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BARYON INHOMOGENEITY FROM THE COSMIC QUARK-HADRON PHASE TRANSITION

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We discuss the generation of inhomogeneity in the baryon-number density during the cosmic quark-hadron phase transition. We use a simple model with thin-wall phase boundaries and ideal-gas equations of state. The nucleation of the phase transition introduces a new distance scale into the universe which will be the scale of the generated inhomogeneity. We review the estimate of this scale. During the transition baryon number is likely to collect onto a layer at the phase boundary. These layers may in the end be deposited as small regions of very high baryon density.

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According to Hogan¹¹, the nucleation rate can be expected to have the general functional form

$$p = CT_c^4 e^{-A}. \quad (2.1)$$

where

$$A = \frac{a}{(1 - \bar{T})^\alpha}. \quad (2.2)$$

We have defined

$$\bar{T} \equiv \frac{T}{T_c}. \quad (2.3)$$

The unknown parameters C , a , and α are determined by the microphysics and may vary with temperature. The important point is that the nucleation happens very rapidly at a temperature (probably a few percent below T_c), which is not special from the microphysical point of view. Therefore these parameters must be essentially constant during the nucleation.

Hogan's treatment is general, and gives one an upper limit for the new distance scale. To get instead an estimate, we now follow KK and FMA, and use as a model classical thermal nucleation with ideal gas equations of state for the two phases. Then¹²

$$A = \frac{w_0}{(1 - \bar{T}^4)^2} \approx \frac{w_0}{16(1 - \bar{T})^2} \quad (2.4)$$

with

$$w_0 = \frac{16\pi}{3} \left[(g_g - g_h) \frac{\pi^2}{90} \right]^{-2} \left(\frac{\sigma}{T_c^3} \right)^3. \quad (2.5)$$

We are assuming the supercooling $1 - \bar{T}$ is small.

The nucleation rate is zero at $T = T_c$ but grows rapidly as the supercooling

$1 - \bar{T}$ increases. The nucleated bubbles grow fairly slowly by deflagration, or 'slow combustion'^{13,1}, each bubble sending a compression wave into the surrounding plasma. The spherical deflagration was discussed by Kurki-Suonio¹⁴ and Miller and Pantano¹⁵. The outer edge of the compression wave is a very weak spherical shock which propagates slightly faster than the speed of sound,

$$v_{sh} \simeq c_s \simeq \frac{1}{\sqrt{3}}. \quad (2.6)$$

After the shock has passed, the density keeps increasing in the wave, most of the compression being done after the shock. This compression heats up the plasma suppressing further nucleation. Since the nucleation rate is very sensitive to the plasma temperature, a very small amount of heating is sufficient to effectively shut down further nucleation.

The fraction of space affected is in the first approximation

$$F(t) = \int_{t_c}^t dt' p(t') V(t, t'), \quad (2.7)$$

where

$$V(t, t') = \frac{4\pi}{3} v_{sh}^3 (t - t')^3 \quad (2.8)$$

is the volume at time t of the compression wave nucleated at time t' . In terms of the decreasing temperature T of the unaffected supercooled plasma this is

$$F(\bar{T}) = \frac{4\pi}{3} v_{sh}^3 C T_c^4 \left(\frac{dt}{d\bar{T}} \right)^4 \times \int_{\bar{T}}^1 d\bar{T}' (\bar{T}' - \bar{T})^3 \exp \left\{ -\frac{w_0}{16(1 - \bar{T}')^2} \right\}. \quad (2.9)$$

The cooling time scale $dt/d\bar{T}$ is of the order T_{Pl}/T_c^2 (actually about one-tenth of this).

In the integral (2.9) most of the contribution comes when $\bar{T}'$ is much closer to $\bar{T}$ than 1, and thus we can approximate⁶

$$\begin{aligned} & \int_{\bar{T}}^1 d\bar{T}' (\bar{T}' - \bar{T})^3 \exp\left\{-\frac{w_0}{16(1-\bar{T}')^2}\right\} \\ & \approx \exp\left\{-\frac{w_0}{16(1-\bar{T})^2}\right\} \times \\ & \times \int_0^{1-\bar{T}} x^3 \exp\left\{-\frac{w_0}{8(1-\bar{T})^3}x\right\} dx, \end{aligned} \quad (2.10)$$

where $x \equiv \bar{T}' - \bar{T}$. Extending the latter integral to infinity produces negligible error, and thus

$$\begin{aligned} F(\bar{T}) & \simeq \frac{4\pi}{3} v_{sh}^3 C T_c^4 \left(\frac{dt}{d\bar{T}}\right)^4 \frac{6 \cdot 8^4}{w_0^4} (1-\bar{T})^{12} \\ & \times \exp\left\{-\frac{w_0}{16(1-\bar{T})^2}\right\}. \end{aligned} \quad (2.11)$$

The nucleation ends at a maximum supercooling to temperature T_f , when the region heated by the compression waves fills the plasma.

$$F(\bar{T}_f) = 1. \quad (2.12)$$

Setting (2.11) equal to 1, the supercooling $\eta_f \equiv 1 - \bar{T}_f$ can be solved,

$$\eta_f = \sqrt{\frac{w_0}{16(L + 12 \ln \eta_f)}}, \quad (2.13)$$

where

$$L = \ln \left[\frac{4\pi}{3} v_{sh}^3 C T_c^4 \left(\frac{dt}{d\bar{T}}\right)^4 6 \cdot 8^4 \frac{1}{w_0^4} \right] \quad (2.14)$$

depends only logarithmically on the various parameters. Assuming C and w_0 are close to unity,

$$L \approx \ln \frac{T_{Pl}^4}{T_c^4}. \quad (2.15)$$

One gets a rough estimate for the supercooling η_f by ignoring the recursive logarithmic term $\ln \eta_f$ in (2.13) (Hogan¹¹ does this). For a more precise number one should iterate a few times. For $T_c = 200 \text{ MeV}$ and $w_0 = 1$ we now get $L \approx 182$, $L + 12 \ln \eta_f \approx 136$, and $\eta_f = 0.021$.

The number density of bubbles is

$$\begin{aligned} N &= \int_{t_c}^{t_f} dt' p(t') \\ &\approx C T_c^4 \frac{dt}{d\bar{T}} \exp\left\{-\frac{w_0}{16\eta_f^2}\right\} \times \\ &\times \int_0^\infty \exp\left\{-\frac{w_0 x}{8\eta_f^3}\right\} dx \\ &= C T_c^4 \frac{dt}{d\bar{T}} \frac{8\eta_f^3}{w_0} \exp\left\{-\frac{w_0}{16\eta_f^2}\right\} \\ &\simeq \frac{3}{4\pi v_{sh}^3} \left(\frac{dt}{d\bar{T}}\right)^{-3} \frac{w_0^3}{6 \cdot 8^3 \eta_f^9}, \end{aligned} \quad (2.16)$$

where in the last step we used the expression (2.11) for $F(\bar{T}_f) = 1$. Therefore the typical separation between nucleation sites is

$$\begin{aligned} l_n &\equiv N^{-1/3} \\ &\simeq (8\pi)^{1/3} \frac{dt}{d\bar{T}} v_{sh} \frac{8\sqrt{w_0}}{[16(L + 12 \ln \eta_f)]^{3/2}}. \end{aligned} \quad (2.17)$$

Since $dt/d\bar{T}$ is of the order of Hubble scale, the new distance scale l_n is roughly the Hubble length divided by the $3/2$ power of the logarithm $L = \ln T_{Pl}^4/T_c^4$.

Compare this to Hogan's result¹¹ for the ratio of this scale to the Hubble length,

$$\Delta \simeq \frac{1}{B_f L}, \quad (2.18)$$

where Hogan argues $B_f \equiv -t_f(\dot{A}/A)_f$ is greater than unity. In our model

$$B_f \simeq \frac{2}{\eta_f} \frac{d\bar{T}}{dt} t_f \simeq \frac{1}{\eta_f}, \quad (2.19)$$

which thus contains the missing half power of the logarithm L .

Inserting the parameter values of our ideal gas model⁴ into (2.17) gives the numerical result

$$l_n = 0.7m \left(\frac{T_c}{200\text{MeV}} \right)^{-2} \left(\frac{\sigma}{T_c^3} \right)^{3/2}. \quad (2.20)$$

This scale is for the beginning of the phase transition. During the phase transition the universe expands by a linear factor of $(g_q/g_h)^{1/3} \approx 1.4$, so at the end of the transition the scale is

$$l_n = 1.0m \left(\frac{T_c}{200\text{MeV}} \right)^{-2} \left(\frac{\sigma}{T_c^3} \right)^{3/2}. \quad (2.21)$$

This agrees with FMA eq. (28b), when one corrects for a numerical error: in FMA eq. (26) the number 1.4 should be¹⁰ about 0.4, and since eq. (28b) contains its third power, it should be divided by about 50. This error has propagated into the review paper by Reeves¹⁶.

Recent estimates from finite- T lattice QCD calculations^{17,18} give $\sigma/T_c^3 \lesssim 0.3$, implying that the scale l_n is probably less than a meter. A more detailed calculation of the spectrum of separation distances has been done by Meyer *et al.*¹⁰.

Note that $1m$ at $T = 100\text{MeV}$ (after the phase transition) corresponds to one light-hour today. Thus the structures generated in this transition are smaller than the solar system. There are no obvious observable remnants of this scale in the present universe. Indeed, some ingenuity is required to preserve a presently observable signature of these events of the first microseconds.

FIGURE 1

A crude simulation of the nucleation process. Bubbles ($\bullet$) are nucleated at random locations with rate $p(t)$. A shock sphere expands around them at speed $1/\sqrt{3}$. Collisions are ignored here. Further nucleation is not allowed within the spheres. This is a snapshot at time t_f , when in the first approximation (ignoring overlap) the spheres fill the space. This actually happens somewhat later, and sites of further nucleation after t_f in this simulation are denoted by ($\times$). Since the bubbles in the figure lie in a plane, the average separation appears as roughly $\sqrt{2/3}l_n$.

3. COALESCENCE AND PERCOLATION

Once this nucleation stage of the phase transition is over, most of the space is still in the Q phase. The nucleated H bubbles will now grow slowly, as allowed by the expansion of the universe¹. I will not address here the question of possible instability of the phase boundary¹⁹.

Eventually the bubbles will come in

contact with each other. The size distribution of the bubbles at this stage will not be exactly the same as that of their shock spheres at the end of nucleation. A bubble that nucleated early has a large region around it reheated by its compression wave, where there are no other bubbles, and thus it will grow larger. But otherwise there is no memory of which parts were reheated by which bubbles, and a bubble next to this region that nucleated late, may also expand into this region. This will concentrate the size distribution somewhat tighter around the average size.

If the bubbles are small enough when they come in contact with each other, surface tension will cause them to coalesce into larger bubbles. Witten¹ estimated this coalescence scale. In his order of magnitude estimate he used the density $\rho \sim T_c^4$ both for the mass to be moved and for the timescale available for this. Since the density is actually an order of magnitude larger, Witten's result 1–10 cm is a slight overestimate. We estimate the coalescence scale to be

$$l_c = 3\text{mm} \left(\frac{\sigma}{T_c^3} \right)^{1/3} \left(\frac{T_c}{200\text{MeV}} \right)^{-5/3}. \quad (3.1)$$

Comparing our estimates (2.21) for the nucleation scale l_n and (3.1) for the coalescence scale l_c , we conclude that it is likely that $l_n > l_c$, and thus no coalescence takes place.

After the H bubbles percolate, the remaining regions of quark-gluon plasma separate into Q droplets. (We call them droplets, since they have a higher density than the surrounding H phase). If indeed $l_c \ll l_n$, this also means that these droplets will remain aspherical for most of the duration of the phase transition. We will here assume that they become spherical in the

end, before they finally disappear. This assumption may be questionable and requires further study.

4. BARYON NUMBER

Before the phase transition the quark-gluon plasma had an almost equal number of quarks and antiquarks, since the baryon number of the universe is so small,

$$\frac{n}{n_\gamma} \sim 10^{-9}. \quad (4.1)$$

This baryon number is too small to affect the dynamics of the early universe; but since it makes up the present world, after all the antimatter annihilated almost all of the matter, it is important to follow it through these events.

The baryon number in the Q phase is carried by quarks, some of which have masses well below T_c . In the H phase it is carried by nucleons, whose masses are well above T_c . Therefore the baryon number prefers to reside in the Q side during the phase transition¹.

We can estimate the baryon number contrast

$$R \equiv \frac{n_q s_h}{n_h s_q} \quad (4.2)$$

at the phase boundary by assuming chemical equilibrium. Here n_q and n_h are the baryon number densities on the Q and H sides of the phase boundary, and s_q and s_h are the entropy densities. According to Kapusta and Olive²⁰, $R > 6$, and if T_c is low, $O(100 \text{ MeV})$, R can be $O(100)$. Note that if we assume overall equilibrium during the entire process, we end up with a homogeneous n after the transition is over. Since we have a dynamical situation, where the universe is expanding, and the H phase is

growing at the expense of the Q phase causing the phase boundary to move, we have to ask, how fast are the processes that try to restore equilibrium, compared to the expansion timescale. Whereas heat transport is efficient enough to maintain the universe close to thermal equilibrium, it is not clear that baryon number transport is as successful in maintaining chemical equilibrium.

Whether chemical equilibrium is maintained across the phase boundary depends on the baryon penetrability of the boundary. If the penetrability is small, we may have an even larger contrast R at the boundary, than estimated from chemical equilibrium⁹.

As the phase boundary moves into the Q phase, most of the baryon number does not pass through the boundary, but gets pushed by it. Unless the diffusion rate of baryon number is very high, this excess remains close to the boundary, and piles up into a layer on the Q side. This raises n_q , and eventually n_q/R , which gets passed through, becomes as high as the n deep within the Q phase. Then we have reached a steady state at the boundary, valid as long as it can be treated as planar. No further baryon number gets added to the layer.

The planar steady-state solution of the diffusion equation with a moving wall (velocity v) boundary is

$$n(z, t) = n(z - vt) \propto e^{-v(z-vt)/D}. \quad (4.3)$$

This gives the thickness of the baryon layer as

$$d = \frac{D}{v} \sim \frac{Dt}{l}, \quad (4.4)$$

where t is the duration of the transition, and l is the distance the boundary has to move. The fraction of the total baryon number collected in these layers will be approximately

$$f_n \sim R \frac{d}{l} \sim \frac{RDt}{l^2}. \quad (4.5)$$

The above discussion is valid if $f_n \ll 1$.

The microscopic diffusion coefficient is the mean free path of quarks in the quark-gluon plasma, which should be of the order $1/T_c \sim 1\text{fm}$. With *e.g.* $R = 10^2$, $t = 10\mu\text{s} = 3\text{km}$, $l = 1\text{m}$, we get

$$f_n \sim 3 \times 10^{-9}. \quad (4.6)$$

Thus only a very small fraction of the total baryon number would be affected.

However, Mathews *et al.*²¹ argue that baryon number may be transported much more effectively by turbulence. Indeed we know that in many familiar situations like *the atmosphere or the Sun*, microscopic diffusion is irrelevant, since transport is completely dominated by turbulence with an effective diffusion coefficient easily more than 10 orders of magnitude larger than the microscopic one. Thus it is possible that $f_n > 1$, *i.e.*, most of the baryon number gets moved around. It would be important to study possible causes of turbulence during the phase transition to get an estimate for the effective D . One possibility is the remnants of the compression waves from the nucleation stage of the transition.

When the Q droplets finally disappear they leave behind the baryon number they had collected. If the droplets become spherical, we have a geometry effect in the end: as the surface area of the droplet shrinks, the density of baryon number has to increase. Thus the droplet leaves behind a peak in baryon number density with a profile⁷

$$n \propto \frac{1}{r^2} \quad (4.7)$$

(however smoothed in scale d). This could possibly lead to a very high baryon number density, which could modify the picture at

the end stage of these events. We do not discuss here the formation of quark nuggets, but if they are formed, this is where it would happen.

In the end we mention some things we have not treated here: In the case $l_n < l_c$ the coalescence of the bubbles will leave a signature on the baryon number density profile, since after the coalescence diffusion will be less effective, due to longer distances to be traversed⁷.

Another effect is the radiation loss from the Q droplet which will increase n/s within it². This could lead to a larger effect r/s measured by the total baryon number moved into an inhomogeneity, but it would not lead to sharp peaks like the effects discussed here.

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