle \sigma v \rangle_{ion} - \tau_e^{-1}) , \quad (4.1)$$

where n_o is the neutral particle density and $\langle \sigma v \rangle_{ion}$ is the ionization rate coefficient. An electron loss rate is defined,

$$\tau_e^{-1} = \left(\frac{1}{\tau_{De}} + \frac{1}{\tau_{\delta e}} \right) \exp \left\{ \frac{|e|\phi_{AMB}}{kT_e} \right\} , \quad (4.2)$$

which weights the relative importance of curvature and error field drift losses and reflects the fact that only a fraction of the electrons which reach the ambipolar potential can actually penetrate through to the limiter or wall. As seen in Eq. (3.3), the value of $e\phi_{AMB}$ depends on the global ion loss rate. In ISX-B, ion confinement is limited to the average drift time by the presence of horizontal rail limiters. Such limiters are not found in FED. Nevertheless, the ion drift time approximation is still used in order to ensure conservative estimates of preheating power for FED (Section 6).

4.2 Energy Balance

The ion temperature is determined by a balance between heating due to collisions with electrons and cooling associated with charge exchange and curvature drift transport. Parallel losses are negligible for the ions except in the case where δB is large (discussed in Section 4.4). The electron temperature is determined by rf heating which balances losses resulting from: the ionization of neutrals; collisions with ions, Q_{ei} ; impurity ionization, p_{ion} ; and radiation, p_{rad} ; cooling by limiter-emitted secondary electrons; and $\vec{V}_B$ and δB drifts. The appropriate balance equations are given by

$$\frac{dE_i}{dt} = \left[\underbrace{\frac{3}{2} \frac{m_e}{m_i} \frac{n_e k(T_e - T_i)}{\tau_{ei}}}_{\equiv Q_{ei}} - \frac{3}{2} n_e n_o \langle \sigma v \rangle_{cx} k(T_i - T_o) - \frac{3}{2} n_e k T_i \left(\frac{1}{\tau_{Di}} + \frac{1}{\tau_{\delta i}} \right) \right] V_p \quad \text{and} \quad (4.3)$$

$$\frac{dE_e}{dt} = \left[p_{rf} - n_e n_o \langle \sigma v \rangle_{ion} k W_{ion} - Q_{ei} - (p_{ion} + p_{rad})_{oxygen} - \delta n_e k \frac{(T_e - T_{e2})}{\tau_e} - \frac{\gamma n_e k T_e}{\tau_e} \right] V_p. \quad (4.4)$$

Here $\tau_{ei}(m_i/m_e)$ is the electron-ion equilibration time, T_o is the neutral temperature, $\langle \sigma v \rangle_{cx}$ is the charge exchange rate coefficient, p_{rf} is the microwave heating power per unit volume absorbed by electrons over a heating width Δ_H (defined in Section 4.3), V_p is the toroidal volume of the vertical heating zone $[= 2\pi R(\approx R_{UH} + \Delta_H/2) \cdot 2b \cdot \Delta_H]$, $E_{e(i)} [= 3/2 \times n_e k T_{e(i)} V_p]$ is the electron (ion) thermal energy content, and kT_{e2} ($\approx e\phi_{AMB}$) is the energy of the secondary electron following acceleration through the sheath potential. The parameter W_{ion} is an aggregate energy loss attributed to hydrogen dissociation,

ionization, and radiation; it has been assigned a value of 30 eV [53].

4.3 Electron Preheating Assumptions

The analysis of the UHR preheating process has relied on several key assumptions. The first is the existence of a currentless toroidal plasma in which the vertical electric field has been significantly reduced due to the combined presence of an ambipolar electric field and a conducting limiter/vacuum vessel arrangement. In this configuration currents flowing over the conducting walls help to equalize differences in potential [54], while poloidal $\vec{E}_{AMB} \times \vec{B}$ drift of confined electrons prevents charge buildup and improves confinement. Although not conclusively supporting the above assumption, electron temperature measurements in ISX-B do not confirm a fatal $\vec{E}_V \times \vec{B}$ drift. The following example demonstrates this fact.

In ISX-B [$a = 27$ cm, $b = 50$ cm, B ($R_O = 93$ cm) = 13 kG] electron temperatures of ~ 50 eV have been measured near the UHR layer ($R_{UH} = 108$ cm and $\Delta_H \approx 4$ cm) at densities of $\sim 3 \times 10^{12}$ cm $^{-3}$. If the plasma cannot neutralize itself, $\vec{E}_V \times \vec{B}$ drift will cause an outward plasma displacement of Δ in a time given by [55]

$$\tau_{E/B} = \left\{ \frac{\Delta R m_i}{k(T_e + T_i)} \right\}^{1/2} \quad (4.5)$$

Using the ISX-B parameters along with $\Delta \approx (R_O + a) - R_{UH} \approx 12$ cm, $m_i = 1.673 \times 10^{-24}$ g and $T_e/T_i \sim 2$, Eq. (4.5) indicates $\tau_{E/B} \approx 4.3$ μ s. With $3n_e kT_e/2 \tau_{E/B} \approx 8.4$ W/cm 3 and $(2\pi R_{UH} \cdot 2b \cdot \Delta_H) \approx 2.7 \times 10^5$ cm 3 , an estimated 2.3 MW of ECRH power is required just to balance plasma transport; this is ~ 30 times the available ECRH power used in the ISX-B experiments and therefore suggests significantly better confinement.

The second assumption is that plasma equilibrium is maintained through the interaction of the toroidal magnetic field and the vertical plasma currents arising from curvature and error field drifts. These currents can form a closed electrical circuit by flowing along

magnetic field lines to the conducting limiter and vacuum vessel [56-58]. In the FED the role of a conducting limiter is played by the conducting vacuum vessel itself, which provides the necessary circuit path.

A final assumption involves the method used for estimating Δ_H , the radial extent of the heated electron region. Because of efficient UHR wave absorption and poor cross-field thermal equilibration, large electron temperature gradients can be produced. Such gradients can lead to possible drift-interchange instabilities and enhanced radial transport, which subsequently broadens the heating region. The thickness of this broadened layer is estimated by assuming that Bohm diffusion "dispersed" the heated electrons over a distance Δ_H in a time that is comparable to the electron drift time (the smaller of either τ_{De} or $\tau_{\delta e}$). Setting τ_{BOHM} equal to τ_{De} and $\tau_{\delta e}$ leads to the expression

$$\frac{\Delta_H^2 B}{6.25 \times 10^6 T_e} \approx \begin{cases} b RB / (2 \times 10^8 T_e) & ; \tau_{De} \ll \tau_{\delta e} \\ (bB/\delta B) / (3.3 \times 10^7 T_e^{1/2}) & ; \tau_{\delta e} \ll \tau_{De} \end{cases} \quad (4.6)$$

from which one obtains

$$\Delta_H \approx \begin{cases} 0.177(bR)^{1/2} & ; \tau_{De} \ll \tau_{\delta e} \\ 0.433(b/\delta B)^{1/2} T_e^{1/4} & ; \tau_{\delta e} \ll \tau_{De} \end{cases} \quad (4.8)$$

$$(4.9)$$

Here, b is the vertical half-height of the vacuum chamber at $R = R_{UH}$. As indicated above, the value of Δ_H depends primarily on size and on the level of magnetic field error present in the device.

4.4 Transport Regimes of Interest

The magnitude of the ambipolar potential that arises during pre-heating will depend sensitively on whether transport is dominated by curvature drift ($\tau_\delta \gg \tau_D$) or parallel drift loss ($\tau_\delta \ll \tau_D$). Magnetic

perturbations of the toroidal field structure can cause field lines to spiral out of the chamber where they may intercept the limiter or vessel walls. Error fields of only a few gauss can significantly impact plasma confinement in both the bulk and UHR regions. Equations (3.1) and (3.2) indicate that if δB satisfies

$$\delta B \geq 1.98 \times 10^{14} \frac{\sqrt{m_j T_j}}{R} ; j = e, i , \quad (4.10)$$

losses due to error fields cannot be neglected. For a hydrogen plasma, Eq. (4.10) simplifies to

$$\delta B \geq \left\{ \begin{array}{l} 6 T_e^{1/2}/R \\ 256 T_i^{1/2}/R \end{array} \right\} \quad \text{for} \quad \left\{ \begin{array}{l} V_{\delta e} \geq V_{De} \\ V_{\delta i} \geq V_{Di} \end{array} \right\} . \quad (4.11)$$

Because $V_{\delta e}/V_{\delta i} > \sqrt{m_i/m_e}$, large levels of magnetic field error are required to affect ion transport.

Various transport regimes may exist in the ISX-B tokamak (Fig. 3.2b) depending on the magnitude of the error fields present in the device. In this study it is assumed that the major component of δB is in the vertical direction (i.e., parallel to the major axis of the torus) and that parallel losses add to those associated with the toroidal drift. Potential sources of vertical error fields are the geomagnetic fields and the residual magnetization of the ISX-B iron core. Because ISX-B is not shielded against such vertical error fields, it is expected that ISX-B will operate in regime II. For $R_0 = 93$ cm, $B(R_0) = 13$ kG, $b = 50$ cm, $R_{UH} \approx 108$ cm, and $T_e \approx 50$ eV. Eqs. (3.1) and (3.2) indicate that $\tau_{De} \approx 6$ ms and $\tau_{\delta e} \approx (2.75/\delta B)$ ms. In larger, D-shaped, reactor-grade plasmas such as FED [$R_0 = 480$ cm, $B(R_0) = 36.2$ kG, $a = 130$ cm, $\sigma = 1.6$, and $b(R_{UH} = 590 \text{ cm}) \approx 100$ cm], $\tau_{De} \approx 87$ ms and $\tau_{\delta e} \approx (9/\delta B)$ ms for electrons preheated to ~ 100 eV. It is apparent that increases in size and magnetic field strength can lead to significant improvements in confinement provided effective error field control can be realized. For example, requiring that

$\tau_{De} = \tau_{\delta e}$ would dictate designing a large and complex coil system to a tolerance level of $\delta B/B \lesssim 3 \times 10^{-6}$. Such levels may be difficult to achieve in FED-size devices. Error field levels as low as $\delta B/B \sim 0.5 \times 10^{-4}$ have been achieved in the closed field line EBT device [59] using a combination of control techniques. These techniques include electron beam alignment of the toroidal mirror coil geometry and the use of error field compensation coils during operation.

5. COMPARISON WITH ISX-B EXPERIMENTAL RESULTS

During recent preheating experiments on ISX-B (Fig. 3.2a), a 35-GHz gyrotron was used to inject ~70-80 kW of microwave power into the plasma for up to 15 ms [28,35]. The ECRH power, which contained equal mixtures of ordinary and extraordinary mode waves, was launched from the high-field side at an angle of 45° to the major radius.

A variety of diagnostic techniques was used to study the characteristics of the preheated plasma. The following list summarizes the experimental findings of Ref. 28:

1. Plasma formation is initiated in a narrow, vertical layer located at the cyclotron resonance [$R_{EC} = 97$ cm for $B(R_o = 93$ cm) = 13 kG].
2. Within a fraction of a millisecond, some of the plasma reaches the top of the ISX-B chamber.
3. As the density builds, the vertical layer moves outward to a location of lower magnetic field corresponding to the upper hybrid resonance (at $n_e = 3 \times 10^{12}$ cm $^{-3}$, $R_{UH} \approx 108$ cm). This spatial transition is observed in both time-resolved density profiles and high-speed movies of the preheating process.
4. After ~5 ms of rf heating, a flat density profile is measured with an average density of $n_e \approx 3 \times 10^{12}$ cm $^{-3}$. Measurements are made using on-axis vertical and horizontal microwave interferometers.
5. The electron temperature in the bulk plasma is estimated to be 11-13 eV based on spectroscopic measurements of oxygen impurity. A lower estimate of 8-10 eV is obtained using a particle energy analyzer (PEA) which is inserted into the center of the plasma.

6. An electron temperature of ~ 50 eV is measured (using the PEA) over a 4-cm-thick region centered at $R_{UH} = 108$ cm.
7. The preheated plasma appears macroscopically stable 5 ms after the microwave power is turned on. It remains quiescent during the remainder of the 15-ms rf pulse.
8. Once the rf power is terminated, the density decays to half its value in about 10 ms. This half-life indicates a bulk plasma temperature of ~ 20 eV if the decay is due to curvature drift.
9. Spectroscopic and radiometric data prove that radiation cooling is not a dominant loss mechanism in the preheated plasma. Of the 5 kW of radiated power measured, only about 20% is from impurities and the remaining 80% is from hydrogen.

This last experimental observation is most interesting because it contradicts the theoretical analysis of Ref. 28, which suggests that oxygen radiation is responsible for most (~ 75 kW) of the injected microwave power.

5.1 Theory vs Experiment

In an effort to explain the preheating experiments, the ISX-B results have been reanalyzed using the theoretical model for UHR heating presented in the previous sections. With $T_e \approx 10$ eV, $n_e \approx 3 \times 10^{12} \text{ cm}^{-3}$, and $f_{RF} = 35$ GHz, Eq. (2.1) indicates that wave absorption will occur primarily at the UHR layer. According to Eq. (2.3), this layer is located at $R_{UH} \approx 108$ cm for $\alpha \approx 0.2$ and $R_{FC} = 97$ cm. The significant electron heating expected at R_{UH} is also substantiated by the 50-eV electron temperatures measured at $R = 106$ and 110 cm. The hot plasma measured outboard of the UHR layer is interpreted as evidence supporting a broadening of the heated electron region. Because T_e measurements were not made beyond 110 cm, the actual extent of the heating region is unknown. However, by assuming that error field losses dominate electron transport, the width of

the heated electron region can be estimated using Eq. (4.9). Calculations indicate that Δ_H can vary from ~ 8 to 4 cm for $T_e = 50$ eV and $\delta B = 1-4$ G.

Because of the apparent leveling off of the plasma density and temperature after several milliseconds of preheating [28], the equations for particle [Eq. (4.1)] and energy balance [Eqs. (4.3) and (4.4)] have been solved as steady-state equations. Before one can estimate the ECRH power required to balance the various electron energy losses, it is necessary to first determine the confining ambipolar potential. The procedure used for estimating ϕ_{AMB} is to first eliminate the neutral density (n_0) from the ion energy balance using Eq. (4.1). With the plasma density held fixed at $n_e = 3 \times 10^{12} \text{ cm}^{-3}$, one then solves Eq. (4.3) for ϕ_{AMB} as a function of T_i . Estimates of ϕ_{AMB} are made in both the bulk and UHR regions using a range of electron temperatures and error field values believed to be characteristic of that region. However, only those self-consistent sets of parameters (corresponding to a single value of ϕ_{AMB}) that satisfy Eq. (3.3) constitute a bonafide "zero net current" solution determining the global ambipolar potential. Once ϕ_{AMB} is known, the neutral density and ECRH power requirements for each region can be obtained using Eqs. (4.1) and (4.4).

Steady-state solutions, representing a number of ambipolar equilibria, have been generated for comparison with experiment. The results are summarized in Tables 1 and 2. Using the plasma parameters measured in ISX-B, Table 1 shows the required ECRH power as a function of error field value and secondary electron emission.

5.1.1 Effect of Error Fields

As Table 1 indicates, increasing values of δB lead to a decrease in electron confinement and a rise in the ambipolar potential. Error fields have little effect on ion transport which is dominated by curvature drift losses. In the absence of secondary electron emission, the ambipolar potential is very effective in confining the bulk electrons. Such increased confinement leads to low values of neutral density in the bulk plasma. This fact, together with the lengthy

Table 1. Estimates of key plasma parameters and required ECRH power in ISX-B for $n_0 = 3 \times 10^{12} \text{ cm}^{-3}$ and several values of δB without and with secondary electron emission

$\delta_c = 0.0, \delta_H = 0.0$	$\delta B = 1 \text{ G}$		$= 2 \text{ G}$		$= 4 \text{ G}$	
T_e (eV)	10	50	10	50	10	50
T_i (eV)	9.87	25.9	9.87	23.2	9.85	19.3
τ_{De} (ms)	30.2	6.05	30.2	6.05	30.2	6.05
$\tau_{\delta e}$ (ms)	6.15	2.75	3.08	1.38	1.54	0.69
τ_{Di} (ms)	30.6	11.7	30.6	13.0	30.7	15.5
$\tau_{\delta i}$ (ms)	265	164	133	86.6	66.4	47.2
ϕ_{AMB} (V)	-30.0		-47.4		-54.3	
n_0 (10^{10} cm^{-3})	0.068	0.89	0.054	1.27	0.051	2.01
Δ_H (cm)	-	8.1	-	5.8	-	4.1
V_H (10^6 cm^3) ^a	-	0.56	-	0.40	-	0.28
V_C (10^6 cm^3) ^b	2.59	-	2.75	-	2.87	-
P_{ECRH} (kW)	1.3	12.9	1.3	12.8	1.5	13.8
$\delta_c = 0.1, \delta_H = 0.5$	$\delta B = 1 \text{ G}$		$= 2 \text{ G}$		$= 4 \text{ G}$	
T_e (eV)	10	50	10	50	10	50
T_i (eV)	9.76	23.6	9.76	20.5	9.74	16.9
τ_{De} (ms)	30.2	6.05	30.2	6.05	30.2	6.05
$\tau_{\delta e}$ (ms)	6.15	2.75	3.08	1.38	1.54	0.69
τ_{Di} (ms)	31.0	12.8	31.0	14.7	31.0	17.9
$\tau_{\delta i}$ (ms)	267	172	133	91.9	66.8	50.7
ϕ_{AMB} (V)	-22.9		-30.1		-36.2	
n_0 (10^{10} cm^{-3})	0.74	1.23	0.30	1.80	0.31	2.89
Δ_H (cm)	-	8.1	-	5.8	-	4.1
V_H (10^6 cm^3)	-	0.56	-	0.40	-	0.28
V_C (10^6 cm^3)	2.59	-	2.75	-	2.87	-
P_{ECRH} (kW)	3.2	16.3	3.4	15.9	3.9	16.8

^a $V_H = 2\pi R_{UH} \cdot 2b \cdot \Delta_H$.

^b $V_C = V_P - V_H$, where $V_P = 2\pi R_0 \cdot 2b \cdot 2a = 3.15 \times 10^6 \text{ cm}^3$.

Table 2. Estimates of the same key parameters and ECRH power for several different cold/hot electron temperatures

$\delta_c \approx 0.1, \delta_H \approx 0.7$	$\delta B = 1 \text{ G}$		$= 2 \text{ G}$		$= 4 \text{ G}$	
T_e (eV)	20	70	20	70	20	70
T_i (eV)	17.9	23.3	17.9	20.1	17.7	16.5
τ_{De} (ms)	15.1	4.32	15.1	4.32	15.1	4.32
$\tau_{\delta e}$ (ms)	4.35	2.33	2.18	1.16	1.09	0.58
τ_{Di} (ms)	16.9	13.0	16.9	15.0	17.1	18.4
$\tau_{\delta i}$ (ms)	197	173	98.5	92.9	49.5	51.4
Φ_{AMB} (V)	-32.0		-43.5		-54.4	
n_o (10^{10} cm^{-3})	0.39	1.41	0.39	1.97	0.42	3.01
Δ_H (cm)	-	8.9	-	6.3	-	4.4
v_H (10^6 cm^3)	-	0.62	-	0.43	-	0.30
v_c (10^6 cm^3)	2.53	-	2.72	-	2.85	-
P_{ECRH} (kW)	11.0	25.9	13.3	23.8	16.0	27.6
$\delta_c \approx 0.1, \delta_H \approx 0.8$	$\delta B = 1 \text{ G}$		$= 2 \text{ G}$		$= 4 \text{ G}$	
T_e (eV)	20	100	20	100	20	100
T_i (eV)	17.8	18.4	17.8	15.0	17.6	11.3
τ_{De} (ms)	15.1	3.02	15.1	3.02	15.1	3.02
$\tau_{\delta e}$ (ms)	4.35	1.95	2.18	0.97	1.09	0.49
τ_{Di} (ms)	17.0	16.4	17.0	20.2	17.2	26.8
$\tau_{\delta i}$ (ms)	198	194	98.8	108	49.7	62.1
Φ_{AMB} (V)	-33.8		-45.4		-56.4	
n_o (10^{10} cm^{-3})	0.36	1.92	0.35	2.75	0.38	4.32
Δ_H (cm)	-	9.7	-	6.8	-	4.8
v_H (10^6 cm^3)	-	0.68	-	0.47	-	0.33
v_c (10^6 cm^3)	2.47	-	2.68	-	2.82	-
P_{ECRH} (kW)	10.4	52.5	12.8	49.3	15.3	51.2

confinement time for ions (~ 30 ms), results in a negligible power loss from the bulk plasma.

In the UHR region, however, $e\phi_{AMB}/kT_{eH}$ is of order unity, and a greater percentage of heated electrons escape to the limiter. Transport losses from this region account for ~ 60 to 70% of the calculated ECRH power. Collisional cooling of electrons on ions and neutral hydrogen ionization (listed in order of their importance) account for the remaining power.

5.1.2 Secondary Electron Emission

Those heated electrons which succeed in hitting the stainless steel limiters in ISX-B can possess sufficient energy to cause secondary electron emission. Data showing the secondary electron yield (δ) for stainless steel as a function of primary electron energy has been given by Hall [60]. This data has been extrapolated to a lower range of electron temperatures ($10 \text{ eV} \leq T_e \leq 100 \text{ eV}$) in order to examine the effects of secondary emission on particle and energy transport in ISX-B. As Table 1 shows, secondary electrons can cause a substantial reduction in the sheath potential (by as much as 60 to 70%). However, at $T_{ec} \approx 10 \text{ eV}$ and $T_{eH} \approx 50 \text{ eV}$, the resulting deterioration in confinement does not translate into a large increase in hot plasma transport because $\sim 50\%$ of the sheath sustaining energy is automatically returned to the plasma by the secondary electrons. The additional power requirements in the UHR region are due mainly to increased neutral ionization losses and collisional electron cooling on colder, secondary electrons. These secondary electrons equilibrate rapidly ($< 5 \mu\text{s}$) with the heated electrons and account for roughly 1.5 to 2.0 kW of additional ECRH power. In the bulk plasma, the higher ECRH power needs are due to increased neutral ionization and transport. Potential heating of the bulk electrons by the more energetic secondary electrons has been neglected in the bulk plasma power estimates.

5.1.3 Suggestion of Higher Electron Temperatures

Even with secondary electron emission included in these theoretical estimates, one can, at best, account for only 25% of the injected rf power using the electron temperatures reported in Ref. 28. The lack of agreement between theory and experiment at these temperatures has prompted consideration of a higher range of values for both the bulk and UHR region. Indications of higher electron temperatures ($T_{ec} = 20$ eV and $T_{eh} \leq 100$ eV) can, in fact, be inferred from several of the experimental findings (5, 6, and 8) mentioned at the beginning of this section. The sensitivity of the ECRH preheating power to different cold/hot electron temperature combinations is shown in Table 2. Here, one notices that there is a significant increase in the ECRH power requirements. At $T_{eh} = 70$ eV, $P_{ECRH} \lesssim 40$ kW, and at $T_{eh} = 100$ eV, $P_{ECRH} \lesssim 66$ kW. Enhanced transport accounts for most of the observed increase in the UHR region (~ 15 kW at $T_{eh} = 70$ eV and ~ 35 kW at $T_{eh} = 100$ eV). Because the percentage of secondary emission increases with electron temperature, the ambipolar potential remains roughly constant (for a given error field value), despite the rising temperatures. Under these conditions a larger number of hot electrons will be lost from the UHR layer and, in their place, cold secondary electrons will be accelerated back in. This exchange of cold for hot electrons satisfies the ambipolarity constraint, but it also leads to a sizeable electron energy loss. The equilibration of the secondary electrons with their hot counterparts causes a further energy drain amounting to ~ 10 kW (for $T_{eh} \approx 100$ eV). In the bulk plasma, the nonresonant absorption of rf power balances losses due to electron-ion collisions (~ 5 to 7 kW), transport (~ 3 to 7 kW), and neutral ionization (~ 2 to 3 kW). Because the rf power requirements in the heated plasma exceed the 40 kW of extraordinary mode radiation, some absorption of the ordinary mode (~ 10 kW) is also indicated. Despite the fact that the preheated plasma is optically thin to the 0-mode at the plasma parameters being discussed, effective absorption should still be possible because of multiple reflections and the potential for repolarization at the vacuum vessel walls.

Before concluding this section, a comparison between the theoretical results (at $T_{ec} = 20 \text{ eV}$ / $T_{eh} = 100 \text{ eV}$) and the ISX-B experimental observations is in order. According to the theoretical model, the formation of an ambipolar potential is a direct consequence of ECRH. The radial electric field ($\vec{E}_{AMB}$), which forms between the neutral plasma and the vacuum vessel, enables the plasma to neutralize itself via poloidal $\vec{E}_{AMB} \times \vec{B}$ drift. The ambipolar equilibrium which results may explain the quiescent nature of the preheated plasma and the absence of large-scale $\vec{E}_v \times \vec{B}$ drift. This ambipolar potential improves bulk electron confinement and allows significant electron-ion temperature equilibration to occur. Using $\Sigma \dot{N}_i = n_i V_p / \tau_{pi}$ to define a global ion confinement time, one finds that $\tau_{pi} \approx 16\text{-}13 \text{ ms}$ for $\delta B = 1\text{-}4 \text{ G}$. These values compare favorably with the experimental estimate of $\sim 15 \text{ ms}$. It also suggests that plasma decay in ISX-B is controlled mainly by ion transport. Because oxygen radiation is negligible in the ISX-B experiment, it has not been included in the power balance calculations. However, it has been estimated that $\sim 8 \text{ kW}$ (total for both regions) is required for neutral hydrogen ionization. By assuming that roughly half of the 30 eV needed for ionization is associated with line radiation, one finds that the resulting 4 kW is consistent with spectroscopy and radiometry measurements for hydrogen. Finally, the estimates for Δ_H ($\sim 5\text{-}10 \text{ cm}$) and P_{ECRH} ($\lesssim 66 \text{ kW}$) appear to be in reasonably good agreement with the experimental data ($\Delta_H \gtrsim 4 \text{ cm}$ and $P_{ECRH} \approx 70\text{-}80 \text{ kW}$). In order to be fair in this comparison, it should be pointed out that the theoretical results are sensitive to the values of δ selected. Reference 60 indicates that $\delta \lesssim 1.0$ at $T_e = 100 \text{ eV}$. The author has, however, specified that δ is not to exceed ~ 0.8 , a value which corresponds to saturated emission [49] in a hydrogen plasma. As a final consideration, it has been assumed that unipolar arcing [61] is unimportant. Although estimates of the triggering potential for arcing range from a few tens [61] to several hundred eV, [62] the signature of arcing (significant impurity production) has not been observed in the experiments.

The results just presented provide a plausible interpretation of the available ISX-B data. However, in order to rigorously test the

UHR heating model, the experimental data base should be expanded to include measurements of all key parameters (T_e , T_i , n_e , n_o , δB , and Δ_n). For example, by monitoring the particle current to the ISX-B limiters and utilizing infrared camera measurements, valuable data concerning particle and energy transport during preheating could be obtained.

6. APPLICATION TO THE FED TOKAMAK

Encouraged by the reasonably good agreement, to date, between theory and experiment, the UHR heating model has been applied to FED to assess the ECRH power requirement and its sensitivity to uncertainties in temperature, error field, δB , ambipolar potential, ϕ_{AMB} , and impurity content. The results, summarized in Table 3, are based on several assumptions. These include (1) total absorption of the EO-mode wave energy at the UHR layer with very little nonresonant heating of the inner bulk plasma and (2) a dynamic density startup. In the latter, it is assumed that preheating commences from a prefill pressure of $\sim 10^{-5}$ torr. As electrons heat and the ionization rate becomes substantial, the plasma density increases to 10^{19} m^{-3} in the UHR region as neutrals are pumped from the remaining chamber volume. Inherent in this second assumption is a controlled neutral gas feed capability and the absence of significant sources or sinks on the chamber wall.

Examining Table 3, it is apparent that increasing levels of oxygen contaminant can have a strong impact on the rf power requirement before the electron temperature is raised beyond the radiation barrier (occurring at $T_e = 20 \text{ eV}$). It is also noted that these results may be somewhat optimistic because the radiation model used [63] assumes coronal equilibrium. Once this critical barrier temperature is surpassed, the required ECRH power drops sharply and is determined primarily by error field transport and collisional cooling of electrons on ions. The ions in turn lose energy mainly through charge exchange with cold neutrals ($T_0 \approx 0 \text{ eV}$ is assumed).

Table 3 also illustrates that the presence of a low error field (2 G) is sufficient to dominate electron transport. Error field levels higher than those shown can significantly reduce the confinement time and compete with cross-field Bohm diffusion to bring about a large decrease in the heated volume. This results in only a modest

Table 3. Dependence of ECRH preheating power requirement P_{ECRH} on poloidal error field δB , T_e , the ambipolar potential ϕ_{AMB} , and the impurity content in FED

Parameters	Poloidal error field $\delta B = 2G$			Poloidal error field $\delta B = 0$		
T_e (eV)	20	100	200	20	100	200
T_i (eV)	~ 20	92	144	~ 20	94	154
$\tau_{\delta e}$ (ms)	10	4.4	3.1	-	-	-
τ_{De} (ms)	430	87	43	430	87	43
Δ_H (cm)	6.5	10	12	42	42	42
V_H (m ³)	4.8	7.5	8.7	31	31	31
ϕ_{AMB}/T_e	-3.1 ± 0	-2.7 ± 0	-2.7 ± 0	~ 0	~ 0	~ 0
P_{ECRH} (MW)						
(1% oxygen)	0.44 ± 0.46	0.15 ± 0.50	0.48 ± 1.65	2.8	0.31	0.98
(3% oxygen)	1.30 ± 1.32	0.16 ± 0.51	0.50 ± 1.68	8.4	0.34	1.07

decrease in loop voltage during the subsequent current startup phase. In contrast to the small ISX-B tokamak ($\tau_{\delta e} \leq \tau_{De}$), rapid electron loss in FED ($\tau_{\delta e} \ll \tau_{De}$) results in large ambipolar potentials ($e\phi_{AMB}/kT_e \sim 3$), which can exceed the triggering potential for unipolar arcing. Secondary electron emission from the walls can also lead to a deterioration of the ambipolar potential and an increase in P_{ECRH} . The actual value of this increase (a factor of 2 to 3) has been exaggerated by assuming $\phi_{AMB} \rightarrow 0$. In reality, for electron temperatures ≥ 100 eV, secondary emission will saturate at $\delta \sim 0.8$, causing the sheath potential to fall to a minimum value of order $e\phi_{AMB}/kT_e \sim 1$. With no error fields present, there is an improvement in confinement leading to $T_e \approx T_i$, $\phi_{AMB} \approx 0$ and a factor of ~ 4 increase in $\Delta_H [\propto (bR)^{1/2}]$. Because the volume of the heated plasma has increased by a factor of ~ 6 , oxygen losses can become quite prohibitive at $T_e \approx 20$ eV. Finally for $\phi_{AMB} \sim 0$, a continuous conducting limiter or vacuum vessel becomes essential to prevent the buildup of a large vertical electric field.

These results suggest the following approach to electron heating prior to current initiation. In the presence of a low poloidal error field (~ 2 G), ECRH power up to 2 MW is injected to burn through the impurity radiation barrier in the UHR region, achieving $T_e \lesssim 100$ eV. The initial heating volume, estimated to be about 5 m^3 would be produced in a time scale < 50 ms. The error field would then be eliminated near the heating zone (e.g., via field-cancelling coils), thereby allowing the heated volume to increase to about 30 m^3 . The ECRH power can then be reduced to about 1 MW, sustaining an electron temperature of ~ 100 eV at a density of about 10^{19} m^{-3} .

7. PHYSICS CONSTRAINTS DURING CURRENT STARTUP

In present-day tokamak experiments, gas breakdown and current startup are achieved by applying a toroidal voltage to the plasma via a series of preprogrammed current ramps in the OH coils. Because the duration of a tokamak discharge is presently in the range of 0.1 to 1.0 s, the plasma current ramp rates have a tendency to be large ($\dot{I}_p \cong 5\text{-}20 \text{ MA/s}$) in order to maximize the period of current flattop. The current-rise phase is also somewhat turbulent in existing tokamaks with experimental observations indicating a series of "minor" disruptions. Characteristically, there is a brief loss of equilibrium accompanying each event. These disruptions are thought to be responsible for the introduction of impurities into the plasma through enhanced plasma-limiter contact. Even with impurity ingestion, the ohmic heating power density is sufficiently large in these small tokamaks that a favorable plasma energy balance can usually be maintained during the startup phase.

Difficulties arise, however, when this scenario is extrapolated to large tokamaks such as FED. Attempts at full-size startup with $\dot{I}_p \cong 15 \text{ MA/s}$ and a plasma inductance of $L_p \lesssim 12 \text{ }\mu\text{H}$ would require a sizeable inductive voltage ($\lesssim 200 \text{ V}$). The ability to maintain an acceptable energy balance would also be in question due to the fall-off in the ohmic power density with increasing major radius. There is, therefore, a strong incentive to examine startup scenarios for large tokamaks which can eliminate many of these potential problems. In this and the remaining chapters of this study, an rf-assisted expanding radius approach to current startup is examined. Modest quantities of ECRH power (up to 5 MW) are utilized for both preheating and also as an auxiliary heat source for electrons during the expansion phase.

As discussed previously, preheating can lead to substantial electron temperatures ($\sim 50 - 100 \text{ eV}$) prior to current initiation. At such temperatures, the large resistive voltage requirement (indicated

in Fig. 1.2) is significantly reduced and current startup can be accomplished with a much lower applied voltage. A low-voltage start-up, however, implies reduced current ramp rates ($\sim 0.5 - 1.0$ MA/s), extended startup times, and a gradual buildup of ohmic input power to the plasma. With energy confinement scaling like $\tau_E \propto na^2$ in ohmic discharges, supplemental ECRH power is necessary to maintain a high plasma conductivity. This auxiliary heating is particularly important during the current initiation phase, when both the plasma density and minor radius are at their minimum values. Despite this apparent drawback, starting up at reduced radius is actually advantageous. Smaller rf power levels (< 0.5 MW) are sufficient to sustain a desired heating level and a small radius can also augment the current penetration process.

In large part, the feasibility of the expanding radius approach is dependent on active control of the plasma position and shape, the fueling and current ramp rates, and the level of electron heating. Because the tokamak concept has been continually improving due to advances in all of these areas, a broad operating regime is anticipated for current startup. A number of key physics constraints must be observed, however, if the initial discharge is to evolve properly to an ignition-size plasma. These constraints are important because they limit the rate of current rise, along with the fueling and heating rates. A discussion of these constraints and their impact on the expanding radius approach are outlined below.

7.1 Current Ramp Rate and Skin Current Formation

Transport simulations of the current-rise phase in tokamaks indicate the formation of a sizeable skin current, when the current ramp time is significantly less than the plasma skin time. Figure 7.1 from Hogan [64] shows the time evolution of the current density profile in ISX-B using a neoclassical resistivity model. The current ramp rate in this case is considerable ($\dot{I}_p = 20$ MA/s) and leads to a pronounced edge peaking of the current.

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Fig. 7.1 Current penetration calculated for ISX-B without the MHD relaxation model. The attempt to produce a 200-kA current in 10 ms leads to a pronounced skin current, unlike that of the experimental observation. Figure courtesy of J. T. Hogan [64].

In actual experiments, however, the distinct skin distribution is absent. In its place, a series of minor disruptions are observed [65]. These disruptions, which can begin shortly after plasma formation, are attributed to the nonlinear evolution of magnetic islands induced by double tearing modes [31,66,67]. These tearing modes are driven unstable whenever the $q_I(r)$ profile is double-valued, as in the case of a hollow current profile (see Fig. 1.3b). The subsequent growth and interaction of these islands can lead to field line reconnection and a relaxation of the skin distribution.

A phenomenological model has been developed for use in transport codes which can reproduce key features of the relaxation process observed in experiments [68,69]. The same ISX-B startup conditions from Fig. 7.1 are shown in Fig. 7.2 using the MHD relaxation model [64]. Instead of a noticeable skin current, a series of small relaxations occurs which causes the current profile to "fill in" and the dimensionless internal inductance to increase (see Fig. 7.2). Each jump in inductance leads to a momentary loss of equilibrium as the plasma pushes out against the applied vertical field.

Experimental observations [32] indicate that the resulting contact between the plasma and the limiter can lead to limiter overheating and the release of gas and material impurities into the plasma. If the impurity influx is sufficiently large, radiative edge cooling can contract the current channel and possibly trigger a major disruption.

During such major disruptions, magnetic flux surfaces can be destroyed over a sizeable portion of the plasma causing a drastic reduction in confinement and large plasma energy losses. Ultimately, a severe major disruption can terminate the discharge. The resulting plasma energy dump to the limiter can cause surface ablation and impurity contamination in subsequent discharges. In extrapolating to an FED-size plasma, the ability to sustain several severe disruptions during current rise can be costly in terms of its depleting effect on the ohmic power supply and its potential damage to limiters and walls.

The above discussion indicates that current ramp rate, plasma-wall interactions and enhanced impurity content are strongly correlated.

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Fig. 7.2 Current penetration for the same case as Fig. 7.1 but with the MHD reconnection model. The reconnection process gives rise to a plasma shift (proportional to $\Delta\lambda_i$). This shift leads to the enhanced probability of arcing with the high- T_e plasma contacting the wall or limiter. Figure courtesy of J. T. Hogan [64].

It therefore seems prudent to avoid this entire operating regime in order to obtain a quality (low impurity content) ohmic discharge. In this study, the current ramp rates are sufficiently low (because of the low voltage requirement) that skinless current profiles result even with a neoclassical resistivity model. Experimental studies on the ALCATOR-A tokamak [65] have shown that reducing $\dot{I}_p$ is a viable technique for minimizing, and possibly even eliminating, disruptive activity during current startup.

Because skin formation is most likely to occur during the current initiation phase of the startup scenario, the current ramp times have been constrained to satisfy the following inequality:

$$\tau_I (= I_p / \dot{I}_p) > \tau_s \quad . \quad (7.1)$$

Here,

$$\tau_s (s) = \frac{\mu_o a_p^2}{\eta_{SPITZER}} \cong \frac{767 [T_e (keV)]^{3/2} [a_p (m)]^2}{Z_{eff} \ln \Lambda_{ei}} \quad (7.2)$$

is the resistive skin time across the plasma [70]. The enhancement of τ_s with increasing plasma temperature and size makes it difficult to satisfy Eq. (7.1) in large tokamaks with no rf assist because τ_I is characteristically small. Starting up at reduced radius (discussed in Section 8) eases the severity of this constraint and is also more compatible with low current ramp rates.

7.2 Density Limitations

The maximum plasma density at which stable tokamak operation is possible is determined by the onset of a disruptive instability. In ohmically heated discharges with typical operating conditions ($q_I(a) \cong 5.0 - 3.0$), Murakami [71] has observed an empirical scaling for the maximum electron density

$$(\bar{n}_e)_{max} = k \times (B/R_o); k \propto (q_I(a) Z_{eff})^{-1} \quad , \quad (7.3)$$

which is sensitive to the current density profile and the plasma impurity content. Exceeding this critical density can lead to enhanced edge cooling brought on by high levels of neutral gas and impurity radiation. The subsequent shrinkage of the current channel can then trigger a disruption. Controlling the gas feed rate and impurity level of the plasma has led to increases in this density ceiling. In beam heated discharges, the beam power tends to broaden the current density and can further extend the operational range of $\bar{n}_e$.

Based on data from various ohmic and beam heated discharges, the maximum achievable density is given by [72]

$$(\bar{n}_e)_{\max} (\text{m}^{-3}) = k(B/R_0) = 4 \times 10^{19} \frac{I_p (\text{MA})}{[a_p (\text{m})]^2} \quad (7.4)$$

The expanding radius simulations discussed in Sections 10 and 11 have complied with this density scaling limitation.

7.3 Beta Limitations During Current Startup

A measure of the maximum plasma pressure that can be confined by a magnetic field is provided by the parameter beta which is defined as^{*}

$$\beta = \frac{2\mu_0 \int p dV_p}{V_p B^2} = \frac{2\mu_0 \bar{p}}{B^2} \quad (7.5)$$

Here, $\bar{p}$ is the volume-averaged plasma pressure, V_p is the plasma volume, and B is the toroidal magnetic field at the center of the plasma.

High- β operation is desirable in a fusion reactor because the fusion power density increases as β^2 for a fixed magnetic field strength ($P_F \propto \beta^2 B^4$). In principle, therefore, operating at a high fusion power density should lead to smaller and less expensive

^{*} In Sections 7 through 11, mks units are used and temperatures are given in kiloelectron volts.

fusion reactors. At present, the maximum attainable β is thought to be limited by either equilibrium or stability considerations.

During an rf-assisted expanding radius startup, substantial amounts of ECRH power (up to 5 MW) will be used to maintain a high conductivity plasma. In order to ensure that equilibrium and stability limits are not violated, it will be necessary to carefully monitor the plasma density, temperature, and current.

7.3.1 Equilibrium Beta Limits

In view of the lack of present experimental evidence supporting flux-conserving tokamak operation [73] (which allows $\beta_p > A$), a separatrix equilibrium limit [74] is generally used to constrain poloidal beta at

$$\beta_p = \frac{2\mu_0 \bar{p}}{\bar{B}_p^2} \lesssim f_A A. \quad (7.6)$$

Here, $\bar{B}_p$ is the average poloidal magnetic field value at the plasma surface (defined in Section 8.2), $A (= R/a)$ is the plasma aspect ratio, and f_A is an aspect ratio multiplication factor determined by experiment (typically $f_A \lesssim 1$). For low- β hydrogen plasmas, with circular cross section, Eq. (7.6) simplifies to

$$\beta_p \approx 10^{-8} \frac{a^2 \overline{n(T_e + T_i)}}{I_p^2} \lesssim f_A A. \quad (7.7)$$

Equations (7.6) and (7.7) relate the volume-averaged plasma pressure to the confining poloidal magnetic field and to the plasma current which produces it. Complying with Eq. (7.7) is particularly important during rf-assisted current startup. Because the plasma current levels are characteristically low during the current initiation phase, the introduction of large amounts of rf power must be avoided in order to satisfy the equilibrium condition of Eq. (7.7).

One can proceed a step further in this analysis by introducing the plasma safety factor which relates the toroidal magnetic field to

the plasma current. The flux definition of the edge safety factor, q_I , is given by [75]

$$q_I = \frac{RB}{2\pi} \int \frac{dl_p}{R^2 B_p}, \quad (7.8)$$

where the line integral is evaluated at the boundary of the plasma cross section. For a large aspect ratio circular plasma, at low β , Eq. (7.8) reduces to

$$q_I = \frac{aB}{RB_p} = \frac{2\pi a^2 B}{\mu_0 I R}. \quad (7.9)$$

Combining Eqs. (7.5), (7.6), and (7.9), a simple equation relating β and β_p is obtained. This relation, which again is valid at low β and high aspect ratio, is given by

$$\beta = \frac{\beta_p}{q_I^2 A^2}. \quad (7.10)$$

Because of the equilibrium ceiling on β_p , given by Eq. (7.7), β will be constrained to a level given by

$$\beta \lesssim (\epsilon/q_I^2) f_A, \quad (7.11)$$

where $\epsilon (=A^{-1})$ is the inverse aspect ratio.

7.3.2 Stability Beta Limits

At sufficiently high plasma pressures, theory predicts [76] that ballooning modes can be driven unstable when

$$\beta \gtrsim \beta_{\text{crit}} = \frac{r_p}{R_0 q_I^2}. \quad (7.12)$$

Here, r_p is the pressure gradient scale length, $p/(dp/dr)$. For sufficiently broad pressure profiles, $r_p \cong a$, and it is observed that the equilibrium and stability beta limits are roughly equivalent.

Recent ISX-B operations with neutral beam injection heating have produced high-beta equilibria with β values in excess of the predicted stability limits for ideal ballooning modes [77]. However, an apparent saturation of poloidal beta with increasing beam power is also observed at $\beta_p \cong 2$ [78]. With ISX-B having an aspect ratio of ~ 3.4 , this saturation implies a possible β_p limit at $\beta_p \cong 0.6 A$.

Based on the theoretical estimates and experimental evidence discussed above, the following set of conservative beta limits has been selected for use in this study:

$$(1) \quad \beta_p \lesssim 0.5 A ,$$

$$(2) \quad \beta \lesssim \beta_{\text{crit}} = \epsilon/q_I^2 .$$

Of the two, the β_p limit is the most restrictive. No credit has been taken in (1) and (2) for elongation effects which are expected to provide some beta enhancement.

8. EXPANDING RADIUS OPTIONS FOR CURRENT STARTUP

8.1 Advantages of the Expanding Radius Approach

During current startup in large tokamaks, a pronounced skin current distribution is theoretically predicted. This skin effect arises because ramp times are typically much shorter than the diffusion time for the poloidal magnetic field. These skin currents can prevent the applied electric field from penetrating into the plasma center, thereby inhibiting the development of the desired current distribution. The technique most frequently suggested to prevent the occurrence of this skin effect is the expanding radius startup [17,79]. Beginning with a small radius plasma, the size and current are simultaneously increased. With proper synchronization, current can be added to the outside edge of the plasma in a manner which leads to a uniform current density distribution.

In addition to eliminating the skin effect, additional physics benefits can be realized. These include: (1) the potential for controlling the evolution of the current profile during expansion; (2) reduced volume losses (ionization and radiation) associated with full-bore startup; and (3) a decrease in plasma-limiter interaction and subsequent impurity contamination (a manifestation of skin current relaxation). The expanding radius startup technique is also well suited to the UHR preheating process (refer to Section 8.3 of this paper).

A number of expanding radius options are available for improving current startup. These include the use of a retracting limiter or the shifting of the discharge toward the center of the chamber from either the inboard or the outboard regions of the vacuum chamber. Although growth of the plasma minor radius off the inner wall effectively uses the ohmic power density $P_{OH} \propto [B/R]^2$, it prevents the utilization of UHR preheating. Therefore, this option will not be considered in this study.

8.2 Current Rise During the Expansion Phase

The relationship between plasma current, current density, and poloidal magnetic field is provided by Ampere's law:

$$\mu_0 I_p = \mu_0 \iint \vec{j} \cdot d\vec{A} = \iint \vec{\nabla} \times \vec{B}_p \cdot d\vec{A} = \oint \vec{B}_p \cdot d\vec{\ell} . \quad (8.1)$$

The calculations performed in this study have focussed on plasmas with elliptical cross sections as shown in Fig. 8.1. For this geometry

$$\mu_0 I_p \cong \bar{B}_p \oint d\ell = \bar{B}_p 2\pi a \left(\frac{1 + \sigma^2}{2} \right)^{1/2} , \quad (8.2)$$

where

$$\bar{B}_p = \oint B_p d\ell / \oint d\ell \quad (8.3)$$

is the average value of the poloidal field on the plasma surface, and $\oint d\ell$ is the circumference of the elliptical plasma derived in Appendix A. The parameter $\sigma (\equiv b/a)$ is the plasma elongation and a and b are the horizontal and vertical plasma minor radii, respectively.

For an elliptically shaped plasma, the edge safety factor, given by Eq. (7.8), becomes

$$q_I = \frac{aB(R)}{R\bar{B}_p} \left(\frac{1 + \sigma^2}{2} \right)^{1/2} . \quad (8.4)$$

Combining Eqs. (8.2) and (8.4), one obtains the following expression:

$$I_p = \frac{2\pi a^2 B(R_0) R_0}{\mu_0 q_I R^2} \left(\frac{1 + \sigma^2}{2} \right) , \quad (8.5)$$

linking the plasma current to the major tokamak parameters, $B(R)$, R , a , σ , and q_I . In the above equation, the toroidal field on-axis, $B(R_0)$, is assumed constant and $B(R) = B(R_0)R_0/R$.

$$\sigma \equiv \frac{b}{a} \geq 1$$

$$x = \rho \cos \theta$$

$$z = \sigma \rho \sin \theta$$

Fig. 8.1 Toroidal coordinate system for plasma with an elliptical cross section.

During the expansion phase, the total current and size of the plasma are related by

$$I_p(t) = \iint J(t) dA(t) , \quad (8.6)$$

where $J(t)$ and $A(t)$ are the time-dependent current density and cross-sectional area, respectively. For an elliptical plasma, $A(t) = \pi a^2(t)\sigma(t)$ (see Appendix A). By defining an average current density

$$\bar{J}(t) \equiv \frac{1}{A(t)} \iint J(t) dA(t) = I_p(t)/A(t) , \quad (8.7)$$

the time rate of change of the total plasma current is given by

$$\frac{dI_p}{dt} = \bar{J} \frac{dA}{dt} + A \frac{d\bar{J}}{dt} . \quad (8.8)$$

Written explicitly in terms of the major tokamak parameters of Eq. (8.5), Eq. (8.8) becomes

$$\dot{I}_p = 2I_p \left[\frac{\dot{a}}{a} - \frac{\dot{R}}{R} + \left(\frac{\sigma}{1 + \sigma^2} \right) \dot{\sigma} - \frac{\dot{q}_I}{2q_I} \right] . \quad (8.9)$$

Similarly, the time derivative of the current density is given by

$$\dot{\bar{J}} = -2\bar{J} \left[\frac{\dot{R}}{R} + \left(\frac{1 - \sigma^2}{1 + \sigma^2} \right) \frac{\dot{\sigma}}{2\sigma} + \frac{\dot{q}_I}{2q_I} \right] . \quad (8.10)$$

In a large tokamak, the success of an expanding radius startup is contingent upon the technical feasibility of controlling the plasma current and size in time. Assuming such a control capability, Eq. (8.8) indicates that it should be possible to tailor the current distribution by regulating the time evolution of $I_p(t)$ and $A(t)$. The relationship between $\bar{J}(t)$, $I_p(t)$, and $A(t)$ during an expanding radius startup is illustrated in Figs. 8.2 - 8.4 for the FED tokamak.

A constant q_I expansion with fixed major radius and $I_p(t)$ increasing linearly in time is shown in Fig. 8.2. The plasma elongation is assumed constant with $\sigma = 1.6$. Prior to expansion, the current rises linearly at a rate of $\dot{I}_p = 0.5 \text{ MA/s}$ in a fixed-size, small radius plasma (specified here at $a_0 = 0.4 \text{ m}$). The resulting linear rise in the current density is due to the decreasing value of the plasma safety factor. At $q_I = 2.36$, the expansion phase begins. With both current and cross-sectional area increasing linearly in time, the safety factor at the limiter remains fixed. This amounts to adding a constant current density at the plasma edge. Finally, it should be noted that $\dot{a}$ increases with time in accordance with Eq. (8.9).

A constant q_I expansion involving major radius compression is illustrated in Fig. 8.3. With $a_0 = 0.4 \text{ m}$, σ fixed at 1.6 and $I_p(t)$ increasing linearly in time, Fig. 8.3 is similar to Fig. 8.2, but with one important exception. The small radius plasma is formed near the outside wall of the vacuum chamber in a region of weakened toroidal field. Because $\bar{J} \propto (B(R_0)R_0/R^2 q_I \sigma)(1 + \sigma^2)$ for an elongated plasma at fixed $B(R_0)$, expansion begins (with $q_I = 2.36$) at lower values of I_p and $\bar{J}$. To achieve the linear current ramp depicted in Fig. 8.3, $\dot{a}$ must now vary in time in order to maintain a constant q_I . This time variation in $\dot{a}$ causes $\dot{\bar{J}}$ to also increase in time as shown.

To accommodate minor radius expansion, the center of the discharge must be shifted to smaller major radius by vertical field variation (this process is described in more detail in Section 8.3). Because the toroidal field at the plasma axis increases with decreasing major radius, R-compression produces two effects. In the inner plasma region, the minor radius contracts ($a \propto R^{1/2}$) due to conservation of toroidal flux. Concomitantly, the plasma current and current density increase as $1/R$ and $1/R^2$, respectively. (The scaling laws for adiabatic compression of an elongated plasma are covered in detail in Section 9.4). In the expansion region, the average current density added to the plasma edge is also increased by $1/R^2$. This technique, therefore, produces results similar to the fixed major radius expansion, namely a current density profile with no skin distribution

Fig. 8.2 Constant q_I minor radius expansion in FED with fixed R_0 and $I_p(t)$ increasing linearly in time.

Fig. 8.3 Constant q_I minor radius expansion in FED with R_O -compression and $I_p(t)$ increasing linearly in time.

Fig. 8.4 Minor radius expansion in FED with $a(t)$ increasing and $q_I(t)$ decreasing linearly in time.

Although Figs. 8.2 and 8.3 have assumed a linear current rise at constant q_I , it is also possible for the minor radius and q_I to vary linearly in time. The effect on current density of a varying edge q_I (specifically q_I decreasing linearly with increasing radius) is shown in Fig. 8.4. These cases will be analyzed and discussed in more detail in Section 11.2.5.

8.3 Constant q_I Expansion With R-Compression – The Proposed Startup Scenario for FED

The UHR electron heating discussed previously leads naturally to the formation of a small radius current channel centered at the UHR layer. It is predominantly in this region of high T_e that the plasma current will be induced once the toroidal loop voltage is applied. The ensuing rotational transform is expected to "smear out" the energy content of the axisymmetric vertical absorption layer as flux surfaces begin to form. This process will lead to substantial electron cooling over most of the plasma cross section except for a small inner region of radius $a_0 \geq \Delta_H$ localized at R_{UH} .

In FED such a small radius plasma is formed by applying a relatively low voltage using programmed current ramps in the ohmic heating coils. This applied voltage drives the plasma current to a level consistent with the specified edge safety factor, $q_I \cong 2.36$. During this current initiation phase, the small radius plasma (a_0) maintains its radial location against fixed startup limiters located at the outboard midplane. Throughout this time period, the current ramp time satisfies the physics constraint $\tau_I (= I_p / \dot{I}_p) > \tau_s$ (the plasma skin time). This prevents the formation of skin currents and minimizes the potential hazards of plasma disruptions prior to expansion. To examine the sensitivity of current penetration with size, calculations have been performed with a_0 varying between 0.2 and 0.4 m. Because $\tau_s (= \mu_0 a^2 / \eta_{||})$ is proportional to $a^2 (T_e)^{3/2} / Z_{eff}$, the $a_0 = 0.4$ m case will require roughly a factor of 4 increase in the current ramp period (compared to the $a_0 = 0.2$ m case) for the same level of electron preheating. About 1 MW of extraordinary mode, ECRH (at 87 GHz for $a_0 = 0.2$ m and 90 GHz for $a_0 = 0.4$ m), is maintained to ensure a proper electron temperature during current initiation.

Having achieved the desired value of q_I at the plasma-limiter edge, the major radius $R_{\text{initial}} (= R_{UH})$ is shifted inward and a new layer of plasma is added to the warm core through ionization of a regulated gas feed (Fig. 8.5). This major radius compression permits minor radius expansion (to $a = 1.3$ m and $\sigma = 1.6$) and the simultaneous increase in plasma current (to $I_p = 4.8$ MA), while maintaining a constant q_I . The R-compression is achieved through appropriate feedback control of the vertical magnetic field [sensing on $I_p(t)$].

With the onset of major radius compression, additional rf power (~ 4 MW) is applied to the plasma. Because of the modest plasma parameters ($n_e \geq 10^{19} \text{ m}^{-3}$, $T_e \approx 0.1 - 0.2$ keV, and $I_p \approx 0.04 - 0.18$ MA) that exist at this time, wave absorption and heating are most efficiently achieved using EO-mode, ECRH. As the plasma current and density are ramped up, cyclotron damping will increase [38], shifting the location of wave absorption back to the cyclotron layer ($R_{EC} \approx 5.6$ m for $f_{RF} = 87$ GHz and ≈ 5.4 m for $f_{RF} = 90$ GHz). Because of this effect, a single rf frequency should provide adequate plasma heating during the compression from R_{UH} ($= 5.9$ m for $a_0 = 0.2$ m and 5.7 m for $a_0 = 0.4$ m) to R_{EC} . To continue with ECRH beyond this point requires going to higher frequencies (~ 100 GHz for $R_{EC} = R_0 = 4.8$ m) for better on-axis heating. Although a 100-GHz capability is being provided for FED, ion cyclotron resonance heating (ICRH) of a deliberately introduced proton minority [80] is a convenient option with which to continue heating. The required frequency range is approximately 47-54 MHz and low-field side injection is possible. Furthermore, minority heating would be initiated at a point where the plasma size is comparable to PLT and the plasma currents are of order ~ 0.5 MA, ensuring some confinement of the energetic protons.

Because of the substantial starting densities ($n_e \approx n_i \approx 10^{19} \text{ m}^{-3}$), good Coulomb coupling between the electrons and ions should ensure approximate equality of their respective temperatures. The purpose of the ECRH/ICRH power is to supplement the ohmic heating and eliminate the need for a fast current rise. By producing and maintaining a reasonably high plasma conductivity and extending the startup

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Fig. 8.5 A constant q_1 minor radius expansion with major radius compression and rf assist.

times ($\sim 5-6$ s), the initial voltage demands can be significantly reduced (to ~ 25 V), compared to the purely inductive current startup (≥ 100 V). The additional ECRH/ICRH power can also reduce the impact of impurities on resistive volt-second consumption during the current ramp.

8.4 Constant q_I Expansion With Fixed Major Radius – An Alternative Option for FED

A constant q_I plasma expansion involving R-compression is the proposed scenario for current startup in the FED [81]. The reason for selecting this option is one of practicality, for this approach will be tried on the Tokamak Fusion Test Reactor (TFTR) [82] over the next several years. Given the feasibility of high-field injection on TFTR, a 60-GHz demonstration test of this entire expanding radius option (from $a_0 \approx 0.2$ m and $n_e \approx 10^{19}$ m $^{-3}$) is possible under low toroidal field operation, $B(R_0 = 2.65$ m) = 2.5 T. A disadvantage, however, of the R-compression scenario is that the plasma axis is continually forced into regions of higher magnetic field during the expansion. This compression shifts the location of cyclotron resonance and necessitates multiple frequency operation with its added technological complexity.

An alternative to the compressed major radius approach involves a constant q_I expansion from a fixed major radius (Fig. 8.6). Once the desired q_I value is achieved, the minor radius and plasma current are simultaneously increased to their full-size ohmic values. The actual radius expansion is performed in a controlled manner using a retractable limiter. The moving limiter concept was originally proposed by Kadomtsev [79] and has been subsequently analyzed by Duchs et al. [17], Girard et al. [83], and Houlberg et al. [84] for circular plasmas. Achieving a constant q_I expansion requires synchronization between the limiter withdrawal rate and the plasma current rise to ensure that $a^2(t)/I_p(t)$ remains invariant with time (for fixed σ). This leads to a $J(t)$ profile with no skin distribution since the average current density added to the expansion region is also constant in time. Although the initial ECRH frequency required for preheating is higher, this approach offers the advantage that a single frequency can be used

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Fig. 8.6 A constant q_I minor radius expansion with fixed major radius and rf assist.

for on-axis heating during the entire plasma expansion. This is particularly true for the higher field tokamak designs like ETF/INTOR [85,46], where the UHR and ECR layers are in close proximity to each other.

In terms of technological requirements, a moveable limiter is needed to define the plasma size during the expansion phase. Such retractable limiters are planned for use on the large Japanese tokamak, JT-60 [86]. The information gained from this experiment and TFTR should help to delineate the potential advantages (or disadvantages) between these two expanding radius options.

9. SPATIAL- AND TIME-DEPENDENT SIMULATIONS OF CURRENT STARTUP IN A TOKAMAK

Containing kilovolt hydrogen plasmas within material walls has been particularly successful in the toroidal magnetic confinement device called the tokamak. In this axially symmetric device, a weak poloidal field is set up around the minor cross section of the plasma by means of an electric current induced along the plasma column. In combination with a strong externally applied toroidal field, helical field lines are formed which map out a set of nested magnetic flux surfaces. In low- β , ohmically heated plasmas, these flux surfaces are nearly concentric. In high- β plasmas, with substantial auxiliary heating (neutral beams or rf), this is not the case. With increasing β , the plasma evolves through a sequence of equilibrium states involving motion of the magnetic axis and changes in the flux surface shape. Tokamaks with elongated, noncircular cross sections are currently under active study as fusion reactors [87] because of their potential for stable operation at high β ($>5\%$).

Plasma simulations under ohmic- and ignition-type conditions have traditionally been done using time-dependent transport codes. The simplest codes are spatially independent and are referred to as global models. (Analysis of current startup using a global plasma model is discussed in Chapter 10). In realistic plasmas, however, particles and energy can escape from the tokamak via transport from one magnetic surface to another. This cross-field transport is driven by radial density and temperature gradients found within the plasma. By contrast, the flow of particles and heat along the helical field lines is extremely rapid and tends to produce equilibrium density and temperature profiles which are nearly uniform on a flux surface. In order to model the radial diffusion of particles and energy, one-dimensional (1-D) transport codes were developed which treated the plasma as a circular cylinder (spatial dependence in the r -coordinate). Vertically

elongated plasmas were also represented as circular plasmas having cross-sectional areas and volumes equivalent to their noncircular counterparts. The geometric effects or toroidicity and noncircularity entered through modified radial transport coefficients.

State-of-the-art transport codes handle noncircular geometries self-consistently by solving a set of coupled differential equations which describe the time evolution of the fluid plasma properties, the electromagnetic fields, and the changing magnetic geometry. Simulation models with this capability are categorized as 1 1/2-dimensional (1 1/2-D) transport codes (i.e., 2-D flux surface geometry with 1-D transport across the flux surfaces). One such 1 1/2-D transport code is called WHIST. Although a detailed discussion of the WHIST code has been presented elsewhere by Houlberg et al. [34], a brief description is included here to familiarize the reader with the physical model and the numerous plasma processes important to the study of ohmic current startup.

9.1 The Transport Model

In order to model the time evolution of an ohmically heated, tokamak plasma, the WHIST code solves a set of equations involving particle and energy balance and poloidal magnetic field diffusion. The basic equations describing particle and energy transport are obtained by taking velocity moments of the Boltzmann equation [88]. The use of a Fokker-Planck collision operator, in toroidal geometry, permits a calculation of needed transport coefficients in a self-consistent manner. These coefficients linearly relate the cross-field fluxes of particles and energy, and the parallel electric current to the thermodynamic forces (the radial density and temperature gradients, and the parallel electric field). When combined with the conservation laws for particle and energy balance, and Maxwell's equations, a closed system of equations is obtained (referred to as neoclassical transport theory [89]).

9.1.1 Particle and Energy Balance Equations

Due to the rapid helical transport within a flux surface, the two-dimensionality of the transport equations (describing noncircular geometries) can be reduced to a single spatial coordinate by flux surface averaging. For an ohmically heated, hydrogen plasma, the flux-surface-averaged equation for particle balance is given by

$$\frac{\partial}{\partial t} \left[V'(\rho) \langle n_i \rangle_f \right] = - \frac{\partial}{\partial \rho} \left[V'(\rho) \langle \Gamma_i \rangle_f \right] - V'(\rho) \frac{\langle \Gamma_{\parallel i} \rangle_f}{L_{\parallel}} + V'(\rho) \langle \sum_n \rangle_f \quad (9.1)$$

where

$V(\rho)$ = plasma volume contained within a flux surface,

$V'(\rho) = \partial V / \partial \rho$ represents an effective surface area,

ρ = radial coordinate associated with a flux surface ψ ,

n_i = ion density,

Γ_i = radial ion particle flux,

$\Gamma_{\parallel i}$ = ion particle flux parallel to the field lines in the scrape-off layer,

$L_{\parallel}$ = effective parallel flow length from the plasma edge to the limiter/divertor surface.

The electron density and particle flux are treated as dependent variables and are determined by invoking charge neutrality and ambipolar diffusion:

$$n_e = n_i + \sum_{j=1}^{Z_I} j n_I^j,$$

$$\Gamma_e = \Gamma_i + \sum_{j=1}^{Z_I} j \Gamma_I^j,$$

$$\Gamma_{\parallel e} = \Gamma_{\parallel i} + \sum_{j=1}^{Z_I} j \Gamma_{\parallel I}^j.$$

Here, n_I^j and Γ_I^j represent the density and flux of impurity ions stripped of j electrons and Z_I is the ion's atomic number. The bracket $\langle \rangle_f$ represents the flux-surface-average operator defined as [90]

$$\langle A \rangle_f \equiv \frac{\int_0^{2\pi} \sqrt{g} A d\theta}{\int_0^{2\pi} \sqrt{g} d\theta} = \frac{\int_0^{2\pi} \sqrt{g} A d\theta}{2\pi \langle \sqrt{g} \rangle_\theta}, \quad (9.2)$$

where $\sqrt{g}$ is the 3-D Jacobian of the transformation from $\vec{x} = (R, Z, \phi)$ to (ρ, θ, ϕ) (see Appendix B). It is not only through the transport coefficients, but also through the various surface averages (e.g., $\langle \dots \rangle_f$, $V'(\rho)$, etc.) that the flux surface geometry appears in the conservation equations.

The energy balance equation for the ions and electrons can be written in the following general form:

$$\begin{aligned} & \frac{\partial}{\partial t} \left[(V')^{5/3} \langle \frac{3}{2} n_{i(e)} k T_{i(e)} \rangle_f \right] \\ &= - (V')^{2/3} \frac{\partial}{\partial \rho} \left[V'(\rho) \left(\langle Q_{i(e)} \rangle_f + \langle \frac{5}{2} k T_{i(e)} \Gamma_{i(e)} \rangle_f \right) \right] \\ &- (V')^{5/3} \frac{\langle Q_{\parallel i(e)} \rangle_f}{L_{\parallel}} + (V')^{5/3} \langle \sum E_{i(e)} \rangle_f, \end{aligned} \quad (9.3)$$

where

- $T_{i(e)}$ = Maxwellian ion (electron) temperature,
- $Q_{i(e)}$ = radial ion (electron) conduction heat flux,
- $Q_{\parallel i(e)}$ = ion (electron) conduction and convection heat flux along field lines in the scrape-off layer.

The second term on the right side of Eq. (9.3) corresponds to convective heat flow and the third term represents heat flow to limiters or divertors in the scrape-off layer. The quantities Σ_n and $\Sigma_{Ei(e)}$

represent aggregate source/sink terms for particles and energy.

In the large aspect ratio, circular cross-section ($V' \propto \rho = r$) limit, with fixed magnetic geometry, Eqs. (9.1) and (9.3) simplify to the familiar transport equations for a circular cylinder:

$$\frac{\partial n_i}{\partial t} = -\frac{1}{r} \frac{\partial}{\partial r} (r \Gamma_i) - \frac{\Gamma_{\parallel i}}{L_{\parallel}} + S_w(r, t) + S_{\text{ext}}(r, t), \quad (9.4)$$

$$\frac{\partial}{\partial t} \left(\frac{3}{2} n_i k T_i \right) = -\frac{1}{r} \frac{\partial}{\partial r} \left[r (Q_i + \frac{3}{2} k T_i \Gamma_i) \right] - \frac{Q_{\parallel i}}{L_{\parallel}} - P_{\text{cx}} + Q_{\text{ei}}, \quad (9.5)$$

$$\begin{aligned} \frac{\partial}{\partial t} \left(\frac{3}{2} n_e k T_e \right) = & -\frac{1}{r} \frac{\partial}{\partial r} \left[r (Q_e + \frac{3}{2} k T_e \Gamma_e) \right] - \frac{Q_{\parallel e}}{L_{\parallel}} - Q_{\text{ei}} - P_{\text{ion}} \\ & - P_{\text{rad}} + P_{\text{rf}} + P_{\text{oh}}. \end{aligned} \quad (9.6)$$

In the above, the individual source/sink terms are explicitly indicated. The quantity S_w is the source of ions due to ionization of neutrals which originate from the walls and limiters, and S_{ext} is the source of ions due to ionization of neutrals from gas puffing.

(Additional external sources in WHIST include pellets and neutral beam injection, but these have not been considered in this study).

In the energy balance equations, P_{cx} is the ion energy loss due to charge exchange events, Q_{ei} is the collisional energy transfer from electrons to ions, P_{ion} is the electron energy loss due to impact ionization of neutral atoms, and P_{rf} is the auxiliary ECRH power provided to the electrons. (The quantities, P_{cx} , Q_{ei} , P_{ion} , and P_{rf} have been defined previously in Section 4.2).

The term P_{rad} accounts for power losses due to various types of radiation. In a clean plasma, the primary losses, bremsstrahlung [91] and synchrotron [92] radiation are given by

$$P_{\text{brem}} \left(\text{W/m}^3 \right) = 4.80 \times 10^{-37} Z_{\text{eff}} n_e^2 T_e^{1/2}, \quad (9.7)$$

$$P_{\text{sync}} \left(\frac{\text{W}}{\text{m}^3} \right) = 1.94 \times 10^{-10} (1 - r_w) \left[1 + 0.18 \left(1 - \frac{A}{5} \right) \right]^3 \cdot \left\{ \frac{B^{5/2} T_e^{2.1} n_e^{1/2}}{a^{1/2}} \right\}, \quad (9.8)$$

where r_w is the wall reflectivity, and $A(= R/a)$ is the plasma aspect ratio. In the above equations and in those which follow, the units used are T_e (keV), n_e (m^{-3}), B (T), and a (m). The effective ion charge

$$Z_{\text{eff}} = \sum_j Z_j^2 n_j / n_e$$

is an indicator of the plasma impurity concentration ($Z_{\text{eff}} = 1$ for a clean, hydrogen plasma) and the level to which the impurity ions are stripped. When impurities are present, line radiation (from atomic transitions of electrons bound to the impurity ion) and recombination radiation (from electron recombination processes) become important. Their contribution to the overall radiative losses is calculated using a "coronal equilibrium" model [63].

The resistive dissipation of the plasma current, via collisions between electrons and ions, provides a power input to the electrons known as ohmic heating, P_{oh} . Including neoclassical corrections [93] to the Spitzer resistivity [14], the ohmic heating power density is given by

$$P_{\text{oh}} \left(\frac{\text{W}}{\text{m}^3} \right) = \eta_{\parallel} J^2, \quad (9.9)$$

$$\eta_{\parallel}(r) = \eta_{\text{SPITZER}} \left[1 - \frac{1.95 \left(\frac{r}{R} \right)^{1/2} - 0.95 \left(\frac{r}{R} \right)}{1 + \nu_{*e}} \right]^{-1}, \quad (9.10)$$

$$\eta_{\text{SPITZER}} (\text{ohm-m}) = \left(\frac{m_e \nu_{ei}}{n_e e^2} \right) \gamma,$$

$$\gamma = \left(0.29 Z_{\text{eff}} + \frac{0.457 Z_{\text{eff}}}{1.077 + Z_{\text{eff}}} \right) ,$$

where

$$\nu_{ei} (\text{s}^{-1}) = 9.1 \times 10^{-17} n_i \ln \Lambda_{ei} / T_e^{3/2}$$

is the electron-ion collision frequency, $\ln \Lambda_{ei}$ is the Coulomb logarithm, and ν_{*e} is the dimensionless electron collisionality parameter [8]. The correction term (in square brackets) takes into account the fraction of electrons which are mirror trapped at low collisionalities ($\nu_{*e} < 1$). These electrons increase the overall plasma resistance.

Finally, losses due to radial particle and energy transport are calculated using a diagonal model of the following form:

$$\Gamma_i = -D_i \frac{\partial n_i}{\partial r} , \quad (9.11)$$

$$Q_{i(e)} = -K_{i(e)} \frac{\partial T_{i(e)}}{\partial r} = -n_{i(e)} \chi_{i(e)} \frac{\partial T_{i(e)}}{\partial r} .$$

Here, D is the particle diffusion coefficient, K is the thermal conductivity, and χ is the thermal diffusion coefficient. Added to the neoclassical transport coefficients [89] are a set of "empirical" coefficients which mock up the anomalous losses of particles and electron energy observed in present-day experiments [94]. This study uses the reference transport model used in the INTOR [46] and FED [81] studies, namely,

$$D_i = 3.0 D_i^{\text{NC}} + D_i^{\text{ANOM}} ,$$

$$\chi_i = 3.0 \chi_i^{\text{NC}} ,$$

$$\chi_e = 3.0 \chi_e^{\text{NC}} + \chi_e^{\text{ANOM}} ,$$

where

$$\chi_e^{ANOM} \left(\text{m}^2/\text{s} \right) = 5.0 \times 10^{19} / n_e(r) , \quad (9.12)$$

$$D_i^{ANOM} = 0.2 \chi_e^{ANOM} .$$

The anomalous contributions to the particle diffusion and electron heat conduction dominate over the neoclassical terms. Ion heat conduction, on the other hand, is generally in good agreement (within a factor of 3) with neoclassical theory. The approximate forms for χ_i^{NC} , as a function of decreasing ion collisionality, are given by [95]

$$\chi_i^{NC} = \begin{cases} (1.0 + 1.6q^2) v_{ii} \rho_i^2 & \text{(collisional)} \\ 1.9(kT_i/m_i)^{1/2} \frac{q\rho_i^2}{R} & \text{(plateau)} \\ 0.66 \left(\frac{r}{R} \right)^{1/2} v_{ii} \rho_{ip}^2 & \text{(banana)} \end{cases} , \quad (9.13)$$

where $v_{ii} = (m_e/2m_i)^{1/2} (T_e/T_i)^{3/2} v_{ei}$ is the ion-ion collision frequency, $\rho_i = (2m_i kT_i)^{1/2} / eB$ is the ion gyroradius, and $\rho_{ip} = \rho_i (B/B_p)$ is the ion gyroradius in the poloidal magnetic field. When using this hybrid neoclassical-empirical model, the convection loss terms in Eqs. (9.5) and (9.6) use a factor of 3/2 instead of 5/2.

In simulating transport in the scrape-off region, the WHIST code uses the model of Mense and Emmert [96]. In this 1-D model, the radial diffusion of particles and energy is assumed to be Bohm-like with

$$D = \chi_i = \chi_e = F \frac{kT_e}{16eB} = 62.5 \frac{FT_e}{B} \left(\text{m}^2/\text{s} \right) , \quad (9.14)$$

where F is an arbitrary multiplication factor. Full Bohm diffusion is obtained for $F = 1$. The selection of Bohm diffusion corresponds to

the assumption that low-frequency drift waves (which can cause $\vec{E} \times \vec{B}$ drifts) are the primary mechanism for cross-field transport. Along the magnetic field lines, particle and energy losses are evaluated self-consistently in terms of parallel flow rates and the buildup of a sheath potential at the limiter/divertor plate. For a Maxwellian velocity distribution, the ion and electron particle fluxes are given by

$$\Gamma_{\parallel i} = \int_{-\infty}^{+\infty} dv_x dv_y \int_0^{\infty} dv_z v_z f_i(v) = n_i(r) \sqrt{\frac{kT_i(r)}{2\pi m_i}} , \quad (9.15)$$

$$\begin{aligned} \Gamma_{\parallel e} &= \int_{-\infty}^{+\infty} dv_x dv_y \int_{v_{e,\max}}^{\infty} dv_z v_z f_e(v) \\ &= n_e(r) \sqrt{\frac{kT_e(r)}{2\pi m_e}} \exp \left\{ \frac{|e| \phi_{AMB}}{kT_e} \right\} . \end{aligned} \quad (9.16)$$

Only electrons with energy greater than the sheath potential ($1/2 m_e v_{e,\max}^2 = -|e|\phi_{AMB}$) can penetrate through to the limiter surface. Equating $\Gamma_{\parallel i}$ to $\Gamma_{\parallel e}$, the potential necessary to ensure an ambipolar removal of particles can be determined with

$$\phi_{AMB} = - \frac{kT_e}{2|e|} \ln \left(\frac{m_i T_e}{m_e T_i} \right) . \quad (9.17)$$

Making use of Eqs. (9.15) and (9.17), the parallel energy fluxes to the limiter are given by

$$Q_{\parallel i} = \iiint_{-\infty}^{+\infty} dv_x dv_y \int_0^{\infty} dv_z \frac{1}{2} m_i v^2 v_z f_i = 2kT_i \Gamma_{\parallel i} , \quad (9.18)$$

$$Q_{\parallel e} = \iiint_{-\infty}^{+\infty} dv_x dv_y \int_{v_{e,\max}}^{\infty} dv_z \frac{1}{2} m_e v^2 v_z f_e = \gamma k T_e \Gamma_{\parallel i}, \quad (9.19)$$

where

$$\gamma = 2 + \frac{1}{2} \ln \left(\frac{n_i T_e}{n_e T_i} \right).$$

The parameter γ (discussed previously in Section 3.2) is the electron energy coefficient and the quantity $\gamma k T_e$ represents the average energy transported to the limiter by each collected electron.

The characteristic distance along a field line that particles must travel before being intercepted by the limiter ($L_{\parallel}$) depends on the magnetic field topology. For the FED tokamak, which has a full toroidal startup limiter, $L_{\parallel}(r) = \pi R q(r)$.

9.1.2 Toroidal Current and Poloidal Field Equations

By combining the flux-surface-averaged equations [34] for Faraday's law,

$$\frac{\partial}{\partial t} \left(\frac{I(\rho)}{G_{\theta}} \right) = \frac{\partial}{\partial \rho} \left(\frac{I(\rho)}{G_{\theta}} \frac{\partial \Phi / \partial t}{\partial \Phi / \partial \rho} \right) + \frac{4\pi^2}{\mu_0} \frac{\partial}{\partial \rho} \left(\frac{\langle \sqrt{g} \rangle_{\theta} \langle \vec{E} \cdot \vec{B} \rangle_f}{\partial \Phi / \partial \rho} \right), \quad (9.20)$$

Ohm's law,

$$\langle \vec{E} \cdot \vec{B} \rangle_f = \eta_{\parallel} \langle \vec{J} \cdot \vec{B} \rangle_f - \langle \vec{J}_s \cdot \vec{B} \rangle_f, \quad (9.21)$$

and Ampere's law,

$$\langle \vec{J} \cdot \vec{B} \rangle_f = \frac{1}{4\pi^2 \langle \sqrt{g} \rangle_{\theta}} \left[\frac{(\partial \Phi / \partial \rho)^2}{G_{\phi}^2} \right] \frac{\partial}{\partial \rho} \left(\frac{I(\rho) G_{\phi}}{\partial \Phi / \partial \rho} \right), \quad (9.22)$$

an equation describing the time evolution of the toroidal plasma current, $I(\rho)$, can be obtained:

$$\frac{\partial}{\partial t} \left(\frac{I(\rho)}{G_\theta} \right) = \frac{\partial}{\partial \rho} \left[\left(\frac{I(\rho)}{G_\theta} \right) \left(\frac{\partial \Phi / \partial t}{\partial \Phi / \partial \rho} \right) \right] + \frac{1}{\mu_0} \frac{\partial}{\partial \rho} \left[\eta \left\{ \frac{\partial \Phi / \partial \rho}{G_\phi^2} \frac{\partial}{\partial \rho} \left(\frac{I(\rho) G_\phi}{\partial \Phi / \partial \rho} \right) \right. \right. \\ \left. \left. - \frac{4\pi^2 \langle \sqrt{g} \rangle_\theta}{\partial \Phi / \partial \rho} \langle \vec{J}_s \cdot \vec{B} \rangle_f \right\} \right]. \quad (9.23)$$

In the above, $I(\rho)$ is the integrated toroidal plasma current, Φ is the toroidal magnetic flux, $\langle \sqrt{g} \rangle_\theta$ is poloidal-angle-averaged Jacobian, and G_θ and G_ϕ are dimensionless flux-surface-averaged geometric factors defined later in this section. The averaged quantities $\langle \vec{E} \cdot \vec{B} \rangle_f$, $\langle \vec{J} \cdot \vec{B} \rangle_f$ and $\langle \vec{J}_s \cdot \vec{B} \rangle_f$ represent the parallel projections of the electric field, total current density and noninductive current density along the magnetic field.

An alternative to using $I(\rho)$ as an independent variable is to use the poloidal magnetic field. Because this approach is used in WHIST, Faraday's law is recast as

$$\frac{\partial \tilde{B}_p}{\partial t} = \frac{\partial}{\partial \rho} \left(v_\phi \tilde{B}_p \right) + \frac{\partial \tilde{E}}{\partial \rho}, \quad (9.24)$$

by making use of the following definitions [97]:

$$\tilde{B}_p \equiv \frac{\mu_0 I(\rho)}{2\pi R_0 G_\theta} = \frac{\psi'(\rho)}{2\pi R_0}, \quad (9.25)$$

$$v_\phi \equiv \frac{\partial \Phi / \partial t}{\partial \Phi / \partial \rho}, \quad (9.26)$$

$$B \equiv \frac{\mu_0 F(\rho)}{2\pi R} = \frac{\phi'(\rho)}{2\pi R G_\phi}, \quad (9.27)$$

$$V'(\rho) \equiv 4\pi^2 \langle \sqrt{g} \rangle_\theta, \quad (9.28)$$

and

$$\tilde{E} \equiv \frac{V'}{\mu_0 F(\rho) G_\phi} \left(\frac{\langle \vec{E} \cdot \vec{B} \rangle_f}{2\pi R_0} \right). \quad (9.29)$$

Here, $\tilde{B}_p$ and $\tilde{E}$ are the effective poloidal magnetic field and electric field, respectively, V_ϕ is the velocity of a surface of constant toroidal flux relative to a surface of constant ρ , ψ is the poloidal flux, F is the poloidal current external to flux surface ρ , and $V'(\rho)$ is the effective surface area associated with a flux surface.

In a similar fashion Eq. (9.21) can be multiplied through by $V'[\mu_0 F(\rho) G_\phi (2\pi R_0)]^{-1}$ to form an effective Ohm's law given by

$$\tilde{E} = \eta_{||} [\tilde{J} - \tilde{J}_s], \quad (9.30)$$

where

$$\begin{aligned} \tilde{J} &\equiv \frac{V'}{\mu_0 F(\rho) G_\phi} \left(\frac{\langle \vec{J} \cdot \vec{B} \rangle_f}{2\pi R_0} \right) \\ &= \frac{1}{2\pi R_0} \left(\frac{F(\rho)}{G_\phi} \right) \frac{\partial}{\partial \rho} \left(\frac{I(\rho)}{F(\rho)} \right), \end{aligned} \quad (9.31)$$

$$\tilde{J}_s \equiv \frac{V'}{\mu_0 F(\rho) G_\phi} \left(\frac{\langle \vec{J}_s \cdot \vec{B} \rangle_f}{2\pi R_0} \right). \quad (9.32)$$

Noninductive sources of current, such as those utilizing particle [98] and wave drivers [99], have not been considered in this study ($\tilde{J}_s = 0$).

In general, the total current density $\tilde{\mathbf{J}}$ is not used to determine the ohmic power density in Eq. (9.6). Instead, the resistive heating term given by Hinton and Hazeltine [89] is used as the starting point with

$$\langle \tilde{\mathbf{J}} \cdot \tilde{\mathbf{E}} \rangle_f = \langle \tilde{\mathbf{J}} \cdot \tilde{\nabla}\phi \rangle_f \langle \tilde{\mathbf{E}} \cdot \tilde{\mathbf{B}} \rangle_f \left(\frac{2\pi \langle \sqrt{g} \rangle_\theta}{\mu_0 F G_\phi} \right) - \langle v_\phi \rangle_f \frac{\partial p}{\partial \rho} . \quad (9.33)$$

In the above expression, the second term on the right is related to the work done by the moving magnetic surface against the plasma pressure gradient (this term is generally small). The averaged quantity

$$\langle \tilde{\mathbf{J}} \cdot \tilde{\nabla}\phi \rangle_f = \frac{1}{\mu_0 V'(\rho)} \frac{\partial}{\partial \rho} \left[4\pi^2 G_\theta R_0 \tilde{B}_p \right] \quad (9.34)$$

is the toroidal current density term and $\tilde{\nabla}\phi (= \hat{e}_\phi / R_0)$ is the reciprocal basis vector [100] which is normal to the $\phi = \text{constant}$ surface ($\hat{e}_\phi$ is a unit vector in the ϕ direction). Equation (9.33) can be simplified by defining an effective ohmic current density [34],

$$\tilde{\mathbf{J}}_{\text{oh}} \equiv R_0 \langle \tilde{\mathbf{J}} \cdot \tilde{\nabla}\phi \rangle_f = \frac{(2\pi R_0)^2}{\mu_0 V'(\rho)} \frac{\partial}{\partial \rho} (G_\theta \tilde{B}_p) , \quad (9.35)$$

and using Eqs. (9.28) and (9.29). The result obtained is

$$p_{\text{oh}} = \langle \tilde{\mathbf{J}} \cdot \tilde{\mathbf{E}} \rangle_f = \tilde{\mathbf{J}}_{\text{oh}} \cdot \tilde{\mathbf{E}} . \quad (9.36)$$

In WHIST, Eqs. (9.24), (9.25), (9.30) and (9.31) are combined to form a diffusion equation for the poloidal magnetic field:

$$\frac{\partial \tilde{B}_p}{\partial t} = \frac{\partial}{\partial \rho} (v_\phi \tilde{B}_p) + \frac{1}{\mu_0} \frac{\partial}{\partial \rho} \left[\frac{\eta_{\parallel} F}{G_\phi} \frac{\partial}{\partial \rho} \left(\frac{G_\theta \tilde{B}_p}{F} \right) \right] , \quad (9.37)$$

with boundary conditions

$$\tilde{B}_p(\rho = 0) = 0$$

and

$$\tilde{B}_p(\rho = a) = \frac{\mu_o I(a)}{2\pi R_o G_\theta(a)} . \quad (9.38)$$

An alternative edge boundary condition is to use a fixed safety factor with

$$\tilde{B}_p(\rho = a) = \frac{\mu_o F(a) G_\theta(a)}{2\pi R_o q(a)} \quad (9.39)$$

and

$$q(\rho) \equiv \frac{\Phi'}{\psi} = G_\theta G_\phi \frac{F(\rho)}{I(\rho)} . \quad (9.40)$$

Once $\tilde{B}_p$ has been evolved in time, the dependent variables, namely, the toroidal current, current density, toroidal electric field, and plasma safety factor, are determined using the appropriate equations. All the effective quantities discussed above have been chosen such that they reduce to the usual cylindrical approximation in the low-beta, circular cross-section limit. For the case of concentric ellipses with constant elongation, the averaged quantities G_θ and G_ϕ are given by

$$G_\theta \equiv \langle g_{\theta\theta} / \sqrt{g} \rangle_\theta = \frac{a}{R_o} \left(\frac{1 + \sigma^2}{2\sigma} \right) , \quad (9.41)$$

$$G_\phi \equiv \langle \sqrt{g} / g_{\phi\phi} \rangle_\theta = \frac{a\sigma}{R_o} ,$$

for $\rho = a$ (see Appendix B). It should be noted here that the average poloidal field value ($\bar{B}_p$) at the plasma surface ($\rho = a$), discussed in Section 8.2, is related to $\tilde{B}_p$ through the following relationship:

$$\bar{B}_p \equiv \langle B_p^2 \rangle_f^{1/2} = \left(\frac{1}{V'(\rho)} \frac{\mu_0^2 I(\rho)^2}{G_\theta} \right)^{1/2} = \left(\frac{4\pi^2 R_0^2 G_\theta}{V'(\rho)} \right)^{1/2} \tilde{B}_p \quad (9.42)$$

Finally, Eqs. (9.1), (9.3), and (9.37) form a set of coupled, partial differential equations for the plasma density, temperature and poloidal magnetic field. These equations are finite-differenced [101] and are then cast into the form of a matrix equation. At each timestep, the matrix is inverted to give the time-evolved parameters.

The magnetic geometry, which can be held fixed or be allowed to evolve in time, enters the transport and field equations parametrically through $V'(\rho)$ and the various geometric factors. An evolving magnetic geometry, represented by surfaces of constant poloidal magnetic flux, $\psi(R, Z)$, must satisfy the Grad-Shafranov equation [102]:

$$\begin{aligned} \nabla^2 \psi(R, Z) &\equiv \frac{\partial^2 \psi}{\partial R^2} - \frac{1}{R} \frac{\partial \psi}{\partial R} + \frac{\partial^2 \psi}{\partial Z^2} \\ &= -4\pi^2 \left(\mu_0 R^2 \frac{dP}{d\psi} + \frac{\mu_0^2}{8\pi^2} \frac{dF^2}{d\psi} \right), \end{aligned} \quad (9.43)$$

where R is the distance from the symmetry axis and Z is the distance from the equatorial plane. The parameter $P(\psi)$ ($= P_e(\psi) + P_i(\psi)$) is the total plasma pressure and $F(\psi)$ ($= 2\pi RB/\mu_0$) is the poloidal current enclosed between the axis of symmetry and the magnetic surface of interest. By specifying $P(\psi)$ and $F(\psi)$, Eq. (9.43) can be solved for $\psi(R, Z)$. When coupled to a transport code, however, multiple solutions of Eq. (9.43) can significantly increase the computer time and storage requirements. The WHIST code uses a computationally fast, variational moments method [90] to find approximate solutions to the Grad-Shafranov equation using $P(\psi)$ and $I(\psi)$ as driving functions.

9.2 Voltage and Flux Equations

In order to induce and sustain current flow in a tokamak plasma, the poloidal field coil system is required (1) to produce a specified total magnetic flux (ϕ_T) through the plane of the plasma and (2) to

change this flux at a specified rate ($\dot{\phi}$). This changing flux induces a plasma voltage (Faraday's law) which is oriented in such a direction as to produce a current and resulting flux change that counteracts the original flux change responsible for the induction (Lenz's law).

For a coupled poloidal field coil-plasma circuit, a general voltage equation can be written which is of the following form:

$$V_L = \sum_{\substack{k=OH \\ EF \\ \text{etc.}}} \frac{d}{dt} (M_{p-k} I_k) = \underbrace{\frac{d}{dt} (L_p^{\text{tot}} I_p)}_{= V_{\text{ind}}} + \underbrace{2\pi R \eta_{\parallel} J}_{= V_{\text{res}}} \quad (9.44)$$

Here, V_L is the externally applied loop voltage, M_{p-k} is the mutual inductance between the plasma and a particular PF coil set [103], and $L_p^{\text{tot}} (= L_p^{\text{ext}} + L_p^{\text{int}})$ is the total plasma self-inductance. The currents I_k and I_p are those existing in the various coil groups and the plasma, respectively, and the voltages V_{ind} and V_{res} are the inductive and resistive loop voltage components discussed previously in Section 1.3. (When using averaged plasma parameters or flat profile models, the resistive voltage can be rewritten as $V_{\text{res}} = R_p I_p$ where R_p is the plasma resistance and $I_p (= \bar{J} \pi a^2 \sigma)$ is the current carried by the plasma.)

Integrating Eq. (9.44) over time, one obtains a flux balance equation,

$$\phi_{\text{ext}} = L_p^{\text{ext}} I_p + L_p^{\text{int}} I_p + \int_0^t 2\pi R \eta_{\parallel} (0) J(0) dt' \quad (9.45)$$

relating the externally applied flux from the PF system to the volt-seconds required to induce the plasma current. By convention [7], the resistive volt-second loss on-axis is defined to be the difference between the sum of the external ($L_p^{\text{ext}} I_p$) and internal ($L_p^{\text{int}} I_p$) inductive fluxes and the externally applied volt-seconds.

The plasma external inductance is an important quantity during startup, for it directly affects the energy, power, and flux requirements which must be provided by the PF system. For a circular plasma with major radius R and minor radius a , the external self-inductance is given by [104]:

$$L_p^{\text{ext}} = \mu_0 R \left[\left(\frac{2}{k} - k \right) K(k) - \frac{2}{k} E(k) \right] , \quad (9.46)$$

where

$$k^2 = \frac{4R(R - a)}{(2R - a)^2} \quad (9.47)$$

and $K(k)$ and $E(k)$ are the complete elliptic integrals of the first and second kind defined by [105]

$$K(k) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - k^2 \sin^2 \theta}} ,$$

$$E(k) = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \theta} \, d\theta .$$

During the current initiation phase of a small radius plasma, the configuration is characterized by a large aspect ratio ($A = R/a \gg 1$). In this large A limit, Eq. (9.47) shows that k tends toward unity and the elliptic integrals can be approximated by

$$K(k) \cong \ln \left(\frac{4}{\sqrt{1 - k^2}} \right) ,$$

$$E(k) = 1$$

Using these results in Eq. (9.46) yields the following expression for external inductance:

$$L_p^{\text{ext}} (\text{henrys}) = \mu_0 R \left[\ln \left(\frac{8R}{a} \right) - 2 \right] . \quad (9.48)$$

For noncircular elliptically shaped plasmas, the external inductance is modified by the introduction of a $\sqrt{\sigma}$ into the denominator of the logarithmic argument [19].

While the external inductance is a function of the configurational geometry of the plasma, the plasma internal inductance depends upon the poloidal field profile established in the plasma. For a circular plasma, the internal inductance is given by

$$L_p^{\text{int}} = \mu_0 R \frac{\ell_i}{2} = \frac{2\pi R}{I_p(a)} \int_0^a B_\theta(r) dr \quad (9.49)$$

The value of the dimensionless internal inductance ℓ_i is an indicator of the poloidal field or current profile existing in the plasma. It is defined by Eq. (9.49). Using the equations

$$I_p(r) = \int_0^r J(r') 2\pi r' dr' ,$$

$$B_\theta(r) = \mu_0 I_p(r) / 2\pi r$$

$$= \frac{\mu_0}{r} \int_0^r J(r') r' dr' ,$$

one can write Eq. (9.49) in the form

$$L_p^{\text{int}} = \mu_0 R \frac{\ell_i}{2} = \frac{2\pi R \int_0^a \frac{\mu_0}{r} dr \int_0^r J(r') r' dr'}{2\pi \int_0^a J(r) r dr} , \quad (9.50)$$

which emphasizes the dependence of the internal inductance and the internal flux ($L_p^{\text{int}} I_p$) on the current profile. For a current density distribution of the form $J(r) = J_0 (1 - r^2/a^2)^{\alpha_j}$, $\ell_i = 1.0, 1.5$, and ~ 2.1 for $\alpha_j = 0, 1$, and 3 , respectively.

In order to determine the startup voltage and flux requirements in noncircular geometries, a metric formulation for the above quantities has been developed and incorporated into the WHIST code. The rigorously correct starting point for this formulation is Faraday's Law of Induction. For an arbitrary but closed path, this law states that

$$V_{\phi} = \oint \vec{E} \cdot d\vec{\ell}_{\phi} = - \frac{d}{dt} \int_{S_{\theta}} \vec{B} \cdot d\vec{S}_{\theta} \quad (9.51)$$

In the above equation, which has been specifically applied to Fig. 9.1, V_{ϕ} is the induced toroidal plasma voltage, $d\vec{\ell}_{\phi}$ is the differential distance in the toroidal direction, $\vec{E}$ and $\vec{B}$ represent the total electric and magnetic fields in the plasma, and dS_{θ} represents an element of the planar area extending from the minor (or magnetic) axis of the plasma out to a specified flux surface and encircling the major axis. As in the previous sections, a curvilinear coordinate system (ρ, θ, ϕ) is used where ρ is the radial coordinate associated with a flux surface and $\hat{e}_{\rho}$, $\hat{e}_{\theta}$, and $\hat{e}_{\phi}$ represent unit vectors in the radial, poloidal, and toroidal directions, respectively.

To evaluate Eq. (9.51), the contour shown in Fig. 9.1 is used which encloses the planar area, S_{θ} , located between the flux surface coordinates ρ_1 and ρ_0 . This usage leads to

$$\begin{aligned} V_{\phi}(\rho_1) - V_{\phi}(\rho_0) &= \oint \vec{E} \cdot d\vec{\ell}_{\phi} \\ &= - \frac{d}{dt} \int_{S_{\theta}} \vec{B} \cdot d\vec{S}_{\theta} \quad (9.52) \end{aligned}$$

The normal to the planar area $d\vec{S}_{\theta}$ can be expressed as

$$d\vec{S}_{\theta} = d\vec{\ell}_s \times d\vec{\ell}_{\phi} = 2\pi R d\vec{\ell}_s \times \hat{e}_{\phi} \quad (9.53)$$

where $d\vec{\ell}_s$, a line element connecting ρ_0 to ρ_1 , is given by

$$d\vec{\ell}_s = \hat{e}_{\rho} d\rho + \hat{e}_{\theta} d\theta + \hat{e}_{\phi} d\phi \quad (9.54)$$

Fig. 9.1 Toroidal flux geometry used in calculating the loop voltage and flux quantities.

Here, $\vec{e}_\rho$, $\vec{e}_\theta$, and $\vec{e}_\phi$ are covariant basis vectors defined as [100]

$$\vec{e}_\rho \equiv \frac{\partial \vec{r}}{\partial \rho}, \quad \vec{e}_\theta \equiv \frac{\partial \vec{r}}{\partial \theta}, \quad \vec{e}_\phi \equiv \frac{\partial \vec{r}}{\partial \phi},$$

and $\vec{r}$ is a position vector extending from the origin of the coordinate system to a point P. The covariant basis vectors are tangent to their respective coordinate curve through P. In general, the covariant vectors are not orthogonal (i.e., $\vec{e}_\rho \cdot \vec{e}_\theta \neq 0$), nor are they unit vectors (i.e., $\vec{e}_\rho \cdot \vec{e}_\rho \neq 1$). By contrast, contravariant basis vectors are defined as vectors normal to their respective coordinate surface:

$$\vec{e}^\rho \equiv \vec{\nabla}_\rho, \quad \vec{e}^\theta \equiv \vec{\nabla}_\theta, \quad \vec{e}^\phi \equiv \vec{\nabla}_\phi.$$

The relationships between $\hat{e}_\rho$, $\vec{e}_\rho$, and $\vec{e}^\rho$, and their counterparts in θ and ϕ , can be expressed in general form as follows:

$$\hat{e}_i \equiv \frac{\vec{e}_i}{\sqrt{\vec{e}_i \cdot \vec{e}_i}} = \frac{\vec{e}_i}{\sqrt{g_{ii}}}, \quad (9.55)$$

$$\vec{e}_i = \sqrt{g} \epsilon_{ijk} \vec{e}^j \times \vec{e}^k, \quad (9.56)$$

$$\vec{e}^i = (\epsilon_{ijk} \vec{e}_j \times \vec{e}_k) / \sqrt{g}, \quad (9.57)$$

$$\vec{e}^i \cdot \vec{e}_j = \delta_{ij}, \quad (9.58)$$

$$\vec{e}_i \cdot \vec{e}_j = g_{ij}, \quad (9.59)$$

where

$$\delta_{ij} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \text{ for } \begin{cases} i \neq j \\ i = j \end{cases} ,$$

$$\epsilon_{ijk} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} \begin{cases} \text{when any two indices are alike} \\ \text{for any cyclic permutation of } ijk = \rho\theta\phi \\ \text{for any cyclic permutation of } ijk = \phi\theta\rho \end{cases} ,$$

and $\sqrt{g}$, g_{ij} are the Jacobian and metric elements, respectively (see Appendix B).

Making use of Eqs. (9.55) through (9.59), one can proceed to evaluate $d\vec{S}_\theta$ as follows:

$$\begin{aligned} d\vec{S}_\theta &= \frac{2\pi R}{\sqrt{g_{\phi\phi}}} (\vec{e}_\rho d\rho + \vec{e}_\theta d\theta + \vec{e}_\phi d\phi) \times \vec{e}_\phi \\ &= \frac{2\pi R}{\sqrt{g_{\phi\phi}}} [(\vec{e}_\rho \times \vec{e}_\phi) d\rho + (\vec{e}_\theta \times \vec{e}_\phi) d\theta] \\ &= 2\pi R \left(\frac{\sqrt{g}}{\sqrt{g_{\phi\phi}}} \right) [-d\rho \vec{e}^\theta + d\theta \vec{e}^\rho] . \end{aligned} \quad (9.60)$$

Similarly, the total magnetic field can be transformed from

$$\vec{B} = B_\theta \hat{e}_\theta + B_\phi \hat{e}_\phi$$

to

$$\vec{B} = \frac{B_\theta}{\sqrt{g_{\theta\theta}}} \vec{e}_\theta + \frac{B_\phi}{\sqrt{g_{\phi\phi}}} \vec{e}_\phi . \quad (9.61)$$

Now using Eqs. (9.60) and (9.61) together with Eq. (9.58), one finds that

$$\begin{aligned}\vec{B} \cdot d\vec{S}_\theta &= \left(\frac{B_\theta}{\sqrt{g_{\theta\theta}}} \vec{e}_\theta + \frac{B_\phi}{\sqrt{g_{\phi\phi}}} \vec{e}_\phi \right) \frac{2\pi R \sqrt{g}}{\sqrt{g_{\phi\phi}}} (-\rho \vec{e}^\theta + d\theta \vec{e}^\rho) \\ &= - \left(\frac{B_\theta}{\sqrt{g_{\theta\theta}}} \frac{2\pi R \sqrt{g}}{\sqrt{g_{\phi\phi}}} d\rho \right) .\end{aligned}$$

This leads to the following form for Faraday's law:

$$V_\phi(\rho_1) - V_\phi(\rho_0) = \frac{d}{dt} \int_{\rho_0}^{\rho_1} \frac{B_\theta 2\pi \sqrt{g}}{\sqrt{g_{\theta\theta}}} d\rho , \quad (9.62)$$

where $\sqrt{g_{\phi\phi}} = R$ has been used (see Appendix B). A final step remains. By defining the poloidal magnetic flux as

$$\psi(\rho, t) = \int \vec{B}_\theta \cdot d\vec{S}_\theta = 2\pi \int_0^\rho \frac{B_\theta}{\sqrt{g_{\phi\phi}}} \sqrt{g} d\rho , \quad (9.63)$$

Eq. (9.62) reduces to

$$V_\phi(\rho_1) - V_\phi(\rho_0) = \frac{d}{dt} [\psi(\rho_1) - \psi(\rho_0)] . \quad (9.64)$$

Now stipulating the boundary conditions

$$V_\phi(\rho_1) = V_\phi(\rho = a) = V_s \text{ (plasma surface voltage),}$$

$$\psi(\rho_1) = \psi(\rho = a) = \psi(a) \text{ (poloidal flux within the plasma),}$$

$$V_\phi(\rho_0) = V_\phi(\rho = 0) = V_{res} \text{ (plasma voltage on-axis),}$$

$$\psi(\rho_0) = \psi(\rho = 0) = 0,$$

Eq. (9.64) can be arranged into the same format as Eq. (9.44), namely,

$$V_s = V_{res} + \frac{d}{dt} \psi(a) . \quad (9.65)$$

Here, $\frac{d}{dt} \psi(a)$ is the internal inductive voltage. By adding $\frac{d}{dt} (L_p^{ext} I_p)$ to Eq. (9.65), the expression for the applied loop voltage is obtained:

$$V_l = \frac{d}{dt} (L_p^{ext} I_p) + V_s . \quad (9.66)$$

In the actual WHIST code, an effective poloidal field $\tilde{B}_p$ and not B_θ is evolved in time. The internal flux is therefore determined throughout the course of the simulation by integrating Eq. (9.25) to obtain

$$\psi(a) = \mu_0 \int_0^a \frac{I(\rho')}{G_\theta(\rho')} d\rho' . \quad (9.67)$$

A word or two is in order here regarding the details of the resistive voltage term. By defining the toroidal voltage as

$$V_\phi \equiv \frac{\langle \vec{E} \cdot \vec{B} \rangle_f}{|B|} 2\pi R_M , \quad (9.68)$$

one can make use of Eqs. (9.21) and (9.31) to obtain

$$V_\phi = 2\pi R_M (2\pi R_O)^2 \eta_{||} \tilde{J} \left(\frac{G_\phi}{V'} \right) . \quad (9.69)$$

Here, R_M and R_O are the magnetic and geometric axes, respectively ($R_M = R_O$ for the case of concentric ellipses). In the limit as $\rho \rightarrow 0$, one finds $\tilde{B} \rightarrow B_T \hat{e}_\phi$, $B_p \rightarrow 0$, and $E_{||} \rightarrow E_T$ on-axis. Applying this limit to Eq. (9.69) and using the result

$$\lim_{\rho \rightarrow 0} \left(\frac{G_\phi}{V'} \right) = \frac{\partial G_\phi / \partial \rho}{\partial V' / \partial \rho} = \frac{\sigma / R_O}{4\pi^2 R_O \sigma} = \frac{1}{(2\pi R_O)^2} ,$$

one obtains

$$\begin{aligned}
 \lim_{\rho \rightarrow 0} V_{\phi} &= 2\pi R_M (2\pi R_o)^2 \lim_{\rho \rightarrow 0} \eta_{\parallel} \tilde{J} \left(\frac{G_{\phi}}{V'} \right) \\
 &= 2\pi R_M (2\pi R_o)^2 \eta_{\parallel}(0) \tilde{J}(0) \lim_{\rho \rightarrow 0} \left(\frac{G_{\phi}}{V'} \right) \\
 &= V_{\text{res}} = 2\pi R_M [\eta_{\parallel}(0) \tilde{J}(0)] . \quad (9.70)
 \end{aligned}$$

[For the case of concentric ellipses $\tilde{J}(0) = \tilde{J}_{\text{oh}}(0)$.]

Finally, by integrating Eq. (9.66) with respect to time, one can determine the total flux (ϕ_T) required for starting up the plasma current:

$$\underbrace{\int_0^{t_{\text{SU}}} V_{\ell} dt'}_{= \phi_T} = \underbrace{L_p^{\text{ext}} I_p}_{= \phi_{\text{ind}}^{\text{ext}}} + \underbrace{\psi_a(t_{\text{SU}})}_{= \phi_{\text{ind}}^{\text{int}}} + \underbrace{\int_0^{t_{\text{SU}}} 2\pi R_M [\eta_{\parallel}(0) \tilde{J}(0)] dt'}_{= \phi_{\text{res}}} . \quad (9.71)$$

The dimensionless internal inductance associated with the particular current profile can also be estimated by using the familiar relation

$$L_p^{\text{int}} I_p = \mu_o R I_p \frac{\ell_i}{2} = \psi_a(t) . \quad (9.72)$$

9.3 Modelling the Expansion Phase in WHIST

In WHIST the finite-differenced forms of the transport equations are solved on a nonuniform, two-region spatial grid used to model the plasma and scrape-off zones [34]. In the plasma zone, the magnetic field lines form closed magnetic surfaces. The scrape-off region is characterized, however, by open field lines which connect to a material limiter or magnetic divertor. A separatrix or limiter edge separates the two regions.

The WHIST code has the capability of handling up to 61 spatial mesh points or nodes. The allocation of all or part of these nodes, in either a uniform or nonuniform mesh arrangement, is a user option. A simplified illustration of the spatial mesh notation used to model a moveable limiter expansion (fixed major radius) is shown in Fig. 9.2. The number of nodes between the minor axis of the plasma and the separatrix is given by NSS. The subscripted variable RMID(I) represents the boundaries of unit cells located midway between the main grid points [34]. The variable RMID(NSS) is used to specify the plasma minor radius. Similarly, there are a total of N nodes between the plasma axis and the wall with NSO(= N-NSS) nodes in the scrape-off layer. It is only in the scrape-off region [where $\text{RMID}(I) > \text{RMID}(\text{NSS})$] that the parallel loss terms ($\Gamma_{\parallel i,e}/L_{\parallel}$ and $Q_{\parallel i,e}/L_{\parallel}$) are evaluated and added to the overall particle and energy balance. The inclusion of sink terms in the scrape-off layer is an important feature of the WHIST code. By reducing the influence of boundary conditions applied at the chamber wall, the interior and edge plasma can evolve in a more self-consistent manner. This is important when studying the expanding radius process.

9.3.1 Fixed Major Radius Expansion Model

In order to simulate a fixed major radius expansion (retractable limiter case), the WHIST code has been modified so that nodes can be added to the plasma over a specified period of time. The technique used there is similar to that employed by Mense et al. [106] in examining circular plasmas.

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Fig. 9.2 Illustration of the spatial mesh used in WHIST to model a moveable limiter expansion.

Because of the finite size of a unit cell, the addition of individual nodes to the plasma zone is equivalent to expanding the minor radius in discrete size increments. The mesh size is determined by inputting the initial dimensions of the minor radius and scrape-off layer and indicating the allocation of nodes to each region. The final minor radius is also specified and NSSF is calculated internally by the code. The total number of nodes used in the simulation satisfies the constraint $N = NSS_i + NSO_i = NSS_f + NSO_f = \text{constant}$. At a predetermined time, a new node is added to the plasma ($NSS = NSS + 1$). Because N remains fixed during the expansion, the number of nodes in the scrape-off NSO decreases by 1. By transferring a node from the scrape-off region to the plasma zone, the parallel loss terms originally evaluated in that cell are removed. When NSS equals $NSSF$, the plasma is fully expanded [$RMID(NSSF) = a_f$].

Because of the rapid attenuation of energy within the limiter shadow, the plasma temperatures near the first wall can be exceedingly low (less than 1 eV). Such low temperatures can make the numerical solutions of transport coefficients and balance equations difficult or even impossible. To eliminate these potential problems, the transport equations are solved over a truncated portion of the scrape-off region. Here, plasma temperatures are sufficiently high that numerical solutions are possible. Beyond this region, gradient boundary equations are solved [106].

After the minor radius has been expanded by a new node, an increment of current is added to the unit cell in accordance with the prescribed rate of current rise given by Eq. (8.9). This rate is determined by specifying the time-dependent forms for the total plasma current, $I_p(t)$, and cross-sectional area, $A(t)$. Prescribing the temporal behavior of $I_p(t)$ and $A(t)$ is also required in order to determine when the limiter should be withdrawn past the next node. To illustrate this, consider the general relationship between $\bar{J}$, $I_p(t)$, and $A(t)$ provided by Eqs. (8.5) and (8.7). For a fixed $q_1(a)$ and constant elongation (σ not varying in minor radius or time),

$$\begin{aligned} \bar{J} &= \frac{2B(R_o)}{\mu_o q_I R_o} \left(\frac{1 + \sigma^2}{2\sigma} \right) = \text{const} = \frac{I_p(t)}{\pi a^2(t)\sigma} \\ &= \frac{I_f}{\pi a_f^2 \sigma} = \frac{I_i}{\pi a_i^2 \sigma} \end{aligned} \quad (9.73)$$

The subscripts i and f refer to parameter values at the beginning and end of the expansion phase. Equation (9.73) provides a synchronization relationship between the rising plasma current and the instantaneous plasma radius given by

$$a_p(t) = \left[\left(\frac{\mu_o q_I R_o I_p(t)}{2\pi B(R_o)} \right) \left(\frac{2}{1 + \sigma^2} \right) \right]^{1/2} \quad (9.74)$$

In an actual experiment, the plasma expansion can proceed along two paths. The first path assumes a programmed withdrawal of the limiter in time. To maintain a constant q_I , the ohmic heating system must be capable of increasing the current ramp rate in time [see Eq. (8.9)]. The second path assumes the current rise is programmed in time. This approach involves coupling the limiter withdrawal to the plasma current via active feedback control (e.g., a Rogowski coil [107] signal proportional to the plasma current can be used to control the positioning of the limiter).

To study each approach, two basic options have been incorporated into the fixed major radius algorithm. The first option assumes that the minor radius increases linearly in time from a_i to a_f during the time interval $t_i \leq t \leq t_f$ (see Fig. 9.3). The variation of plasma radius with time is given by

$$a_p(t) = a_i + \dot{a} (t - t_i) \quad , \quad (9.75)$$

where

$$\dot{a} = \frac{da}{dt} = \left(\frac{a_f - a_i}{t_f - t_i} \right)$$

Fig. 9.3 Constant q_I minor radius expansion in FED with fixed R_0 and $a(t)$ increasing linearly in time. The effect of a time-dependent elongation (option 3) on $I_p(t)$ is also shown.

is the limiter withdrawal rate (or equivalent expansion rate). Equation (9.75) is used to calculate the plasma expansion time,

$$t_{\text{exp}} = t_i + \left(\frac{a(t) - a_i}{a_f - a_i} \right) (t_f - t_i) , \quad (9.76)$$

which is used like an independent variable. When the simulation time exceeds the expansion time, the limiter is withdrawn past the next outermost node. If $\text{RMID}(\text{NSS}) = a(t) < a_f$, t_{exp} is recalculated using $\text{RMID}(\text{NSS} + 1)$. The simulation is continued until $\text{NSS} = \text{NSSF}$. With the addition of the new node, the plasma current increases to

$$I_p(t) = I_i + \pi \bar{J} (a^2(t) - a_i^2) \sigma .$$

This total current is then applied as a boundary condition on the poloidal field equation. Examples of the incremental jumps in size and current made during the expansion phase are shown in Fig. 9.4.

The second option consists of linearly increasing the plasma current in time. Here, the transient behavior of the current and minor radius is given by

$$I_p(t) = I_i + \left(\frac{I_f - I_i}{t_f - t_i} \right) (t - t_i) , \quad (9.77)$$

$$a_p(t) = a_i \left[1 + \frac{1}{I_i} (I_f - I_i) \left(\frac{t - t_i}{t_f - t_i} \right) \right]^{1/2} . \quad (9.78)$$

Using Eq. (9.73) to eliminate I_f from Eq. (9.78), one obtains

$$a_p(t) = \left[a_i^2 + \left(\frac{a_f^2 - a_i^2}{t_f - t_i} \right) (t - t_i) \right]^{1/2} . \quad (9.79)$$

The corresponding value for t_{exp} given by

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Fig. 9.4 WHIST simulation of the last second of Fig. 9.3. The plasma size and current levels are incremented in discrete steps.

$$t_{\text{exp}} = t_i + \left(\frac{a^2(t) - a_i^2}{a_f^2 - a_i^2} \right) (t_f - t_i) \quad (9.80)$$

is smaller than that obtained using Eq. (9.76) (for the same t_i). This is to be expected, however. With $I_p(t)$ increasing linearly in time, the limiter withdrawal rate must be increased in order to ensure the addition of a constant $\bar{J}$ at the plasma edge.

Variations on the above two options, which involve the addition or nonuniform current densities ($d\bar{J}/dt \neq 0$), have also been examined. Option 3 involves increasing both the minor radius and elongation linearly in time, while in option 4, plasma current and elongation are increased linearly in time. A fifth option examines expansions with varying edge q_I .

For option 3, the time evolution of the plasma elongation is given by

$$\sigma(t) = \sigma_i + (\sigma_f - \sigma_i) \left(\frac{a(t) - a_i}{a_f - a_i} \right), \quad (9.81)$$

while $a_p(t)$, $I_p(t)$, and t_{exp} are described by Eqs. (9.75), (9.82), and (9.76), respectively. In option 4, the parameters $I_p(t)$, $a_p(t)$, and $\sigma(t)$ are related through

$$\begin{aligned} \frac{I_p(t)}{a^2(t)} \left(\frac{2}{1 + \sigma^2(t)} \right) &= \frac{I_f}{a_f^2} \left(\frac{2}{1 + \sigma_f^2} \right) \\ &= \frac{I_i}{a_i^2} \left(\frac{2}{1 + \sigma_i^2} \right) = \frac{2\pi B(R_o)}{\mu_o q_I R_o} \end{aligned} \quad (9.82)$$

For $I_p(t)$ increasing linearly in time,

$$a^2(t) = \frac{a_i^2}{I_i} \left(\frac{1 + \sigma_i^2}{1 + \sigma^2(t)} \right) \left[I_i + \left(\frac{I_f - I_i}{t_f - t_i} \right) (t - t_i) \right]. \quad (9.83)$$

Using Eq. (9.82) to eliminate I_f , an equation is obtained for the plasma radius which depends on the time-dependent form of the elongation

$$a^2(t) (1 + \sigma^2(t)) = \left[a_i^2 (1 + \sigma_i^2) + \left(\frac{a_f^2 (1 + \sigma_f^2) - a_i^2 (1 + \sigma_i^2)}{t_f - t_i} \right) (t - t_i) \right]. \quad (9.84)$$

Equation (9.84) is identical to Eq. (9.79) in the limit of constant elongation. Upon substituting a linear form for elongation

$$\sigma(t) = \sigma_i + \left(\frac{\sigma_f - \sigma_i}{t_f - t_i} \right) (t - t_i) \quad (9.85)$$

into Eq. (9.84), one obtains a quadratic equation in $(t - t_i)$ given by

$$\begin{aligned} & a^2(t) \left(\frac{\sigma_f - \sigma_i}{t_f - t_i} \right)^2 (t - t_i)^2 \\ & + \left[2a^2(t) \sigma_i \left(\frac{\sigma_f - \sigma_i}{t_f - t_i} \right) - \left(\frac{a_f^2 (1 + \sigma_f^2) - a_i^2 (1 + \sigma_i^2)}{t_f - t_i} \right) \right] (t - t_i) \\ & + \left(a^2(t) - a_i^2 \right) (1 + \sigma_i^2) = 0. \end{aligned} \quad (9.86)$$

The negative root of Eq. (9.86) is used in determining t_{exp} . In the fifth and final option, $q_I(a)$ decreases linearly with increasing minor radius while σ remains constant. The time-dependent form for

q_I is given by

$$q_I(t) = q_i + (q_f - q_i) \left(\frac{a(t) - a_i}{a_f - a_i} \right). \quad (9.87)$$

9.3.2 Major Radius Compression Model

The algorithm developed for studying plasma growth off of a fixed outboard limiter differs significantly from the fixed major radius model. In the latter algorithm, the major radius location and spatial grid dimensions are invariant in time (assuming a constant elongation). That is not the case, however, when plasma expansion occurs in conjunction with major radius compression. In order to accommodate the addition of nodes to the plasma region, the major radius must be simultaneously compressed in discrete size increments.

R-compression is achieved by increasing the applied vertical field. The resulting $\vec{J}_T \times \vec{B}_V$ force nudges the plasma inward in major radius. As with the retractable limiter option, the rising vertical field can be preprogrammed (to allow the plasma size to increase linearly in time) or it can be coupled to the plasma current via feedback control. As the plasma moves into a region of higher toroidal field, the minor radius contracts in order to conserve toroidal magnetic flux within the plasma cross section. Simulating this effect involves compressing the plasma spatial mesh [out to RMID(NSS-1)] by a finite amount following the addition of a new node. Once the grid has been compressed, the plasma density, temperature, pressure, and current are updated using the scaling laws for adiabatic compression discussed in the next section.

The instantaneous major radius of the expanding plasma is given by

$$R(t) = \frac{R_{LIM}}{[1 + \epsilon(t)]}, \quad (9.88)$$

where

$$\epsilon(t) \equiv \frac{a(t)}{R(t)} = \left[\left(\frac{\mu_0 q_I I_p(t)}{2\pi B(R_0) \cdot R_0} \right) \left(\frac{2}{1 + \sigma^2(t)} \right) \right]^{1/2} \quad (9.89)$$

and

$$R_{LIM} = 6.1 \text{ m} = R(t) + a(t)$$

is the major radius location of the outboard startup limiters. The initial minor radius is specified to be $a(t_i) = a_i = 0.4 \text{ m}$ for $R(t_i) = R_i = 5.7 \text{ m}$ and $a_i = 0.2 \text{ m}$ for $R(t_i) = 5.9 \text{ m}$. Using Eqs. (9.88) and (9.89), the corresponding instantaneous plasma radius is given by

$$a(t) = R_{LIM} \left(\frac{\epsilon(t)}{1 + \epsilon(t)} \right) \quad (9.90)$$

The same options discussed in the fixed major radius algorithm have been examined in the R-compression model. The results for each option are summarized below.

Option 1:

$$R(t) = R_i + \left(\frac{R_f - R_i}{t_f - t_i} \right) (t - t_i) \quad (9.91)$$

with

$$\dot{R} = \left(\frac{R_f - R_i}{t_f - t_i} \right) = \dot{a} \quad (9.92)$$

and

$$t_{comp} = t_{exp} = t_i + \left(\frac{R(t) - R_i}{R_f - R_i} \right) (t_f - t_i) \quad (9.93)$$

The plasma current is given by

$$I_p(t) = \left[\frac{2\pi B(R_0) \cdot R_0}{\mu_0 q_I} \left(\frac{1 + \sigma^2}{2} \right) \right] \epsilon^2(t) . \quad (9.94)$$

Option 2:

$$I_p(t) = I_i + \left(\frac{I_f - I_i}{t_f - t_i} \right) (t - t_i) ,$$

$$\epsilon^2(t) = \epsilon_i^2 + \left(\frac{\epsilon_f^2 - \epsilon_i^2}{t_f - t_i} \right) (t - t_i) , \quad (9.95)$$

$$t_{\text{comp}} = t_i + \left(\frac{\epsilon^2(t) - \epsilon_i^2}{\epsilon_f^2 - \epsilon_i^2} \right) (t_f - t_i) . \quad (9.96)$$

Option 3:

$$\sigma(t) = \sigma_i + (\sigma_f - \sigma_i) \left(\frac{R(t) - R_i}{R_f - R_i} \right) . \quad (9.97)$$

Option 4:

$$\begin{aligned} \epsilon^2(t) (1 + \sigma^2(t)) &= \epsilon_i^2 (1 + \sigma_i^2) \\ &+ \left[\frac{\epsilon_f^2 (1 + \sigma_f^2) - \epsilon_i^2 (1 + \sigma_i^2)}{t_f - t_i} \right] (t - t_i) , \end{aligned} \quad (9.98)$$

$$\sigma(t) = \sigma_i + \left(\frac{\sigma_f - \sigma_i}{t_f - t_i} \right) (t - t_i) , \text{ and}$$

$$\epsilon^2(t) \left(\frac{\sigma_f - \sigma_i}{t_f - t_i} \right)^2 (t - t_i)^2$$

$$\begin{aligned}
& + \left[2 \epsilon^2(t) \sigma_i \left(\frac{\sigma_f - \sigma_i}{t_f - t_i} \right) - \left(\frac{\epsilon_f^2(1 + \sigma_f^2) - \epsilon_i^2(1 + \sigma_i^2)}{t_f - t_i} \right) \right] (t - t_i) \\
& + (\epsilon^2(t) - \epsilon_i^2) (1 + \sigma_i^2) = 0 .
\end{aligned} \tag{9.99}$$

Option 5:

$$q_I(t) = q_i + (q_f - q_i) \left(\frac{R(t) - R_i}{R_f - R_i} \right) . \tag{9.100}$$

9.4 Scaling Laws for Adiabatic Compression of an Elliptical Plasma

In the R-compression model discussed previously, the plasma remains in contact with the fixed outboard limiter as it expands to full size. In this configuration, a new node can be added to the plasma only after the major radius has undergone an equivalent compression in size [$R(t) = R_{LIM} - a(t)$]. Because each discrete compression satisfies the time scale ordering

$$\tau_{ei} \ll \tau_c \ll \tau_E < \tau_s ,$$

the plasma obeys the law of adiabatic compression,

$$pV^\gamma = pn^{-\gamma} = \text{const} , \tag{9.101}$$

with the addition of each new node. In the above inequality, τ_{ei} is the electron-ion collision time, τ_c is the compression time [which is different from t_{comp} calculated in Eq. (9.93)], τ_E is the energy confinement time, and τ_s is the resistive skin time of the plasma. The parameter V is the specific volume ($\propto n^{-1}$), γ is the gas constant, and p is the plasma kinetic pressure ($= nkT$).

For collisional compression ($\gamma = 5/3$), Eq. (9.101) can be written as

$$\tau_n^{-2/3} = \text{const} . \quad (9.102)$$

Toroidal and poloidal magnetic fluxes within the plasma should also be conserved during compression since $\tau_c \ll \tau_s$. For an elliptical plasma, flux conservation provides the following constraints:

$$\Phi = \pi a^2 \sigma B(R) = \text{const} \quad (9.103)$$

and

$$q_I = \frac{2\pi a^2 B(R)}{\mu_0 I_p R} \left(\frac{1 + \sigma^2}{2} \right) = \text{const} . \quad (9.104)$$

Equation (9.103) can be more conveniently expressed in terms of the major (R) and minor (a) radii and the elongation (σ) by using $B(R) = B(R_0) \cdot R_0/R$. The result is

$$\frac{a^2(t) \sigma(t)}{R(t)} = \frac{a_{\text{old}}^2 \sigma_{\text{old}}}{R_{\text{old}}} = \text{const} . \quad (9.105)$$

On the basis of Eqs. (9.102), (9.103), and (9.104), the following scaling laws are obtained:

$$n \propto V^{-1} \propto a^{-2} \sigma^{-1} R^{-1} \propto R^{-2} ,$$

$$T \propto n^{2/3} \propto a^{-4/3} \sigma^{-2/3} R^{-2/3} \propto R^{-4/3} ,$$

$$B \propto a^{-2} \sigma^{-1} \propto R^{-1} ,$$

$$I_p \propto \frac{a^2 B}{R} \left(\frac{1 + \sigma^2}{2} \right) \propto \frac{a^2}{R^2} \left(\frac{1 + \sigma^2}{2} \right) \propto R^{-1} \left(\frac{1 + \sigma^2}{2\sigma} \right) ,$$

$$B_p \propto \frac{I_p}{a} \left(\frac{1 + \sigma^2}{2} \right)^{-1/2} \propto \frac{a}{R} \left(\frac{1 + \sigma^2}{2} \right)^{1/2} \propto R^{-3/2} \left(\frac{1 + \sigma^2}{2\sigma} \right)^{1/2} ,$$

$$\beta \propto \frac{nT}{B^2} \propto \left(\frac{a^2 \sigma}{R^5} \right)^{1/3} \propto R^{-4/3},$$

$$\beta_p \propto \frac{nT}{B_p^2} \propto \left(\frac{R^7}{a^{16} \sigma^5} \right)^{1/3} \left(\frac{1 + \sigma^2}{2} \right)^{-1} \propto R^{-1/3} \left(\frac{2\sigma}{1 + \sigma^2} \right),$$

$$J \propto \frac{I_p}{a^2 \sigma} \propto \frac{a^2}{R^2} \left(\frac{1 + \sigma^2}{2} \right) \left(\frac{1}{a^2 \sigma} \right) \propto R^{-2} \left(\frac{1 + \sigma^2}{2\sigma} \right).$$

By defining several important ratios,

$$C \equiv \frac{R_{\text{old}}}{R_{\text{new}}}, \quad E \equiv \frac{\sigma_{\text{new}}}{\sigma_{\text{old}}}, \quad S \equiv \left[\frac{1 + \sigma^2}{2\sigma} \right]_{\text{new}} / \left[\frac{1 + \sigma^2}{2\sigma} \right]_{\text{old}},$$

the above scaling laws can be generalized to those shown in Table 4. In the limit of a circular plasma ($E = S = 1$), the "Type B" compression scaling of Furth and Yoshikawa [108] is obtained.

Table 4. Adiabatic compression scaling for an elliptical plasma

Quantity	R-compression constant $B(R_0)$
Minor radius, a	$C^{-1/2} \cdot E^{-1/2}$
Major radius, R	C^{-1}
Plasma density, n	C^2
Plasma temperature, T	$C^{4/3}$
Plasma pressure, p	$C^{10/3}$
Toroidal field on axis, $B(R_0)$	constant
Toroidal field @ plasma axis, $B(R)$	C
Plasma current, I_p	$C \cdot S$
Poloidal field, B_p	$C^{3/2} \cdot S^{1/2}$
Toroidal plasma beta, β	$C^{4/3}$
Poloidal plasma beta, β_p	$C^{1/3} \cdot S^{-1}$
Current density, J	$C^2 \cdot S$

10. STARTUP VOLTAGE AND VOLT-SECOND REQUIREMENTS - "ZERO-DIMENSIONAL" ANALYSIS

10.1 Electron Temperature Estimates During Current Startup

In order to assess the voltage (V_e) and flux (ϕ) requirements during startup, the time history of the electron temperature is needed. A conservative estimate of $T_e(t)$ can be obtained by using a simplified set of global particle and power balance equations which are spatially independent (zero-dimensional). Approximating an elongated plasma by a cylinder of length L ($= 2\pi R$), the spatial dependence is eliminated by utilizing volume-averaged particle and energy densities,

$$\bar{n} \equiv \frac{L \int_0^{a_{eff}} n(r) 2\pi r dr}{\pi a_{eff}^2 L} \quad (10.1)$$

and

$$\frac{3}{2} \bar{n} kT \equiv \frac{L \int_0^{a_{eff}} 3/2 n(r) kT(r) 2\pi r dr}{\pi a_{eff}^2 L}, \quad (10.2)$$

together with average particle and energy transport confinement times given by

$$\tau_p \equiv \frac{\bar{n} V_p}{\int_{V_p} (\vec{\nabla} \cdot \vec{\Gamma}) d^3 r} \equiv \frac{\bar{n} V_p}{\int_S \vec{\Gamma} \cdot d\vec{S}} \cong \frac{a_{eff}^2}{4D(\bar{n}, \bar{T})} \quad (10.3)$$

and

$$\tau_E \equiv \frac{\frac{3}{2} \overline{nkT} V_p}{\int_{V_p} \vec{\nabla} \cdot (\vec{Q} + \frac{3}{2} kT \vec{r}) d^3r} = \frac{\frac{3}{2} \overline{nkT} V_p}{\int_S (\vec{Q} + \frac{3}{2} kT \vec{r}) \cdot d\vec{S}} \approx \frac{a_{\text{eff}}^2}{4\chi(\bar{n}, \bar{T})} \quad (10.4)$$

By taking the volume average of Eqs. (9.4) - (9.6) (and dropping the overbar for convenience), the following global balance equations for a hydrogen plasma are obtained (mks units with T in keV):

$$\frac{dn_e}{dt} = n_e n_o \langle \sigma v \rangle_{\text{ion}} - \frac{n_e}{\tau_p}, \quad (10.5)$$

$$\frac{dn_o}{dt} = -n_e n_o \langle \sigma v \rangle_{\text{ion}} + \frac{n_e}{\tau_p} f_R + S_{\text{ext}}, \quad (10.6)$$

$$\frac{dW_p}{dt} = P_{\text{oh}} + P_{\text{rf}} - P_{\text{ion}} - P_{\text{cx}} - P_{\text{rad}} - \frac{W_p}{\tau_E}, \quad (10.7)$$

where

$\tau_E = (\tau_{Ei}^{-1} + \tau_{Ee}^{-1})^{-1} [= 5 \times 10^{-21} n_e a_{\text{eff}}^2 (a_{\text{eff}}^2 = a^2 \sigma)]$ is the energy confinement time (assuming $n_e = n_i$ and $T_e \cong T_i$),

$\tau_p (= 3 \tau_E)$ is the particle confinement time,

$W_p (= 3 n_e k T_e V_p)$ is the plasma thermal energy,

$V_p (= 2\pi^2 R a^2 \sigma)$ is the plasma volume,

$R_p [= 2R\eta_{\parallel} / a^2 \sigma (\eta_{\parallel} = \eta_{\text{SPITZER}} \times \alpha_{\parallel})]$ is the plasma resistance,

$P_{\text{oh}} (= I_p^2 R_p)$ is the ohmic heating power,

$P_{\text{ion}} (= n_e n_o \langle \sigma v \rangle_{\text{ion}} kW_{\text{ion}} V_p)$ is the power lost in ionizing the hydrogenic fuel gas,

$P_{cx} (= \frac{3}{2} n_e n_o \langle \sigma v \rangle_{cx} kT_i V_p)$ is the power loss associated with charge exchange events occurring between cold neutrals and warm ions,

$P_{rad} [= 2 \times 10^{-12} n_e^2 \xi R a^2 \sigma L_z(T_e)]$ is the radiation power loss due to oxygen impurity.

The coronal equilibrium values for $L_z(T_e)$, the impurity emission rate per ion per free electron and $\langle Z_{oxy} \rangle$, the average charge state of an oxygen ion at a given T_e , are calculated from the model developed by Post et al. [63]. In Eq. (10.6), f_R is the effective recycling coefficient and S_{ext} is the particle source resulting from the ionization of cold hydrogenic neutrals introduced into the vacuum chamber. The parameter $a_{eff} (= a\sqrt{\sigma})$ is the effective circular plasma radius for an elongated plasma, $\eta_{||}$ is the parallel plasma resistivity, α_{η} is a resistivity enhancement factor, and $\xi (= n_{oxy}/n_e)$ is the percentage of oxygen impurity in the plasma. The remaining parameters were defined earlier.

During the first 6 s of current startup, τ_E ranges from 2 ms to 0.4 s ($a \approx 0.2$ m to 1.3 m, $\sigma = 1.0$ to 1.6, and $n_e \approx 10^{19} \text{ m}^{-3}$ to $3 \times 10^{19} \text{ m}^{-3}$). Because the current-rise time is much larger than the rapid energy and particle response time of the plasma, one can neglect the time derivatives in Eqs. (10.5) to (10.7), thereby obtaining the following quasi-steady-state particle and power balance equations:

$$n_o = (\langle \sigma v \rangle_{ion} \tau_p)^{-1}, \quad (10.8)$$

$$n_e n_o \langle \sigma v \rangle_{ion} = \frac{n_e}{\tau_p} f_R + S_{ext}, \quad (10.9)$$

$$\begin{aligned}
& \frac{3.3 \times 10^{-9} \alpha_{\eta} \ln \Lambda_{ei} Z_{eff} R I_p^2}{a^2 \sigma T_e^{3/2}} + P_{rf} = 2.1 \times 10^5 W_{ion} R \\
& + 3.2 \times 10^5 \left(\frac{\langle \sigma v \rangle_{cx}}{\langle \sigma v \rangle_{ion}} \right) T_e R + 2.0 \times 10^{-12} n_e^2 \xi R a^2 \sigma L_z(T_e) \\
& + 1.9 \times 10^6 T_e R, \tag{10.10}
\end{aligned}$$

where

$$Z_{eff} = 1 + \xi \langle Z_{oxy} \rangle (\langle Z_{oxy} \rangle - 1).$$

The explicit parametric dependence of each of the source and sink terms found in Eq. (10.7) is indicated in Eq. (10.10). Using Eq. (10.8), the neutral density has been eliminated in Eq. (10.10). Also, it has been assumed that $T_e \approx T_i$, and this assumption leads to pessimistic estimates of T_e during the initial stages of current rise when $\tau_{EQ}^{ei} = \tau_{ei}(m_i/m_e) \geq 2\tau_E$ and $T_e > T_i$. At the end of current startup ($T_e \approx 1$ keV, $n_e \approx 3 \times 10^{19} \text{ m}^{-3}$, $a = 1.3$ m, and $\sigma = 1.6$), $\tau_{EQ}^{ei} \approx 0.1 \tau_E$, and the assumption is justified.

The second term on the right-hand side of Eq. (10.10) is the power loss due to charge exchange. With a purely inductive current startup, the plasma is initially cold ($T_e \approx T_i \lesssim 2$ eV), and the mean free path for neutral penetration can be comparable to the plasma radius ($\lambda_{mfp} \gtrsim a_0 \approx 0.2$ m). At these low temperatures, $\langle \sigma v \rangle_{cx} / \langle \sigma v \rangle_{ion} \approx 1200$, indicating that charge exchange events can be a major energy loss mechanism early in the discharge [13]. However, with successful electron preheating ($T_e \approx T_i \approx 0.1$ keV, $n_e \approx 10^{19} \text{ m}^{-3}$, and $a_0 \approx 0.2$ m - 0.4 m), the plasma is much more effective in screening out neutrals ($\lambda_{mfp} < 5$ cm). The result is a significant reduction of charge exchange, as well as ionization and radiation losses.

Finally, because n_e is treated as an independent variable in these calculations, the quasi-steady-state value of n_0 is determined from Eq. (10.8) for a given T_e . In turn, Eq. (10.9) can be used to estimate the required fueling rate, S_{ext} . Hawryluk et al. [13] have pointed out, however, that control of S_{ext} may be difficult because of

the uncertainties in recycling and particle-induced desorption processes in the chamber wall.

10.2 Calculating the Applied Loop Voltage and Its Components

In estimating the applied voltage requirements, the controllable parameters are assumed to be the ECRH/ICRH power and the plasma current, density, and elongation. By specifying the plasma current, rather than the applied voltage, the differential equation describing the current evolution is used to determine the time history of the plasma loop voltage. Specifically,

$$V_L = V_{res} + V_{ind} = R_p I_p + \frac{d}{dt} (L_p I_p) , \quad (10.11)$$

where $V_{res} = P_{oh}/I_p$, and the plasma inductance is approximated by

$$L_p = \mu_0 R \left[\ln \left(\frac{8R}{a\sqrt{\sigma}} \right) + \frac{\ell_i}{2} - 2 \right] .$$

The dimensionless parameter ℓ_i is the flux-related definition of the internal inductance [7] discussed previously and has a value of ~ 1 for broad current profiles.

For constant q_1 expansion with R-compression, the plasma inductance varies in accordance with the time-dependent behavior of the major radius [Eq. (9.88)], minor radius [Eq. (9.90)], elongation, and dimensionless internal inductance. The general form for $I_p dL_p/dt$ can be written as

$$I_p \dot{L}_p = I_p \left\{ \frac{\partial L_p}{\partial R} \dot{R} + \frac{\partial L_p}{\partial a} \dot{a} + \frac{\partial L_p}{\partial \sigma} \dot{\sigma} + \frac{\partial L_p}{\partial \ell_i} \dot{\ell}_i \right\} . \quad (10.12)$$

It has been assumed that $\dot{\ell}_i \approx 0$ since a uniform current profile is maintained during minor radius expansion and there are no skin currents. With $\dot{a} = -\dot{R}$, Eq. (10.12) can be written explicitly as

$$I_p \dot{I}_p = \mu_0 I_p \left\{ \ln \left(\frac{8R}{a\sqrt{\sigma}} \right) + A - 0.5 \right\} \dot{R} - \mu_0 I_p \left(\frac{R}{2\sigma} \right) \dot{\sigma}, \quad (10.13)$$

where $A (= R/a)$ is the plasma aspect ratio. For a specified $\dot{I}_p$, $\dot{\sigma}$, and fixed q_I , the major radius compression rate is determined from

$$\dot{I}_p = 2I_p \left\{ \frac{-\dot{R}}{R} + \frac{\dot{a}}{a} + \left(\frac{\sigma}{1 + \sigma^2} \right) \dot{\sigma} \right\}, \quad (10.14)$$

where

$$I_p(t) = \frac{2\pi [a(t)]^2 B(R_0) \cdot R_0}{\mu_0 q_I [R(t)]^2} \left(\frac{1 + [\sigma(t)]^2}{2} \right).$$

10.3 Results and Discussion

Starting with a reasonable time history for $I_p(t)$, Eqs. (10.10) and (10.11) have been used to study the time evolution of T_e , P_{oh} , V_{res} , and V_ℓ . The resistive flux expenditure during startup, $\Delta\phi_{res}$, has also been estimated by numerically integrating the area under the representative voltage curve.

Several expanding radius startup cases have been generated to illustrate the sensitivity of the startup requirements to variations in the initial minor radius, oxygen impurity content, and electron preheating level. The basic physics assumptions used in these cases include:

1. flat temperature, density, and current density profiles during startup,
2. uniform and total absorption of both ECRH and ICRH power by the plasma,
3. $T_e \approx T_i$ during the current-rise phase,
4. INTOR scaling for energy confinement,
5. a relatively low $q_I \approx 2.4$,

6. a controlled density buildup satisfying the "Marakami limit"[71] (a linear density ramp is assumed),
7. a limitation on the maximum equilibrium and ballooning stability beta values specified at $\epsilon\beta_p \approx 0.5$ and $\beta_{crit} \approx \epsilon/q_I^2$, respectively,
8. uniform distribution of oxygen contaminant in those cases where impurity effects are examined,
9. coronal equilibrium of oxygen impurity during startup, and
10. a factor of 2 enhancement ($\alpha_\eta = 2$) in the Spitzer resistivity.

The results of the analysis are summarized in Table 5, and some typical examples are shown in Figs. 10.1-10.5. A varying major radius expansion (from $a_0 = 0.2$ m) with rf assist and no impurities ($Z_{eff} = 1$) is shown in Fig. 10.1. Here the electrons have been pre-heated to ~ 70 eV, a value believed to be comparable to that already achieved in the ISX-B tokamak. By $t = 50$ ms, the plasma current has risen to ~ 10 kA, and all of the physics constraints mentioned previously are satisfied. Supplemental ECRH (~ 1 MW at 87 GHz) helps to maintain this electron temperature, as the plasma current and q_I value evolve to their specified values of ~ 42 kA and 2.36 at $t = 200$ ms. Over the next 5.8 s, the plasma current is ramped-up linearly to the full ohmic heating current of 4.8 MA, while the plasma expands to full size ($a = 1.3$ m) and shape ($\sigma = 1.6$). Following 6 s of intense ICRF heating (50 MW at 54 MHz), the plasma enters the burn phase (lasting ~ 100 s) where 180 MW of fusion power is sustained by 36 MW of ICRH (a plasma $Q \sim 5$) [81].

Assessing the overall voltage and flux requirements during start-up, Fig. 10.1 indicates that the resistive voltage reaches a maximum value (~ 15 V) at $t = 0.2$ s. It then drops rapidly when the 5 MW of ECRH/ICRH power is applied during the expansion phase. The maximum V_ℓ is ~ 21 V. During the burn phase, the loop voltage is decreased to ~ 0.03 V, which is sufficient to maintain 5.4 MA of plasma current at 10 keV against resistive dissipation. The additional 0.6 MA of pressure-driven current is produced during bulk heating and must be compensated for by current increases in the equilibrium field coils [109]. The inductive volt-seconds at full current are estimated to be

Table 5. Dependence of startup requirements on initial plasma minor radius a_0 , oxygen content n_{ox}/n_e , and electron temperature before current initiation $T_e(t = 0)$

<u>Assumptions</u>							
a_0 (m)	0.2^a	0.2^b	0.2^c	0.2^d	0.4^e	0.4^f	0.4^g
$\xi = n_{ox}/n_e$ (%)	0	3	3	3	0	3	3
$T_e(t = 0)$ (eV)	~ 70	~ 70	~ 10	~ 10	~ 35	~ 70	~ 10
<u>Requirements</u>							
Maximum V_L (V)	21	25	66	120	24	26	101
at time (s)	0.2	0.2	0.04	0.04	0.3	0.3	0.08
Resistive induction flux loss $\Delta\phi_{res}$ (Wb)	10	18	19	42	10	17	22
at $t = 6$ s							

^aWith electron preheating and $P_{ECRH} = 1.0$ MW for $0 \leq t \leq 0.2$ s.

^bWith preheating and a phased input of ECRH power given by $P_{ECRH} = 1.0(1 + 2.5 t)$ MW for $0 \leq t \leq 0.2$ s.

^cSame conditions as *b* but without electron preheating.

^dWithout electron preheating or rf assist during the current-rise phase.

^eWith preheating and $P_{ECRH} = 0.5$ MW for $0 \leq t \leq 0.3$ s.

^fWith electron preheating and a phased input of ECRH power given by $P_{ECRH} = 1.0(1 + 2t)$ MW for $0 \leq t \leq 0.3$ s.

^gSame conditions as *f* but without electron preheating.

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Fig. 10.1 Time evolution of key parameters in FED during an expanding radius startup (from an initial minor radius $a_0 = 0.2$ m) with electron preheating, rf assist, and no oxygen impurity ($Z_{eff} = 1$).

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Fig. 10.2 Time evolution of the same key parameters during an expanding radius startup of a clean ($Z_{eff} = 1$), 0.4-m minor radius plasma with preheating and rf assist.

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Fig. 10.3 Time evolution of key parameters in FED during an expanding radius startup (from $a_0 = 0.2$ m) with electron preheating, rf assist, and 3% oxygen impurity.

ORNL-DWG 81-17120AR FED

Fig. 10.4 Time evolution of the same key parameters as in Fig. 10.3 but without preheating. The time behavior is identical to that shown in Fig. 10.3 after 50 ms.

Fig. 10.5 Time evolution of key parameters in FED during an expanding radius startup (from $a_0 = 0.4$ m) without electron preheating but with rf assist and 3% oxygen impurity.

$L I_p \approx 54$ Wb, assuming a flat current distribution. The accumulated loss of resistive volt-seconds during 6 s of current startup are calculated to be ~ 10 Wb. Also shown in Fig. 10.1 is the behavior of the ohmic heating power during this period. Figure 10.1 indicates that the resistive heating power rises monotonically to about 0.6 MW at $t = 0.2$ s and about 4.0 MW at $t = 6$ s.

For an initial minor radius of 0.4 m, Fig. 10.2 indicates that a maximum V_ℓ of ~ 24 V is required in the absence of impurities. In going to a larger starting minor radius, it is necessary to increase the initial rate of current rise (from ~ 0.2 MA/s to ~ 0.6 MA/s) while reducing the level of electron preheating (from ~ 70 eV to ~ 35 eV). These changes minimize the time duration ($0 \leq t \leq 70$ ms) over which the various physics constraints cannot be satisfied. Preheating the plasma to just beyond the hydrogen ionization and radiation barrier (≥ 30 eV) enhances current penetration and reduces the possibility for skin current formation. A higher rate of current rise can also support increased plasma temperatures ($T_e \propto I_p^2 / n_e a^2$ for fixed A and $T_e = T_i$) and more quickly satisfy the equilibrium and stability beta limits during current initiation.

With an oxygen impurity content of 3%, V_ℓ increases to ~ 25 V for $a_0 = 0.2$ m (Fig. 10.3) and ~ 26 V for $a_0 = 0.4$ m (Table 5). Because the $a_0 = 0.4$ m case employs a greater rate of current rise during the initiation phase, it also requires a greater combined power input (1.6 MW of ECRH and 2.1 MW of P_{oh} at $t = 300$ ms) to minimize the resistive portion of the total loop voltage. In comparison, the $a_0 = 0.2$ m case utilizes 1.5 MW of ECRH and ~ 0.8 MW of P_{oh} at $t = 200$ ms.

Without electron preheating, small levels of oxygen impurity can dominate the plasma energy balance. At $T_e \approx 20$ eV, $n_e \approx 10^{19} \text{ m}^{-3}$ and $\xi = 3\%$, oxygen's radiative power density is $\sim 0.27 \text{ MW/m}^3$. This large cooling rate (~ 1.26 MW for $a_0 = 0.2$ m and ~ 4.86 MW for $a_0 = 0.4$ m) suppresses the electron temperature during the beginning stages of current initiation. Calculations for the $a_0 = 0.2$ m (0.4 m) case indicate that if the electrons are not heated beyond 20 eV prior

to current initiation, the maximum V_{ℓ} requirement increases to ~ 65 V (~ 100 V), as indicated in Figs. 10.4 and 10.5. An initiation voltage of ~ 25 V would then fail to ramp-up the plasma current as shown. These calculations quantify the importance of successful preheating to "burn through" the impurity radiation barrier prior to current initiation. The impact of no rf assist prior to and during plasma current rise is to raise further the maximum loop voltage (to > 100 V for $a_0 = 0.2$ m) and more than double the resistive flux consumption during the 6-s current startup phase.

11. 1-D SIMULATIONS OF EXPANDING RADIUS CURRENT STARTUP

In Section 10, the primary emphasis was on determining the voltage and flux requirements for FED under various operating conditions. A flat profile model was used to provide a point of comparison with the more "realistic" radial transport calculations presented in this section. The importance of profile effects [110] in transport simulations has been recognized for some time. Their inclusion in various reactor calculations (in either an assumed form or else calculated self-consistently) can lead to a relaxation of the ignition criteria [110] and an increase in the average fusion power density for a given beta limit and reactor configuration. In a similar fashion, profile effects are expected to be beneficial during the current startup process.

At present, theoretical [87] and experimental [111] studies are underway to determine ways of controlling the evolution of the current density during startup. Of particular interest are methods for obtaining broad current profiles. Such profiles are consistent with low- q_I operation and can lead to high- β burns in tokamak reactors. In addition to enhancing the high- β possibilities in a reactor, broad current profiles are desirable because they can be established with a lower internal flux expenditure than can more peaked current profile shapes. The resulting savings in transformer volt-seconds can thus be used in prolonging the burn pulse or for increasing the current level.

In this section, the effects of evolving profiles on the voltage and flux requirements for startup are investigated. Potential techniques useful for current profile control are also examined. The machine parameters and physics model used in simulating the current startup process in FED are summarized in Table 6. The options which have been analyzed include constant and varying q_I expansions with (1) fixed major radius (retractable limiter) and (2) R-compression

Table 6. FED current startup parameters and models

PARAMETERS

Major radius (m)	$R_i \rightarrow R_f$	5.70 $\rightarrow$ 4.80
Minor radius in midplane (m)	$a_i \rightarrow a_f$	0.40 $\rightarrow$ 1.30
Scrape-off thickness in midplane (m)	a_{so}	0.10
Elongation ($\sigma = b/a$)	$\sigma_i \rightarrow \sigma_f$	1.0 $\rightarrow$ 1.6
Triangularity	c/a	0.0 (concentric ellipses)
Vacuum toroidal field at R (T)	$B_i \rightarrow B_f$	3.05 $\rightarrow$ 3.62
Plasma current at full radius (MA)	I_p	4.80
Safety factor at a_p during expansion	$q_I(a)$	5.0 $\rightarrow$ 2.36
ECRH power (MW)	P_{RF}	0.25 $\rightarrow$ 5.0
Required ECRH frequency (GHz)	f_{RF}	90 $\rightarrow$ 100

MODELS

Scrape-off	Retractable and fixed toroidal limiter .
Fueling for hydrogen plasma	Gas puffing from limiter radius
Electron energy confinement	$\chi_e = 3.0 \chi_e^{NC} + \chi_e^{ANOM^a}$
Ion energy confinement	$\chi_i = 3.0 \chi_i^{NC}$
Particle confinement	$D = 3.0 D^{NC} + 0.2 \chi_e^{ANOM}$
Impurities	Clean ($Z_{eff} = 1$) and oxygen impurity

$$^a \chi_e^{ANOM} \text{ (m}^2/\text{s)} = 5.0 \times 10^{10} \gamma_e(r).$$

(fixed limiter location). The plasma elongation has been either fixed or allowed to increase linearly in time with σ constant in radius. For the majority of cases (except those in Section 11.3), the plasma has been modelled as a series of concentric ellipses. The plasma fueling technique has been gas puffing from the limiter radius. The ECRH power has been applied both in a step-function fashion and linearly in time. A detailed rf physics model has not been used in this study. Rather, a reference heating model has been constructed which uses various heating profiles and deposits 100% of the ECRH power to the electrons. The neoclassical transport model used in these simulations is that of Hinton and Hazeltine [89]. Anomalous contributions to the electron energy confinement and particle diffusion terms have been added to it. This composite transport model has been employed [112] previously as a reference for various physics studies on both INTOR and FED.

11.1 Comparison of 0-D and 1-D Results

The reference case selected for comparison is a constant q_I ($= 2.36$) varying major radius expansion, with I_p and σ increasing linearly in time. The initial plasma radius is specified at $a_0 = 0.4$ m (PLT-size plasma). The plasma is assumed to be free from impurities ($Z_{\text{eff}} = 1$) and the initial electron temperature profile is assumed to be flat with an averaged value of $\bar{T}_e = 35$ (modest electron preheating). The current rises linearly to ~ 180 kA in 300 ms (corresponding to an $\dot{I}_p = 0.6$ MA/s) at which time $q_I = 2.36$ and expansion begins. ECRH power at the 0.5-MW level is maintained (with uniform deposition being used) during the current initiation phase. Over the next 5.7 s, the plasma current and elongation increase linearly to their full-bore values of 4.8 MA and 1.6 (from an initial circular plasma) while the plasma expands to full size ($a = 1.3$ m). As the expansion commences, the rf heating is stepped up to a constant 5.0-MW level. A controlled linear ramp in density is specified which has n_e increasing from its initial value of $1.0 \times 10^{19} \text{ m}^{-3}$ to 1.1 and then to $3.0 \times 10^{19} \text{ m}^{-3}$ at the ends of the current initiation and expansion phases, respectively. (The case discussed here is the same as that presented in Fig. 10.2.)

Shown in Figs. 11.1 - 11.3 is the time evolution of various plasma parameters associated with the expanding radius scenario outlined above. Figure 11.1 depicts the evolving profiles of the electron and ion temperatures together with their peak and volume-averaged values. The initial small radius plasma is originally centered at $R_i = 5.7$ m, and leans against startup limiters located at $R_{LIM} = 6.1$ m. After 300 ms, the major radius is compressed inward to accommodate the expanding plasma. This expansion is clearly seen in the evolving electron and ion temperature profiles. The electron density (for a hydrogen plasma) and pressure profiles accompanying this expansion are shown in Fig. 11.2 along with the peak, volume- and line-averaged densities. The jaggedness associated with $\bar{n}_e$ is due to the combined effects of expansion and density feedback control. With the addition of each new spatial node to the plasma size, there is a drop in $\bar{n}_e$. This, in turn, is followed by a density climb as the feedback counters the drop plus increases the density in accordance with the specified ramp rate. The evolving current density, along with its radial distribution at 6.1 s (end of the simulation), is displayed in Fig. 11.3.

In comparing the 0-D results of Fig. 10.2 with the corresponding 1-D analysis, one notices a significant decoupling of the electron and ion temperatures immediately following the application of 5.0 MW of ECRH at $t = 350$ ms. This decoupling persists for roughly 2.2 s into the expansion phase. By contrast, the electron and ion temperatures have been assumed to be equal in Fig. 10.2. This assumption leads to pessimistic estimates of T_e during the initial stages of current ramp-up when the electron-ion equilibration time $\tau_{EQ}^{ei} [= \tau_{ei}(m_i/m_e)] \gg \tau_E$ and $T_e > T_i$. At the end of current startup ($T_e \cong 1$ keV, $n_e \cong 3 \times 10^{19}$ m $^{-3}$, $a = 1.3$ m and $\sigma = 1.6$), $\tau_{EQ}^{ei} \cong 0.1 \tau_E$ and the assumption of $T_e = T_i$ is justified. This observation is born out in the 1-D results, which show both peak and volume-averaged temperatures being approximately equal at $t = 6.0$ s. The final average temperatures for the two cases are in remarkably good agreement, however, with $\bar{T}_{e,i}$ (0-D) $\cong 0.95$ keV and $\bar{T}_e/\bar{T}_i$ (1-D) = 0.97 keV/0.96 keV.

One reason for this good agreement is a larger 0-D value for r_h . This increase is due in part to (1) an assumed resistivity

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FED

Fig. 11.1 Time evolution of the plasma temperature in FED during an expanding radius startup (from $a_0 = 0.4$ m) with electron preheating, rf assist, and no impurities ($Z_{\text{eff}} = 1$). The rf power levels used here are the same as those used in Fig. 10.2.

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Fig. 11.2 The evolving densities and pressure associated with the expanding radius startup of Fig. 11.1.

(a)

(b)

Fig. 11.3 The current density profiles in time (a) and at $t = 6.1$ s (b) associated with the expanding radius startup of Fig. 11.1.

enhancement factor of $\alpha_\eta = 2$ and (2) lower average electron temperatures from the 0-D analysis. In the 1-D calculations, Spitzer resistivity is used (corresponding to $\alpha_\eta = 1$). Assuming cardinal points in Fig. 10.2 at $t = 0.3$ s (end of current initiation) and at $t = 6$ s (end of plasma expansion), the matching 0-D (1-D) values at these points are 1.7 (0.5) MW and 4.4 (2.5) MW, respectively. At $t = 0.3$ s the ratio of 0-D/1-D ohmic power is 3.4, indicating an additional factor of 1.7 directly attributable to the enhanced resistivity at low T_e . This ratio decreases to ≤ 1.8 at $t = 6$ s, denoting the increased dependence of P_{oh} on I_p^2 rather than on the plasma resistance during the later stages of current startup.

Turning now to a comparison of the 0-D and 1-D power balance, one finds some interesting and important differences. In the 1-D simulation, an electron input power level ($P_{rf} + P_{oh}$) of ~ 7.5 MW (at $t = 6$ s) balances losses due to electron-ion collisions (~ 3.5 MW), electron impact ionization of neutrals (~ 0.6 MW), transport (~ 3.2 MW), and bremsstrahlung radiation (~ 0.2 MW). The 3.5 MW of power collisionally transferred to the ions, in turn, balances charge exchange (~ 1.5 MW) and transport (~ 2.0 MW) losses. The 0-D calculations [refer to Eq. (8.6)] neglect bremsstrahlung radiation and use a crude approximation of the neutral ionization power loss (< 0.05 MW). The principal loss mechanisms are found to be transport ($P_{tr}^e + P_{tr}^i = 8.7$ MW, $P_{tr}^e = P_{tr}^i$) and charge exchange (~ 0.7 MW) losses. (As the temperature and density increase and expansion begins, the plasma becomes much more effective in screening out "deep" neutral penetration. This screening effect has been approximated using a simple model similar to that developed by Emmert [113].) While the 0-D model basically neglects ionization and bremsstrahlung losses (amounting to ≤ 1 MW), it is within a factor of 2 of the 1-D results in the key areas of transport and charge exchange loss. An important point to note here is that the 1-D transport losses are a factor of ~ 2 less than the 0-D results. This reduction implies a reduced level of heat loading to the startup limiters.

The final point of comparison is the voltage and flux requirements needed for inductive startup. As hinted at in the beginning of this

section, profile effects can help to reduce the magnitude of the resistive loop voltage during startup. This reduction leads to lower accumulated levels of resistive flux consumption. The differences between the 0-D and 1-D cases are found in the profile dependent quantities – the resistive and internal inductive voltage and fluxes.

For the reference case considered here, a peak voltage demand (at $t = 0.3$ s) of 16.5 V (24V) is predicted using the 1-D (0-D) model. The corresponding 1-D (0-D) estimates of the resistive and total inductive fractions have values of ~ 2 V (10 V) and 14.5 V (14.0 V), respectively. The inductive voltages obtained using the 1-D and 0-D models are in fairly close agreement during the initiation phase because the current density profile (shown in Fig. 11.3a) has had little time to evolve from its initial flat profile. With $\dot{I}_p \cong 0.6$ MA/s (during current initiation), and $L_p^{\text{ext}} = 19.6$ μH , the external inductive voltage is $V_{\text{ind}}^{\text{ext}} \cong 11.8$ V. Assuming a flat current density profile ($\ell_i/2 = 0.5$), $L_p^{\text{int}} = 3.6$ μH , and $V_{\text{ind}}^{\text{int}} \cong 2.2$ V. Because the external inductive voltage is the same for both the 0-D and 1-D models, the 0.5-V disparity is due to a slight profile difference in the 1-D internal voltage component.

The departure from the flat profile condition can be significant, depending on the details of the discharge. Values of $\ell_i/2$ as high as 2 (an extremely peaked profile) have been observed on the Doublet III tokamak [111] shortly after breakdown. During the reference case expansion, the addition of 5 MW of ECRH power helps to freeze in the broad current density profile (with $\ell_i/2 \cong 0.63$) shown in Fig. 11.3. Potential profile peeling is thereby prevented.

The 1-D (0-D) estimates of the resistive, internal, and external flux requirements for a 6-s startup have values of 3.9 V-s (10 V-s), 18.1 V-s (14.5 V-s), and 33.3 V-s, respectively. These results indicate the importance of profile effects in estimating the flux needs for tokamak startup. The resistive flux loss can be significantly less than the 0-D predictions (by more than a factor of 2). However, profile control appears necessary in order to maintain the internal flux at or near the minimum flat profile limit.

In addition to the reference case discussed above, a number of further examples have been analyzed. These results are summarized in

Table 7 and provide further points of comparison with other 0-D cases shown in Table 5. Three of the cases in Table 7 (b, f, and g) are shown in Figs. 11.4 - 11.7. Case b (shown in Figs. 11.4 - 11.5) attempts to reduce the large initial temperature transient of the reference case by ramping up the rf power level in a more judicious manner. Between 300 and 350 ms, the rf power level is increased linearly from 250 kW to 1 MW. This is followed by a second power ramp (again linear) to the 5-MW level which is reached at $t = 6$ s. Because of the reduced initial heating levels, the electron and ion temperatures (both peak and volume-averaged) are rather well equilibrated. The lower starting temperatures also allow faster current penetration and some profile peaking to occur (see Figs. 11.3 and 11.5). Because the average energy confinement time ($\tau_E \propto \bar{n}a^2$) is small at the start of expansion (~ 8 ms for $\bar{n}_e \sim 10^{19} \text{ m}^{-3}$ and $a_0 = 0.4$ m), the application of large amounts of rf power can lead to potentially serious limiter damage. The above scenario may be able to avoid this problem by postponing the introduction of large amounts of rf power until later in the expansion when the energy confinement has increased to an acceptable level. There is, however, a penalty to be paid. By lowering the electron temperature during the expansion phase, the resistive flux more than doubles (from ~ 4 to 9.4 V-s).

Case f (shown in Fig. 11.6) is the 1-D equivalent of the expanding radius scenario presented in Fig. 10.1. It represents an extreme case of rf "overkill." ECRH power at the 1-MW level is maintained during the current initiation phase. This heating level is followed by a step up in power (to the 5-MW level) when expansion begins. With a minor radius of just over 0.2 m and an electron density of $\sim 1.1 \times 10^{19} \text{ m}^{-3}$, the average energy confinement time at the start of expansion is < 5 ms. Because little equilibration is possible under these conditions, a large electron temperature spike is observed. Even with kilovolt electron temperatures and low current levels (≤ 100 kA at the start of expansion), the β_p limit specified by Eq. (7.7) is not violated because of the sizeable initial aspect ratio ($A = R_i/a_i \geq 25$). The ability to maintain proper plasma equilibrium under these conditions, along with earlier concerns of excessive limiter heat loading are problems which still must be addressed, however. Case g (shown in Fig. 11.7) uses the same rf ramping procedure described for case b to

Table 7. Results of radial transport simulations showing the dependence of startup requirements on initial plasma minor radius a_0 , oxygen content n_{ox}/n_e , and electron temperature before current initiation T_e ($t = 0$)

Assumptions								
a_0 (m)	0.4^a	0.4^b	0.4^c	0.4^d	0.4^e	0.2^f	0.2^g	0.2^h
$\xi = n_{ox}/n_e$ (%)	0	0	0	3	0	0	0	3
T_e ($t = 0$) (eV)	35	35	17	70	35	70	70	70
Requirements								
Maximum V_ℓ (V)	16.5	17.0	25.0	18.0	18.0	7.5	11.5	9.0
at time t (s)	0.3	0.3	0.03	0.3	0.3	0.2	0.2	0.2
Resistive induction flux loss $\Delta\phi_{res}$ (Wb)	3.9	8.4	15.7	6.6	7.4	3.0	8.4	5.4
at $t = 6$ s								

^aWith preheating and $P_{ECRH} = 0.5$ MW for $0 \leq t \leq 0.3$ s.

^bWith preheating and $P_{ECRH} = 0.25$ MW for $0 \leq t \leq 0.3$ s.

^cWith no preheating or rf assist during the current-rise phase.

^dWith electron preheating and a phased input of ECRH power given by $P_{ECRH} = 1.0 (1+2t)$ MW for $0 \leq t \leq 0.3$ s.

^eSame conditions as a , but with $\eta_{||} = 2 \times \eta_{SPITZER}$.

^fWith electron preheating and $P_{ECRH} = 1.0$ MW for $0 \leq t \leq 0.2$ s.

^gWith preheating and $P_{ECRH} = 0.25$ MW for $0 \leq t \leq 0.2$ s.

^hWith preheating and a phased input of ECRH power given by $P_{ECRH} = 1.0 (1 + 2.5 t)$ MW for $0 \leq t \leq 0.2$ s.

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Fig. 11.4 Plasma temperatures for the same startup scenario as in Fig. 11.1 but with reduced levels of rf heating power.

(a)

(b)

Fig. 11.5 The current density profiles in time (a) and at $t = 6.1$ s (b) associated with Fig. 11.4.

Fig. 11.6 Time evolution of the plasma temperature in FED during an expanding radius startup (from $a_0 = 0.2$ m) with electron preheating, rf assist, and no impurities ($Z_{\text{eff}} = 1$). The rf power levels used here are the same as those used in Fig. 10.1.

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Fig. 11.7 Plasma temperatures for the same startup scenario as in Fig. 11.6 but with reduced levels of rf heating power.

circumvent the temperature spiking observed in Fig. 11.6. Because of the high central electron temperatures which exist in case f during the expansion phase, only 3 Wb is expended in resistive flux loss during a 6-s startup. This value is in contrast to 8.4 Wb for case g. These results indicate the effectiveness of ECRH power in reducing resistive volt-second consumption.

11.2 Current Profile Modification During the Expansion Phase

11.2.1 Size Ramping vs Current Ramping

Current profile control can be accomplished using a variety of external controls. For example, techniques such as gas puffing [114] and local ECRH can modify the current profile by altering the plasma conductivity in a particular region. The manner in which the current is ramped-up, in either a fixed or varying minor radius plasma, can also affect the evolution of the current density profile. As discussed in Section 7.1, a rapid current rise can lead to the formation of a skin distribution. Figure 11.8 shows the current density profiles for two fixed major radius expansion cases, each having FED characteristics. The details of the expansion are not important here. What is important is the evolution of the current density profile during the initiation phase. For an initial small radius plasma of $a_0 = 0.4$ m, a sizeable skin current is observed at a ramp rate of $\dot{I}_p = 2.0$ MA/s. By reducing $\dot{I}_p$ to 0.5 MA/s (i.e., extending the current initiation period by a factor of 4 to approximately 0.9 s), the current can penetrate classically. In the remaining simulations discussed in this section, a current ramp rate of $\dot{I}_p = 0.5$ MA/s will be assumed. The FED parameters shown in Table 6 will also be used.

During an expanding radius current startup, either the size or current can be programmed to increase in time (see Section 9.3). Figures 11.9 - 11.11 show two constant $q_1 (= 2.36)$ minor radius expansions with R-compression, constant elongation ($= 1.6$), and no rf heating. In one case, the minor radius is increasing linearly in time, while in the other, it is the plasma current which increases linearly. Although the two cases appear, at first glance, to be roughly

(a)

(b)

Fig. 11.8 A comparison of current density profiles with two different current ramp rates during the initiation phase. In (a) $\dot{I}_p = 0.5 \text{ MA/s}$, while in (b) $\dot{I}_p = 2.0 \text{ MA/s}$.

(a)

(b)

(c)

(d)

Fig. 11.9 Time evolution of the electron temperature profile (a) together with peak and average electron and ion temperatures (b) for the case of linear size ramping. The same parameters for the case of linear current ramping are shown in (c) and (d).

Fig. 11.10 Time evolution of the current density profile (a) together with its radial distribution at $t = 6.1$ s (b) for the case of linear size ramping. The same parameters for the case of linear current ramping are shown in (c) and (d).

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Fig. 11.11 Time evolution of the ohmic heating power for the case of linear size (a) versus linear current (b) ramping.

equivalent, the linear current ramping case is clearly superior. A quick look at Figs. 8.2 and 9.3 indicates the reason for this superiority. With linear current ramping, both the current and minor radius increase faster in time. The enhanced plasma growth is due to the fact that $\dot{a}$ increases with time in order to maintain a constant q_I [see Eq. (8.9)]. Because $P_{oh} \propto I_p^2/T_e^{3/2}$ and $\tau_E \propto na^2$, a hotter plasma exists earlier in the expansion phase with linear current ramping (see Fig. 11.9b and d) than with linear size ramping. The jaggedness observed in the curve of Fig. 11.11b is directly attributable to this rapid current rise. With the addition of each new node and current increment, there is a jump in the ohmic power level. This is followed by a rise in the electron temperature and a subsequent drop in the plasma resistivity.

Because of the higher peak and volume-averaged temperatures associated with linear current ramping, less resistive volt-seconds are consumed (16.3 Wb vs 19.8 Wb for the case of linear size ramping) and broader current density profiles are obtained during the expansion. With linear current ramping, the applied voltage requirements decrease in time as the plasma resistivity and external inductances are reduced. With linear size ramping, however, the current ramp rate increases in time [see Eq. (8.9)] and the voltage can actually increase.

11.2.2 Controlled Gas Puffing and Current Channel Contraction

During the initial stages of current startup in present-day tokamaks, hollow temperature profiles [115,116] are frequently observed. Such profiles are thought to be a manifestation of a temporary skin current distribution which relaxes through a series of minor disruptions. With skin and other broad current profile shapes, the edge temperature can be relatively high, resulting in increased plasma-limiter interaction. Experiments on PLT [114] have shown that it is possible to contract the current channel by properly controlling the gas influx during the startup phase. The relatively low edge temperatures associated with peaked current profiles can lead to reduced plasma-limiter interaction and lower concentrations of limiter material in the plasma.

Figure 11.12 shows two fixed major radius expansion cases with constant $q_I (= 2.36)$ and elongation ($= 1.6$). In (a) and (b) a linear

Fig. 11.12 Time evolution of the electron and ion temperatures (both peak and average) and the current density profile for two different density ramps. The ramp rate in (a) and (b) is twice as fast as that in (c) and (d) during the first 0.9 s of current startup.

density ramp, from 1 to $2 \times 10^{19} \text{ m}^{-3}$, occurs during the current initiation phase ($0 \leq t \leq 0.9 \text{ s}$). This increase is followed by a second slower ramp to $3 \times 10^{19} \text{ m}^{-3}$ at $t = 6 \text{ s}$. By comparison, there is only a 10% increase in the density rise (to $1.1 \times 10^{19} \text{ m}^{-3}$ at $t = 0.9 \text{ s}$) in (c) and (d). Both cases also benefit from the addition of 3.0 MW of ECRH power at the start of the expansion. As is apparent, the density ramp in (b) helps to contract the current density profile. This peaked profile persists (relatively unchanged) throughout the remainder of the expansion phase due to the freezing effect of the rf power. In (d), the rf power retards contraction (despite a second strong density ramp from 1.1 to $3 \times 10^{19} \text{ m}^{-3}$ during the expansion phase). The increased density ramp also helps equilibration in (a).

11.2.3 Modifications of the Current Distribution Using RF Power

As mentioned previously, ECRH can be used to modify the current profile by increasing the plasma conductivity in a localized region. In order to assess the effectiveness of this external control technique, a number of heating profiles have been examined. Figures 11.13 and 11.14 show the same fixed major radius expansion as in Fig. 11.12d but with two different heating profiles. In Fig. 11.13, a Gaussian heating profile of the form

$$H(r) \sim e^{-(4r/a)^2}$$

is selected to represent a case of strong localized heating. Because of the high concentration of current on-axis, $q_{\parallel}(0)$ can drop below unity. Under these conditions, an $m = 1$, $n = 1$ resistive tearing mode [117] becomes unstable. This so-called "sawtooth" or "internal" disruption flattens the temperature and current profiles over the center of the plasma. After this brief cooling period, the center of the plasma heats up again due to a thermal instability [118] and another sawtooth oscillation occurs. Because of the strong on-axis heating, the resistive flux loss is held to a minimum ($< 7 \text{ Wb}$ for a 6-s startup). In Fig. 11.14, the ECRH power is deposited uniformly in the outer half-radius of the plasma. Such a heating profile may occur once the plasma begins to heat up. Because absorption increases with temperature [38],

Fig. 11.13 Evolution of temperature and current density in FED using 3.0 MW of ECRH power and a Gaussian rf heating profile, (a) $T_e(r,t)$, (b) peak and average electron and ion temperatures, (c) $J(r,t)$, and (d) $J(r)$ at $t = 6.1$ s. Sawtooth disruptions driven by the thermal instability are observed in (b) and (c).

Fig. 11.14 The current density profiles in time (a) and at $t = 6.1$ s (b) resulting from the use of 3.0 MW of ECRH power deposited uniformly in the outer half-radius of the plasma. The rf power, turned on at $t \geq 0.9$ s, is maintained during the expansion.

much of the ECRH power could be deposited in the outer portions of the plasma before reaching the resonance layer.

11.2.4 Varying the Minor Radius Expansion Time

During the expansion phase, the plasma is subject to a thermal instability which tends to concentrate the current density in the hot, central core of the plasma [118]. With a sufficiently long expansion time, the current density can become peaked on-axis. The uniform deposition of ECRH power at the MW level (accomplished by using multiple frequencies) can retard this peaking effect by increasing the plasma conductivity, thereby slowing the diffusion of poloidal flux. One can further counter the effects of the thermal instability by increasing the rate of current rise. This is equivalent to decreasing the expansion time.

Figure 11.15 shows the effect of a varying expansion time on the current density profile evolution. As the expansion times become progressively shorter, there is less time available for internal plasma heating and current penetration. This shortening of the expansion time leads to the formation of broad current profiles (see Fig. 11.15c and d) as current is essentially added to the plasma boundary in a uniform fashion.

In terms of resistive flux consumption, cases a, b, and c have requirements of 10, 9, and 6 Wb, respectively (these estimates are obtained with 3.0 MW of ECRH power applied at the start of the expansion phase). However, with increasing current ramp rates (up to $\dot{I}_p \cong 6$ MA/s for case c), the inductive voltage demands can be large.

11.2.5 Minor Radius Expansion With Varying q_I

A potentially more viable approach for obtaining broad profiles involves minor radius expansion with decreasing q_I as shown in Fig. 11.16. Referring to Fig. 8.4, one sees that the average current density can increase substantially depending on the details of the expansion. For the particular case shown in Fig. 11.16c, the current ramp rate varies from $\dot{I}_p \cong 0.2$ MA/s at $t \cong 0.6$ s to $\cong 1.65$ MA/s at $t = 6$ s.

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Fig. 11.15 The effect of a varying expansion time on the current density profile evolution. Here, $a(t)$ is increasing linearly in time and $q_I (= 2.36)$ is constant. Expansion begins at 0.9 s and terminates (a) 5.1 s, (b) 3.1 s, and (c) 1.1 s later. The radial profiles shown in (d) are those existing at $t = 6.1$ s.

Fig. 11.16 Time evolution of current density profile with $a(t)$ increasing and $q_1(t)$ decreasing linearly in time. During the expansion, the edge safety factor decreases to 2.36 from initial values of (a) 3.0, (b) 4.0, and (c) 5.0. The radial profiles shown in (d) are those existing at 6.1 s. A constant $q_1 (= 2.36)$ expansion is included for comparison.

Initially, when $\dot{i}_p$ and the rf power levels are small, the plasma skin time is sufficiently short that local peaking of the current density can occur on a heating time scale ($\tau_h \cong 3/2 n_{eo} T_{eo} / \eta_o J_o^2$). However, in the latter stages of the expansion, the ohmic power increases dramatically. This heating, in conjunction with a phased input of ECRH power (reaching 3.0 MW at $t = 6$ s), increases the plasma conductivity sufficiently that a skin effect occurs. The subsequent concentration of poloidal field in the outer plasma radius tends to broaden the current profile.

For comparison with Fig. 11.15, the resistive volt-second loss on-axis for cases a, b, and c are 10.1, 9.4, and 8.7 Wb, respectively. The applied voltage needs are lower, however, than in Fig. 11.15c because the maximum current ramp rate is $\dot{i}_p = 1.65$ MA/s.

11.3 MHD and Neoclassical Correction Effects

The results presented in this work have used, almost exclusively, the Spitzer formulation for the plasma resistivity which is appropriate for use in moderate and high collisionality plasmas (with $v_{*e} > 1$). However, at low collisionality [$v_{*e} < 1$ in Eq. (9.10)], it is necessary to take into account mirror-trapped electrons which can increase the plasma resistance. The consequences of including neoclassical corrections for electron trapping are shown in Figs. 11.17 and 11.18. By increasing the resistivity in the outer periphery of the plasma, additional current is forced to flow on-axis (where the trapping effect is small). This concentration of current on-axis can drive $q_I(0) < 1$, thereby triggering the sawtooth disruptions shown in Fig. 11.17b. During the current startup process, the neoclassical term is not expected to be significant.

Finally, Fig. 11.19 shows the effects of magnetic axis shift and a radial dependent elongation on the current density distribution. In general, for startup analyses, the concentric ellipse approximation yields results which are in good agreement with the MHD calculated current distributions.

(a)

(b)

Fig. 11.17 Time evolution of the current density profile, under the same expansion conditions, (a) without and (b) with neoclassical corrections to the Spitzer resistivity.

Fig. 11.18 The current density distributions for the two cases shown in Fig. 11.17 at $t = 6.1 \text{ s}$.

Fig. 11.19 MHD equilibrium effects on the current density distribution. The profiles shown are those existing at $t = 6.1$ s. During the expansion $a(t)$ is increasing linearly in time, q_I is constant and there is no rf heating.

12. SUMMARY AND CONCLUSIONS

The purpose of this study has been to examine the rf-assisted, expanding radius approach to tokamak current startup and to assess its merits relative to the conventional startup technique. The specific areas of investigation included: currentless electron preheating, expanding radius plasma modelling, methods for reducing the voltage and flux requirements during startup, and the prospects for current profile control.

In order to quantify the potential benefits of preheating, a near-classical particle and energy transport model was developed for estimating the efficiency of electron heating in a currentless toroidal plasma. MHD instabilities are presumed absent due to the combined presence of an ambipolar sheath and a conducting limiter/vacuum vessel arrangement that "short-circuit" potential vertical charge separation. The electrons are preferentially heated at the UHR layer by an extraordinary mode wave introduced from the high-field side of the torus. The resulting loss of electrons via curvature and error field drifts leads to the formation of an ambipolar potential and poloidal $\vec{E}_{\text{AMB}} \times \vec{B}$ drift. This drift introduces an effective rotational transform which should improve particle confinement. The benefits of this electrostatic confinement are tempered, however, by the possibility of significant secondary electron emission from the limiters and vessel wall. This is an important area that merits further investigation. Using the UHR preheating model, a reexamination of the ISX-B experimental results was conducted, and reasonably good agreement between the theoretical estimates and the diagnostic data was obtained.

On this basis, estimates for FED have been made assuming the presence of a local poloidal error field (~ 2 G), a plasma density of $n_e = 10^{19} \text{ m}^{-3}$, and up to 3% oxygen content. The results indicate that

no more than 2 MW of ECRH power at ~ 90 GHz is needed to achieve $T_e \lesssim 100$ eV in a volume of ~ 5 m³ near the UHR layer. Once the ECRH power burns through the oxygen radiation barrier in the UHR region, field-cancelling coils would be used to eliminate the error field over most of the plasma cross section. This elimination would allow the heating zone to increase to $\lesssim 30$ m³ in volume. By controlling the existing level of error fields in this manner, a substantial reduction in the amount of rf power required for preheating is anticipated.

Next, a set of numerical algorithms was developed for modelling the plasma expansion phase of current startup. These algorithms were then coupled to the WHIST transport code, which was used to examine the expansion to full size of small radius plasmas formed both on-axis and near the outboard wall of the vacuum chamber. In order to determine the voltage and flux requirements during startup in noncircular geometries, a compatible metric formulation for the above quantities was also developed and incorporated into the WHIST code.

To provide a point of comparison with the radial transport (1-D) simulations, a 0-D model was first developed and used to determine the plasma characteristics, as well as the voltage and flux needs during startup. The 0-D results indicate that $\lesssim 1$ MW of ECRH power can maintain an average electron temperature of around 70 eV during the current initiation phase without introducing serious skin currents. For an initial minor radius of 0.2 m (0.4 m), a time scale of about 0.2 s (0.3 s) is required to reach $I_p \approx 0.04$ MA (0.18 MA) and $q_1 \approx 2.4$. The auxiliary heating power is then raised to about 5 MW to assist the relatively slow buildup of the plasma cross section to $a = 1.3$ m, $\sigma = 1.6$, and the plasma current to 4.8 MA in about 5.8 s.

The 0-D estimates also indicate a maximum V_ℓ and $\Delta\phi$ requirement of ~ 20 V and ~ 58 Wb ($\Delta\phi_{\text{res}} \approx 10$ Wb), respectively, under relatively ideal conditions ($Z_{\text{eff}} = 1$) for $a_0 = 0.4$ m. To take into account uncertainties in impurity content and to allow for larger initial minor radii ($a_0 = 0.4$ m), the V_ℓ requirements for FED have been specified to be ~ 25 V. A resistive induction flux capability of $\lesssim 20$ Wb can also allow a slow current rise with up to 3% oxygen impurity content.

By contrast, the 1-D simulation results indicate a substantial

easing of the voltage and flux requirements for startup. Considering a clean ($Z_{\text{eff}} = 1$), 0.4-m minor radius plasma as the reference case, the 1-D (0-D) estimates indicate a maximum resistive voltage and flux requirement of ≤ 2 V (10 V) and ~ 4.0 Wb (10 Wb), respectively. The internal flux rises (to ~ 18 Wb) from the flat profile value of 14.5 Wb due to flux diffusion and a thermal instability which tends to concentrate the current on-axis. These results indicate that profile effects are important when determining the flux needs for tokamak startup. This is particularly true for the resistive voltage and flux components which can be significantly less (by more than a factor of 2) than the 0-D predictions.

Finally, the prospects for tailoring the current profile during startup were examined using external controls such as gas puffing, localized rf power deposition, and various expansion techniques. For the same startup conditions, linear current ramping was found to be superior to linear size ramping. It provided hotter plasmas with broader current profiles and with reduced resistive flux consumption. Locally, modifications to the plasma conductivity by gas puffing and ECRH can alter the current profile. Obtaining truly broad current profiles, however, necessitates the use of expansion techniques which can inhibit or counter the centralized peaking of the current density. Shortening the length of the expansion phase can delay the effect of the thermal instability, but only at the expense of increasing the current ramp rate and inductive voltage. An alternate method, with a lower voltage requirement, involves minor radius expansion with decreasing q_1 . This technique essentially utilizes a controlled skin effect to broaden the current profile. When used in conjunction with ECRH, it appears to be a particularly attractive option.

To summarize, auxiliary rf heating of electrons (before and during current rise) can be an effective method for improving the basic problem of conventional current startup, namely, a low ohmic heating efficiency. When coupled to an expanding radius plasma, modest amounts of rf power (< 5 MW) can maintain a reasonably high plasma conductivity even with a slow current rise. As a result, the initial voltage

demands in large tokamaks can be significantly reduced (to ~ 20 V or less) compared to the purely inductive current startup. The combination of rf heating and current profile control can also lead to a substantial savings in volt-seconds by (1) minimizing the resistive-flux consumption and (2) maintaining the internal flux at or near the flat profile limit.

Before concluding this effort, a word or two is in order regarding directions for additional research and model development. One potential area of future study involves the coupling of an electromagnetics model [119] to the radial transport model. At present, the modified WHIST code requires one to specify either the size or current in time, rather than the applied voltage. The differential equation describing the current evolution is then used to determine the time history of the plasma loop voltage. Modifications which would allow the plasma to be inductively driven by arbitrarily placed external field coils could provide valuable information on the technical feasibility of the various expansion options examined in this work. The implementation of a realistic ECRH model is also another logical area for improvement.

A final comment deals with efforts to compare the expanding radius transport model predictions with actual experimental results. While the ISX-B experiment has provided a valuable, although limited, data base for benchmarking purposes, little data is available on expanding radius plasma experiments. In this sense, the development of an expanding radius transport model at this time is opportune. The initial operation of TFTR and the Joint European Torus (JET) will provide a unique opportunity to study the current startup process over a wide range of plasma shapes, sizes, magnetic fields, and current levels.

APPENDICES

Appendix A

Circumference and Area of a Plasma With an Elliptical Cross Section

The elliptical cross-section plasma shown in Fig. 8.1 has major radius R_0 , with half-axes b and a , and elongation $\sigma \equiv b/a$. With $x = a \cos \theta$ and $z = b \sin \theta$, the circumference of the ellipse is given by

$$\begin{aligned}
 \oint dl &= 4 \int_0^{\pi/2} \sqrt{(dx)^2 + (dz)^2} & (A.1) \\
 &= 4 \int_0^{\pi/2} \sqrt{a^2 \cos^2 \theta + b^2 \sin^2 \theta} \, d\theta \\
 &= 4 \int_0^{\pi/2} \sqrt{a^2 (1 - \sin^2 \theta) + b^2 \sin^2 \theta} \, d\theta \\
 &= 4 \int_0^{\pi/2} \sqrt{a^2 - (a^2 - b^2) \sin^2 \theta} \, d\theta \\
 &= 4a \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \theta} \, d\theta
 \end{aligned}$$

where $k = \sqrt{a^2 - b^2}/a$ has been used. The plasma circumference is given exactly in terms of the complete elliptic integral of the second kind:

$$\oint dl = 4a E(k) \quad (A.2)$$

A good approximation for the circumference of an elliptical cross section is [105]

$$\oint dl \cong 2\pi a [(1 + \sigma)/2] (1 + m^2/4) \cong 2\pi a \sqrt{\frac{1 + \sigma^2}{2}} , \quad (\text{A.3})$$

where $m = (\sigma - 1)/(\sigma + 1)$.

The cross-sectional area of the ellipse is given by

$$A = \iint_A dx dz = \iint_A J\left(\frac{x, z}{\rho, \theta}\right) d\rho d\theta , \quad (\text{A.4})$$

where

$$J = \frac{\partial(x, z)}{\partial(\rho, \theta)} = \begin{vmatrix} \cos \theta & -\rho \sin \theta \\ \sigma \sin \theta & \sigma \rho \cos \theta \end{vmatrix} \quad (\text{A.5})$$

$$= \sigma \rho (\cos^2 \theta + \sin^2 \theta)$$

is the Jacobian. Evaluating the integrals, the result is

$$A = \int_0^{2\pi} \int_0^a \sigma \rho d\rho d\theta = \pi a^2 \sigma = \pi ab . \quad (\text{A.6})$$

In the above calculation, it is assumed that σ does not vary with minor radius.

Appendix B

Evaluation of the Geometric Quantities G_θ and G_ϕ

For Low-Beta, Elliptical Cross-Section Plasmas

The magnetic geometry for an axisymmetric tokamak equilibrium can be represented using the rectangular (R,Z) coordinate system shown in Fig. B.1. The R-axis provides a measure of the horizontal distance from the major axis of the torus and the Z-axis a measure of the vertical distance from the plasma midplane. A Fourier series expansion of the (R,Z) coordinates in poloidal angle θ yields a set of parametric equations which describe the flux surface contours of the plasma [90]:

$$R(\rho, \theta) = R_0(\rho) - R_1(\rho)\cos\theta + \sum_{n=2}^{n_R} R_n(\rho)\cos n\theta, \quad (B.1)$$

$$Z(\rho, \theta) = E(\rho) \sum_{n=1}^{n_R} R_n(\rho)\sin n\theta. \quad (B.2)$$

The flux surface label ρ runs from the magnetic axis ($\rho = 0$) to the plasma minor radius ($\rho = a$). The flux surfaces are assumed to have up-down symmetry, $\rho(R, Z) = \rho(R, -Z)$, about the toroidal midplane, and the poloidal angle is chosen such that $\theta = 0^\circ$ is located on the inside midplane of the torus. The Fourier amplitudes $R_0(\rho)$, $R_1(\rho)$, $E(\rho)$, and $R_2(\rho)$ describe the shift, minor radius, elongation, and triangularity of the flux surfaces, respectively. A value of $R_2(\rho) = 0$ corresponds to an elliptically shaped plasma, while $R_2(\rho) > 0$ corresponds to a D-shaped plasma. With $\rho = a$, $R_0(a)$ corresponds to the major radius of the torus.

In order to evaluate the geometric quantities, $G_\theta (\equiv \langle g_{\theta\theta} / \sqrt{g} \rangle_\theta)$, and $G_\phi (\equiv \langle \sqrt{g} / g_{\phi\phi} \rangle_\theta)$, it is necessary to first determine the Jacobian $\sqrt{g}$ and the metric elements $g_{\theta\theta}$ and $g_{\phi\phi}$. The 3-D Jacobian of

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Fig. B.1 Coordinate representation of toroidal flux surfaces. Figure courtesy of L. L. Lao [90].

the transformation from Cartesian coordinates to curvilinear coordinates (ρ, θ, ϕ) (with ϕ the ignorable coordinate) is given by [90]

$$\sqrt{g} = R\tau, \text{ where} \quad (\text{B.3})$$

$$\tau = \frac{\partial R}{\partial \rho} \frac{\partial Z}{\partial \theta} - \frac{\partial R}{\partial \theta} \frac{\partial Z}{\partial \rho} . \quad (\text{B.4})$$

For $(\mu, \nu) = (\rho, \theta)$, $g_{\mu\nu}$ are the elements of the metric tensor

$$g_{\mu\nu} = \frac{\partial R}{\partial \mu} \frac{\partial R}{\partial \nu} + \frac{\partial Z}{\partial \mu} \frac{\partial Z}{\partial \nu} \quad (\text{B.5})$$

with

$$g_{\theta\theta} = \left(\frac{\partial R}{\partial \theta} \right)^2 + \left(\frac{\partial Z}{\partial \theta} \right)^2 , \quad (\text{B.6})$$

$$g_{\phi\phi} = R^2 \quad (\text{B.7})$$

and all other metric elements equal to zero. In terms of the parametric equations (B.1) and (B.2), Eqs. (B.3) through (B.6) can be evaluated for the case of concentric elliptical flux surfaces with constant elongation [$E'(\rho) = 0$]. For $\rho = a$, $E(\rho) = \sigma$, and $R_0(\rho) = R_0$, this leads to

$$\sqrt{g} = R_0 a \sigma ,$$

$$\langle \sqrt{g} / g_{\phi\phi} \rangle_\theta = \int_0^{2\pi} \frac{d\theta}{2\pi} \left(\frac{\sigma a}{R_0} \right) = \frac{a\sigma}{R_0} ,$$

$$g_{\theta\theta} = (a\sigma)^2 + a^2(1 - \sigma^2)\sin^2\theta$$

and

$$\langle g_{\theta\theta} / \sqrt{g} \rangle_\theta = \int_0^{2\pi} \frac{d\theta}{2\pi} \left[\frac{a\sigma}{R_0} + \frac{a}{\sigma R_0} (1 - \sigma^2) \sin^2\theta \right]$$

$$= \frac{a\sigma}{R_0} + \frac{a}{\sigma R_0} (1 - \sigma^2) \frac{1}{2\pi} \int_0^{2\pi} \sin^2 \theta d\theta$$

$$= \frac{a\sigma}{R_0} + \frac{a}{R_0} \left(\frac{1 - \sigma^2}{2\sigma} \right)$$

$$= \frac{a}{R_0} \left(\frac{1 + \sigma^2}{2\sigma} \right) .$$

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