COLLIDER PERFORMANCE WITH IDEAL COLLISIONS

M. Month

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MASTER

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Summary

Performance is estimated for head-on bunch collisions. The luminosity is written as a distribution in time and length along the collision path, using a Gaussian distribution in all space dimensions. Computations are made for the Tevatron. Sources of luminosity reduction are listed in the case of non-ideal conditions.

Luminosity

The luminosity, L , is a measure of the event rate for a given particle interaction cross section. If $\dot{n}$ is the number of events per unit time for a given type of collision, described by the cross section σ , then we have

$$\dot{n} = \sigma L .$$

The luminosity is a relativistic invariant, and can be expressed by the overlap integral,

$$L = c(|\vec{\beta}_1 - \vec{\beta}_2|^2 - |\vec{\beta}_1 \times \vec{\beta}_2|^2)^{1/2} \int \rho(1)\rho(2)dV ,$$

where $\rho(1)$ and $\rho(2)$ are volume particle densities, $\vec{\beta}_1$ and $\vec{\beta}_2$ are particle velocities in units of the velocity of light, c , and dV is the space differential,

$$dV = dx dy dz .$$

Luminosity with Head-on Collisions

Figure 1: Head-on Collision Geometry.

For a Gaussian beam in all three space dimensions and in a coordinate system with the bunches moving in the z -direction, the densities can be written,

$$\rho_{1,2} = \frac{N_B}{(2\pi)^{3/2} \sigma_h \sigma_v \sigma_\ell} e^{-\left[\frac{x^2}{2\sigma_h^2} + \frac{y^2}{2\sigma_v^2} + \frac{(z-\delta_{1,2})^2}{2\sigma_\ell^2} \right]},$$

where N_B is the number of particles in each bunch (taken for simplicity to be the same), σ_h , σ_v , σ_ℓ are horizontal, vertical and longitudinal rms bunch sizes at the crossing point, and $\delta_{1,2}$ are beam offsets for bunches 1 and 2 respectively.

To specify $\delta_{1,2}$, let bunch 1 be centered at the "collision point" at $t = 0$, and let bunch 2 be delayed by the time t_0 . Then,

$$\delta_1 = ct \quad , \quad \delta_2 = -c(t - t_0) \quad .$$

Substitute $\rho_{1,2}$ and $\delta_{1,2}$ into the general expression for the luminosity to obtain the instantaneous luminosity, $L(t)$. Then, integrate over time as the bunches pass through each other to find the luminosity per bunch collision, L_B (cm^{-2}):

$$L_B = \int_{-\infty}^{\infty} L(t) dt .$$

Problem: Assuming that the σ 's are independent of z , show that L_B is independent of the offset, ct_0 .

Problem: Show, in the round beam case, $\sigma_h = \sigma_v = \sigma$, that L_B is independent of σ_L and is given by,

$$L_B = \frac{N_B^2}{4\pi\sigma^2} .$$

Define the transverse emittance in each dimension as the phase space area containing 95% of the particles in the beam. Denote this emittance by ϵ , which can be written in terms of the normalized emittance E , by

$$\epsilon = E/\beta\gamma .$$

For a Gaussian beam, σ^2 at the crossing point can be expressed in terms of E by

$$\sigma^2 = \frac{E\beta^*}{6\pi\gamma} ,$$

where β^* is the β value at the crossing point and the velocity, β , has been taken to be 1.

Proof: In terms of the normalized coordinates, $\eta = x/\sqrt{\beta^*}$, $\dot{\eta} = \sqrt{\beta^*} x'$, a Gaussian phase space density is given by

$$\rho(\text{phase space}) = \frac{\beta^*}{2\pi\sigma^2} e^{-(\eta^2 + \dot{\eta}^2)/(2\sigma^2/\beta^*)} ,$$

where,

$$\int \rho d\eta d\eta = 1 .$$

Transforming to Polar coordinates: r, ϕ , we have

$$\rho(r^2) = \frac{\beta^*}{2\pi\sigma^2} e^{-r^2/(2\sigma^2/\beta^*)} .$$

Let R be the "95% radius," then

$$\int_0^{2\pi} \int_0^R r dr d\phi \rho(r^2) = 0.95 .$$

Integrating leads to the equation

$$[1 - e^{-R^2/(2\sigma^2/\beta^*)}] = 0.95 .$$

Thus, the area,

$$\pi R^2 = \frac{E}{\gamma} = \frac{2\pi\sigma^2}{\beta^*} \ln(20) \approx \frac{6\pi\sigma^2}{\beta^*} .$$

Using the expression for σ^2 , we have the luminosity per bunch collision,

$$L_B = \frac{3}{2} \frac{N_B^2 \gamma}{E \beta^*} \cdot (\text{units cm}^{-2}) .$$

The average luminosity is L_B times the bunch collision frequency, f_B :

$$L_{AV} = L_B f_B ,$$

or,

$$L_{AV} = \frac{3}{2} \frac{N_B^2 f_B \gamma}{E \beta^*} \cdot (\text{units cm}^{-2} \text{ sec}^{-1}) .$$

Time and Space Distribution of Luminosity

In the case where σ_h and σ_v are independent of z in the crossing region (i.e. $\beta_{h,v}^* \gg \sigma_\ell$), then the instantaneous luminosity can be expressed as a function of length along the bunch collision path, z , and time, t . Using the expressions for $\delta_{1,2}$ in terms of t and integrating over the transverse cross section (i.e. over x and y), we can show that

$$L(t, z) = \frac{c N_B^2}{(2\pi)^2 \sigma_\ell^2 \sigma_h \sigma_v} e^{-(z - \frac{1}{2} c t_0)^2 / \sigma_\ell^2} e^{-c^2 (t - \frac{1}{2} t_0)^2 / \sigma_\ell^2} .$$

Introducing the dimensionless variables

$$s = z / \sigma_\ell \quad \text{and} \quad \tau = c t / \sigma_\ell \quad \text{and}$$

writing the dimensionless time offset, $\tau_0 = c t_0 / \sigma_\ell$, we have for the specific instantaneous luminosity

$$\frac{d\mathcal{L}(\tau, s)}{d\tau ds} = \frac{N_B^2}{4\pi^2 \sigma_h \sigma_v} e^{-(s - \frac{1}{2} \tau_0)^2} e^{-(\tau - \frac{1}{2} \tau_0)^2} ,$$

where $\mathcal{L}(\tau, s)$ is a measure of the luminosity per bunch collision. For ideal collisions, that is, with zero offset,

$$\frac{d\mathcal{L}(\tau, s)}{d\tau ds} = \frac{N_B^2}{4\pi^2 \sigma_h \sigma_v} e^{-s^2} e^{-\tau^2}$$

Integrating over space and time gives the luminosity per bunch collision,

$$L_B = \frac{N_B^2}{4\pi\sigma_h\sigma_v} .$$

Thus,

$$\frac{d\mathcal{L}}{d\tau ds} = \frac{L_B}{\pi} e^{-s^2} e^{-\tau^2} .$$

Luminosity Definitions

$$\frac{d\mathcal{L}(\tau, s)}{d\tau ds} \quad \text{Specific instantaneous luminosity}$$

$$\frac{d\mathcal{L}}{d\tau} \quad \text{Instantaneous luminosity}$$

$$\frac{d\mathcal{L}}{ds} \quad \text{Specific luminosity}$$

$$L_B \equiv \mathcal{L} \quad \text{Luminosity per bunch collision}$$

$$L_{AV} = \mathcal{L} f_B \quad \text{Average luminosity}$$

(f_B is the frequency of bunch collisions) .

Ideal Tevatron Luminosity

To compute the Tevatron performance with ideal collisions, we need the characteristics of the colliding bunches. Ideal collisions are characterized by having the proton and antiproton bunches identical. However, it is quite possible that as the Tevatron collider gets started, the antiproton bunch intensity will be significantly lower than the proton bunch intensity. To modify our expressions to take account of this difference, we simply take

$$N_B^2 \longrightarrow N_p N_p^- .$$

The Tevatron parameters for the assumed identical bunches are as follows:

Longitudinal

rms bunch length: $\sigma_L = 40 \text{ cm}$

95% bunch length: $l_B = 2\sqrt{6} \sigma_L$

rms momentum spread: $\sigma_p/p = 1.2 \times 10^{-4}$

Transverse

rms width at crossing point:

$$\sigma^* = \left[\frac{\beta^* E}{6\pi\gamma} \right]^{1/2} = 6.1 \times 10^{-5} \text{ m} .$$

Taking, β at the crossing point: $\beta^* = 1 \text{ m}$

Energy in mass units: $\gamma = 1066.1 \text{ (1 TeV)}$

and the normalized 95% emittance: $E = 24\pi \times 10^{-6} \text{ rad-m}$

rms horizontal and vertical widths: $\sigma_v = \sigma_h = \sigma^* = 6.1 \times 10^{-5} \text{ m} .$

Thus, assuming the number of particles per bunch is the same for protons and antiprotons,

$$N_p = N_{\bar{p}} = 6 \times 10^{10}$$

we obtain for the luminosity per bunch collision,

$$L_B = \frac{N_p N_{\bar{p}}}{4\pi\sigma_h\sigma_v} = 0.76 \times 10^{25} \text{ cm}^{-2} .$$

Since the revolution frequency at relativistic velocities is $f_0 = 4.77 \times 10^4 \text{ Hz}$, we have, taking $n_B = 3$ bunches in each beam, for the frequency of bunch collisions,

$$f_B = n_B f_0 = 143.1 \text{ kHz} .$$

Thus the average luminosity,

$$L_{AV} = f_B L_B = 1.1 \times 10^{30} \text{ cm}^{-2} \text{ sec}^{-1} .$$

Also, it should be noted that with a total interaction cross section of $\sigma_{\text{tot}} = 100 \text{ mb}$, the mean number of events per bunch collision is

$$n_{\text{mean}} = L_B \sigma_{\text{tot}} = 0.76 .$$

The specific instantaneous luminosity is given by

$$\frac{d\mathcal{L}}{d\tau ds} = \frac{L_B}{\pi} e^{-\tau^2} e^{-s^2} ,$$

where we have that

$$s \text{ is a normalized length: } s = z/\sigma_\ell$$

$$\tau \text{ is a normalized time: } \tau = ct/\sigma_\ell .$$

Thus, the luminosity distribution is characterized by:

rms length of luminosity along collision path

$$= \frac{1}{\sqrt{2}} \sigma_\ell = 28.3 \text{ cm}$$

rms time distribution of luminosity within a bunch collision

$$= \frac{1}{\sqrt{2}} \frac{\sigma_\ell}{c} = 0.94 \text{ nsec}$$

(note that the period of bunch collisions

$$= \frac{1}{f_B} = 7.0 \text{ usec}) .$$

Sources of Luminosity Reduction

The luminosity calculated with an ideal collision configuration will in practice be reduced because actual conditions may not conform to the ideal. A list of some of the sources of luminosity reduction is as follows:

1. If $\sigma_\ell \gtrsim \beta^*$, we must multiply the overlap density by the factor

$$\left[\left(1 + \frac{s^2 \sigma_\ell^2}{\beta_h^{*2}} \right) \left(1 + \frac{s^2 \sigma_\ell^2}{\beta_v^2} \right) \right]^{-1/2}.$$

This is related to the fact that even when the bunch centers meet at the collision point, if the bunches are "long," then the ends of the bunch are simultaneously at points of higher beta and therefore points of lower overlap density.

2. If there is a delay of bunch 2 with respect to bunch 1, it will, if uncorrected, result in a luminosity reduction, again because of an effective reduction in overlap density.

3. A reduction of luminosity will result if there is an offset of the minimum of β_h^* with respect to β_v^* , even if they are equal in magnitude.

4. An increase of emittance of beam 1 with respect to beam 2 will result in a luminosity reduction and should be considered. In the SPS, it appears that the low intensity beam (the weak beam) cannot sustain an emittance larger than the strong beam due to the electromagnetic beam-beam effect. Thus in the weak-strong case, the emittances tend to equalize if the weak beam emittance is initially greater than the strong beam emittance. Of course the number of particles in the weak beam is reduced.

5. If the dispersion function is not zero in the collision region, the overlap density is reduced.

6. There are phenomena which decrease the luminosity lifetime by either increasing beam bunch sizes (σ_h , σ_v , or σ_ℓ) or by decreasing the number of particles in the bunches. Size increase can be caused by scattering off the residual gas in the vacuum chamber; intrabeam scattering; blowup due to the beam-

beam interaction; and blowup due to the interaction of the environment with intense bunches. The most important of these are probably intrabeam scattering in the intense proton bunches and beam-beam blowup of the "weak" antiproton bunches. Direct particle loss will occur mainly from events arising from the total nuclear $p\bar{p}$ cross section. As was seen earlier, using a total cross section of 100 mb, the mean event rate is 0.76 events per bunch collision. In the case of high luminosity together with low bunch current, this could become a limiting factor. In the Tevatron, however, this should not be a problem.