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SHOCK WAVES IN LUMINOUS EARLY-TYPE STARS*

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ABSTRACT. Shock waves that occur in stellar atmospheres have their origin in some hydrodynamic instability of the atmosphere itself or of the stellar interior. In luminous early-type stars these two possibilities are represented by shocks due to an unstable radiatively-accelerated wind, and to shocks generated by the non-radial pulsations known to be present in many or most OB stars. This review is concerned with the structure and development of the shocks in these two cases, and especially with the mass loss that may be due specifically to the shocks. Pulsation-produced shocks are found to be very unfavorable for causing mass loss, owing to the great radiation efficiency that allows them to remain isothermal. The situation regarding radiatively-driven shocks remains unclear, awaiting detailed hydrodynamics calculations.

1. GENERAL PROPERTIES OF SHOCKS

In order to understand how shock waves behave in a stellar atmosphere, it is first necessary to recall the Rankine–Hugoniot shock jump conditions, and then consider the various relaxation processes that take place near the shock. What we call a 'shock' when we view it in the large is really a complex non-equilibrium flow when examined in detail. The details must be considered before choosing the correct way of embedding the shock in the larger problem.

1.1. Jump Conditions

The effects of a shock are all consequences of the jumps in density, pressure, temperature and velocity that are dictated by the laws of conservation of mass, momentum and energy. If ρ, p, T, and v are, respectively, the density, pressure, temperature and flow velocity relative to the shock, then the jump conditions for a strong shock are

$$\frac{\rho_1}{\rho_0} = \frac{v_0}{v_1} = 4, \quad p_1 = \frac{3}{4}\rho_0 v_0^2, \quad and \quad T_1 = \frac{3}{16}\frac{\mu v_0^2}{\mathcal{R}}.$$

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(μ is the mean atomic weight, $\mathcal{R}$ is the gas constant, and subscripts 0 and 1 refer to pre- and post-shock, respectively.) A shock is strong when the upstream flow velocity relative to the shock, v_0, is large compared with the upstream sound speed $a = \sqrt{p_0/\rho_0}$. Many of the interesting effects of shocks are due to the elevated temperature of the post-shock gas; the numerical value of T_1 is given by

$$T_1 = 1.51 \times 10^5 \left(\frac{v_0}{100\,\mathrm{km\,s^{-1}}}\right)^2 \mathrm{K}.$$

Since the speeds that have been suggested for shocks in OB stars are of order 200–600 km s^{-1}, post-shock temperatures in the range $T_1 = 6 \times 10^5$–5×10^6 K are expected.

1.2. Internal Structure of Shocks

The 'discontinuity' of flow variables at a shock is an idealization—the atomic nature of the gas, and the transport properties due both to atoms and radiation, smooth out the jumps and give them a finite width. Atomic transport gives shocks a width about equal to the gas-kinetic mean free path in the shocked gas (with a correction for the electron/proton mass ratio). (See Zel'dovich and Raizer [1967], § VII.2.) This thickness corresponds roughly to a particle column thickness (*i.e.*, $N = \int n\,dx) \approx 1 \times 10^4 T_1^2/\ln\Lambda$, where $\ln\Lambda$ is the usual Coulomb logarithm. For $T \approx 10^6$ K, this thickness is $\approx 10^{16}\,\mathrm{cm^{-2}}$. As we will see shortly, this is quite narrow compared with the broader parts of the shock's internal structure. A very useful approximation is that there is a true discontinuity—the 'gas-dynamic' shock—embedded in a broader region of radiation transport effects and excitation and ionization relaxation.

It may be helpful to picture the whole structure of the shock as containing four regions, from upstream to downstream: (A) cold unshocked gas; (B) hot shocked gas—region of ionization run-up; (C) radiative cooling, ionization about in balance; and (D) cold dense gas. The 'gas-dynamic' shock separates regions (A) and (B). Some of the radiation produced as the gas cools in region (C) may be absorbed in region (A), leading to a 'radiative precursor' (Zel'dovich and Raizer [1967], §§ VII.14–18). The length scale of this precursor, if the gas flows into the shock too quickly for it to be able to come to thermal equilibrium with the precursor radiation, is a mean free path of the predominant radiation. This radiation is mostly in the He II Lyman continuum, and the mean free path corresponds to a column N of $\approx 10^{19}\,\mathrm{cm^{-2}}$, or more if the predominant photon energy is above 100 eV. This scale is great enough that it belongs to the outer structure of the stellar atmosphere, rather than to the internal structure of the shock. That is, for our purpose region (A) can be considered transparent, and whatever preheating and preionization occur have taken place before the gas flows into region (A).

The Mach number (v/a) of the flow is small throughout regions (B), (C) and (D), with the result that the pressure is nearly constant ($= \rho_0 v_0^2$). The temperature in region (D) has cooled to the level determined by radiative energy balance with the ambient radiation field, which also determines the temperature in region (A). (Regions [A] and [D] view the same radiation unless region [A] is opaque.) Thus the jump from region (A) to region (D) can be called an 'isothermal shock'. Although there is no net temperature jump, there is a large density jump

$$\frac{\rho(\mathrm{D})}{\rho_0} = \left(\frac{v_0}{a_0}\right)^2 \gg 1,$$

an equal pressure jump, and a reciprocal velocity jump. Notice that the density jump can become arbitrarily large instead of being limited to 4, as in an adiabatic shock.

The processes that occur in regions (B) and (C) are these: The gas enters region (B) quite hot, since the pre-shock kinetic energy has been converted to enthalpy. But the state of ionization and excitation of the material is unchanged by the shock. This situation is very much out of equilibrium, and a process of relaxation begins, in which thermal energy is used up in ionizing and exciting the atoms. As long as the degree of ionization of the atoms is low, the ionization rate is very rapid; as the ionization increases, the rate slows, and a steady state of ionization balance would be reached if time and space permitted and if radiative processes did not intervene. This is about the state of the matter as it exits region (B) for region (C). The ionization and recombination rates around the state of ionization equilibrium are comparable with the rates of emission of radiation, such as free-bound emission and collision-induced resonance line emission. These processes convert thermal and excitation energy into radiation, which leaves the region of the shock. This cooling is what occupies region (C). Since ionization rates are somewhat larger than the cooling rate, the material stays relatively near the condition of ionization equilibrium as it cools. Region (C) ends when the radiative cooling rate is balanced by the heating rate due to the ambient radiation.

There are a variety of characteristic column thicknesses N associated with regions (B) and (C). The thickness of the ionization layer for an ion depends on its ionization potential, χ, with the layer being very thin if χ is small, and thick if χ is large:

$$N \approx \begin{cases} 10^{14}\left(\frac{v_0}{100\,\mathrm{km\,s^{-1}}}\right)^4 \mathrm{cm}^{-2}, & \chi \approx kT; \\ 4\times 10^{17}\left(\frac{v_0}{100\,\mathrm{km\,s^{-1}}}\right)^4 \mathrm{cm}^{-2}, & \chi \approx 5kT. \end{cases}$$

The typical value of χ when the degree of ionization is highest is about $5kT$. The column thickness of the ionization region for the low ionization potential case is similar to, and perhaps smaller than, the thickness of the gas-kinetic shock. This means that low ionization potential species may be ionized within the shock. However the peak ionization is attained only at a downstream distance that is many times greater than the shock width.

The processes that contribute to cooling of the gas are many, and a proper cooling calculation must include many different ionic species with all their possible resonance excitations. One such set of calculations was made by Raymond, Cox and Smith (1976). The cooling rate per unit volume is expressed as $n^2\Lambda$, where n is the total number density of nuclei and Λ (not to be confused with the Coulomb logarithm) is a function of T and also weakly a function of the radiation environment and the past history of the material. (See Fig. 1 of Raymond, Cox and Smith.) Λ has a broad maximum near 10^5 K, where the cooling is dominated by collisional excitation of abundant lithium-like ions. Above about 3×10^5 K Λ declines, with bumps due to other collisional excitations. This decline can very roughly be fitted to a power law: $\Lambda \propto T^{-1/2}$. From this formula it is easy to calculate the column thickness needed for cooling to remove all the enthalpy of the shocked gas. This result has been given by Krolik and Raymond (1985):

$$N_{\mathrm{cool}} = \text{total column to return to ambient } T \approx 7\times 10^{17}\left(\frac{v_0}{100\,\mathrm{km\,s^{-1}}}\right)^4 \mathrm{cm}^{-2}.$$

This is comparable with the thickness needed to reach peak ionization, so we conclude that the mean ionization starts downward, as the temperature declines, just as the ionization and recombination rates are coming into balance. We can regard N_{cool} as the thickness of region (C), and, to a fair approximation, of the entire internal structure of the shock.

The bulk of the emission from the shock is produced in region (C). The net flux emitted is, in total, about equal to the kinetic energy flux into the shock, $\frac{1}{2}\rho_0 v_0^3$. How this flux is distributed in the spectrum depends on the actual temperature structure in the shock, which in turn is primarily sensitive to v_0. As the observations of x-rays from OB stars with the *Einstein Observatory* have shown (Cassinelli and Swank [1983]), the source of the x-rays must be material with a temperature of order 3×10^6 K, which requires $v_0 \approx 450\,\mathrm{km\,s^{-1}}$. The corresponding N_{cool} is of order $10^{20}\,\mathrm{cm^{-2}}$, which exceeds the radiation mean free path estimate given earlier; however, the typical photon energy is about 300 eV at this temperature, which increases the He II Lyman continuum mean free path to the equivalent of $N \approx 7 \times 10^{21}\,\mathrm{cm^{-2}}$.

2. SHOCKS FROM PERIODIC PULSATION

The first mechanism for shock production that I want to consider is the one in which the root instability originates in the stellar interior, so that the whole star is pulsationally unstable and can be supposed to pulsate in some normal mode with a well-defined period, P. From the atmosphere's point of view, it is being driven by an oscillating piston characterized by P and the velocity amplitude, U. If P is comparable with the period of the radial fundamental mode, about 4 hours for main-sequence OB stars, then the atmosphere is being driven at a period below its acoustic cut-off frequency (see Lamb [1945], § 309). Thus the motion of the atmosphere tends to be a standing wave, and lacks the running-wave character that most easily leads to shock formation. If the piston velocity is moderate, however, non-linear effects still produce a shock at a certain height above the piston that increases with P and decreases with U.

Once a shock forms, its strength, measured by the jump in velocity across it, increases as the shock runs upward through material of lower and lower density. When the shock is sufficiently strong, the density and velocity of the material into which the shock runs will be affected considerably by the *previous* similar shock that passed through that material. The passage of each shock delivers an upward impulse to the material, and the effect is a 'shock levitation' of the atmosphere, partially offsetting gravity. As a result, the scale height of the atmosphere is expanded, which also diminishes the tendency of the shock strength to increase with height. In this way, the shock strength finds a stable limiting value, which is a definite function of height.

The picture just described is possible only so long as the shock is effectively isothermal—the shocked gas can cool within a time shorter than, or at most comparable with, the period. The cooling time behind the shock increases as the pre- (and post-) shock density declines, so the isothermal condition inevitably fails above some height in the atmosphere. Above this point the temperature remains near the post-shock temperature, T_1, and the shocks, now weak, provide the energy deposition that maintains the temperature. This temperature is comparable with the 'escape temperature' at which the sound speed equals half the escape velocity, thus a stellar wind of the kind described by Parker (1958) results.

2.1. The Height of Shock Formation

An accurate calculation of the height at which a shock is formed when an atmosphere is driven by a periodically oscillating piston can only be done numerically. A simple estimate of the scaling of this height can, however, be made by the following argument. Assuming for the moment, as will be verified shortly, that the sound-travel time up to the height of shock formation is a small fraction of the period, then the shock should form relatively soon after the beginning of the outward-acceleration phase of the piston motion. One possible idealization, therefore, is to consider a piston moving into an atmosphere at rest with a position $R(t)$ that is constant for $t \leq 0$ and has a constant positive acceleration for $t > 0$. This has the objection that the atmosphere will respond to the piston acceleration (the standing-wave effect) so that the matter has just the same acceleration as the piston, and no shock is produced. That is, the shock creation is tied to the *change* in piston acceleration. This leads to a model like the previous one, but with a constant positive d^3R/dt^3 for $t > 0$.

Let a be the isothermal speed of sound. Then at each instant t_0 a sound wave of speed a leaves the piston at $R(t_0)$. These sound waves cover the region above the piston, and, since the later-departing waves have a larger absolute velocity, they in fact cross each other. Thus the wave trajectories form an *envelope* in the r–t diagram, with a cusp where the shock forms. (See § 49 of Courant and Friedrichs [1948].) The cusp forms in this simple model at the height

$$h \approx \frac{1}{3}\sqrt{\frac{a^3}{d^3R/dt^3}}.$$

If now d^3R/dt^3 is related to U using the assumption of sinusoidal oscillation, and taking H to be the static scale height of the atmosphere, a^2/g, the height expressed in scale heights is

$$\frac{h}{H} \approx \frac{1}{6\pi}\frac{Pg}{\sqrt{aU}}.$$

The velocity amplitude is thought to be comparable to a, and in this case h/H ranges from a few to 15 for a plausible range of P. (The sound-travel time is then about 1/3 radian of pulsation phase, justifying the earlier assumption.)

2.2. The Growth of Shock Strength with Height

For cases of interest, the shock first forms in a layer dense enough that the shock is quite isothermal, *i.e.*, the picture advanced in § 1.2 is applicable. In general, the evolution of the shock as it moves upward is governed by the pre-shock density and velocity, and all the conditions in the post-shock region. If the Mach number is large, however, the dependence on post-shock conditions becomes weak, and an approximation due to Whitham (1958) can be used. The pre-shock conditions are taken as input, and the jump conditions are solved for the post-shock flow variables with the Mach number as a parameter. These relations are substituted into the differential equation that is valid along an outward-running characteristic (sound wave), *neglecting the difference between the path of the outward characteristic and the shock*. The result is a differential equation for the Mach number:

$$\left(1 + \frac{1}{M}\right)^2 \frac{dM}{dr} = -\frac{d\ln\rho_0}{dr} - \frac{g}{a\left(v_0 + \left(M - \frac{1}{M} + 1\right)a\right)}.$$

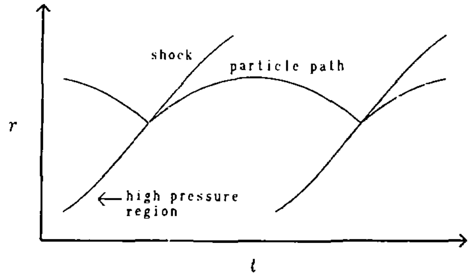

Figure 1. Space–time diagram showing the periodic shock trajectory and the path of a parcel of material. The shading indicates the high-pressure post-shock region.

If the density distribution is close to hydrostatic, then the first term on the right side is $\approx 1/H$ and is much larger than the second term, provided $M \gg 1$. This simple relation results:

$$M \approx \text{constant} - \ln \rho_0.$$

The interpretation of this relation is that it gives the Mach number of the shock as it moves upward in terms of the current density just in front of the shock. This relation breaks down when the density gradient starts to differ substantially from the hydrostatic one, which happens when the 'shock levitation' becomes significant. The dynamics then passes to the other limit, in which the Mach number no longer varies rapidly with height, and the two terms on the right of the equation above balance each other. In this case the density scale height has been extended by just a factor M.

If the shock were to become adiabatic before the levitation effect reduced the density gradient, then the development of the Mach number with height would be quite different. For an exponential distribution of density, the variation of Mach number is given by Zel'dovich and Raizer (1967, § XII.25):

$$M \propto \rho_0^{-1/\alpha},$$

where $\alpha = 4.90$ for $\gamma = 5/3$. The growth of Mach number in the adiabatic case is much stronger than in the isothermal case, once the Mach number is large. The two cases are illustrated by Castor (1970, Fig. 8).

2.3. The Limiting Strength for Periodic Shocks

The shock growth decribed in the previous section can be thought of as a transient effect before the shock attains the strength that gives full levitation of the atmospheric material. 'Full levitation' means that all the outward force on a parcel of material is exerted either in one shock jump each period, or in a relatively thin high-pressure zone behind the shock, and that for the rest of the period the parcel is essentially in free fall. (See Fig. 1.) This simple situation is amenable to both numerical and analytic treatment, and has been studied by Hill (1972), Hill and Willson (1979), Willson and Hill (1979), Willson and Bowen (1985), and Bertschinger and Chevalier (1985).

High Mach number allows the pressure-gradient force to be neglected altogether (apart from the shock jump), which leads to even greater simplification, the ballistic limit. This is justified when $Pg \gg a$, since the flow velocity scales as Pg. Values of Pg/a range from 40 to 150 for the non-radial pulsations observed in OB stars (from the periods quoted by Smith [1986]), so the approximation should be excellent.

In the ballistic model, the parcel of material is exactly in free fall between shock passages. Periodicity dictates that the pre-shock and post-shock velocities of the parcel be numerically equal ($u_0 = -u_1$) and the large Mach number also implies that the shock velocity u_s be $\approx u_1$. In order that the period be compatible with these initial and final velocities, the following condition must apply:

$$\frac{PV_{esc}(r)}{2r} \equiv 38.3Q\left(\frac{R_*}{r}\right)^{\frac{3}{2}} = \frac{\beta}{1-\beta^2} + \frac{\sin^{-1}\beta}{(1-\beta^2)^{\frac{3}{2}}},$$

where $V_{esc}(r)$ is the local escape velocity, Q is the pulsation constant in days, R_* is the stellar radius, and β stands for $u_s(r)/V_{esc}(r)$. This equation provides u_s, and indirectly all the other shock properties, as a function of r.

From the point of shock formation, the shock strength increases as it moves upward, according to the relations in § 2.2, until it reaches the limiting value just determined, whereafter the strength slowly decreases with r, following the formula above. Of course, periodic motion as described here precludes the possibility of any mass loss, and, indeed, that is the result of numerical calculations (*e.g.*, those of Hill [1972]), so long as isothermal conditions obtain.

3. MASS LOSS DUE TO PULSATION SHOCKS

As I just noted, shock-induced mass loss is tied to the breakdown of the isothermal shock approximation. Specifically, if the cooling time of the post-shock gas is longer than the pulsation period, then a parcel of material steadily gains heat as it is successively shocked, so that periodicity is impossible. This extra heat is used to do work lifting the parcel upward, producing a net outward flow or mass loss. (See Wood [1979].) The key question is the height at which the isothermal approximation breaks down; the next question is how the transition to a wind then occurs.

3.1. The Post-shock Cooling Time and the Transition to a Wind

The same fit to Λ *vs.* T quoted earlier from Krolik and Raymond (1985) can be used to find the flow time through region (C),

$$t_{cool} \approx 2 \times 10^{-14}\frac{(v_0/100\,\mathrm{km\,s^{-1}})^3}{\rho_0}\,\mathrm{s} \quad \text{for } 100 \le v_0 \le 1000\,\mathrm{km\,s^{-1}}.$$

The dynamic range of the density is much greater than that of the shock speed, v_0, so the critical condition $t_{cool} = P$ essentially fixes ρ_0. With P in the range 10^4–10^5 s and v_0 in the range 100–300 $\mathrm{km\,s^{-1}}$, ρ_0 lies between 10^{-19} and $10^{-17}\,\mathrm{g\,cm^{-3}}$. This is quite a low density, due to the fact that the radiating efficiency of a moderately hot, ionized plasma is excellent.

We may define r_{ad}, the adiabatic radius, as the place where $t_{cool} = P$. Above the adiabatic radius the shock may be treated as adiabatic. Since the pre-shock temperature is now high, the shock is no longer strong, (*i.e.*, $M \approx 1$). The nature of the transition to a wind depends on the value of β at r_{ad}. If $\beta \approx 1$, then the temperature at r_{ad} is already comparable to the Parker temperature and the sonic point of a Parker wind will be at or near r_{ad}. If $\beta \ll 1$, however, the temperature at r_{ad} is less than the Parker temperature by about a factor β^2. In the latter case the sonic point lies a modest distance outside r_{ad}, and the intervening region is approximately hydrostatic with a temperature that increases outward determined by a balance between shock heating and adiabatic expansion. Simple estimates of the mass loss rates that result in these two cases give

$$\dot{M} \approx \begin{cases} 4\pi r_{ad}^2 \rho_0(r_{ad}) u_s(r_{ad}), & \beta \approx 1; \\ 4\pi r_{ad}^2 \rho_0(r_{ad}) \dfrac{(u_s(r_{ad}))^2}{V_{esc}(r_{ad})}, & \beta \ll 1. \end{cases}$$

It should be noted that the mass loss rates scale directly with the density at the adiabatic radius, and thus inversely with the cooling efficiency, Λ.

As an example, consider a non-radially pulsating B star with the properties

$$M = 15\,M_\odot, \quad R_* = 6.4\,R_\odot, \quad V_{esc} = 944\,\mathrm{km\,s^{-1}},$$

and with a pulsation period equal either to 3 hours (in an $l = 8$ mode, say, with $Q = 0.03\,\mathrm{d}$) or to 12 hours ($l = 2, Q = 0.12\,\mathrm{d}$). The results of applying the ballistic theory are that $\beta = 0.43$ and 0.75, and $u_s = 400$ and $700\,\mathrm{km\,s^{-1}}$, for $P = 3$ hours and 12 hours, respectively, assuming $r = R_*$. However, it can be seen that the limiting amplitude is never attained while the shock is isothermal. If it is assumed that the density at the photosphere (the 'piston' location) is $10^{-9}\,\mathrm{g\,cm^{-3}}$ and that $U = a \approx 14\,\mathrm{km\,s^{-1}}$, then the densities at the height of shock formation in the two cases are $\rho \approx 2 \times 10^{-11}$ and $5 \times 10^{-17}\,\mathrm{g\,cm^{-3}}$. (The low value of ρ for $P = 12$ hours is due to the difficulty of forming a shock from a low-frequency wave.) If these densities are used in the shock-growth relation between density and Mach number, it is found that the velocities 400 and 700 $\mathrm{km\,s^{-1}}$ are attained only when ρ_0 is less than the critical value at r_{ad}; the shocks become adiabatic in the growth region. A simultaneous solution of the shock growth relation and the adiabatic radius condition leads to these data at r_{ad}:

$$\rho = 1 \times 10^{-17}\,\mathrm{g\,cm^{-3}} \quad u_s = 190\,\mathrm{km\,s^{-1}} \quad \text{for } P = 3\,\mathrm{h},$$
$$\rho = 2 \times 10^{-19}\,\mathrm{g\,cm^{-3}} \quad u_s = 80\,\mathrm{km\,s^{-1}} \quad \text{for } P = 12\,\mathrm{h}.$$

The estimates for the mass loss rate turn out to be $\dot{M} \approx 2 \times 10^{-12}\,M_\odot\,\mathrm{y^{-1}}$ for $P = 3$ hours, and $\dot{M} \approx 1 \times 10^{-14}\,M_\odot\,\mathrm{y^{-1}}$ for $P = 12$ hours.

These rates of mass loss are considerably less than the smallest rates that might be observed in OB stars, and therefore this mass loss mechanism, for these stars, does not appear to be significant. Two factors are responsible for this: One is the high cooling efficiency, which forces the density to be very low before adiabatic shocks are possible. The second factor, which is important for $P = 12$ hours, is that the long period means that the density is already quite low where the shock forms. These factors are much more severe for OB stars than for red giant variables, such as those studied by Willson and Wood, for which shock-driven mass loss appears to be quite important.

4. RADIATION-DRIVEN SHOCKS

The luminous early-type stars all have winds that are thought to be driven by the force due to resonance-line scattering (Lucy and Solomon [1970], Castor, Abbott and Klein [1975]). That these winds are unstable was suggested by Lucy and Solomon (1970); the later work on the radiatively-driven instability is reviewed by Rybicki elsewhere in these proceedings.

There is also an abundance of observational indicators of instability: x-ray emission seen with the *Einstein Observatory*; strong UV absorption by 'superionized' species like O VI and N V; non-thermal radio emission (Abbott, Bieging and Churchwell [1984]); and variable features in the UV absorption lines (Henrichs 1986). These may very well all be a result of shock waves passing through the stellar wind, and the shock waves may very well be due to the instability of the radiative driving. In this section I will discuss two such models that have been proposed.

4.1. Lucy's Periodic Shock Model

Lucy (1982) proposed a model involving a train of shock waves moving outward, spaced in time by an amount τ which turns out to be a few tens of seconds. This imparts a 'saw-tooth' structure to the flow velocity (Fig. 2). On the assumption that the cooling length is negligible compared with the shock separation $l = V\tau$, isothermal hydrodynamics is used. The shock velocity V and the shock jump U are supposed to be slow functions of r. The flow velocity in the shock frame, w, is supposed to be a strong function of the distance behind the shock front, but a slow function of r. The result is that in the accelerating frame of the shock there is a nearly-steady inward flow, with density obeying the continuity equation $\rho w \approx$ constant, and with w obeying a momentum equation that includes the outward radiation force and the inward forces of gravity and the reaction to the shock acceleration, in addition to the pressure-gradient force.

Since w must be subsonic just behind the shock, and is definitely supersonic in front of the next shock, the flow in the shock's frame must pass through a sonic point. This is possible only if the radiation force decreases inward, so that the net force is outward on the outer side of the sonic point (*i.e.*, just behind the shock), and inward on the inner side of the sonic point. This modulation of the radiation force can come only from the 'velocity shadowing' effect of the shock(s) at smaller r. In particular, the radiation force can change substantially near the sonic point $w = a$ only if the drop in V to the previous shock, $w_\dagger$, is also comparable with a. Lucy argued that stabilizing effects exist to ensure that this will be the case: A shock will 'clone sisters' if the gap to the next shock is too great, since in that case it will act like an isolated shock, which is known to be unstable. A shock will 'eat its sisters' if they get too close, since all the intervening matter will fall into the 'velocity shadow' and the outer shock will be unable to stay ahead of the inner one.

The details of the model are fairly easily worked out. The actual value of $w_\dagger$ is a parameter, which must be comparable with a. The value of the shock time interval is then determined, $\tau = w_\dagger/(V dV/dr)$. This takes values in the range 10–100 s. The shock strength and temperature are then given by

$$U \approx 300\sqrt{\frac{w_\dagger}{20}\frac{V}{2000}}\,\mathrm{km\,s^{-1}}, \quad \text{and} \quad T_1 \approx 116\frac{w_\dagger}{20}\frac{V}{2000}\,\mathrm{eV}.$$

The shock velocity V and the mean density obey the equations of steady flow, as if there were no shocks and V were the flow velocity, except that the radiation force becomes a

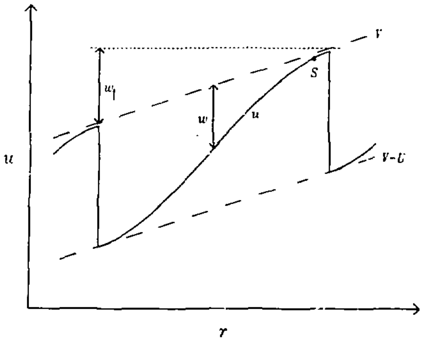

Figure 2. A small section of the velocity distribution u *vs.* r. $V(r)$ is the shock velocity; $U(r)$ is the velocity jump at the shock; $w = V - u$ is the inward flow velocity in the shock frame; $w_\dagger$ is the step in V between successive shocks. The point marked 'S' is the sonic point of the flow in the shock's frame.

suitable average over the inter-shock region. Lucy argued that the radiation force may also be about the same as it would be without shocks, so that the mean flow is in fact identical to that calculated by Castor, Abbott and Klein (1975).

In order for the x-ray output of the shocks to agree with the *Einstein* observations, T_1 should be about 300 eV, which requires $w_\dagger$ to be $60\,\mathrm{km\,s^{-1}}$, 3–4 times larger than the sound speed. It is difficult to understand why this should be the stable value of $w_\dagger$. Other possibilities were discussed by Cassinelli and Swank (1983). The shock period τ is quite short—shorter than any natural time scales of the stellar photosphere or interior—and therefore if the shocks originate from some noise source in those regions there must be considerable 'cloning' to reduce the period to the required value. And, of course, the periodic structure cannot exist if the mechanisms that stabilize $w_\dagger$ do not work.

The column density between shocks in Lucy's model is

$$N_1 \approx 5 \times 10^{20} \left(\frac{\dot{M}}{10^{-6}}\right) \left(\frac{w_\dagger}{20}\right) \left(\frac{2000}{V_\infty}\right)^2 \left(\frac{R_*}{10^{12}}\right) \mathrm{cm}^{-2}.$$

Equating this to N_{cool} for a shock speed equal to U gives

$$\dot{M} = \dot{M}_{\mathrm{min}} \approx 1 \times 10^{-8} \left(\frac{V_\infty}{2000}\right) \left(\frac{w_\dagger}{20}\right) \left(\frac{R_*}{10^{12}}\right) M_\odot\,\mathrm{y}^{-1}.$$

If $\dot{M}$ is below $\dot{M}_{\mathrm{min}}$ the shocked gas in Lucy's model never cools between shocks. Such a hot wind ($T \approx 10^6\,\mathrm{K}$) is possible, and it can still be driven by radiation, but the force is reduced by the high temperature—the driving ions are largely stripped—so less mass loss is produced for a given star than if the wind were cool.

4.2. The Krolik and Raymond 'Shell' Model

In a recent paper, Krolik and Raymond (1985) have proposed a model of radiatively-driven shocks that has some different aspects from Lucy's. They consider a single shock (although there may be others some distance away), and treat in detail the ionization, recombination and cooling behind the shock, as described in § 1.2. From the resulting structure of velocity, temperature and density of various ions, they calculate how much momentum is absorbed within the shock from the photospheric radiation field. The shock is considered to have a definite column thickness, N, which is an unknown of the problem. A simple dynamical model is then used to estimate how N and the shock velocity evolve with time.

This model is, in effect, one of a pancake-like shell of shocked gas that is confined in front by the ram pressure of the pre-shock material, and driven from the rear by a radiation pressure which is the momentum deposition calculated for the shock. This is basically an *episodic* rather than a *periodic* model. It is not unlike Lucy's model, but for two key differences: In the Krolik and Raymond model all the gas swept up by the shock remains confined in the shell, while in Lucy's model gas streams out the back of the shock to balance the gas entering at the front. Lucy's model also accounts in a more consistent way for the dynamics of the post-shock flow. The hydrodynamic boundary conditions at the back of the shell in the Krolik and Raymond model are unclear.

The numerical estimates obtained by Krolik and Raymond for the typical shock strength and x-ray emission are quite similar to the requirements of the *Einstein* data. Since the spacing of shocks is not constrained in this model, in contrast to Lucy's, the shock strength can become greater, giving the desired shock temperature.

Further work by Krolik and Raymond will account for the global dynamics of the wind including such shells of shocked gas.

5 SUMMARY

The discussion I have given above of pulsation-driven shocks and radiation-driven shocks has raised and partially answered several of the interesting questions about shocks in OB stars:

- Are shocks formed by pulsation?
  The answer is "yes" if the mass flux ρu_s of the shock is $\gg$ the radiatively-driven mass flux; otherwise the radiation force overwhelms pulsation as the cause of shocks. For the B star of § 3.2 the answer is "yes" for $P = 3$ h, and "no" for $P = 12$ h.
- Are the periodic shock dynamics unaffected by radiation pressure?
  "Yes", but only if the shock mass flux at the *adiabatic radius* is $\gg$ the radiatively-driven mass flux. This limits the radiatively-driven $\dot{M}$ to $\leq 10^{-11}\, M_\odot\, \mathrm{y}^{-1}$.
- Is there pulsation-produced mass loss, without assistance from radiation?
  Only if the radiatively-driven $\dot{M} \leq 10^{-12}\, M_\odot\, \mathrm{y}^{-1}$.
- Do radiatively-driven shocks have a sawtooth or a shell structure?
  - If Lucy's shock cloning and eating mechanisms work: a sawtooth structure results, with or without pulsation.
  - Without Lucy's mechanisms: shells result with pulsation, and no shocks result without pulsation.

Further insight into the morphology of radiatively-driven shocks awaits the detailed hydrodynamic modeling of stellar winds now in progress, such as the effort by Owocki, Rybicki and myself, some preliminary results of which are described elsewhere in these proceedings.

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