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DEVELOPMENT OF A MINICOMPUTER SYSTEM FOR ON-LINE PROCESSING OF GAMMA-GAMMA COINCIDENCE EVENTS AND MEASUREMENTS OF E2/M1 MIXING RATIOS IN ^{110}Cd AND ^{134}Ba

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(Ph. D. Thesis)

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Development of Minicomputer System for On-Line Processing of
Gamma-Gamma Coincidence Events and Measurements of E2/M1
Mixing Ratios in ^{110}Cd and ^{134}Ba

Abstract

A megachannel pulse-height analysis system using a 32,000 word PDP-8/E minicomputer and two moving-head disk memories has been developed. The system has a storage capacity of 2^{18} events at any of 2^{20} data locations, is capable of processing 1,040 events/s, and provides on-line sorting and disk storage. An X- or Y- pulse-height spectrum in coincidence with one to four arithmetically combined pulse-height windows can be assembled in core for scope display and spectral analysis within 2 to 20 seconds. The software for the system was written extensively in machine language to provide rapid and efficient data acquisition and sorting, and rapid data recovery from the disk memory.

Excellent energy and timing resolution were achieved using electronics with unique capabilities; that is two 40 cc coaxial Ge(Li) detectors, direct coupled recycling preamplifiers, pileup rejectors, and constant fraction timing discriminators. An energy resolution of 2.3 keV FWHM and a timing resolution of 8.5 ns FWHM was obtained for ^{60}Co at a singles rate of 10,000 counts/s. The prompt timing peak remained Gaussian down to the FWHM by gating the TAC with SCAs which discriminated against low-energy events. The coincidence electronics also allowed on-line subtraction of accidental coincidence events.

Alignment of the detectors and tests of the system's performance were made by measuring solid-angle correction factors for the Ge(Li)

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detectors through the correlation measurements of the $0^+-2^+-0^+$ cascade in ^{106}Pd , and by measuring the angular spread of positron annihilation radiation coincidences. These experimental results were in excellent agreement with theoretical solid-angle correction factors calculated for the detector and source sizes used. Directional correlation data were obtained for gamma-ray cascades in ^{134}Ba and ^{110}Cd . Analyses of these data gave E2/M1 mixing ratios which are in excellent agreement with results obtained by other investigators. The improved resolution and data processing capabilities of this system gave new results for the 563-keV transition in ^{134}Ba and the 1505-keV transition in ^{110}Cd . E2/M1 mixing ratios of $13.3^{+2.3}_{-1.8}$ for the 563-keV transition and -1.24 ± 0.20 for the 1505-keV transition were measured.

I. Introduction

I.1. History of Coincidence Methods

When an unstable nucleus decays, sometimes two radiations are emitted in succession. These can be nuclear α -, β -, or γ -radiations, or secondary x-ray emissions from the atomic electron shells. Measurements of the coincidences between these various radiations provide information about the nucleus from which they are emitted. In many cases, the purpose of these studies is the determination of the 'coincidence intensity', i.e., the number of transitions of one kind of nuclear radiation following or preceding the other. This information can be very useful in the construction of nuclear decay schemes. In addition, spatial or time correlations, which examine the time delay¹ between two successive nuclear radiations, can also contribute to understanding the nucleus. The first attempts at applying the coincidence method to the investigation of the characteristics of radioactive decay were made by Bothe and Geiger² in 1925.

The principle of the coincidence method is to measure the "simultaneous" emission of two types of radiation, for example a beta ray and a gamma ray, two gamma rays, or a charged particle and a gamma ray, with the help of two detectors and a coincidence circuit for measuring the simultaneity of these events to within a resolving time τ . In making these measurements, one is interested not only in the time relationship between the emission of the two radiations, but also in the energy distribution of the radiations correlated in time. In the earliest period of coincidence measurements, energy discrimination was not practical for gamma radiation. For other radiation types, the energy resolution was poor and the spectrometers available were nearly always single-channel devices. Coincidence experiments used a single scaler to record

the number of events observed in coincidence : two detectors. Although the experimental equipment assembled was simple, the procedure became very time consuming (in fact, impossible) if coincidence information was desired for the entire spectrum. The development of the coincidence technique using Geiger-Müller and proportional counters, before the advent of scintillation detectors, is described by Mitchell³ and by Mandeville and Scherb⁴.

With the advent of scintillation and semiconductor radiation detectors the coincidence method results were greatly improved. These detectors allowed energy selection by pulse-height discrimination and were readily adaptable to multichannel recording. Initially, because of the cost of multidimensional analyzers and the lack of computers to reduce multidimensional data, coincidence spectrometry was done by selecting a narrow energy interval within the pulse-height distribution from one detector. Then, pulses falling within this interval were used to gate a multichannel analyzer which recorded the pulse-height distribution from one or more additional detectors. Such a procedure was very time consuming, because a new experiment was required for each gating energy interval of interest. As the length of time required to do the experiment increases, the likelihood of calibration shifts increases. Therefore, it is desirable to collect as much data as possible in the shortest period of time.

1.2. Multiparameter Analyzers

An enormous savings in effort and an accompanying improvement in the quality of data result if all coincidence events associated with a particular experimental configuration are recorded at the same time. A system which performs this function has been called a "three-dimensional" pulse-height analyzer because it records the number of events for each

pair of coincident pulse heights. These systems are also called "two-parameter" analyzers after the two pulse heights which are correlated. The latter term is more common and will be used here. A number of approaches have been taken over the past fifteen years to provide multiparameter analysis. Many advances have been made during this time, mainly due to reductions in the costs of various pieces of hardware; but also due to the improved energy resolution offered by the lithium compensated germanium detector (denoted Ge(Li) detector).

One of the first devices developed to provide multiparameter analysis was a pulse-height analyzer with a large ferrite core memory (1961). A typical analyzer for this purpose contained a 20,000 channel memory with a storage capacity of 10^6 counts per channel.⁵ The analyzer included two analog-to-digital converters (ADC) and two address scalars, arranged so that the basic sorting and storage array was 200×100 channels. With the energy resolution available from NaI(Tl) detectors, 100 channels for a pulse-height coordinate was marginally adequate. This system did provide a distinct advantage in that it allowed rapid access to the information, so that the progress of an experiment could be evaluated and correct operation could be verified. Experiments which involve extensive setup time and require repeated source changes need the capability to assess the situation from setup to completion not just as a matter of convenience, but also as a matter of time and concomitant cost.

The improved resolution in gamma-ray spectroscopy brought on by Ge(Li) detectors soon found 20,000 channel two-parameter analyzers inadequate. Ge(Li) detectors could still be used by selecting several windows within the pulse-height spectrum of one of the detectors while storing the coincident spectra from the other detector in the 20,000

channel memory. Several spectra corresponding to different windows could be stored simultaneously with this approach. An alternative approach developed at about the same time (1961) which allowed unlimited resolution for both parameters was the buffer-tape technique.

This technique involves storing the γ - γ coincidence data on magnetic tape in the order in which the coincidence events occurred. This event-by-event storage is accomplished by storing the two energy channel numbers of a particular coincidence event in a core buffer. When one buffer region is filled; the event-pairs from that region are written sequentially onto magnetic tape while data collection continues in a second buffer region.^{6,7} One experiment may require from 10 to 50 or more 30 cm diameter data tapes in order to accumulate sufficient statistics.

After the experiment is completed, the data stored on the tapes must be sorted. The experimental information is made comprehensible to the experimenter by sorting the data in such a way as to form the spectrum from one detector in coincidence with events from the other detector which satisfy certain conditions. One method is to scan all the tapes on a computer (one tape at a time) for a limited set of windows and store the coincident spectra in core memory and repeat for each new set of windows. Accessibility of the recorded data is perhaps most important in the subsequent examination of the data; and continual end-to-end scanning of tapes in order to sort data for spectrum presentation can be extremely time consuming.

The necessity for handling, changing, and processing of tapes periodically can be eliminated by sorting and storing coincidence data on-line with a small computer equipped with a magnetic disk memory. The disk is a much more accessible storage medium during the examination

phase, since the total bulk of data is now subdivided onto tracks on the disk that can be selected electronically. The immediate availability of sorted data is an advantage in initially setting up an experiment and in monitoring it. The principle advantage, however, is in the subsequent detailed examination of the data. Such systems have been developed with fixed-head/track disks⁸ and moving-head disks⁹. These systems have been based on 18-bit/word and 24 bit/word computers, respectively, and are capable of processing 400 coincidence events/s with 2^{20} data locations.

This thesis describes a megachannel pulse-height analyzer system for on-line sorting and storage of coincidence data using a moving-head disk. The basic system includes a 32,000, 12-bit/word, PDP-8/E minicomputer; two moving-head disks, one with a removable cartridge; two analog to digital converter's (ADCs); scope display with function box controls; one line printer; and teletype.¹⁰ The system is shown diagrammatically in Figure I-1.

By coupling dynamic allocation of disk-track buffers with efficient use of the disk, this system is capable of processing 1040 coincidence events/s. The system can store up to 2^{18} coincidence events in any one of 2^{20} data locations. An X- or Y-pulse-height spectrum in coincidence with one or several arbitrary pulse-height windows can be assembled from data on the disk into the minicomputer core for scope display within 2 to 20 seconds. The system provides convenient setup and monitoring of coincidence experiments, and it allows convenient and detailed examination of the data upon completion of an experiment. These capabilities meet the demands of gamma-gamma coincidence experiments in general and directional correlation measurements in particular. The latter is described in the next section.

This system will allow 2-parameter data acquisition

- 220 18-bit/channel capacity
- Capable of processing 1040 coincidence events/sec

Figure 1-1. Diagram of computer system components.

I.3. Angular Correlation

The probability of a particle or quantum being emitted in a particular direction depends on the angle between the emission direction and the nuclear spin axis. Normally, radiation observed from a radioactive sample is isotropic because the nuclei are randomly oriented. Anisotropic distributions can be observed only if the ensemble of nuclei have been oriented or aligned. One method to achieve this, which we will be concerned with here, consists of selecting only those nuclei whose spins happen to have a preferred orientation. This method is realized if the nuclei decay through the successive emission of two radiations. Observation of the first radiation in a fixed direction selects a set of nuclei that have a preferred orientation or spin direction. Then, the succeeding coincident radiation may show an angular correlation with respect to the direction of the first radiation.

A schematic representation of a directional correlation experiment is shown at the top of Figure I-2. A nucleus with excited states I_1 and I_2 and ground state I_3 emits a gamma ray, γ_2 , which is in coincidence with γ_1 . We want to measure the relative probability, $W(\theta)d\Omega$, that γ_2 is emitted into the solid angle $d\Omega$ at an angle θ with respect to the direction of γ_1 .

Experimentally, as shown at the bottom of Figure I-2, the number of coincidences between γ_1 and γ_2 is recorded as a function of the angle ϕ subtended by the axes of the two detectors. Because of the finite solid angles subtended by the detectors A and B, the number of coincidences is an integral average of the true correlation $W(\theta)$ over small angles distributed around ϕ . Therefore, $N(\phi)$ must be corrected and normalized to give $W_{\text{exp}}(\theta)$. Finally, comparison of $W_{\text{exp}}(\theta)$ with theory

Figure I-2. Schematic representation of a directional correlation experiment and its results

provides the desired information about the properties of the nuclear levels and the radiations.

The information that can be obtained from angular correlation work depends on the type of radiation observed (α , β , γ , e^-), on the properties that are singled out by the experiment (direction, polarization, energy), and on any extranuclear fields acting on the nucleus. We assume in the above discussion that, apart from the direct interaction with the radiation field, the decaying nuclei are free, i.e., that no extranuclear fields act on the nucleus and disturb its orientation in the intermediate state. A perturbation of the angular correlation may occur, if there is an interaction between internal or external electromagnetic fields and the nuclear magnetic and electric moments during the time between formation and decay of the intermediate state of the cascade. The magnitude of the perturbation will depend on the strength of the fields and the lifetime of the intermediate state. Detailed discussions of perturbed angular correlation may be found in the works of Abragam and Pound¹¹ and Steffen and Frauenfelder.¹²

In some instances angular correlation measurements involve the observation of the linear or circular polarization of the radiation as well. Polarization, like angular momentum, is a rotational property whose measurement provides a better understanding of the nuclear processes involved. Correlation measurements also sensitive to linear polarization can determine the parity change in γ -transitions. Most γ -correlation experiments are polarization insensitive. These experiments are referred to as "directional correlation measurements", since the measured distribution is dependent only on the angle of the relative propagation directions. "Polarization correlation" is the term used to refer to observations involving the measurement of linear or circular

polarization. Both techniques fall under the general classification of "angular correlation".

The idea that two gamma rays emitted successively in a cascade might have a correlation between their directions is traditionally attributed to Dunworth¹³ and, in that same year 1940, calculations appeared by Hamilton¹⁴ in which the gamma-gamma angular correlations from a radioactive source were presented for the case of dipole and quadrupole radiation. The work was limited; however, because of the sums of large numbers of Clebsch-Gordan coefficients required to specify the angular correlations. This made the calculations appear more difficult and complicated than they need be and also limited their practical value. In 1952 Racah derived expressions for gamma-gamma angular correlations which were simplified by the Racah algebra. Racah also made use of spherical tensor operators and so the resulting theory was essentially in the same form as it remains today. Subsequently, Fano¹⁵ (1952) and Coester and Jauch¹⁶ (1953) independently introduced density matrices and tensor formalism in the angular correlation theory. The remaining history is one of refinements in experimental application and an expanding utilization of the method in nuclear studies.

1.4. Introduction to γ - γ Directional Correlation Theory

Consider a simple γ -cascade as shown in Figure I-2. A density matrix can be used to describe the spin orientation of the initial state.¹⁷ First-order perturbation theory gives the transition probability for the emission of the first γ -ray. This combined with the density matrix for the initial state, yields the density matrix for the intermediate state. Repeating this procedure for the decay of the second γ -ray to the final state yields the desired results, i.e., the probabil-

ity function for directional correlation $W(\theta)$ can be obtained. A full description of the above method is given in reference 18.

As the theory of angular correlations was developed over the years by many authors, ambiguities arose in the correlation formulae for the case of two different multipoles contributing coherently. In any gamma-ray transition, this coherence makes it possible to observe the relative signs of multipole matrix elements or, equivalently, the sign of the multipole mixing ratio. Both the magnitude and sign of this latter quantity may be compared with theoretical expectations based upon the knowledge of the structure of the levels involved in the transition. In the past a comparison of the experimentally determined mixing ratios with results of nuclear model calculations was difficult, since in most cases the measured results were not expressed in terms of explicitly defined mixing ratios. In particular, the signs of the amplitude ratios of the multiple components of the emitted radiation have been rarely defined. Furthermore, in many theoretical papers, the γ -transition operators, and hence the transition matrix elements that are used in the expressions for the angular distributions, are not explicitly given.

In this work the mixing ratio of two multipole components, $\pi'L'$ and πL , where emission operators are used and $\pi=E$ or M for electric or magnetic multipoles is given by:

$$\delta = \frac{\langle I_f || \pi'L' || I_i \rangle}{\langle I_f || \pi L || I_i \rangle}, \quad (I-1)$$

where I_i and I_f represent the initial and final states, respectively.

Note that the initial state of the transition is written on the right hand side in the reduced matrix elements. The reduced γ -emission matrix elements are defined by the Wigner-Eckart theorem:

$$\langle I_{f m_f} || (\pi L, M)^* || I_i m_i \rangle = (-)^{L-M+1} (-)^{I_f - m_f} \begin{pmatrix} I_f & L & I_i \\ -m_f & -M & m_i \end{pmatrix} \times \langle I_f || \pi L || I_i \rangle \quad (I-2)$$

The mixing ratio defined in Eq. I-1, is related to that of Rose and Brink¹⁹ by a phase term,

$$\delta = (-)^{L'-L} \delta_{RB}$$

The phase difference is due to Rose and Brink defining the mixing ratio in terms of reduced matrix elements involving absorption operators.

Note that in Rose and Brink's paper the initial state for the emission process is always written on the left side of the interaction operator.

I.5. The Directional Correlation Function

The directional correlation of two mixed multipole γ -radiations emitted according to the general scheme:

$$I_1 \xrightarrow{\gamma_1} I_2 \xrightarrow{\gamma_2} I_3 \longrightarrow \dots \longrightarrow I_n \xrightarrow{\gamma_n} I_{n+1}$$

is given by

$$W(\theta) = \sum_{k \text{ even}} B_k(\gamma_1) U_k(\gamma_2) \dots U_k(\gamma_{n-1}) A_k(\gamma_n) P_k(\cos\theta) \quad (I-3)$$

where it is assumed γ_2 through γ_{n-1} are not observed. The normalized orientation coefficient $B_k(\gamma_1)$ is given by

$$B_k(\gamma_1) = \frac{F_k(L_1 L_1 I_1 I_2) + 2(-)^{L_1 - L_2} \delta_1 F_k(L_1 L_1 I_1 I_2) + \delta_1^2 F_k(L_1 L_1 I_1 I_2)}{1 + \delta_1^2} \quad (I-4)$$

and the directional distribution coefficient $A_k(\gamma_n)$ is given by

$$A_k(\gamma_n) = \frac{F_k(L_n L_n' I_{n+1} I_n) + 2\delta_n F_k(L_n L_n' I_{n+1} I_n) + \delta_n^2 F_k(L_n L_n' I_{n+1} I_n)}{1 + \delta_n^2} \quad (I-5)$$

The F coefficients $F_k(LL'I'I)$ are defined and tabulated in the work of Frauenfelder and Steffen.²⁰ Note the phase for the multipole interference term in the orientation parameters, as well as the different order for the initial and final state spins appearing in the F coefficients in the expressions for $B_k(\gamma_1)$ and $A_k(\gamma_n)$.

The reorientation parameters $U_k(\gamma_n)$ for the n^{th} transition depend on I_n , I_{n+1} , and on the multipole intensities (but not the interference terms) of the unobserved radiation:

$$U_k(\gamma_n) = \frac{\sum_{L_n \pi_n} (-)^{L_n} \begin{Bmatrix} I_n & I_n & K \\ I_{n+1} & I_{n+1} & I_n \end{Bmatrix} \langle I_{n+1} || \pi_n L_n || I_n \rangle^2}{\sum_{L_n \pi_n} (-)^{L_n} \begin{Bmatrix} I_n & I_n & 0 \\ I_{n+1} & I_{n+1} & L_n \end{Bmatrix} \langle I_{n+1} || \pi_n L_n || I_n \rangle^2} \quad (I-6)$$

The reorientation coefficients are normalized to unity, i.e., $U_0(\gamma_n) = 1$.

If the sequence of angular momenta $I_1, \dots, I_{n+1}$ of the nuclear levels of a cascade forms a monotonic sequence, and if the angular momentum of each radiation takes only its lowest possible value, then the set of transitions $\gamma_1, \dots, \gamma_n$ are said to be pure transitions. Wenner and Hamilton²¹ have derived the following result for directional correlations involving pure transitions: The directional correlation between any two radiations in a sequence of pure transitions is independent of the number or character of the intervening unobserved transitions and of the angular momenta of the intermediate states.

A useful corollary of the above Theorem is that a directional correlation between a gamma ray of unrestricted character and a member

of a sequence of pure transitions has the same form for any of the pure transitions. In particular, the first gamma transition may be of mixed multipole order.

If the transitions are mixed, it is assumed that only the two lowest possible multipole orders contribute in each transition, i.e., for $i = 1$ to n .

$$L_i = |I_{i+1} - I_i|$$

$$= 1 \quad \text{if } I_{i+1} = I_i \quad (\text{I-7})$$

$$\text{and } L'_i = L_i + 1 \quad (\text{I-8})$$

Conservation of parity requires that the competing radiations be of different character, i.e. mixed electric and magnetic multipoles.

Therefore,

$$\pi'_i \neq \pi_i \quad (\text{I-9})$$

The directional correlation function (Eq. I-3) is generally written as:

$$W(\theta) = \sum_{k \text{ even}} A_{kk} P_k(\cos\theta) \quad (\text{I-10})$$

where the directional correlation coefficients are given by

$$A_{kk} = B_k(\gamma) U_k(\gamma_2) \dots U_k(\gamma_{n-1}) A_k(\gamma_n).$$

The maximum value of k is determined by the nonvanishing of the Clebsch-Gordan and Racah coefficients contained in the F and U coefficients. In most experiments, $k_{\max} \leq 4$.

Due to the finite size of the source and detectors, the experimentally determined directional correlation function is given by

$$W^i(\theta) = A_{00}^i + A_{22}^i P_2(\cos\theta) + A_{44}^i P_4(\cos\theta) \quad (\text{I-11})$$

from which the theoretical correlation coefficients A_{kl} can be extracted on the basis of

$$A_{kk} = \frac{A'_{kk}}{A'_{00}} \frac{1}{\bar{Q}_k(\gamma_1) \bar{Q}_k(\gamma_2)} \quad (I-12)$$

where $\bar{Q}_k(\gamma_1) = Q_k(\gamma_1)/Q_0(\gamma_1)$ are the normalized geometrical correction coefficients for the detector that observes γ_1 .

I.6. History of Directional Correlation Measurements.

The first successful directional correlation measurements by Brady and Deutsch²² [1947] were in response to the developments by Hamilton¹⁴ [1940] in directional correlation theory. They used Geiger counters to examine the γ - γ correlation following the β -decay of ^{60}Co . Their experiment was severely limited by the poor resolving time and consequently low coincidence rates, and also by the lack of any γ -ray energy resolution.

The introduction of NaI(Tl) detectors, with their moderate energy resolution, high detection efficiency, and good time definition, meant that the angular correlation experiments could be widely used for spin and multipole mixing ratio measurements. It is difficult today to realize how tedious γ - γ correlation experiments with Geiger counters were. A typical run that takes a day with scintillation detectors would require more than a thousand years of work with Geiger counters. Scintillation detectors offered three major advantages over Geiger counters: high counting efficiency, speed, and energy sensitivity. Most of the nuclear data that provided the impetus for nuclear model developments were obtained using NaI(Tl) detectors. However, their limited energy resolution of typically 60 keV at 1.33 MeV did restrict the measurements when γ -rays could not be adequately resolved or when large coincident background contributions were present.

The accuracy of angular correlation measurements were further improved by using Ge(Li) detectors. Their good energy resolution allows the vast majority of transitions to be clearly resolved with well defined full energy peaks.

Any contribution of Compton continuum background to the coincidence spectrum can, because of the good energy resolution, be assessed to within a few keV of the peak. These advantages outweigh the poor detection efficiency of Ge(Li) detectors when compared to a NaI(Tl) detector. Typically their efficiencies are 5 to 45 percent of of a 7.6×7.6 cm NaI(Tl) detector at 1.33 MeV.

In early measurements, it was customary to select peaks by using single channel analyzers. This method has been referred to as a "direct" γ - γ measurement in the literature.²³ The contribution due to background events falling within the SCA windows was measured by setting an SCA window on one side (usually the high energy side) of a peak and again recording the coincident correlation. This procedure is time consuming for one gamma ray and more so if the correlations of several γ -rays are to be studied. The simultaneous collection of the data for several γ -rays is expensive in terms of the electronic modules required. The background corrections can be done more easily by accumulating the total coincidence spectrum in a multichannel analyzer. Such a method is referred to as an "indirect" γ - γ measurement. The method may be extended to two dimensional analysis and many correlations and their backgrounds can be measured simultaneously. Once an analyzer forms part of the data storage system it is then possible to perform logic functions on the data by using the facilities that are common on most commercial analyzers. For example, gating of the add or subtract mode of an analyzer allows true or chance events to be stored appropriately thus automatically

correcting for accidental events. The extension of the above concepts to an on-line computer is a natural development and offers great advantages in data handling and analysis.

The computer system described in this thesis provides these advantages. The PDP-8/E was used in conjunction with two Ge(Li) detectors to do γ - γ directional correlation measurements. An enormous savings in effort and an accompanying improvement in the quality of the data is realized and demonstrated by the directional correlation measurements performed with this system. The determination of E2/M1 mixing ratios for transitions in selected nuclei by directional correlation measurements provided an excellent way to test the performance of the entire system. Directional correlation measurements are sensitive to the subtraction of accidental and background coincidence events as well as to the overall stability of the system. If the subtraction of accidental and/or continuum background coincidence events is done improperly, then the gated spectra may contain false lines and incorrect intensities. In simple γ - γ coincidence experiments performed for the purpose of placing transitions in a nuclear decay scheme, inaccurate subtractions cannot be easily detected and continuum coincidence peaks may result in the incorrect placement of transitions in a decay scheme. Directional correlation measurements provide a better test of the system, since inaccuracies in data handling will be reflected in the measured $W(\theta)$ dependences which are well known for particular cascades.

II. Computer System

II.1. Introduction

Among the most important concerns with any multiparameter analyzer are: data capacity, data acquisition rate, and data accessibility. To meet these concerns the ideal system would combine the storage capacity of a magnetic tape system (buffer-tape technique) and the accessibility of a core memory system. With the solid state memories now under development such a system may become economically feasible in the near future.

An improvement in data capacity is realized over core memory systems by storing the data on a magnetic disk. The disk provides a more economical form of mass storage than does magnetic core memory. The disk improves data accessibility over that provided by a magnetic tape system, since data can be organized onto tracks on the disk that can be selected electronically rather than scanning numerous tapes from end to end. The organization of the data for storage on the disk can be effectively handled by a minicomputer.

The minicomputer should have a large enough core memory, so that reasonable data acquisition rates are allowed. The size of the core memory necessary for a reasonable acquisition rate is dependent upon the speed at which the disk can accept data. The minicomputer should have an interrupt capability to allow it to "effectively" operate many different peripherals at the same time. Another requirement of the minicomputer is that it have a "direct memory access" (DMA) channel. This becomes necessary when there is a need to transfer data contained in a block of consecutive memory locations out to a high-speed peripheral device such as the disk, or to read sequential words of data from a high-speed device into a specified memory buffer area.

II.2. Hardware

A schematic diagram of the hardware assembled which meets the requirements discussed above was shown in Figure I-1. The features of each piece of hardware and what its role is in the total system are discussed in the following sections.

II.2.1. PDP-8/E

Central to the system is a $32,768(2^{15})$ word PDP-8/E. The PDP-8/E is a member of the family of PDP-8 minicomputers built by the Digital Equipment Corporation. The PDP-8 is a high-speed general purpose digital computer which operates on 12-bit binary numbers.²⁴ It is a single-address parallel machine using two's complement arithmetic. The cycle time of the random access memory is 1.2 microsecond for fetch and defer cycles without auto-indexing and 1.4 microseconds for all other cycles. The 1.2/1.4 microsecond cycle time of the PDP-8/E provides a computation rate of 385,000 additions per second. Standard features include indirect addressing and program interrupts as a function of input/output device conditions. The flexible, high capacity input/output capabilities of the PDP-8/E allow it to operate a variety of peripheral devices.

The PDP-8/E memory organization is shown in Figure II-1. The PDP-8/E memory is divided into 4096-word blocks called memory fields. The memory fields are numbered sequentially from field 0, which is the first 4096 words of memory supplied with the basic system, up to field 7, when a full 32K of memory is installed. Core locations within each field are numbered sequentially, in octal, from 0000 to 7777(4095_{10}).

One twelve-bit word is capable of addressing a maximum of 2^{12} , or 4096, unique locations. However, PDP-8 series computers use two special 3-bit registers which permit 2^{15} , or 32,768 locations to be addressed. These registers are called the "instruction field register" and "data

Figure II-1. PDP-8/E Memory Organization

field register" and must be used in conjunction with the twelve-bit address register to obtain instruction words and data words from fields other than field 0.²⁵

Each memory field is further subdivided into 32 pages of 128 words each. Memory pages are numbered sequentially, in octal, from page 0 to page 37. The computer has no way of recognizing which page of memory it is executing in, and it is not cognizant of executing across a page boundary. Memory pages represent a more or less artificial subdivision of memory that facilitates understanding the PDP-8/E memory reference instruction decoding process.

The individual bits of a PDP-8/E memory word are usually numbered for reference purposes, as shown in Figure II-1. Thus bit 0 is always the high-order bit of a memory word, while bit 11 is the low-order bit.

The PDP-8/E used here has the Extended Arithmetic Element (EAE). This is an option for the PDP-8/E which provides circuitry to perform arithmetic operations that cannot be performed directly using the basic PDP-8/E instruction set. The EAE microinstructions permit multiplication and division of unsigned integers to be performed directly. Other microinstructions perform arithmetic or logical shifts and normalization. EAE microinstructions also permit double precision numbers to be added, complemented, incremented, and stored directly. The EAE option is essentially an increase in instruction capacity. These instructions can effect a significant reduction in core requirements and program execution time by eliminating iterative coding.

II.2.2. Moving-head disks

Two Diallo Series 30 disk drives are interfaced to the PDP-8/E. The direct memory access (DMA) channel is used to transfer data to and from the disks. This type of data transfer is essentially device con-

trolled and allows for direct exchange of large quantities of data between the I/O device and core memory.

The two drives are random access, high density, rotating memories each with a movable read/write head. The rotation speed is 15,000 revolutions per minute or 40 milliseconds per revolution. The average latency for head motion is 20 milliseconds (half-revolution). The typical access time of the head from track-to-track is 10 milliseconds. The typical access time for a random average move is 70 milliseconds. The bit transfer rate of the drives is 1.562 million bits per second.

One drive has a fixed disk cartridge which is used for storing programs and data files while the other drive has a removable disk cartridge on which coincidence data are stored. The cartridges are easy to load and unload, since each disk is permanently mounted inside a protective case that automatically opens when inserted in the disk drive. Each disk is divided into 203 cylinders with two tracks per cylinder. The data are formatted on the disk in 128 word sectors, 32 sectors per track. Each track has 4096 twelve-bit words; therefore, a disk has a storage capacity of 1.6 million words (4096×406). A sector (128 words) is the minimum group of words which can be transferred to or from the disk.

When a transfer requires a head movement to the next sequential cylinder, 15 milliseconds are required for head movement, which forces a wait for sector 0. Rather than wait for a full revolution, the disk is organized so that only one half-revolution wait is required. In other words, the starting point of any one track is 180 degrees away from the starting point of the immediately adjacent track.

II.2.3. Disk Controller

A LUCIFER 8 disk controller interfaces the two disk drives to the PDP-8/E.²⁶ Seven IOT instructions are available to be used in programs

which setup and control the data transfers through the DMA channel. In addition to the normal read/write operations, the controller can initiate both a seek and restore operation. The seek operation moves the head to the desired cylinder, while the restore clears certain disk drive error conditions and moves the head to cylinder 0. After a read operation is complete, new instructions must be given within 140 microseconds to perform an operation in the next sequential sector. Similarly, 130 microseconds are available following a write operation.

II.2.4. LSI-11 Microcomputer

An LSI-11 microcomputer serves as part of the interface between the ADCs and the PDP-8/E. The LSI-11 is a sixteen-bit/word microcomputer with the speed and instruction set of a minicomputer.²⁷ More than 400 instructions make up the LSI-11's extensive instruction set. The processor contains a sixteen-bit buffered parallel input/output bus, a 4096-word random-access memory (RAM), real-time clock input, priority interrupt control logic, and other features to provide stand-alone operation. The entire processor, plus all the above mentioned features, are contained on one 21.6 by 25.4 cm printed circuit board.

The LSI-11 is very efficient for handling eight-bit characters (bytes). (Note: bytes are 8-bits in size for 16-bit computers and 6-bits in size for 12-bit computers.) This provides for rapid bit manipulation of data words. The LSI-11 provides stack processing, which makes it easy to handle structured data, subroutines, and interrupts. Fast interrupt response is achieved by the vectored interrupt interface provided with the LSI-11. Upon recognition of the interrupting device by the LSI-11, the device passes an interrupt vector to the processor which points the processor to the starting address of the interrupt service routine of the device.

These features of the LSI-11 allow rapid handling and manipulation of the coincidence data as it is passed from the ADCs to the PDP-8/E. The details of the manipulation are discussed in a later section. The LSI-11 provides flexibility in the manipulation of the data in comparison to a hardwired interface. This is the primary reason for using an LSI-11 microcomputer as part of the interface between the PDP-8/E and the ADCs.

The LSI-11 does not have external switches with which the processor can be started or halted, nor can the LSI-11 core locations be examined or loaded by external switches. These functions are implemented with microcode routines that communicate with an operator via a teletype. The ASR33 teletype shown in Figure I-1 is used to display error messages, examine core locations, and load programs into the LSI-11 core through its paper tape reader.

II.2.5. ADC - LSI-11 Interface

The electronic path between the ADCs and the LSI-11 consists of two First-In First-Out (FIFO) buffers into which the ADCs load the two digitized results of a coincidence event. The LSI-11 checks the FIFOs until a coincidence pair is found in them. The pair is immediately removed by the LSI-11 and tested against certain criteria. Once the tests have been made and necessary manipulations of the data have been performed, the LSI-11 interrupts the PDP-8.

II.2.6. LSI-11 - PDP-8/E Interface.

A twelve-bit parallel interface (DR8-EA 12-channel buffered digital I/O²⁸) is used for communication between the LSI-11 and PDP-8/E. The unit is used to transfer command functions from the PDP-8/E to the LSI-11 and data from the LSI-11 to the PDP-8/E. Ten bits of data can be passed in one transfer; the remaining two bits are used for input/output flags.

II.2.7. CRT Display

The CRT display uses a Tektronix 604 Monitor to display coincidence spectra during data acquisition and examination. The cathode-ray tube (CRT) has a 13 by 10 cm flat screen and uses electrostatic deflection. The CRT is used to display up to 1024 channels of a spectrum at a time.

The interface between the CRT and the PDP-8/E uses a microcontroller to perform the IOT instructions issued by the PDP-8/E. The microcontroller performs many operations which the PDP-8/E would normally have to be programmed to do such as plotting characters and vectors on the CRT. With this interface the PDP-8/E only has to load one or two registers and issue one IOT instruction to plot a character or vector. This allows the PDP-8/E time for other tasks and also allows the CRT display to be refreshed oftener thus reducing flicker.

The interface is capable of displaying data in the form of 4096 × 4096 dot array on the CRT screen. Under programmed control, a bright spot may be momentarily produced at any selected point in this array. Thus a series of these intensified points may be programmed to produce graphical output on the CRT. The PDP-8/E applies information to the CRT display by means of programmed IOT instructions.

II.2.8. Function Box

The function box consists of twenty pushbutton switches, an array of lights, and a joystick attached to a rheostat whose output is digitized by an analog-to-digital converter (ADC). Pressing a pushbutton or moving the joystick causes the PDP-8/E to change the CRT display in some way or to perform a function on one of the other peripheral devices. When a pushbutton on the function box is pressed, the PDP-8/E is interrupted and bits are set in the 12-bit function box register by the func-

tion box to indicate which pushbutton was pressed. The PDP-8/E services the interrupt by reading the function box register and then proceeds with the programmed action indicated by the value found in the register. The action taken by the PDP-8/E when a particular pushbutton is pressed was determined by the programmer. The lights on the function box are used to indicate what action by the PDP-8/E is in progress.

Movement of the joystick creates an analog signal which is digitized by the ADC and loaded into the function box ADC register. The CRT display program in the PDP-8/E monitors this register and moves markers in the display according to changes in the value of the register.

II.2.9. CRT Terminal

The CRT terminal is a Tele-Tec Data Screen terminal. It is a low-cost, high-speed, alphanumeric CRT display with a teletype keyboard for data entry. It provides communication between the PDP-8/E and an operator. The primary advantage of a CRT terminal is its speed in output compared to a standard Teletypewriter.

II.2.10. Line Printer

The line printer is a Centronics Model 306 printer. It is a medium speed impact printer which uses a standard 5×7 matrix for character generation. The unit prints at a rate of 100 characters per second, which is approximately one full 80-character line per second (including carriage return time). This is approximately ten times faster than the commonly used Teletypewriters (TTY ASR33).

The output speed of the line printer and CRT terminal allow more efficient use of computer time and user time particularly during program development. These devices reduce dramatically the time required to edit and debug programs.

II.3. Program Design

Three programs were developed for the operation of the PDP-8/E system. Each program provides a different mode of operation of the system. They are as follows:

- 1) MEGA8 — operates the system as a gamma-gamma coincidence analyzer;
- 2) SLICE — assembles gated spectra from the sorted coincidence data on the disk; and
- 3) SNGPA — operates the system as two 2048-channel pulse-height analyzers for the purpose of analyzing singles data from each detector.

Each of these programs is discussed separately.

II.3.1. Coincidence acquisition program — MEGA8

This program sorts the gamma-gamma coincidence data as it comes into the computer and stores the data on-line on a moving-head disk. The program's design meets the concerns of data capacity, data acquisition rate, and data accessibility. To meet these concerns special attention was given to the organization of the data on the disk. The following 12 pages describe in detail four important features of the MEGA8 program.

a) Disk layout

One disk provides 2^{20} data locations, each capable of storing 2^{18} events. The relationship between the disk locations and the values of the parameters, in this case pulse heights, is of primary importance in determining how efficiently the disk is used. (Throughout this discussion the values of the pulse heights will be denoted by X and Y.) If both detectors were calibrated, so that the full decay energy, Q, of the nucleus being studied was digitized into 1024-channels on each parameter,

then only one-half of the locations in a 1024×1024 array (2^{19} locations) would have data stored in them. The other half of the array would contain zeroes, since the two coincident pulse heights, X and Y, obtained in gamma-gamma coincidence experiments will always satisfy the condition: $X + Y \leq Q$. By digitizing one of the detectors into 2048-channels we can make use of all of the 2^{20} data locations available. Figure II-2 illustrates how the 2^{20} stored data field relates to the full data field. All real coincidence events are stored below the dashed line as long as the detectors are calibrated so that $Q = 2048$ -channels in the Y-detector and 1024-channels in the X-detector. The X, Y addresses of the coincidence events must satisfy the condition $Y/2 + X \leq 1024$ in order to be stored.

For those coincidence events which give an X-channel number greater than 511, the following algorithm is applied: $X' = 1023 - X$; $Y' = 2047 - Y$. This test on X and the transformation, if necessary, are done by the LSI-11 before the coincidence data are sent to the PDP-8/E. The data are transformed from a 1024×2048 array to a 512×2048 array. Those events within region B of Figure II-2 are stored on the disk in the area corresponding to region A. This approach uses all locations of the 512×2048 array rather than only half of a 1024×1024 array. The concern with data capacity is met effectively by providing 1024×2048 resolution with only 2^{20} data locations.

Since PDP-8 words are twelve-bits in length, data locations with eighteen-bit data capacity imply word-and-a-half manipulation. The PDP-8 computers are not byte (6 bits) oriented, consequently the updating of eighteen-bit data locations requires time consuming bit manipulation. To avoid the word-and-a-half manipulation and concomitantly improve the

Figure 11-2. Data array. Events with $X \geq 512$ are stored in region B of the 512 x 2048 array;

data acquisition rate, each data location is separated into two parts as diagrammed in Figure II-3.

The least significant twelve-bits of a data location (which can store up to 4095 events per X-Y location) reside on one disk track while the remaining most significant six-bits reside on another track. The first 256 tracks of the disk store the least significant 12-bits of each of the 2^{20} data locations while the next 128 tracks store the most significant 6-bits of each data location. One out of every 4096 events for a particular location involve the most significant 6-bits for that location in an update. When the least significant part of a data channel overflows, the location of the overflow is stored in a buffer which, when full, is emptied onto the 128 tracks used for the most significant part of each data location. This arrangement of the more active portions of the data locations into a smaller region on the disk results in less time being lost due to disk-head movement. This leads to a significant improvement in the data acquisition rate.

A disk location must be determined from the X' and Y' addresses received by the PDP-8/E from the LSI-11. A disk location consists of a disk track number and a location on that track. One approach would be as shown in Figure II-4. With this approach the nine-bits of X' determine a half-track number, P , and the eleven-bits of Y' determine a location in that half-track, Q . The tag bit shown in Figure II-4 is used to tag the X-Y coincidence as either a true or chance event. The condition of this bit is determined by the coincidence electronics which are described in Section III.4c.

This relationship between X , Y and P , Q would result in the 2048-channel Y-spectrum in coincidence with $X = 0$ (i.e. the $X = 0$ slice) being stored on the first half of track number 0, the 2046-channel $X = 1$

Figure 11-3. Partial schematic of disk layout. Tracks 0-255 contain least significant 12 bits of each data location. Tracks 256-383 contain most significant 6 bits of each data location.

slice on the second half of track number 0, etc. Each track would store two complete X slices. Since twenty milliseconds are required to read a half-track, an X-slice would be returned very quickly. However, this disk format has a major drawback, namely, the time required to read a Y-slice into core. To assemble the $Y = 0$ slice into core would require reading all 256 tracks of least significant data and the 128 tracks of most significant data, or a total of 384 tracks. This would take at least twenty seconds. This seemingly short time becomes excessive when one considers the problems of not only assembling a spectrum in coincidence with a window that is several channels wide, but also the time required for processing the background slices which must be subtracted from it. The capability to take slices quickly in either direction is desired. This is accomplished by the method shown in Figure II-5.

With this scheme the data are partially randomized, while still allowing for reasonable data-recovery times in both directions. An X-slice requires more time with this method. For example, an $X = 0$ slice requires that a total of 24 tracks be read (approximately 1 second) rather than one-half of a track (20 milliseconds) as was the case with the previous method. So there is a loss in time for an X-slice, but it is more than compensated for by the gain in time realized for a Y-slice. A $Y = 0$ slice with this method requires that only 48 tracks be read rather than 384 tracks. It takes approximately four seconds to read and assemble the $Y = 0$ slice spectrum from the 48 tracks.

The data handling methods described up to this point have been concerned with the organization of the data on the disk. These methods meet the concerns discussed at the beginning of this chapter. The program design must also deal with the handling of the data in the PDP-8/E and how the computer controls the movement of the disk-head.

Figure II-5. Relationship between X'Y' and disk location address which gives reasonable slice times (2s) for both X- and Y-slices.

Since there is only one disk-head, its use must be efficient, i.e., the time lost due to disk-head movement must be minimized.

b) PDP-8/E Organization - List Mode Processor

The coincidence events as they occur in nature correspond to random locations on the disk. To update the disk by moving the head to the appropriate location for each individual coincidence event would be inefficient, since a lot of time would be spent moving the disk-head. Therefore, an organized approach must be taken in updating the disk. In a simplified form the method used is one of rolling the data from the disk through core memory for updating and then back onto the disk in a sequential manner. The sequence is:

Read track N into buffer 1	40 msec
Read track N+1 into buffer 2 while updating buffer 1	40 msec
Write track N while updating buffer 2	40 msec
Write track N+1	40 msec
Cylinder switch	20 msec

This method has two requirements:

- 1) For efficient disposition the incoming data must be organized into lists, each corresponding to a half-track on the disk.
- 2) The time necessary to update one track is 90 milliseconds; therefore, the time required to completely update the 256 tracks which contain the least significant portion of each data location is 23 seconds. Consequently, the size of each list must be variable to allow the data belonging to track T to be saved until the disk returns to track T. Since the data is random there is no way of knowing how much data there will be for track T during the 23s, so it is necessary that the list size be variable.

A method proposed by Gonidec²⁹ and used by Sinclair et al.³⁰ for "list mode processing" the coincident events from the ADCs is used with some modifications. This method satisfies the two requirements given above by defining two memories: a "fixed memory" which satisfies the first requirement and a "free memory" which satisfies the second. Each of the memories is divided into groups of eight words each or "octets".

Figure II-6 is a schematic illustration of this method. The fixed memory is 4096 words in size. It contains 512 octets; each octet is assigned to a half-track on the disk. Each of the octets in the fixed memory consists of 2 control words and 6 data words. The free memory occupies 12,288 words of core. Each of the octets in the free memory contain one control word and 7 data words. The octets in the free memory are initially chained together as follows. Two words in core (free octet pointer) are used to point to the first free octet which in turn points to the next, etc. The free memory occupies three fields of core (see Figure II-1 for organization of PDP-8 memory). Therefore, two words are used to address the first word of any octet in the free memory. One of the words gives the absolute address of an octet in a field while the other word gives the field number.

Two buffers, each 128 words in size, are cyclically opened to the ADCs via the LSI-11 microcomputer. The LSI-11 is used to determine whether the digitized events from the ADCs, X and Y, satisfy the criterion that $Y/2 + X$ is less than 1024, and, if they do, to transform them to X' and Y' . Once this is completed, the LSI-11 interrupts the PDP-8/E and passes the two data words, X' and Y' , to the PDP-8/E. The PDP-8/E converts the two words to a half-track number, P, and a location in that half-track, Q. The resulting P-Q pair is stored in one of the buffers.

Figure II-6. Organization of fixed and free memories. D_j points to the octet that was last filled while D_k points to the octet that is currently being filled. The first control word in a fixed octet is used as either a pointer (PTR) or a counter (CTR).

When one buffer is full, the other is immediately opened to the ADCs and the list mode processor is activated to begin emptying the first.

The P-value of the P-Q pair in the ADC buffer determines the octet in the fixed memory where the processor stores the Q-value. If the first word (control word) of the fixed octet is a counter, then there exists a data word in the fixed octet in which the Q-value may be stored. If the counter reaches 6 and more data belonging to that half-track remains to be stored, then the two words which point to the next available free octet (free octet pointer) are used to form a one word pointer which replaces the counter in the first word of the fixed octet. The one word pointer is formed from the two words by using relative addresses. The pointer is reduced to 12-bits by using 3-bits to give the relative field number, and the remaining 9-bits to store the number of the octet in that field. (Since the free octets occupy three fields only 2-bits are needed to store the relative field number, however; since 3-bits are available, they are used to allow for expansion.) This pointer is used to direct the processor to the free octet currently being used for storing Q-values belonging to that half-track. The first word of the free octet is used as counter until the octet is full. Once it is full, its first word becomes a pointer to the previously filled free octet for that half-track. However, if it is the first free octet to be filled for that half-track, then its counter is set to zero to indicate that it is the first octet in a possible chain of octets. The pointer to the free octet just filled, which resides in the first word of the fixed octet, is moved to the second word of the fixed octet while the pointer to a newly acquired free octet is stored in the first word. In this way once the fixed octet is full, its first word is used to point the processor to the free octet currently being filled with data

belonging to that fixed octet (a half-track). Its second word is used to point to the beginning of the chain of free octets that are already full.

This particular feature differs from the list mode processor concept proposed by Gonidec.²⁹ He used only one word of the fixed octet as a pointer and a counter rather than two as is the case above. The first word of the fixed octet was divided into two parts. Three of the bits in the word served as a counter and the remaining bits were used as a pointer to the chain of free octets already full of data. This is impossible to do with a word size of 12-bits, since 11-bits are required for the pointer. Consequently, a change was made, so that the first two words of the fixed octet are used as described above. For computers with a larger word size (Gonidec's word size was 16-bits) this modification to the fixed octet is not necessary, since more bits per word allow direct addressing of any free octet. Because the free memory was divided into three fields, which have to be addressed separately, this modification was necessary.

The organization of the data into lists of variable size provides for a significant improvement in the rate at which data can be acquired. The rate is determined by the size of the free memory and the speed at which data can be stored on the disk. If the 16,384-word buffer space had been divided into 384 spaces of 42-word buffers, then the system could have processed 466 events/s ($42/.09s$). By allowing the track buffers to vary in size the rate would have increased to 688 events/s ($62/.09s$). By dividing the data locations into two parts, the number of buffers needed was reduced to 256, consequently increasing the buffer size. This gives the system the capability of processing 1067 events/s ($96/.09s$), disregarding the time spent for overflows. If we assume

90 ms to process an overflow/underflow, the maximum rate at which the system can process is $1042 \text{ events/s} \{96 / [0.09s + (96/4096) 0.09s]\}$.

While the ADC buffers are being emptied by the list mode processor, data transfers to and from the disk are occurring through the DMA channel. An entire track is updated before moving the head to the next track; however, only half of a track is transferred at a time. This allows the first half of a track to be updated while the second half is being read in, etc.

After a half-track has been read into a 2048-word disk buffer in core, the computer accesses the Q-values stored in the octets belonging to that half-track. The Q-values give the locations in the disk buffer which are to be incremented or decremented depending upon the condition of the tag bit for each Q-value. Once the information in the free octets has been used, the octets are returned to the chain of free octets and become available for use again. The fixed octet is readied for new Q-values. The updated disk buffer is written back onto the appropriate half-track on the disk.

The half-tracks being updated continuously contain the least significant portions of the data locations. Overflows occur during the update when the number of counts in a location exceeds 4095. (Underflows can also occur when a chance event is subtracted from a data location containing zero events. When this occurs the most significant bits must be changed accordingly.) When an overflow/underflow occurs, two words are stored in a buffer of 256 words along with a tag bit to indicate whether it was an overflow or underflow. These two words give the half-track number and the address in the half-track where the overflow/underflow occurred. Once the overflow/underflow buffer is full, the updating process is switched from updating the least significant tracks to updat-

ing the 128 tracks having the most significant part of each data location. When the list is empty, the processor resumes updating the lower 256 tracks as described.

c) Task scheduling

It is evident from the above discussions that there are many different tasks done by the PDP-8/E computer. Some of these tasks must be done when the computer is interrupted by a peripheral device, other tasks must be done when a buffer is full. Certain tasks must be done before others, so a priority must be assigned to each task. As a result it is necessary that the programming include a priority-level scheme and a program scheduler. This part of the program recognizes hardware and software requests and schedules them according to their priority. Twelve priority-levels above the background level can be accommodated by setting bit flags in single words which define the requested levels and the "active" (i.e., initiated) levels.³¹ Each level has its own stack of registers used to save parameters from that level if it is interrupted by a level of higher priority. Once the task of the higher priority level is completed, the parameters of the level interrupted are restored and processing resumes on that level.

There are three levels of priority used for hardware and three levels for software. The disk and the LSI-11 share the highest priority-level, i.e., if either device is being serviced the other cannot interrupt until the service routine is completed. When the LSI-11 interrupts, the PDP-8/E immediately goes to a routine which completes the transfer of data from the LSI-11 into a PDP-8/E buffer (ADC buffer). During this time another device cannot interrupt. Once the servicing of the LSI-11 is completed, the program scheduler checks to see if any requests have been made for processing at a level higher in priority than the level

that was active when the interrupt occurred. If there are none, the scheduler returns processing to the level the program was at when it was interrupted. If a request exists for a higher level, then the scheduler initiates that level. This is the normal path taken upon completion of a task at any level except the background level.

The completion of reading or writing one-half of a track on the disk causes the disk controller to interrupt the computer. The computer immediately starts a service routine which issues new commands to the disk controller within 33ms. This is a short enough time to allow the disk to begin operations on the next half of the track without losing a revolution. Upon completion of the service routine a request is made for the highest priority-level of software. This level of software determines the next disk operation and assembles the appropriate instructions for the next set of disk commands; and, if necessary, updates the half-track currently in the disk buffer as described earlier. Upon completion of this task the processing takes the normal return path.

The next highest priority-level is the software which is requested when an ADC buffer has been filled up by the LSI-11. This software is the list mode processor described above. If both ADC buffers should be filled up before the request for this software level is recognized, then the LSI-11 interrupt capability is disabled until a buffer is emptied.

The function box and the keyboard on the CRT terminal share the next priority-level. When one presses a pushbutton switch on the function box or a key on the CRT terminal, the task currently being performed is interrupted. A service routine is called to determine which pushbutton or key was pressed and the information is stored. Next, in the case of a function box interrupt, the service routine requests the priority-level used for the function box and CRT terminal. In the case of the

CRT terminal the service routine stores the ASCII value of the key that was pressed in a buffer and the processing which was interrupted is resumed.

The service routine issues a request for a software priority-level to process the CRT terminal input only when the carriage return-key is pressed. Unless a control character-key is pressed, then the service routine issues a request immediately. Once the scheduler is able to initiate this priority-level, the software begins to carry out the action requested by the input on either the function box or CRT terminal.

The next level of priority is assigned to the software task responsible for printing characters on the CRT terminal. Every time a task wants to print a character it must issue a request for this priority-level. The CRT terminal requires a certain amount of time to print a character on the screen, and during this time processing may be done at another level. While the CRT terminal is printing the character, the computer is doing a task of the highest requested priority. When the CRT terminal has printed the character, the terminal issues an interrupt. Once the interrupt is recognized, a service routine checks to see if another character is to be printed. If there is, then a request is made for the output software again. This process continues until the message being printed on the CRT terminal has been completed.

The lowest priority-level or the background program is the display routine. This routine is responsible for displaying the 2048-channel spectrum of those events observed in the Y-detector that were in coincidence with events in the X-detector. The routine displays 1024-channels at a time on the CRT display. The display may be switched to either half of the 2048-channel spectrum by pressing a pushbutton on the function box. The data for the display are stored in a buffer as

double-precision words, i.e., each channel of data takes two 12-bit PDP-8/E words. The display routine scales the double precision data according to a logarithmic or linear mode into 12-bit words and stores the results in a 1024 word buffer. Only 64 channels are scaled on each pass, then the routine returns to make one pass through the scaled data displaying it on the CRT display. This procedure reduces flickering on the display.

d) Display routine

The display form may be changed in a number of different ways by pressing the appropriate pushbutton on the function box. The channel boundaries of the display may be changed by moving the joystick in the appropriate direction along with pressing the correct pushbutton to select either the right or left boundary marker. If one presses both boundary marker pushbuttons and moves the joystick, then the spectrum is rolled across the display screen in the direction of the joystick. If the number of channels between the boundary markers is less than 1024, then the distance between channels may be increased (or the display expanded) by pressing the EXP pushbutton on the function box. A marker is moved across the display when the joystick is moved independently of pressing any pushbuttons. The upper right-hand corner of the display gives the number and counts of the channel where the marker is displayed. Another pushbutton causes the routine to position the marker at the channel between the boundary markers which has the largest number of counts. The display scale changes from logarithmic to linear and vice versa by pressing another pushbutton. The linear display is scaled according to the number of counts in the channel where the marker is located, or if the marker is not being displayed, then the display is scaled to the channel between the boundary markers with the largest number of counts.

The display routine along with the function box provide for detailed and convenient examination of the data. A visual inspection of the spectrum often reveals nuances which are not visible by examining a listing of channel data (A picture is worth a thousand words!). Hardcopy plots provide for visual inspection of the data, but they do not allow the close examination provided by a manipulative CRT scope display. Thus the flexibility of the CRT display described above is helpful for this system.

II.3.2. Data recovery program - SLICE

This program can assemble an X(Y) spectrum in core that is in coincidence with up to four Y(X) spectral regions (windows). These spectral regions may be added or subtracted, thus allowing any particular gamma-ray coincidence spectrum to be examined with the adjacent Compton background region(s) subtracted if desired.

Windows may be set on either the X or Y detector. To specify a window(s) on the Y-detector one types the following on the CRT terminal: SY, $\pm m_i, n_i$ where $i = 1$ up to 4. The algebraic sign indicates whether the window is to be added or subtracted to the coincidence spectrum. The value m gives the starting channel of the window, and the value n gives the last channel in the window if $n > m$. If $n < m$, then n is the number of channels in the window. To specify a window(s) on the X detector one replaces SY with SX.

The program determines from the window information which tracks need to be read into core (One can determine this information from Figure II-5). The tracks are read in a sequence that gives the most efficient use of the disk-head. The tracks are read into alternate 4096 buffers, so that while the disk is transferring a track into one buffer through the DMA channel, the program is getting data from the

appropriate locations in the other buffer. This data is algebraically stored in another 4096 word buffer which is used for storing the data of the gated spectrum. The data for the gated spectra are stored as double precision. Therefore, the 4096 word buffer is just large enough to accommodate the 2048-channel gated Y-spectrum generated for an $X = 0$ window. The extended arithmetic element (EAE) is used to do double precision addition or subtraction giving shorter processing times. The program reads in the tracks with the least significant data first, and then, the corresponding tracks with the most significant data. Simply typing an SX or SY specifies a full X(1024-channel) or Y(2048-channel) window. In this case all 384 tracks on the disk must be read, so the program reads them in consecutive order. This process requires approximately 2.5 minutes to complete. The process takes longer than the 19.2 seconds needed to read in 384 consecutive tracks, since the program cannot process all of the information stored on one track within 40 milliseconds. Consequently, there are many disk revolutions in which no data is read in.

The SLICE program uses the same display routines that are used by the MEGA8 program. However, the SLICE program has two 4096-word data buffers from which it can display data. The data are stored as double precision (2 words/channel = 2^{24} events) giving two 2048-channel spectra. As a result there are four different 1024-channel display fields for the SLICE program. The user may transfer data between the two 4096-word buffers algebraically by typing the appropriate commands on the CRT terminal. This capability allows one to add gated spectra together channel by channel or to subtract one spectrum from another to obtain differences.

Other commands may be typed on the CRT terminal which cause the program to transfer data from one of the 4096-word data buffers to a file on a disk or vice versa. The disk on the drive with the fixed disk cartridge is file structured under the computer's operating system (The PDP-8/E operating system, OS/8, is discussed in a later section), and it is on this disk that the files are stored. After the disk containing the raw coincidence data on the other drive is removed and replaced with a file-structured disk, the data files are transferred to the removable file-structured disk. The removable disk cartridge is then taken to another PDP-8 system with a magnetic tape drive. Here the data files are transferred from the disk to magnetic tape. The tapes are read by a CDC 7600 computer and stored on disk files which may be processed by the spectrum analysis code GAMANAL.³² Programs³³ were also used on the PDP-8 to do limited spectral analysis on slice spectra immediately after obtaining them.

The amount of time required to assemble a spectrum in core depends on the number of windows (≤ 4); the channels per window requested; and on X- or Y-parameter. Typically, to assemble a spectrum in core requires several seconds. Rapid recovery of a desired gate spectrum from the coincidence data is highly advantageous not only for monitoring an experiment but in subsequent data examinations by the user on the CRT display or other read-out peripherals. The capability of returning a gate spectrum very quickly gives the user the time needed to examine carefully differences in coincidence spectra due to variations in the way the windows are set. Small variations in the window settings may reveal doublets or weak transitions.³⁴ These data examination aspects are very important in gamma-gamma coincidence experiments.

II.3.3. Singles acquisition program - SNGPA

This program operates the PDP-8/E as two 2048-channel analyzers.

The singles data from the two ADCs are stored as double precision in two 4096-word data buffers. The system may also be operated as a 4096-channel analyzer by disconnecting the input to one of the ADCs and setting the gain of the other to 4096 channels. The data are displayed on the CRT display in 1024-channel fields. The operation of the system in the singles mode allows for the convenient calibration of each arm of the coincidence electronics. As discussed earlier this must be done to insure that all real coincidences are accepted and stored on the disk.

II.4. PDP-8/E Operating System

The OS/8 Operating System is a sophisticated operating system for the PDP-8/E computer. This system permits use of a wide range of peripherals and all available core up to 32,768 words. OS/8 offers a versatile Keyboard Monitor that supervises a comprehensive library of system programs. A complete description of these programs is given in the OS/8 Handbook published by the Digital Equipment Corporation.³⁵

Three major OS/8 system programs were used in the development of the programs described above. These are: Symbolic Editor, PAL8 Assembler, and Octal Debugging Technique (ODT). All of the programs developed for this work are written in machine language (PAL). The source programs were prepared on the CRT terminal using the OS/8 Symbolic Editor. The Editor is used to create and modify ASCII source files so that these files may be used as input to other system programs such as the PAL8 Assembler.

PAL8 is the assembler for the OS/8 system. PAL8 accepts source files in the PAL language and generates absolute binary files as output.

PAL8 also creates listing files which may be printed out on the line printer for hardcopy listing of the programs.

The programs were debugged with the aid of the system program ODT (Octal Debugging Technique). ODT allows one to run a program on the computer, control its execution, and make alterations to the program by typing instructions at the CRT terminal. ODT features include location examination and modification; and instruction breakpoints to return control to ODT. ODT makes no use of the program interrupt facility and is invisible to the user program.

II.5. LSI-11 Software

The MEGA8 and SNGPA PDP-8/E routines require different programs to run the LSI-11 microcomputer. These programs are stored on paper tape and are loaded into the LSI-11 through the paper tape reader on the ASR33 teletypewriter interfaced to the LSI-11. The programs were written and assembled on another LSI-11 system which had the RT11 Operating System residing on a floppy disk. The RT11 Operating System has available the same type of system programs that are found in the OS/8 Operating System for editing and assembling programs. The programs are written in the LSI-11 machine language (MACRO) and assembled using the MACRO Assembler. The absolute binary output file is punched on paper tape.

Both programs operate the LSI-11 to check the FIFO buffers and send the data found in them to the PDP-8/E. For the coincidence mode the LSI-11 tests the coincidence pair against the condition $Y/2 + X \leq 1024$. If this condition is met, then the transformations, $X' = 1023 - X$ and $Y' = 2047 - Y$, are performed by the LSI-11 if $X > 511$. After this test and transformation, if necessary, the program instructs the LSI-11 to interrupt the PDP-8/E and begins transferring data to the PDP-8/E. Along with the X-Y information that is transferred the original Y-channel

value is also transferred to update the CRT display data buffer. Up to 32 bits of information may have to be transferred to the PDP-8/E, and since only 10 bits may be transferred at one time it is necessary to carry out 4 separate transfers. After the LSI-11 has interrupted the PDP-8/E, it waits until it receives a response back from the PDP-8/E indicating that it has received the 10 bits and is ready for more. The LSI-11 then sends the next 10 bits. This process is repeated until all the bits have been transferred, and then the LSI-11 resumes checking the FIFO buffers. The interrupt mode of transfer used here is not as efficient as the DMA mode; however, lack of manpower and money prevented construction and installation of an interface capable of DMA data transfer.

For the singles mode the program instructs the LSI-11 to alternately check each FIFO. When a digitized pulse height is found in either FIFO it is removed and tagged by the LSI-11 to indicate which ADC it came from, and then transferred to the PDP-8/E. The transfer is carried out in the same manner as described above. This program also enables the livetime clock for one of the ADCs. This clock interrupts the LSI-11 every second which causes the LSI-11 to interrupt the PDP-8/E. A special transfer occurs which indicates a clock interrupt to the PDP-8/E which in turn increments a counter. The number of livetime seconds elapsed may be displayed on the CRT display.

III. Electronic Pulse-Processing Equipment

III.1. Introduction

Certainly one goal with any gamma-gamma coincidence experiment is to collect the maximum amount of data in a minimum period of time, free of any interfering effects and with as little intervention as possible by the experimenter. Such a goal is usually restricted by physical and financial limitations; however, design of the coincidence electronics was undertaken with this in mind.³⁶ A schematic block diagram of the electronic instrumentation is shown in Figure III-1. Each feature of the system will be discussed separately.

III.2. Detectors and Preamplifiers

The detectors of Figure III-1 are lithium-drifted germanium semiconductor detectors referred to as Ge(Li) detectors.³⁷ The most significant advantage of these detectors over NaI(Tl) detectors is the superior energy resolution obtained, enabling the investigation of more complicated decay schemes with far greater accuracy. This improvement is a result of the small amount of energy, nominally 3 electron-volts, required to produce a counting pulse in a Ge(Li) detector, compared to that necessary in NaI(Tl), which amounts to several hundred electron-volts; thus for a given amount of energy lost in the detector, the Ge(Li) detector produces far more counting pulses and as a result better statistics.

The gate detector is true-coaxial of 31 cm^3 active volume with a resolution of 2.1 KeV (FWHM) at 1332 keV. The other detector is true-coaxial of 36 cm^3 active volume with a resolution of 2.2 keV (FWHM) at 1332 keV. The rated efficiency of the larger detector relative to a $7.6 \text{ cm} \times 7.6 \text{ cm}$ NaI(Tl) detector is approximately 6.7% for the 1332-keV ^{60}Co γ -ray counted at a source-detector distance of 25 cm. The smaller

Figure III-1. Coincidence system for angular correlation

detector has an efficiency of approximately 5.8%. The true-coaxial design of the detectors results in the shortest and least varying pulse rise time/unit volume and hence is best for fast-timing in a broad range of γ - γ coincidence applications. The Ge(Li) detectors were mounted in vacuum-sealed cryostats and cooled to liquid nitrogen temperature by a copper cold-finger that is gravity fed from a 10 liter dewar.

Pulses from the Ge(Li) detectors are first amplified by dc-coupled recycling preamplifiers³⁸. The main feature of the preamplifiers is their capability of maintaining a high quality baseline at high singles rates. The preamplifiers require no pole-zero compensation, since each pulse produced is a step-function and upon differentiation the pulse returns exactly to the baseline. As shown in Figure III-1, the output of the preamplifier is split for simultaneous energy and timing analysis.

III.3. Pulse-height analysis (slow signal)

The process of pulse-height analysis requires amplification and shaping of the preamplifier output energy pulses to facilitate their analyses. The linear amplifier provides gaussian shaping with a 700 ns differentiating network on the input.³⁹ The unipolar output pulse has a 1.7 μ s, 10 to 90% risetime, an 8 μ s dwell time, and an excellent return to the baseline with less than 0.02% undershoot. Such a pulse shape provides optimum energy resolution.

The delay amplifier is an inverting passive delay-line. The output of the linear amplifier is negative and the ADCs require positive input, so the delay amplifier inverts the unipolar pulse and delays it by 800 ns. The delay is necessary so that the analog energy signal will arrive at the ADC at the same time as the logic gate pulse from the timing circuitry.

The baseline restorer shown in Figure III-1 does not provide baseline restoration in the normal sense,⁴⁰ i.e., it does not restore pulses quickly to the baseline when undershoot occurs. Undershoot after differentiation is insignificant with the recycling preamplifiers; however, since the system is dc-coupled, some type of baseline restoration is needed to reduce long term drift. The dc-level of the baseline may drift due to temperature changes affecting the electronics and changes in the detector leakage current. To meet these needs a "wrap around" baseline restorer was developed.⁴¹ For small changes in baseline the correction factor for the restorer is $(150)^{-1}$. For example, a 15mV change in the baseline at the restorer's input would result in a 0.1mV change at the output. The response time of the restorer is dependent upon the magnitude of the baseline shift. The larger the shift in the baseline the longer it takes for the restorer to correct the baseline.

The output signals of the baseline restorers are digitized by the analog-to-digital converters (ADCs) provided the ADCs are enabled by a logic gate pulse from the coincidence circuitry. The conditions for the gate pulse are discussed in the next section. The ADCs are Northern Scientific Analog-to-Digital Converters with 100 MHz clocks for digitizing. The Wilkinson⁴² conversion technique is used to convert the analog input signal to a digital output. Once conversion is completed, the ADCs store the digitized results in First-In-First-Out (FIFO) buffers.

The store process is carried out unless a reject signal is generated by either pileup rejector. If either pileup rejector sends out a reject pulse, then both ADCs are cleared without storing and readied for new input. The conditions under which a reject pulse is generated are discussed next.

III.4. Coincidence timing (fast signal)

a. Fast timing amplifiers

Figure III-1 shows that the timing pulses from the preamplifiers are first sent to the fast timing filter amplifiers (LLL model LEA75-1729). These modules shape and amplify the pulses for analysis by the Ortec 463 Constant Fraction Timing Discriminators and the pileup rejectors. Early in the development of the electronics system, Ortec 454 Fast Timing Filter Amplifiers were used; however, they were not able to recover quickly from the large overload pulses created by the recycling preamplifiers and they tended to become unstable when trying to enhance the rise time of the output pulses. They were replaced by a Livermore design⁴³ which overcame these problems and a significant improvement in the FWHM of the timing peak was realized. The gain of the fast timing amplifiers must be adjusted for each nuclide studied to insure that it is appropriate for the range of gamma-ray energies present. Time constants for the shaping circuits in the amplifier can also be selected. The best timing results in all cases was obtained with a 100 ns differentiating network.

b. Constant Fraction Discriminators

The Ortec 463 Constant Fraction Timing Discriminator modules operate on the "amplitude rise-time-compensation" timing technique (ARC timing). As its name implies, it compensates for the timing "walk" which results from variations in preamplifier pulse heights and rise times. The ARC timing technique generally yields better timing results than leading-edge or zero-crossover timing.

Two adjustments are necessary in the setup of the constant fraction timing modules. The timing results are very sensitive to the adjustment of the modules for "walk". Tests for this adjustment are

described in Ortec's operating and Service Manual for the Model 463 Constant Fraction Timing Discriminator.⁴⁴ The second adjustment required is the discriminator level settings. This adjustment must be made each time a change is made in the gain of the fast timing amplifiers. It was found that setting the discriminator levels just above the noise of the input signal from the fast amplifiers gave the best timing results.

c. Time-to-Amplitude Converter (TAC)

The constant fraction discriminator units generate signals which activate the start and stop of a Time-to-Amplitude Converter (TAC) (LBL model 11X5581 P-1) as shown in Figure III-1. Approximately 57 meters of RG 58C/U cable delays the stop signal for 300 ns to insure that it arrives at the TAC after the start signal and that the time difference falls within a linear range of the TAC. The TAC produces pulse amplitudes proportional to the time difference between the start and stop signals.

The TAC is gated by the output of the slow coincidence module (LLL model LEA67-6155). The slow coincidence unit generates a logic pulse whenever the logic pulses from the two single-channel analyzers (SCA) are within its resolving time. Logic pulses are generated by the SCAs, when the amplitudes of the signals from the fast amplifiers fall within the SCA windows. The output of the fast amplifier is negative and the SCA requires a positive input; therefore, an inverter module is necessary between the two. As a result the TAC is allowed to send out a pulse only when the energies of the two gamma rays in coincidence fall within the energy windows of the SCAs. This allows control over the range of pulse heights on which timing is performed. It allows discrimination against low-energy events and is useful in those cases when the low-energy part of the spectrum is of no interest. This discrimination

gives a significant improvement in the full-width at tenth-maximum (FWTM) of the peak in the TAC spectrum as shown in Fig. III-2.

Figure III-2 shows TAC spectra for various settings of the SCA lower level discriminators. As the lower level discriminators are raised from 20 keV to 240 keV a reduction of more than a factor of 2 in the FWTM of the timing peak is observed. There is also a slight improvement in the full-width at half-maximum (FWHM) of the timing peak. These data were taken with a ^{134}Ba source with an activity of 40 μCi . The detectors were 7 cm from the source and singles rates of $\sim 11,000$ counts/s and $\sim 13,000$ counts/s were observed in the two detectors. The coincidence rate varied from 95 counts/s at the lower discriminator setting to 35 counts/s at the upper setting, demonstrating that the majority of coincidences from this decay are Compton-Compton events.

The net area of the single peak in the TAC output spectrum corresponds to those events which are in "true" coincidence, that is, coincidence events which originate from the same nucleus. The flat background is a result of the accidental, or "chance" coincidences, events which originate from different nuclei, but which happen to occur within the time range of the TAC. The two SCAs following the TAC output provide a convenient means of monitoring both the true and chance coincidences. If the windows of the two SCAs are placed corresponding to the regions A and B of the TAC spectrum of Figure III-3, then window A will record the true coincidence events in the peak plus the random chance events under the peak, represented by $T + C$. Window B will accept only chance events, represented by C . If the background is truly flat, and if regions A and B are of equal widths, then $C' = C$, and the true coincidence counting rate is just the difference between the counting rates measured at A and B. In our experimental situation, it was determined

LLD = 20 keV FWHM = 10.9 ns
FWTM = 48.6 ns

LLD = 120 keV

LLD = 240 keV FWHM = 9.0 ns
FWTM = 21.0 ns

LLD = 400 keV

Figure III-2. Time-to-amplitude spectra for various SCA lower level discriminator settings

Figure III-3. Sample Time-to-Amplitude-Converter Spectrum

that the background was flat to within one percent, and thus the above procedure is valid. The SCA window widths were set equal to the FWTM of the timing peak which was typically about 25 ns. The width of this window is usually referred to as the resolving time of the coincidence electronics. The number of chance counts observed is directly proportional to the resolving time.⁴⁵ Thus the discrimination against low-energy events which decreases the FWTM of the timing peak, results in a reduction of the number of chance events observed, and improves the true-to-chance ratio.

The logic pulses, which occur whenever a TAC pulse falls within one of the two SCA windows, A or B, are sent to a true/chance logic module (LLL model LEA67-6158). This unit sends a logic pulse to a gate and delay module (LLL model LEA67-6155), when it receives a logic pulse from either SCA indicating that a coincidence event (true or chance) has occurred. The gate and delay unit stretches the logic pulse before sending it to the pileup rejectors. If the true/chance logic unit receives a logic pulse from the SCA with window B, it sets a bit in the ADC/LSI-11 interface to indicate that the digitized coincidence pair coming from the ADCs is to be tagged as a chance event.

d. Pileup Rejector

The output of the fast amplifiers is also sent to the pileup rejectors as indicated by Figure III-1. The pileup rejectors inspect all pulses from the fast amplifiers for leading and trailing-edge pileup as well as for slow risetime. A pulse with slow risetime may be due to a gamma ray which has created hole-electron pairs in region of the Ge(Li) detector where the electric field strength is low. A piled-up signal occurs when two events are observed in the same detector within

the resolving time of the linear amplifier resulting in a pulse with an amplitude that is not representative of either event.

Once the pileup rejectors receive the logic signals from the gate and delay unit, they must determine whether to enable the linear gates of the ADCs. Enabling the linear gates of the ADCs allows the energy signals of the coincidence to be accepted for conversion. The pileup rejectors will not enable the linear gates if either of the following two conditions are met:

- 1) either energy signal has trailing-edge pileup
- 2) either ADC is busy.

If either of these conditions exist, then the linear gates are not opened and the coincidence event is not processed by the ADCs.

Figure III-1 shows that the busy signal from either ADC is sent to an OR gate. The OR gate sends the busy signal from either ADC to both pileup rejectors. For any coincidence event one ADC may finish digitizing and clear its busy signal before the other. To prevent either ADC from processing another coincidence event before the other ADC is finished with the previous event it is necessary that the pileup rejectors be controlled by the busy signals from both ADCs. This must be done to insure that the digitized results in the two FIFO buffers remain matched.

Even though the pileup rejectors have enabled the linear gates of the ADCs, either pileup rejector may determine that the signal it is inspecting has leading edge pileup or slow risetime. If this occurs the pileup rejector sends out a reject pulse to the reject OR gate as shown in Figure III-1. Consequently, if either pileup rejector rejects an energy signal, both energy signals of the coincidence event are rejected. If only one or the other were rejected, then the digitized results in the FIFO buffers would become unmatched.

The pileup rejectors require a stretched logic signal from the gate and delay unit, so that they can keep the ADC linear gates enabled until the energy signals have reached their peak amplitude. Once peak amplitude is reached, an ADC returns a busy signal to both pileup rejectors which cause the pileup rejectors to disable the linear gates.

After an ADC completes digitizing the energy signal, it stores the digital result in its corresponding FIFO buffer (see Section II.2.5.) and clears the busy signal. The two 64 word FIFO buffers are part of the interface between the ADCs and the LSI-11 microcomputer. When the store process occurs, the interface examines the true/chance logic level and tags the word being stored appropriately. Once the ADCs have stored their digital results, the interface resets the true/chance logic readying it for the next coincidence event. When a reject occurs nothing is stored in the FIFO buffers, and so, the true/chance logic is not reset by the interface. In these cases the reject signal from either pileup rejector is used to reset the true/chance logic.

IV. Directional Correlation Procedure

IV.1. Experimental Setup

Two approximately 40 cc coaxial Ge(Li) detectors are mounted on rack-and-pinion mechanisms which are fixed to aluminum arms. The arms ride on two 1.25 cm bearings which allow the detectors to be moved about the center of a 90 cm diameter aluminum table. The table top is mounted on five-cm angle iron legs which give the table surface a height of 100 cm. Threaded screws are attached to the legs to facilitate leveling of the table surface. Angular positions on the table are marked at every degree, with zero degrees defined as the location of the fixed detector; although either detector could be placed at any angle. A photograph of the detectors on the table surface is shown in Figure IV-1. Various features seen in the photograph are discussed in the following paragraphs.

The rack-and-pinion mechanisms permit the user to adjust the source-to-detector distance. Scales and pointers are mounted on the sides of these mechanisms and when calibrated allow direct reading of the distance between the front surface of the detector and the center of the table. The scales were calibrated by accurately measuring the distance between the table center and the front of the detector chamber and adding the result to the distance between the front of the detector chamber and the detector surface. The distance between the front detector surface and the front of the detector chamber was determined from x-ray images of the detector within its chamber. Both horizontal and vertical views were taken and measurements made on the black-and-white contact prints. Objects in the x-rays whose dimensions were known were used for calibration. The distances measured were determined to be accurate to ± 0.2 mm.

Figure IV-1. Angular correlation table and detectors with the computer system visible in the background.

Pointers, which represented extensions of the axis of each detector out to a distance of 10 cm from the front detector surface, were constructed using measurements from the x-ray images. These devices aided in the alignment of the detectors and in the vertical positioning of the source.

Measurements from the x-ray images were also used in designing lead cones which surround the detectors to reduce the Compton scattering between the detectors. The most troublesome type of Compton scattering is when a gamma ray enters one detector, loses some of its energy through a Compton interaction, and then scatters into the other detector. This is electronically indistinguishable from a true coincidence event. Such events may strongly influence a directional correlation, if the two energies deposited are identical with two transitions in the nucleus being studied. Each lead cone is ~4 millimeters thick which significantly reduces the Compton scatter problem. The conical entrance window in each lead cone is designed to allow all gamma rays from the source to intercept the detector for source-to-detector distances of 10 cm or less.

The aluminum arms supporting the detectors pivot about a 2.0 cm diameter aluminum shaft located at the center of the table. The center of the shaft is drilled out to a 1.27 cm diameter. This hole is used to insert a source holder which is an acrylic rod 1.27 cm in diameter and approximately 11.4 cm long. The center of this rod at one end has a 3.0 mm diameter by 5.0 cm long hole into which an 8 cm long glass rod is inserted. The glass rod has a very thin-walled, nearly spherical glass ampoule attached on the top of it. The glass ampoules were blown from glass tubing and attached to the glass rods at the glass shop at the Lawrence Livermore Laboratory. A photograph of these items is shown in Figure IV-2.

Figure IV-2. Photograph of the protective cap (left), spherical glass ampoule (center), and ampoule in the acrylic source container (right).

Liquid sources were used in all experiments. The sources were made by placing a measured quantity of the appropriate liquid activity in a spherical ampoule. The outer diameters of the ampoules used were approximately 5 millimeters; and the total volume contained in the ampoules was approximately 50 ± 2 microliters. The volume of each ampoule was determined by measuring the volume of water required to fill it. The appropriate amount of liquid activity was pipetted out and then concentrated or diluted to the volume required to fill the ampoule. This volume of liquid was loaded into a 26-gauge hypodermic needle with the aid of a 1/4-cc syringe. The liquid was dispensed into the ampoule by inserting the needle through the glass tube opening of the ampoule. The dispensing process must be done slowly and carefully to insure that the ampoule is completely filled and to avoid getting liquid into the stem or creating bubbles in the cavity. After the ampoule was loaded, it was sealed to prevent evaporation by placing epoxy in the stem opening. This was done for all of the sources except the ^{106}Ru source which is reduced by organics. In this case the ampoule was sealed by melting the top of the stem shut with a small torch. The source itself was placed in an ice bath to remove the heat and reduce the chances of boiling the liquid. All radioactive solutions were handled in a ventilated hood.

Once the ampoule is sealed, the glass rod supporting the ampoule is inserted into an acrylic rod. The outside top of the acrylic rod is threaded, so that a thin-walled acrylic cover may be fastened to the rod. This cover protects the glass ampoule from physical damage and captures the liquid activity in case the ampoule were to break.

The acrylic rod with the source in place is inserted into the aluminum shaft at the center of the table and centered vertically by

aligning the geometrical center of the liquid source with the projected cylinder axes of the detectors as indicated by the pointers.

The vertical centering of the source is less critical than the centering of the two detectors horizontally with respect to the center of the table. Both detectors were also centered horizontally with the aid of the pointers. The criterion for source centering was established as no more than a 1% variation in the singles counting rate as the movable detector is moved from 90° to 270° . The above centering procedure insured that the source was nearly centered relative to the movable detector. It was not certain; however, that the detector axes were colinear. In order to establish this fact, it would be necessary to scan the detectors with a collimated beam of photons. This is tedious and, in fact, there is a quicker way to determine axial colinearity. One can measure the sharply-peaked angular distribution of the radiation produced by positron annihilation. Since the two 511-keV photons produced are emitted in opposite directions, a coincidence experiment using a point source and point detectors would yield a delta-function distribution peaked at 180° . However, the finite angular resolution of real detectors causes the distribution to spread, with the center of the distribution remaining at 180° . This spread can be used to measure the detector angular resolution (discussed in a later section) as well as to test detector alignment.

The positrons were obtained from the decay of ^{22}Na , which was used in the form of NaCl dissolved in 0.5N HCl . Approximately 100 microcuries of activity in solution sufficient to fill one of the spherical glass ampoules was loaded and the ampoule sealed. The source was inserted in an acrylic rod and mounted at the center of the table. The acrylic

cover was not used during this measurement to remove any geometrical effects which might be caused by positron annihilation in or radiation scattered by the acrylic cover.

The centering of the movable detector with respect to the ^{22}Na source was first determined by the singles counting rates. A coincidence experiment was then performed between the two 511-keV photons, and the coincidence counting rates were measured at three symmetric positions on either side of 180° , normally 165° and 195° , 170° and 190° , and 175° and 185° . The movable detector was moved laterally on its mounting bracket until the coincidence rates between these pairs of angles were equal to within normal statistical counting errors. In this situation, the axes of the two detectors are aligned and their centers of rotation are at the source. In making the lateral adjustments to the detector, it was never necessary to move the detector by more than 0.5 millimeter; however, even this small of an adjustment altered the measured counting rates by as much as 50% at the 165° and 195° settings.

The total activity loaded for each source was adjusted to give the largest counting rates possible without seriously degrading energy resolution, and without causing too small a ratio of true coincidences to chance coincidences (true-to-chance ratio). It is known ⁴⁶ that this ratio is inversely proportional to the strength of the source (as well as to the coincidence resolving time). Thus an increase in the source strength causes a decrease in the true-to-chance ratio, and the number of chance coincidences increases faster than the number of true coincidences. It was decided to keep the true-to-chance ratio greater than 10:1. This required source activities less than 100 microcuries for the resolving times employed in this work. An additional upper limit on the

source strength was set by the effect of high count rates on the detector resolution, but no sources were used of high enough activity for this to be a problem.

IV.2. Experimental procedure

A directional correlation experiment is basically a series of coincidence measurements made at several angles. Data were obtained at the nine angles, 90° , 112.5° , 135° , 157.5° , 180° , 212.5° , 225° , 247.5° , and 270° . The measurement at 180° was performed twice, so ten measurements were made for each directional correlation experiment. The measurement at each angle usually had a duration of 22 hours. The number of total coincidence events collected at each angle varied between one and three million. The coincidence data for each angle were stored on a separate disk cartridge. Also the singles count rates of both detectors were recorded at each angle.

The window widths for the TAC SCAs (see Figure III-3) were adjusted for each measurement. The best true-to-chance ratio could be obtained by accepting only a very narrow region near the center of the TAC peak; however, a large number of true coincidences would be rejected in this case. It was decided to set the window just wide enough to accept the full TAC peak at its base (approximately full-width at tenth-maximum). This corresponds to about 92% of all true events (assuming a Gaussian distribution).

IV.3. Data Analysis

The coincidence data accumulated at each angle were treated as follows. A gross coincidence Y-spectrum was obtained by taking a full X-slice using the program SLICE. Examination of the 2048-channel gross coincidence Y-spectrum determined the channel number locations where windows would be set on peaks of interest.

The window on a peak of interest generally includes a distribution due to Compton-scattered events from higher energy gamma rays. These Compton events may also be in coincidence with the transition of interest, but, in general, will have a different angular distribution from the cascade under investigation. Thus these Compton distribution coincidence events must be removed by choosing a window on either side (or perhaps both sides) of the peak of interest and subtracting the resulting spectrum from the spectrum generated by the peak window. This is a simple procedure when the Compton distribution is flat in the region of the peak, and when the peak of interest is sufficiently isolated so that the distribution is accessible. These two conditions are not always in practice.

Figure IV-3a shows the 605-keV peak and the surrounding Compton distribution in the gross coincidence spectrum of ^{134}Ba . The continuum beneath the 605-keV peak is not flat, i.e., the 605-keV peak happens to lie just at the Compton edge of the strong 795-keV transition.

Figure IV-3b shows the spectrum in coincidence with the 605-keV transition without any correction for the Compton distribution. The 605-keV transition itself is strongly present in the spectrum, due to the presence in the 605-keV window of Compton scattered events from higher energy transitions in coincidence with the 605-keV transition. To correct for the Compton distribution when it is not flat one must select background windows on both sides of the peak with appropriate channel combinations which most correctly represent the Compton distribution beneath the peak.

The coincidence system described here, which returns gated spectra in very short times, gives one the ability to select and examine many variations in the background windows without spending a lot of time.

Figure IV-3. ^{134}Ba Coincidence Spectra.

One can examine many fore-aft background window combinations to determine which one gives the "best" results. ("Best" usually implies that considerable knowledge of the decay scheme already exists.) The inset Figure IV-3a shows the windows used for the 605-keV transition. The window for the 605-keV peak is 6 channels wide; while two background windows, each 3 channels wide are taken above and below the peak, and then subtracted from the peak window. The X-spectra in coincidence with these three Y-windows were arithmetically assembled into core by the SLICE program within about 4 seconds. The resulting spectrum is shown in part (c) of Figure IV-3. The input given to the SLICE program to generate this spectrum was the following:

SY,598,6,-594,3,-605,3

(See description of the SLICE program, Section II.3.2., for a more detailed explanation). Note that the 605-keV and 1168-keV ground-state transitions are absent in this spectrum as they should be.

For every cascade studied, windows were set on each of the transitions involved in the cascade. Figure IV-4 illustrates the information obtained from each window. The two gamma rays, γ_1 and γ_2 shown in Figure IV-4, are in coincidence with each other. A spectrum of events observed in the X-detector in coincidence with γ_1 in the Y-detector is obtained by setting a Y-window on γ_1 . The X-spectrum contains a γ_2 peak (shaded) from which the number of coincidences observed between γ_1 and γ_2 can be determined (for simplicity the background windows are not included in this discussion). Setting a window on γ_2 observed in the Y-detector gives an X-spectrum with γ_1 in it (dotted area). From this spectrum another value can be determined for the number of coincidences observed between γ_1 and γ_2 . These two values will not agree, since the two detectors, X and Y, have different efficiencies. However, the

Figure IV-4. Spectral information obtained by slicing on both γ_1 and γ_2 .

variation of these values with angle should agree, i.e., the directional correlation coefficients, A_{22} and A_{44} , for the two different windows on the same cascade should agree within statistics.

This fact provides a method by which one can check the internal consistency of the data. The megachannel analyzer makes this method easy to use, since for each measurement the data for both windows is stored on the same disk. It is only a matter of seconds to obtain the coincidence spectrum for each window, thus little extra effort is required to use this method. In the tables listing the directional correlation coefficients for each cascade there will always be two entries. An entry for the results obtained from each window used in studying the cascade.

The gated spectra have already been corrected for chance coincidence events, since the data is stored with chance events subtracted. The next step in the analysis consisted of integrating the peak intensities above the background in the corrected coincidence spectra. The integrated intensities were computed in either of two ways-by a simple summing of the number of counts in each peak channel or-by fitting the peak using the code GAMANAL⁴⁷. The code uses a Gaussian with an exponential leading edge on a modified step-function background to describe the peak shape. Singles spectra with strong, single peaks were used to define the FWHM and tailing parameters for GAMANAL as a function of energy. The code searches a spectrum, locates peaks or peak groupings, and fits the data with the predetermined shape function to yield peak centroids and areas. Both methods were employed on the ¹³⁴Ba data; however, the summing method provided a better measure of the true intensity because the gated spectrum returned for any Y-slice has only 1024-channel resolution which results in peaks being at most three

channels wide. Fitting a Gaussian to such peaks can introduce systematic errors. In general, the peak area analysis was done without the aid of the computer, because of the great variations in peak intensities, background corrections, and peak location encountered in the measurements. Performing the calculations by hand also gave a better "hands on" feeling for the data.

Normalization of the net peak areas for variations in the singles counting rates was done to eliminate such effects as imprecision in centering the source and decay of the source. The variation in the singles rate for the movable detector due to miscentering was always less than 0.6%. Decay effects on the singles rate were less than 3%, since all measurements involved activities with relatively long half-lives compared to the 10 days required for a directional correlation study.

Each directional correlation study involved ten measurements, two at 180° and one at each of nine other angles. At each angle a coincidence count corrected for Compton and chance coincidences, source miscentering, and decay was determined for each of the transitions observed in the gates taken. Weighted averages of the results for each pair of angles symmetric about 180° were computed, and the probable error of that average was obtained by assuming that the individual peak areas were governed by Poisson statistics. Thus the probable error of a given measured peak area was just the square root of the number of counts. The results for each pair of angles symmetric about 180° should be equal within their statistical error. This fact allows one to again check the data for errors due to incorrect peak area determinations or incorrect normalizations for source miscentering. Differences could also arise from fluctuation or misadjustments in the electronics.

These weighted averages and their probable errors constituted the end result of the data reduction process. The final number for the probable error in the coincidence counts reflected mainly the statistical fluctuations of the data. Other errors such as those in the normalization factors were also folded into the calculations, but had negligible effects on the final results.

Once the reduced data and probable errors at each angle are obtained, that data is analyzed to determine the angular correlation coefficients A_{kk} and the mixing ratio δ . The data were analyzed to determine the mixing ratio directly from χ^2 versus δ plots.

The final results for all the measured cascades were fitted by a weighted least-squares procedure to an expression of the form

$$W(\theta) = b_0 + b_2 P_2(\cos\theta) + b_4 P_4(\cos\theta) \quad (\text{IV-1})$$

where $W(\theta)$ is the corrected number of coincidence counts accumulated at the angle θ and $P_i(\cos\theta)$ is the Legendre polynomial of degree i . This equation can be simplified to

$$W(\theta) = \sum_{j \text{ even}} b_j P_j(\cos\theta) \quad (\text{IV-2})$$

The method of least squares requires that we minimize χ^2 , a measure of the goodness of fit to the data. With the function of Eq. IV-2 the definition of χ^2 becomes

$$\chi^2 = \sum_{i=1}^N \left\{ \frac{1}{\sigma_i^2} [W_i - W(\theta_i)]^2 \right\} \quad (\text{IV-3})$$

where the sum over i runs over the N data points and σ_i^2 is the variance on the i -th data point. We can express the determining equations for the method of least squares, which minimize χ^2 , as a set of $(n/2)+1$ equations

$$\frac{\partial}{\partial b_k} \sum_{i=1}^N \frac{1}{\sigma_i^2} \left\{ w_i - \sum_{j=0}^n [b_j P_j(\theta_i)] \right\}^2 = 0 \quad (\text{IV-4})$$

where the sum over even j runs from $j=0$ to n ; and k takes on all even values from 0 to n . This set of $(n/2)+1$ simultaneous equations can be re-expressed as

$$\sum_{i=1}^N \left[\frac{1}{\sigma_i^2} w_i P_k(\theta_i) \right] = \sum_{j=0}^n \left\{ b_j \sum_{k=0}^n \left[\frac{1}{\sigma_i^2} x_j(\theta_i) x_k(\theta_i) \right] \right\} \quad (\text{IV-5})$$

for all even $k=0$ to n . The solution of these equations yields the values for coefficients for which χ^2 is a minimum. Equations (IV-5) can be expressed in matrix form as the equivalence between a row matrix β and the product of the coefficient matrix b and a symmetric matrix α ⁴⁸

$$\beta_k = \sum_{j=0}^n b_j \alpha_{jk} \quad k = 0, n \text{ (even)} \quad (\text{IV-6})$$

$$\beta = b\alpha$$

where β and α are given in Eq. (IV-5). The symmetric matrix α is called the curvature matrix because of its relationship to the curvature of χ^2 in coefficient space. If we multiply both sides of Eq. (IV-6) by another symmetric matrix ϵ , which is equal to the inverse of the curvature matrix $\epsilon = \alpha^{-1}$, the result gives a solution for the coefficient matrix a directly since $\alpha\alpha^{-1} = 1$.

$$b = \beta\alpha^{-1} = \beta\epsilon \quad (\text{IV-7})$$

Note that since the matrix α is diagonally symmetric, that is,

$\alpha_{jk} = \alpha_{kj}$, the inverse matrix is equally symmetric: $\epsilon_{jk} = \epsilon_{kj}$. An

analytical expression for the elements of the inverse cannot be expressed simply. The matrix ϵ must be determined using standard matrix inversion routines.

The standard deviation σ_{b_j} for the uncertainty of determination of any coefficient b_j is the root sum square of the products of the standard deviation of each data point σ_i multiplied by the effect that data point has on the determination of the coefficient b_j .

$$\sigma_{b_j}^2 = \sum_{i=1}^N \left[\sigma_i^2 \left(\frac{\partial b_j}{\partial W_i} \right)^2 \right] \quad (\text{IV-8})$$

Carrying out the algebra this yields

$$\sigma_{b_j}^2 = \epsilon_{jj} \quad (\text{IV-9})$$

The inverse matrix $\epsilon = \alpha^{-1}$ is called the error matrix because it contains most of the information needed to estimate the errors. A computer program ANGCOR⁴⁹ using the mathematical methods just described, was used on a CDC 7600 to fit all experimental angular correlation data reported in this paper.

The experimental expression can also be written in the general form

$$W(\theta) = 1 + A_{22}' P_2(\cos\theta) + A_{44}' P_4(\cos\theta) \quad (\text{IV-10})$$

where $A_{kk}' = b_k/b_0$. The coefficients are composed of two factors, one for each gamma ray in the cascade, i.e.

$$A_{kk}' = B_k(\gamma_1) A_k(\gamma_2) \quad (\text{IV-11})$$

(See Equations I-4 and I-5)

Before comparison with theoretical values the experimental correlation coefficients were corrected for the effect of the finite solid angles subtended by the two detectors as follows

$$A_{kk}' = A_{kk}/Q_{kk} \quad (\text{IV-12})$$

(The determination of the solid-angle correction factors are discussed in the next section).

The analysis of the spins and mixing ratios was performed by the ANGCOR code directly on the measured correlation rather than indirectly on the A_{kk} values obtained in the above mentioned fit. The method consists of comparing, for a given spin sequence and for given mixing ratios, the predicted and measured correlations by χ^2 analyses.

As part of the input to the ANGCOR program, the user enters the range of spins possible for each nuclear level involved in the cascade. For all possible spin sequences entered, the most likely mixing ratios are evaluated by a χ^2 variation technique. χ^2 is calculated by comparing the input data with normalized calculated data for 49 possible values of each mixing ratio. The calculated data is determined by evaluating $B_k(\gamma_1)$ and $A_k(\gamma_2)$, which are functions of the mixing ratios δ_1 and δ_2 , respectively (See Equations I-4 and I-5). These values, $B_k(\gamma_1)$ and $A_k(\gamma_2)$, determine the A_{kk} 's (Eq. IV-11) which after correction for geometric factors, $A'_{kk} = A_{kk} Q_{kk}$, determine the calculated data

$$W_{\text{calc}}(\theta) = \sum_{k=0}^4 (\text{even}) b_0 A'_{kk} P_k(\cos\theta) \quad (\text{IV-13})$$

where b_0 is the normalization factor used in Eq. IV-10 to give $A_{00} = 1$. The value b_0 normalizes the calculated correlation to the raw correlation data. χ^2 is calculated by evaluating

$$\chi^2 = \frac{\sum_i [W(\theta_i) - W_{\text{calc}}(\theta_i)]^2}{\sigma^2(\theta_i)} \quad (\text{IV-14})$$

where i is the sum over the angles θ .

Results of this analysis are printed out as a 49 by 49 grid with only values of χ^2 less than 10 printed. Values of χ^2 greater than 10 are left blank. The χ^2 grid is also plotted against $\arctan(\delta_1)$ and

$\arctan(\delta_2)$ in the form of a contour map. The $\arctan$ function provides a convenient mapping, since for $\delta = \pm\infty$, $\arctan(\delta)$ is restricted to $\pm 0.48\pi$. This results in values between the limits $\delta = \pm 15.9$ or 99.6% of the $L + 1$ multipole. These outputs allow the user to determine visually the minimum values of χ^2 and qualitatively determine the most likely values for the mixing ratios.

In some cases χ^2 may vary rather sharply with the mixing ratio. As a result the best minimum may not be revealed because of the coarseness of the 49 by 49 grid. The ANGCR program has an option whereby the user may magnify a section of the χ^2 grid in either dimension or both. By requesting a particular value along either mixing ratio axis, the program will search along that slice for the minimum χ^2 and expand the immediate region. Also the user may specify to the program a value on that slice at which he wants the immediate region magnified. The slices of the χ^2 grid are presented both as plots and printouts. Examples of this analysis are given in Section V.

The quantity χ^2 besides allowing a determination of the best values for the mixing ratios δ_1 and δ_2 may also be used to determine the errors in the best values $\vec{\delta}^0$ of $\vec{\delta}$ (Note: the values δ_1 and δ_2 define a vector in the $\delta_1\delta_2$ plane).⁵⁰ If χ^2 is a minimum with respect to δ_k along with the rest of the variables $\vec{\delta}$, it follows that the variation of χ^2 may be written as

$$\chi^2(\vec{\delta}) = \chi^2(\vec{\delta}^0) + 1/2 \frac{\partial^2 \chi^2}{\partial \delta_k^2} (\delta_k - \delta_k^0)^2 + \dots \quad (\text{IV-15})$$

and so χ^2 is locally parabolic with respect to small variations in any one parameter. The correct way to proceed in using $\chi^2(\vec{\delta})$ in determining the standard deviation of the variable δ_k is to examine the variations

in $\chi^2(\vec{\delta})$ around the point $\chi^2(\vec{\delta}^0)$ with respect to δ_k , but at each point the remaining variables must be adjusted in order to obtain a local minimum.⁵¹ The curve obtained will be locally parabolic and the quantity $\chi^2(\vec{\delta}) - \chi^2(\vec{\delta}^0)$ will follow a chi-square distribution with one degree of freedom. Applying the statistical F-test then one gets

$$F(1, n-r, 1-\beta) = \frac{\chi^2(\vec{\delta}) - \chi^2(\vec{\delta}^0)}{\chi^2(\vec{\delta}^0)/(n-r)} \quad (\text{IV-16})$$

where β is the confidence level corresponding to the value δ_k and $n-r$ is the number of degrees of freedom. The value of $\beta = 0.317$ corresponds to one standard deviation. If the value of δ_k is chosen to correspond to $\delta_k^0 \pm \sigma_k$ where σ_k is the standard deviation on δ_k^0 then

$$\left[\frac{\chi^2(\vec{\delta})}{\chi^2(\vec{\delta}^0)} \right]_{\delta_k} = 1 + \frac{F(1, n-r, 1-\beta)}{(n-r)} \quad (\text{IV-17})$$

$$\delta_k = \delta_k^0 \pm \sigma_k$$

Values of F are tabulated in reference 52. Equation IV-17 can be used to determine the errors in any of the δ parameters from the sharpness of the χ^2 function.

A number of the correlations measured involved unobserved intermediate transitions for which the data analyses were somewhat more complex. A computer code GGDC developed by Lent⁵³ was used to analyze these correlations. This code also analyzes the correlation data directly for the mixing ratio by evaluating the reduced chi-square for all possible values of the mixing ratio. The output from this code is similar to the output of the ANGCR code except the χ^2 values given by the GGDC code are reduced, i.e., $\chi^2_{\nu}(\text{reduced}) = \chi^2/\nu$ where ν is the number of degrees of freedom.

The GGDC code also allows the analysis to be extended to consider simultaneously those correlations which involve the same transition.

Generally, this restricts the permissible ranges of the unknown deltas so as to more easily define the possible solutions. Examples of this form of analysis are also given in Section V.

The χ^2 results were used to determine which mixing ratios were acceptable. $P_Y(\chi^2)$ gives the probability that a random set of data (with γ degrees of freedom) when compared to the parent function would yield a value of chi-squared as large or larger than χ^2 , i.e.

$$P_Y(\chi^2) = \int_{\chi^2}^{\infty} p(\gamma, \chi^2) d(\chi^2)$$

where $p(\gamma, \chi^2)$ is the probability distribution of chi-squared. Thus we define $\chi_Y^2(p)$ as the value of χ_Y^2 such that $P_Y(\chi^2) = p$, i.e., the probability of obtaining an experimental value of the reduced chi-squared as large or larger than $\chi_Y^2(p)$ is p . If $\chi_Y^2(A_{kk}) \leq \chi_Y^2(p = 0.1)$, then the fitting function (the measured correlation) is a good approximation to the parent function (Eq. IV.10 with the appropriate A_{kk} values). The criterion that $\chi_Y^2(\delta) \leq \chi_Y^2(p = 0.1)$ determined which mixing ratios had acceptable values. If $\chi_Y^2(A_{kk}) > \chi_Y^2(p = 0.5)$, then the values of $\chi^2(p = 0.1)$ used above is increased by the factor $R = \chi_Y^2(A_{kk}) / \chi_Y^2(p = 0.1)$. This effectively renormalizes the chi-squared probability scale to account for cases where the correlation or its uncertainties are inadequately determined.

IV.4. Geometrical Correction Factors

A precise determination of the "true" directional correlation coefficients A_{kk} from the measured coefficients A'_{kk} depends on the exact knowledge of the geometrical correction factors Q_{kk} . A general expression for these correction factors is given by:

$$Q_{kk} = Q_k^C(\gamma_1) Q_k^D(\gamma_2) \quad (IV-15)$$

where the superscripts refer to detectors C and D. The Q_{kk} are normalized such that Q_{00} is equal to unity.

In general, the $Q_k^C(\gamma_i)$ describe the response of a detector C to gamma ray γ_i and depend on four factors: the dimensions of the detector, its position relative to the source, the energy of the gamma ray, and the dimensions of the source. The first three may be considered to be corrections due to finite detector size, and the last as due to finite source size. These two types of corrections will be considered separately.

1. Corrections Due to Finite Detector Size

(a) Computed values.

Figure IV-5 shows a schematic representation of a coaxial Ge(Li) detector having the form of a right circular cylinder. The source emits a gamma ray at an angle α with respect to the detector axis. The thin outer region of the detector represents the lithium diffused n-region, normally about one millimeter or less in depth. The cross-hatched area represents the region of lithium drifting, which is the "active volume" a gamma ray must enter in order to be detected. The depth of this region is generally from 7 to 20 millimeters. The central "dead" core, the p-type region, does not contribute directly to the detection process; it can, however, scatter gamma rays into the active region. Dimensions for the detectors used in this work are given in Table I and Table II. One of the detectors was manufactured by Ortec and detector parameters were supplied with it. The other detector was fabricated at the Lawrence Livermore Laboratory and detector parameter information was measured from the x-ray images of the chamber except for the "drift depth" and n-layer dimensions which were estimated from well-known detector preparation procedures.

R = Detector
 a = p-core radius
 $R - r$ = Diffusion depth
 $r - a$ = Depletion depth
 L = Detector Length
 ℓ = Source-detector distance

Fig. IV-5. Cross-section of a true coaxial germanium detector.

The correction factor normalized to $Q_{00} = 1$ is given in general by⁵⁴

$$Q_{kk} = \frac{[J_k^C(\gamma_1) J_k^D(\gamma_2) + J_k^C(\gamma_2) J_k^D(\gamma_1)]}{[J_0^C(\gamma_1) J_0^D(\gamma_2) + J_0^C(\gamma_2) J_0^D(\gamma_1)]} \quad (\text{IV-16})$$

This can be written in a more useful form as

$$Q_{kk} = \frac{J_k^C(\gamma_1) J_k^D(\gamma_2)}{J_0^C(\gamma_1) J_0^D(\gamma_2)} \left(\frac{R}{1+R} \right) + \frac{J_k^C(\gamma_2) J_k^D(\gamma_1)}{J_0^C(\gamma_2) J_0^D(\gamma_1)} \left(\frac{1}{1+R} \right) \quad (\text{IV-17})$$

where R is a product of the two detector efficiency ratios and is given by

$$R = \frac{J_0^C(\gamma_1) J_0^D(\gamma_2)}{J_0^C(\gamma_2) J_0^D(\gamma_1)} = \frac{\epsilon_C(\gamma_1) \epsilon_D(\gamma_2)}{\epsilon_C(\gamma_2) \epsilon_D(\gamma_1)}$$

where $\epsilon_C(\gamma_i)$ is the efficiency of detector C at the energy γ_i . The integral J_k is the essential part of the correction factor and is given by

$$J_k(\gamma_1) = \int P_k(\cos\alpha) \left(1 - e^{-\tau\rho X(\alpha)} \right) \sin\alpha \, d\alpha \quad (\text{IV-18})$$

where P_k is the Legendre polynomial of order k , τ is the total absorption coefficient (including photoelectric effects, Compton scattering, and pair production) in germanium for the photon of energy γ_1 , ρ is the density of germanium at 77°K, and $X(\alpha)$ is the distance traversed through the active volume of the detector by a photon which enters at an angle α with the axis of the detector. Theoretical values of τ have been tabulated by Storm, Gilbert, and Israel⁵⁵. The integration must be performed over the entire active volume of the detector.

A code to calculate J_k/J_0 was developed by Camp and Van Lehn⁵⁴ which uses theoretical cross sections in a Monte Carlo calculation. The

Table I
Correction Factors (J_k/J_0) for the Finite Solid Angle
Subtended by the Fixed Ge(Li) Detector

* Detector Dimensions :		Length	40.0 mm			
		Diameter	33.0 mm			
		Drift Depth	12.5 mm			
		Diffusion Depth	0.55 mm			
		Active Volume	30.5 cm ³			
Source Distance (cm)	5.0		7.0		10.0	
E _γ (keV)	J ₂ /J ₀	J ₄ /J ₀	J ₂ /J ₀	J ₄ /J ₀	J ₂ /J ₀	J ₄ /J ₀
60	0.9291	0.7759	0.9620	0.8769	0.9808	0.9370
100	0.9343	0.7918	0.9643	0.8841	0.9817	0.9399
200	0.9466	0.8294	0.9700	0.9024	0.9841	0.9477
300	0.9501	0.8403	0.9717	0.9077	0.9848	0.9500
1000	0.9525	0.8479	0.9728	0.9114	0.9853	0.9516
1500	0.9528	0.8485	0.9729	0.9117	0.9854	0.9517

* Detector dimensions were determined from x-rays of the detector in its vacuum chamber. The drift and diffusion depths were determined at the time of detector fabrication.

Table II

Correction Factors (J_k/J_0) for the Finite Solid Angle

Subtended by the Movable Ge(Li) Detector

Detector Dimensions :		Length		38.0 mm		
		Diameter		37.8 mm		
		Drift Depth		14.9 mm		
		Diffusion Depth		1.1 mm		
		Active Volume		36.8 cm ³		
Source Distance (cm)	5.0		7.0		10.0	
E _γ (keV)	J ₂ /J ₀	J ₄ /J ₀	J ₂ /J ₀	J ₄ /J ₀	J ₂ /J ₀	J ₄ /J ₀
60	0.9151	0.7351	0.9541	0.8523	0.9767	0.9238
100	0.9213	0.7533	0.9568	0.8608	0.9778	0.9272
200	0.9355	0.7959	0.9635	0.8819	0.9806	0.9363
300	0.9394	0.8079	0.9654	0.8878	0.9814	0.9389
1000	0.9421	0.8162	0.9667	0.8919	0.9819	0.9407
1500	0.9424	0.8169	0.9668	0.8922	0.9820	0.9408

* Specifications supplied by manufacturer, Ortec, Inc.

Monte Carlo approach simulates the interactions of the primary photons and all secondary radiation produced in the detectors.

This code was used to calculate at four different distances from the source J_2/J_0 , J_4/J_0 and J_0 as a function of energy. The results of the calculations for the detectors used in this work are listed in Table I and Table II and shown in Figure IV-6.

(b) Measured values

There are two methods which can be used to measure the geometric correction factor Q_{kk} . The first method consists of using a collimated beam of photons to actually measure the response of the detectors for various values of α . The second method consists of measuring a well-known angular correlation and using the measured A'_{kk} and the known A_{kk} to calculate Q_{kk} .

Instead of using a collimated beam of photons, the radiation a result of positron annihilation was used as described in Section IV.2. This method, due to Church and Kraushaar,⁵⁶ consisted of determining the FWHM of the angular resolution curve. From this measurement the unnormalized correction factors Q_{kk} were determined. The results of this measurement are shown in Figure IV-7.

In order to use the second method, it is necessary to choose a directional correlation which can be measured free of any influence of competing cascades, mixing of multipole orders, or external perturbation. The $0^+ - 2^+ - 0^+$ cascade in ^{106}Ru meets these requirements. The gamma rays in this cascade are pure E2 radiations, and the intermediate state is expected to be free of any external influences.

The ^{106}Ru was obtained commercially from International Chemical & Nuclear Corporation of Irvine, California. It was in the form of RuCl_3 dissolved in 0.5N HCl. Fifty microliters were loaded into a glass sphere

Figure IV-6. Energy dependence of geometrical correction factors for the detectors used at various source-detector distances. Vertical lines indicate the lower energy limit of the gamma-ray energies considered in this work.

Figure IV-7. ^{22}Na 511-511 keV coincidences as a function of angle between the two germanium detectors.

as described earlier and sealed. The source had an activity of 100 microcuries. The true-to-chance ratio varied from 12.4 at 90° to 18.4 at 180° . The large variation in the true-to-chance ratio is due to the strong $0^+ - 2^+ - 0^+$ correlation.

Figure IV-8 shows the experimental data for the 622-512 keV $0^+ - 2^+ - 0^+$ cascade in ^{106}Pd . Ten measurements were made; however, the weighted average of the data from each pair of angles symmetric about 180° were taken, reducing the number of data points to five. The uncertainties in the individual points indicated by the horizontal bars were assigned on the basis of statistics. The solid line is the least-squares Legendre polynomial fit to the data by the ANGCOR program. The results from the fit were

$$A_{22} = 0.3340 \pm 0.0058$$

$$A_{44} = 0.346 \pm 0.0071$$

Dividing these results with the known directional correlation coefficients

$$A_{22} = 0.3571$$

$$A_{44} = 1.1429$$

gives the geometric correction factors, Q_{kk} .

The results of the measurements on ^{22}Na and ^{106}Ru along with the calculated values are listed in Table III. The calculated values given are for gamma-ray energies of 500 keV each. The overlap among the results is quite good, keeping in mind that all gamma rays involved in these measurements have energies above 500 keV; therefore, there should be little energy dependence of the correction factors. The calculated values were corrected for finite source size in order that comparison could be made with the measured values.

Figure IV-8. Correlation data for the 622-512 keV cascade in ^{106}Pd .

2. Corrections Due to Finite Source Size

The sources used for these measurements were spherical in shape with approximately 5 millimeter diameters. One would expect that the effect of finite source sizes would be that of a larger solid-angle correction; that is, the Q_{kk} would become smaller. The magnitude of this decrease would depend on the size of the source and its orientation relative to the detectors.

It can be argued that spherical sources are approximately equivalent to circular sources. Consider a point source. If the source distance is increased slightly, the solid-angle corrections decrease slightly while if the source is moved closer, the corrections increase. Any finite thickness of the source tends to introduce corrections which to first order cancel. The effect of radiation from the front half of the source tends to decrease the source-to-detector distance; radiation from the back half of the source has the opposite effect, and the latter tends to cancel the former.

As a first approximation, therefore, a spherical source is seen by any detector as circular, and the percentage reductions for a spherical source of radius r are approximately those for a circular source of radius r . Formulas for the percentage reduction of Q_2 and Q_4 for a circular source are given by Camp and Van Lehn.⁵⁴ The inside diameter of the spherical ampoules was estimated to be 2.4 millimeters which gives a 0.2 percent change in Q_{22} and a 0.5 percent change in Q_{44} . These corrections were applied to the calculated Q_{22} and Q_{44} values given in Table III.

Weighted averages of the results given in Table III were used to arrive at suitable values of the Q_{kk} . The weighted averages reflect geometric corrections at 500 keV. To determine the values at other

Table III

Geometrical Correction Factors

Method	Q_{22}	Q_{44}
^{22}Na	0.934 ± 0.002	0.797 ± 0.005
^{106}Pd	0.935 ± 0.016	0.818 ± 0.006
Calculated	0.937 ± 0.002	0.805 ± 0.006
Weighted Average	0.935 ± 0.001	0.808 ± 0.004

Source-to-detector distances in all cases = 7.0 cm

energies the calculated values were normalized at 500 keV and the normalization factors were used to determine Q_{kk} at other energies. The values at various energies above 300 keV were stored on a CDC 7600 disk file. The program ANGCOR used this file to determine the Q_{kk} values at any energy above 300 keV by interpolation. This approach is satisfactory since the correction factors vary linearly with energy above 300 keV. There were no measurements of cascades with transitions below 300 keV in this work.

V. Results

V.1. Introduction

Directional correlation studies were made on transitions in ^{134}Ba and ^{110}Cd using the procedures described in Section IV. The gamma decay of both isotopes offered unique problems against which the PDP-8/E coincidence pulse-height analysis system was tested. The results, directional correlation coefficients and mixing ratios, were checked by a number of different methods.

For those gamma-ray cascades where all transitions are pure E2 multipole radiations the theoretical directional correlation coefficients are:

$$A_{22} = + 0.1020$$

$$A_{44} = + 0.0091$$

Directional correlation measurements involving a $4^+ - 2^+ - 0^+$ cascade or any two members of a $6^+ - 4^+ - 2^+ - 0^+$ cascade such as the one in ^{110}Cd should yield these directional correlation coefficients (unless one of the transitions, except the $2^+ - 0^+$ transition, contains a M3 multipole admixture). Thus these cascades provide a method by which the performance of the system and the data analysis can be checked.

In some cases a gamma transition is involved in several of the γ - γ directional correlation measurements either as a first, second, or intermediate (unobserved) transition. Hence, in these cases the mixing ratio of a particular transition could be extracted from several different correlations, and the results could be checked for internal consistencies. Agreement between the results from these different correlations also reflects favorably on the performance of the system and on all phases of the precorrelation analysis including the normalization factors for each angle, the accidental and Compton background coincidences, the peak area determinations and their associated errors, and the values used for the solid-angle correction factors.

A third method used to evaluate the system's performance was to compare the results of this work with other published results. In general, our results were in good agreement with the results of others. In a couple of cases; however, significant disagreements were found. This also created a significant test for the system in that it required close examination of our data to determine possible causes for the differences. The results of the directional correlation measurements carried out in this work are discussed in the following sections.

V.2. Measurement of ^{134}Ba

The decay of 2.1y ^{134}Cs to ^{134}Ba has been studied by both gamma-ray singles measurements⁵⁷⁻⁵⁹ and directional correlation measurements.⁶⁰⁻⁶⁵ A level scheme for ^{134}Ba due to Van Hise et al.⁵⁹ is given in Figure V-1.

All of the directional correlation measurements in ^{134}Ba , except for those by Behar et al.⁶⁴, were made using NaI(Tl) detectors⁶⁰ or combinations of NaI(Tl) and Ge(Li) detectors⁶¹⁻⁶³. Behar's directional correlation work was done using two 30cm³ Ge(Li) detectors, each with an energy resolution of 2.5 keV FWHM at 1.332 MeV. Reasonable agreement

$$Q_{\beta^-} = 2058.5 \pm 0.4 \text{ keV}$$

$$Q_{EC} = 1216 \pm 13 \text{ keV}$$

$$t_{1/2} = 2.062 \pm 0.005 \text{ yr}$$

Figure V-1. Decay scheme of ^{134}Ba .

exists between Behar's results and previous results except for the 563-keV $2^{1+} - 2^{+}$ transition.

The present study of ^{134}Ba was undertaken to test the performance of the megachannel coincidence system and to resolve the differing results for the 563-keV transition. ^{134}Ba provides a significant test of the system, because of the Compton background problems which exist for the 563.3-keV, 569.3-keV, and the 604.7-keV peaks. Some aspects of these problems were discussed earlier in Section V.3. Another feature of the gamma-ray transitions in ^{134}Ba is that they have E2 admixtures ranging from 7% E2 in the 569-keV transition to reportedly ~99% E2 in the 563-keV transition.

In the present work, two separate ten-angle directional correlation measurements were carried out, each with a different ^{134}Cs source. In addition, two different methods were used to reduce inter-detector scattering. In the one case the lead shielding cones described in Section IV.1. were used; while in the second case a rectangular piece of one-half inch thick lead was used to shield one detector from the other at all detector angles $\leq 45^\circ$ (Hereafter, these measurements shall be referred to as Run I and Run II, respectively).

The ^{134}Cs radioactivity was obtained from International Chemical & Nuclear Corp. of Irvine, Calif. The cesium activity was delivered in the form of CsCl dissolved in dilute HCl . Both sources prepared from this solution had an activity of approximately 45 microcuries. With the sources at 7 cm a singles rate of ~10,000 counts/s was observed in the fixed or gate detector and a rate of ~12,400 counts/s was observed in the movable detector. The lower level discriminators on the SCAs used to gate the TAC were set at 350 keV. The several gamma-ray transitions below this energy are much too weak to be seen; consequently, there is

no harm in discriminating against them. Adjustments were also made to the discriminators on the Ortec 463 Constant Fraction Discriminator modules to give optimum timing results. Under these conditions the prompt or true γ - γ coincidence peak in the time-to-amplitude spectrum had a FWHM of ~ 9.0 ns and a FWTM of ~ 21 ns. With the 45 μ Ci sources at 7 cm this gave a coincidence count rate of approximately 3000 coincidences/min. The SCA monitoring the prompt timing coincidence peak was set at FWTM, resulting in a mean true-to-chance ratio of 21 to 1.

V.3. Directional Correlation Results for ^{136}Ba

In Tables IV and V are listed the A_{kk} values determined in Runs I and II for the cascades considered in this study. Accurate correlations were not measured for the remaining cascades, because their small coincidence intensities prohibited accurate peak area determinations in the 22 hour/angle coincidence spectra. Tables IV and V show two entries per correlation for the A_{22} and A_{44} coefficients. The two entries are determined by analyzing the resultant coincidence spectrum from a gate on the initial transition; and the resultant coincidence spectrum from a gate on the final transition. The two entries should be equal within their errors.

Comparison of the results from Run I and Run II reveal that for the cascades with the strongest coincidence intensity, the A_{44} coefficients of Run II are higher than those of Run I. The A_{44} coefficient for the 795-605 keV $4^+ - 2^+ - 0^+$ correlation is significantly higher than the theoretical value $A_{44} = 0.009$. It was noted in the examination of the data that the agreement between the data for angles symmetric about 180° was not as good in Run II as in Run I. This may be due to the asymmetric positioning of the rectangular lead shield used in Run II with respect to the source and detectors. Also during this run the

Table IV

Summary of γ - γ Directional Correlation Measurements in ^{134}Ba
Using Lead Cones to Reduce Scattering Between Detectors^a

Cascade (keV)	A_{22}^b	A_{44}^b	$\bar{A}_{22}^c$	$\bar{A}_{44}^c$
604.7-563.3	-0.127 ± 0.015	0.317 ± 0.021	-0.127 ± 0.008	0.329 ± 0.012
	-0.127 ± 0.010	0.334 ± 0.014		
604.7-(795.8)-569.3 ^d	0.108 ± 0.010	0.007 ± 0.014	0.099 ± 0.006	0.007 ± 0.009
	0.094 ± 0.008	0.007 ± 0.011		
604.7-795.8	0.100 ± 0.004	0.011 ± 0.006	0.099 ± 0.003	0.013 ± 0.005
	0.096 ± 0.006	0.017 ± 0.008		
604.7-1038.5	0.317 ± 0.048	-0.121 ± 0.048	0.303 ± 0.030	-0.063 ± 0.042
	0.295 ± 0.038	-0.028 ± 0.053		
604.7-1365.1	0.081 ± 0.026	-0.003 ± 0.036	0.092 ± 0.018	0.015 ± 0.025
	0.103 ± 0.026	0.032 ± 0.036		
569.3-795.8	0.116 ± 0.011	0.019 ± 0.017	0.112 ± 0.007	0.017 ± 0.011
	0.109 ± 0.010	0.016 ± 0.015		
563.3-801.8	-0.003 ± 0.017	0.007 ± 0.024	-0.010 ± 0.010	-0.003 ± 0.014
	-0.013 ± 0.012	-0.008 ± 0.017		
1167.8-801.8	0.132 ± 0.039	0.013 ± 0.055	0.144 ± 0.028	-0.004 ± 0.041
	0.155 ± 0.039	-0.027 ± 0.063		

Table IV (continued)

- a) All coefficients listed have been corrected for the finite solid angles of the detectors.
- b) The first entry is the result obtained from having gated on the first gamma ray listed; while the second entry is the result obtained from having gated on the second gamma ray listed.
- c) Weighted averages of the two A_{22} values and the two A_{44} values.
- d) The transition in parenthesis is unobserved.

Table V

Summary of γ - γ Directional Correlation Measurements in ^{134}Ba Using a Lead Shield to Reduce Scattering Between Detectors^a

Cascade (keV)	A_{22}^b	A_{44}^b	$\overline{A}_{22}^c$	$\overline{A}_{44}^c$
604.7-563.3	-0.127 ± 0.013	0.368 ± 0.019	-0.128 ± 0.008	0.361 ± 0.012
	-0.129 ± 0.010	0.356 ± 0.015		
604.7-(795.8)-569.3 ^d	0.111 ± 0.008	0.012 ± 0.013	0.095 ± 0.006	0.014 ± 0.008
	0.078 ± 0.008	0.015 ± 0.011		
604.7-795.8	0.100 ± 0.004	0.017 ± 0.005	0.100 ± 0.003	0.018 ± 0.004
	0.099 ± 0.005	0.021 ± 0.007		
604.7-1038.5	0.163 ± 0.045	0.129 ± 0.070	0.225 ± 0.030	-0.047 ± 0.045
	0.277 ± 0.041	-0.168 ± 0.058		
604.7-1365.1	0.084 ± 0.026	-0.012 ± 0.038	0.120 ± 0.018	0.007 ± 0.027
	0.151 ± 0.025	0.024 ± 0.039		
569.3-795.8	0.100 ± 0.009	0.007 ± 0.013	0.101 ± 0.006	0.006 ± 0.009
	0.094 ± 0.008	0.006 ± 0.012		
563.3-801.8	-0.013 ± 0.012	0.005 ± 0.018	-0.013 ± 0.008	0.005 ± 0.013
	-0.013 ± 0.012	0.005 ± 0.018		
1167.8-801.8	0.092 ± 0.038	-0.037 ± 0.058	0.089 ± 0.027	0.012 ± 0.042
	0.085 ± 0.041	0.067 ± 0.061		

Table V (continued)

- a) All coefficients listed have been corrected for the finite solid angles of the detectors.
- b) The first entry is the result obtained from having gated on the first gamma ray listed; while the second entry is the result obtained from having gated on the second gamma ray listed.
- c) Weighted averages of the two A_{22} values and the two A_{44} values.
- d) The transition in parenthesis is unobserved.

lead shield was repositioned at each new angle setting of the movable detector. Thus although the lead shield was always centered midway between the two detectors, it was not in the exact same position for each pair of angles symmetric about 180° . Moving the shield, then, may have introduced an asymmetry in the coincidence data which was not removed by normalizing the data to the movable detector singles rate at each angle. Therefore, we chose to use only the results of Run I in the final analysis.

Since the spins of the levels in ^{134}Ba are known,⁶⁴ the data were analyzed only for directional correlation coefficients and possible $(L+1)/L$ multipole admixtures. Table VI compares the results of Run I with previous correlation studies.⁶¹⁻⁶⁵

We consider first, the $4^+ - 2^+ - 0^+$ cascades. They are, in increasing order of coincidence intensity:

801.8 - 1167.9 keV

1365.1 - 604.7 keV

795.8 - 604.7 keV

The 604.7-keV and 1167.9-keV transitions are 2^+ to 0^+ transitions, hence, can only contain E2 multipole. The 795.8, 801.8 and 1365.1-keV radiations are $4^+ - 2^+$ transitions and are probably E2 in multipolarity, but may contain a small M3 multipole admixture. Since the correlation results for these cascades are known, they served as checks on the system's performance.

V.3.1 1168 - 802 keV cascade

The 1168 - 802 keV coincidence intensity was ~100 times weaker than the 795 - 605 keV coincidence intensity. With a 22 hour measurement duration at each angle the statistics obtained on the number of

Table VI

Experimental Results of the Directional Correlation Coefficients

 A_{kk} for the ^{134}Ba γ - γ Cascades and Der

Multipole Mixing Ratio Assignment

Cascades (keV)	$A_{22} \pm \Delta A_{22}$	$A_{44} \pm \Delta A_{44}$	δ	Reference
795-605	0.095 ± 0.009	0.009 ± 0.013	E2	63
$(4^+ - 2^+ - 0^+)$	0.102 ± 0.009	0.008 ± 0.016	E2	64
	0.099 ± 0.003	0.015 ± 0.004	E2	This work
1365-605	0.105 ± 0.004	0.020 ± 0.013	E2	63
$(4^+ - 2^+ - 0^+)$	0.108 ± 0.015	0.001 ± 0.020	E2	64
	0.092 ± 0.018	0.015 ± 0.025	E2	This work
569-(795)-605	0.099 ± 0.006	0.007 ± 0.009	$+0.27 \pm 0.2$	Present
$4^+ - (4^+ - 2^+) - 0^+$				
569-795	0.104 ± 0.010	0.010 ± 0.014	$\begin{vmatrix} \delta \\ \delta \end{vmatrix} > 0.22 \\ < 0.28$	61
$(4^+ - 4^+ - 2^+)$	0.095 ± 0.009	0.009 ± 0.013	$+0.278 \pm 0.019$	63
	0.091 ± 0.006	0.011 ± 0.012	$+0.29 \pm 0.02$	64
	0.111 ± 0.004	0.022 ± 0.006	$+0.240 \pm 0.010$	65
	0.112 ± 0.007	0.017 ± 0.011	$+0.24 \pm 0.04$	This work

Table VI (continued)

Cascades (keV)	$A_{22} \pm \Delta A_{22}$	$A_{44} \pm \Delta A_{44}$	δ	Reference
563-605	-0.079 ± 0.032	0.303 ± 0.047	$ \delta > 15$	61
$(2^+ - 2^+ - 0^+)$	-0.16 ± 0.10	0.27 ± 0.13	$ \delta < \infty$	
			$\delta > 4$	63
			$\delta < -30$	64
			-7.4 ± 0.9	
	0.025 ± 0.011	0.285 ± 0.035	$+2.3$	
	-0.127 ± 0.008	0.329 ± 0.012	-1.8	This work
1038-604	0.285 ± 0.03	-0.002 ± 0.05	0.85 ± 0.12	62
$(3^+ - 2^+ - 0^+)$	0.22 ± 0.016	-0.013 ± 0.014	0.51 ± 0.06	63
			1.35 ± 0.15	64*
			0.82 ± 0.19	This work
	0.157 ± 0.02	-0.034 ± 0.04		
	0.303 ± 0.030	-0.063 ± 0.042		

1168-keV and 802-keV events (coincidence statistics) observed in coincidence at each angle was $\pm 4.1\%$. The directional correlation coefficients obtained from the 1168 - 802 keV correlation data are given in Table IV. The A_{22} coefficient agrees poorly with the theoretical value of 0.1020. This is probably due to the relatively poor statistics of the coincidence data.

V.3.2. 1365 - 605 keV cascade

The coincidence intensity of the 1365 - 605 keV cascade was ~ 50 times weaker than the 795 - 605 keV coincidence intensity. The coincidence statistics obtained for the 1365 - 605 keV cascade were $\sim \pm 2.8\%$. The experimental data and the least-squares Legendre-polynomial fit using the standard equation $N(\theta) = b_0 + b_0 P_2(\cos\theta) + b_4 P_4(\cos\theta)$ is shown in Figure V-2. The values of the resultant A_{22} and A_{44} coefficients ($A_{11} = b_1/b_0$) corrected for the solid angles subtended by the detectors are given in Table VI. The results overlap the theoretical A_{22} and A_{44} coefficients for a $4^+ - 2^+ - 0^+$ cascade. The experimental A_{22} and A_{44} coefficients for the 1365 - 605 keV cascade have errors of 19.6% and 167% respectively.

V.3.3. 795 - 605 keV cascade

A statistical error of $\pm 0.4\%$ was obtained on the 795 - 605 keV cascade coincidence data. The experimental data for the 795 - 605 keV correlation are shown in Figure V-3. The values obtained for the A_{22} and A_{44} coefficients from this correlation are in good agreement with the theoretical values. The error on the A_{22} coefficient is $\sim 3\%$ and on the A_{44} coefficient the error is $\sim 27\%$.

Determination of the 795 - 605 keV correlation by setting a window on the 795-keV peak is made more difficult, since the 795-keV and 802-keV peaks are not very well separated. A certain fraction of the 802-keV

Figure V-2. Correlation data for the 1365-605 keV cascade in ^{134}Ba .

Figure V-3. Correlation data for the 795-605 keV cascade in ^{134}Ba .

low-energy tail events lie beneath the 795-keV peak. These gated 802-keV events will bring back some 605-keV events in the resultant coincidence spectrum, since the 802-keV and 605-keV transitions are in coincidence. The interference introduced in the 795 - 605 keV correlation by the 802 - (563) - 605 keV correlation is small, since the 802 - (563) - 605 keV correlation is nearly angle independent (asymmetry < 3%) and the intensity of the 802-keV γ -ray is about 0.1 the intensity of the 795-keV γ -ray.

The 802-keV events in the 795-keV window were corrected for by including part of the 802-keV peak in the background window used to subtract Compton events from the coincidence spectrum. The 802-keV contribution to the coincidence spectrum was monitored by observing the 1168-keV region in the spectrum. Since the 802-keV transition is in coincidence with the 1168-keV transition and the 795-keV transition is not, any 802-keV events in the 795-keV window not corrected for would bring back 1168-keV events. With the above subtraction technique the intensity of the 1168-keV line was zero.

The good agreement of the directional correlation coefficients for the 795 - 605 keV cascade with the theoretical values indicates that the system is functioning properly. The agreement with the results of the other two $4^+ - 2^+ - 0^+$ cascades was not as good, because of the poor statistics. The $4^+ - 2^+ - 0^+$ correlation also provides a check on the normalization factors and the solid-angle correction factors, since these are the only factors which influence this correlation. The good results obtained for these cascades adds confidence to the results obtained for other cascades where multipole mixing becomes an additional factor.

V.3.4. 569 - 795 keV and the 569 - (795) - 604 keV cascades

The directional correlations of the 569 - 795 keV and the 569 - (795) - 605 keV cascades were used to determine the amount of E2/M1 mixing in the 569-keV $4^{1+} - 4^{+}$ transition. The directional correlation coefficients for the two correlations should be equal, since the unobserved 795-keV transition in the latter is pure E2 (See discussion of unobserved transitions in section I.5.). The 569 - 605 keV correlation had better statistics than the 569 - 795 keV correlation, since the detection efficiency for the 605-keV γ -ray is greater than for the 795-keV γ -ray.

The experimental data measured for these cascades and the least-squares fits indicated by the solid lines are shown in Figure V-4. The directional correlation coefficients determined from the two correlations agree within the limits of their errors. Figure V-5a shows the results of the analysis of the 569 - (795) - 605 keV correlation by the GGDC code. A value of $\delta_{569} = 0.27 \pm 0.02$ was obtained which is in agreement with other published values determined from 569 - 795 keV correlation measurements.^{61, 63-65} Our correlation data for the 569 - 795 keV cascade gave a mixing ratio value of $\delta = 0.24 \pm 0.04$. The results of the analysis of the 569 - 795 keV correlation data by the ANGCOR code are shown in Figure V-5b. This value is low in comparison to the results reported by Hoffmann⁶³ and Behar⁶⁴; however, it is in good agreement with $+0.24^{+0.010}_{-0.011}$ reported by Gardulski⁶⁵ and within the range of values reported by Singh and Taylor⁶¹. A final value for the mixing ratio of the 569-keV transition was obtained from the simultaneous analysis of the 569 - (795) - 605 keV and 569 - 795 keV correlations using the GGDC code. It gave $\delta_{569} = 0.25 \pm 0.02$. Figure V-6 shows the χ^2 results from

Figure V-4. Correlation data for the 569-795 keV and 569-(795)-605 keV cascades in ^{134}Ba .

Figure V-5. χ^2 analyses for the a) 569-(795)-605 keV and b) 569-795 keV cascades in ^{134}Ba .

Figure V-6. Simultaneous χ^2 analysis of the 569-(795)-605 keV and the 569-795 keV correlation data.

this analysis. The mixing ratio value indicates that the 569-keV transition is predominantly an M1 transition with a $5.9 \pm 0.9\%$ E2 admixture.

V.3.5. 563 - 605 keV cascade

The mixing ratio values determined for the 563-keV transition reveal that the transition is predominantly of E2 multipole character.^{61, 63-65}

The determination of the sign of the 563-keV mixing ratio was indicated only by the measurements of Behar, *et al.*⁶⁴ Their results give a negative sign for the mixing ratio; however, their A_{22} coefficient for the 563 - 605 keV correlation differs significantly from other published values.^{61, 63}

The directional correlation for the 563 - 605 keV cascade determined in this work differs significantly from Behar's results as shown in figure V-7. The figure shows the data obtained in the present work with a least-squares fit indicated by the solid line. The directional correlation coefficients and the mixing ratio determined from this data are listed in the lower right-hand corner. The dashed curve indicates the shape of the correlation curve for the mixing ratio of $\delta_{563} = -7.4$ given by Behar *et al.*⁶⁴ The ordinate scale applies only to our data, not to Behar's. The $\delta = -7.4$ correlation curve could be normalized to any point of our data.

Behar was unable to completely resolve the 563-keV and 569-keV peaks and chose to measure the 563 - 605 keV correlation using two different windows. In one case they set a window on the 563-keV peak and determined that the window accepted a 17% contribution of the 569-keV γ -ray. In the other case a window was set to include both the 563-keV and 569-keV γ -rays. In both cases the directional correlation coefficients for the 563 - 605 keV cascade were deduced by subtracting out the 569 - 605 keV contribution. Their results for the 563 - 605 keV cascade

Figure V-7. Comparison of the 563-605 keV correlation data with a theoretical correlation curve given by a mixed E2/M1 transition having a mixing ratio of -7.4.

from these two methods agreed within their errors and they chose a weighted average for their final result.

We analyzed our coincidence data by setting a window on both the 563-keV and 569-keV peaks as well as analyzing the 563 - 605 keV correlation data directly, since we were able to resolve the 563-keV and 569-keV peaks. The resultant coefficients from our measurement of the (563 + 569) - 605 keV correlation were

$$A_{22} = 0.021 \pm 0.007, \text{ and}$$

$$A_{44} = 0.145 \pm 0.011.$$

The A_{22} value given here agrees with Behar's A_{22} result for the 563 - 605 keV correlation with the 569 - 605 keV correlation supposedly removed. Behar does not state results for either of their combined cascade measurements. Next, we subtracted the 569 - 605 keV contribution from our (563 + 569) - 605 keV correlation data using a mixing ratio of $\delta_{569} = + 0.25$ for the 569-keV transition. The resulting correlation gave the following directional correlation coefficients:

$$A_{22} = - 0.132 \pm 0.027$$

$$A_{44} = 0.382 \pm 0.041$$

These values are in reasonable agreement with the directional correlation coefficients obtained by measuring the 563 - 605 keV correlation directly, i.e.,

$$A_{22} = - 0.127 \pm 0.008, \text{ and}$$

$$A_{44} = 0.329 \pm 0.012$$

versus the results as reported by Behar et al. of

$$A_{22} = 0.025 \pm 0.011, \text{ and}$$

$$A_{44} = 0.285 \pm 0.035$$

The results of both our direct and combined minus contaminant measurements clearly disagree with the results reported by Behar et al.⁶⁴

Since Behar's results for the 569 - 605 keV correlation agree with ours, we can only conjecture that either Behar's (563 + 569) - 605 keV correlation data were incorrect or he incorrectly subtracted the 569 - 605 keV cascade contribution from the (563 + 569) - 605 keV correlation data. Attempts to duplicate Behar's 563 - 605 keV correlation result from our (563 + 569) - 605 keV correlation data by varying the amount of the 569 - 605 keV contribution were unsuccessful.

Care must be taken in correcting for the Compton distribution contributions in this correlation. The Compton edge from the 802-keV gamma ray is near 605 keV, so a window set on either the 563-keV or 605-keV photopeaks will accept some Compton events from the 802-keV γ -ray. As a result the 802 - 563 keV cascade will contribute to the 563 - 605 keV correlation. This interference was monitored by looking for the presence of the 1168-keV line in the coincidence spectrum, which appears because the 802-keV Compton events can have associated 802 - 1168 keV coincidence events. The background windows were chosen in such a way that the 1168-keV line was not present in the coincidence spectrum. This procedure minimized any interference in the 563 - 605 keV correlation by the 802 - 563 keV cascade.

The background beneath the 563-keV and 605-keV peaks in the coincidence spectra increases by about 10% as the angle between the detectors is changed from 90° to 180°. If one assumes that the background remained constant, which Behar et al. may have done since their background corrections required separate measurements, then one might obtain results comparable to those of Behar et al.⁶⁴. Figure V-8 shows our results when we assumed the background from 90° to 180° to be constant.

Figure V-8. Correlation data for 563-605 keV cascade determined by assuming coincident background to be constant.

The directional correlation coefficients determined from the data under these assumptions become

$$A_{22} = -0.011 \pm 0.017, \text{ and}$$

$$A_{44} = 0.275 \pm 0.028.$$

These values agree within the errors with the 563 - 605 keV correlation results reported by Behar et al.⁶⁴; therefore, this is at least one possible explanation of Behar's results.

V.3.6. 802 - 563 keV cascade

The correlation for the 802 - 563 keV is very weak, and the data were not used to determine the mixing ratio for the 563-keV transition, but only as a confirmation of the 563 - 605 keV results. The correlation coefficients determined for this cascade are given in Table IV. The results indicate a mixing ratio of between + 10 and + 100. This range of values is in agreement with the $\delta = +13.3^{+2.3}_{-1.8}$ determined from our 563 - 605 keV correlation measurements; however, it disagrees with the result given by Behar et al.⁶⁴ for the 802 - 563 keV cascade.

V.3.7. 1038 - 605 keV Cascade.

The 1038 - 605 keV cascade had the weakest coincidence intensity of all the cascades which we measured. The statistics obtained on the coincidence data for this cascade were $\sim \pm 4.7\%$. A number of values exist for the mixing ratio of the 1038-keV transition as shown in Table VI. Our results are in good agreement with Taylor et al.⁶². A plot of the chi-squared values as a function of the Arctan (δ) is shown in Figure V-9.

The K-conversion coefficient for the 1038-keV transition has been quoted by Brown and Ewan⁵⁷ as $(1.62 \pm 0.18) \times 10^{-3}$ and by Nagpal⁶⁶ as $(1.68 \pm 0.20) \times 10^{-3}$. The theoretical value of α_k is 1.36×10^{-3} for E2 and 1.86×10^{-3} for M1 radiation.⁶⁷ A 40 $\pm$ 9% E2 admixture for the

Figure V-9. χ^2 analysis of ^{134}Ba 1038-605 keV correlation data.

1038-keV transition as indicated by a $\delta_{1038} = 0.82 \pm 0.19$ gives an $\alpha_k = (1.66 \pm .05) \times 10^{-3}$ which is in excellent agreement with the reported experimental conversion coefficients.

V.4. A comment on the ^{134}Ba nucleus

It is interesting to note the similarity between the level structure of ^{134}Ba and ^{192}Pt . In fact, the relative spacings of the levels for these nuclei agree reasonably well with each other except for the second 4^+ level of ^{134}Ba . This state appears to be a two quasi-neutron level⁵⁹ with configuration $\nu(d_{3/2} d_{5/2})$ similar to that observed in all of the $N = 78$ isotones. The single-particle nature of this 4^+ level may explain the large M1 admixture in the 569-keV transition from this level. The remaining levels in ^{134}Ba and ^{192}Pt have been predicted by the collective potential energy surface calculations of Gneuss and Greiner⁶⁸. Their calculations indicate that these nuclei resemble an asymmetric nucleus but differ by having a large degree of γ -softness. The shape of ^{134}Cs might be similar to ^{192}Pt as there are 78 neutrons in ^{134}Cs and 78 protons in ^{192}Pt and ~ 12% of the other particles (or holes) are outside the nearest closed shell. Calculations by Kumar and Baranger⁶⁹ predicted E2/M1 mixing ratios for $^{192}, ^{194}, ^{196}\text{Pt}$ which agree surprisingly well with experimental data.⁷⁰ Their calculations predicted a mixing ratio of + 14.3 for the $2^+ - 2^+$ transition in ^{192}Pt ; the experimental value is $+ 9.1 \pm 0.3$.⁷¹ One can speculate further that there is a similarity between the ^{134}Ba and ^{192}Pt nuclei, since our measured value of $+ 13.3^{+2.3}_{-1.8}$ for the $2^+ - 2^+$ transition in ^{134}Ba agrees quite well with the measured and predicted values for this same transition in ^{192}Pt . Calculations of the Kumar and Baranger type for ^{134}Ba would be very useful.

V.5. Measurement of ^{110}Cd

The present study of ^{110}Cd was undertaken to demonstrate the performance of the $\gamma\text{-}\gamma$ coincidence system through directional correlation measurements. Figure V-10 shows the decay scheme of $^{110\text{m}}\text{Ag}$ to ^{110}Cd . The numbers in parentheses following the gamma-ray energies represent the M1 admixtures measured in this work (expressed as a percent of the total M1 + E2 intensity). The decay of $^{110\text{m}}\text{Ag}$ to ^{110}Cd has been studied by a number of investigators.⁷²⁻⁷⁹ Early studies of the decay scheme by directional correlation techniques^{72, 73} and extensive studies of β -rays and internal conversion electrons⁷⁴⁻⁷⁶ have resulted in a well-established level scheme of ^{110}Cd . Internal-conversion studies⁷⁶ indicate that, of the 15 most intense γ -rays emitted, 8 are expected to show mixed dipole-quadrupole multipolarity. With NaI(Tl) scintillation detectors it had been possible to determine the multipole mixing ratio of only two of these transitions⁷⁵; however, the use of Ge(Li) detectors permitted an investigation of the multipole character of all γ -rays emitted.⁷⁷ More recently, Ge(Li) detectors have been used in nuclear orientation experiments to determine the multipole mixing ratios of transitions in ^{110}Cd .^{78, 79}

The agreement between the multipole mixing ratios determined by $\gamma\text{-}\gamma$ directional methods and those determined by nuclear orientation methods is reasonable except for the 1505-keV ($3^+ - 2^+$) transition. The discrepancy reported for the 1505-keV transition was resolved by this work. ^{110}Cd provides a significant test of the coincidence analysis system, because of the complexity of the spectrum. Many of the ^{110}Cd γ -transitions were involved in several of the $\gamma\text{-}\gamma$ directional correlation measurements either as a first, second, or intermediate (unobserved) transition. Hence, in many cases the mixing ratio of a particular

Figure V-10, Decay Scheme of ^{110m}Ag .

transition could be extracted from several different measurements, and the results compared for internal self-consistency.

A liquid source containing ^{110m}Ag , in the form of AgNO_3 dissolved in dilute HNO_3 was obtained from New England Nuclear of Boston, Massachusetts, and used for the ^{110}Cd directional correlation measurements. The source had an activity of approximately 50 microcuries. A singles rate of $\sim 10,000$ counts/s was observed in the gate detector. The coincidence rate was $\sim 3,000$ coincidences/min. The lower level discriminators on the SCAs used to gate the TAC were set at 350 keV. The low-energy cutoff in the gross coincidence Y-spectrum shown in Figure V-11 is a result of this discrimination. The prompt or true γ - γ coincidence peak in the time spectrum had a FWHM of 11.0 ns and a FWTM of 25.0 ns. The observed true-to-chance ratio was ~ 65 to 1.

V.6. Directional Correlation Results for ^{110}Cd

The results for the directional correlation coefficients A_{22} and A_{44} extracted from the directional correlation measurements are listed in Table VII. The two A_{22} and A_{44} coefficients listed for each cascade are the results of gating on first one transition of the cascade and then the other. The errors shown are due solely to the statistical errors associated with the amount of coincidence data obtained in each gated spectrum.

The results show reasonable internal agreement. For example, the directional correlation measurements involving any two members of the 937.5 - 884.7 - 657.7 keV cascade ($6^+ - 4^+ - 2^+ - 0^+$) yield the expected coefficient $A_{22} = 0.102$ (entries 9, 10, and 22 in Table VII). In addition, all three of these transitions are in coincidence with the 446.8-keV transition; the three results for measurements involving the 447-keV transition agree (entries 1, 18, and 24 in Table VII). The good

Figure V-11. ^{110}Cd gross coincidence spectrum. Note low-energy cutoff due to discrimination against coincidences involving a low-energy event.

Table VII

Summary of γ - γ Directional Correlation Measurements in ^{110}Cd .

	Cascade (keV)	A_{22}	A_{44}	$\bar{A}_{22}$	$\bar{A}_{44}$
1)	658/447	0.025 ± 0.031 0.144 ± 0.018	0.045 ± 0.043 -0.093 ± 0.024	0.114 ± 0.016	-0.060 ± 0.021
2)	658-(885)-620	0.220 ± 0.050 0.193 ± 0.024	-0.086 ± 0.071 -0.014 ± 0.034	0.198 ± 0.025	-0.027 ± 0.031
3)	658-(885)-678	0.245 ± 0.019 0.032 ± 0.028	0.250 ± 0.015 0.001 ± 0.020	0.248 ± 0.012	0.011 ± 0.016
4)	658-(818)-687	-0.062 ± 0.046 0.012 ± 0.056	-0.013 ± 0.026 -0.020 ± 0.038	-0.032 ± 0.036	-0.005 ± 0.021
5)	658-(885-678)-707	-0.303 ± 0.014 -0.004 ± 0.020	-0.303 ± 0.011 -0.021 ± 0.015	-0.303 ± 0.009	-0.015 ± 0.021
6)	658-(818)-744	0.018 ± 0.050 -0.057 ± 0.031	0.030 ± 0.070 0.005 ± 0.038	-0.036 ± 0.026	0.011 ± 0.033

Table VII (continued)

Cascade (keV)		A_{22}	A_{44}	A_{22}	$\bar{A}_{44}$
7)	658/764	0.042 ± 0.014	-0.002 ± 0.017	0.043 ± 0.008	0.002 ± 0.011
		0.043 ± 0.010	0.005 ± 0.014		
8)	658-818	0.397 ± 0.026	0.255 ± 0.034	0.376 ± 0.016	0.232 ± 0.021
		0.365 ± 0.020	0.219 ± 0.026		
9)	658-885	0.099 ± 0.005	0.016 ± 0.008	0.099 ± 0.004	0.008 ± 0.006
		0.099 ± 0.005	0.000 ± 0.008		
10)	658-(885)-937	0.100 ± 0.008	-0.016 ± 0.012	0.097 ± 0.006	-0.004 ± 0.008
		0.094 ± 0.008	0.008 ± 0.012		
11)	658-(885)-1384	-0.337 ± 0.012	-0.005 ± 0.015	-0.331 ± 0.009	-0.017 ± 0.011
		-0.325 ± 0.013	-0.033 ± 0.017		
12)	658-1505	-0.550 ± 0.020	-0.063 ± 0.024	-0.557 ± 0.015	-0.060 ± 0.018
		-0.565 ± 0.022	-0.056 ± 0.026		
13)	764-620	0.090 ± 0.027	0.041 ± 0.036	0.095 ± 0.023	0.048 ± 0.031
		0.108 ± 0.042	0.067 ± 0.058		

Table VII (continued)

Cascade (keV)		A_{22}	A_{44}	$\bar{A}_{22}$	$\bar{A}_{44}$
14)	764-687	-0.191 ± 0.018	-0.014 ± 0.025	-0.187 ± 0.014	-0.037 ± 0.018
		-0.182 ± 0.021	-0.065 ± 0.027		
15)	764-(687)-818	-0.042 ± 0.026	-0.036 ± 0.035	-0.051 ± 0.019	-0.018 ± 0.025
		-0.061 ± 0.027	-0.001 ± 0.035		
16)	764-(620)-885	0.067 ± 0.036	-0.063 ± 0.055	0.070 ± 0.028	-0.001 ± 0.041
		0.074 ± 0.044	0.072 ± 0.060		
17)	764-1505	-0.208 ± 0.022	-0.007 ± 0.030	-0.209 ± 0.018	-0.003 ± 0.025
		-0.212 ± 0.032	0.005 ± 0.043		
18)	885-(937)-447	0.157 ± 0.038	0.046 ± 0.057	0.129 ± 0.018	-0.012 ± 0.028
		0.121 ± 0.020	-0.030 ± 0.032		
19)	885-620	0.179 ± 0.043	0.031 ± 0.059	0.231 ± 0.023	-0.003 ± 0.032
		0.252 ± 0.027	-0.017 ± 0.038		

Table VII (continued)

Cascade (keV)	A_{22}	A_{44}	$\bar{A}_{22}$	$\bar{A}_{44}$
20) 885-678	0.254 ± 0.018 0.268 ± 0.017	0.026 ± 0.025 -0.011 ± 0.022	0.261 ± 0.012	0.005 ± 0.016
21) 885-(678)-707	-0.377 ± 0.018 -0.358 ± 0.015	0.027 ± 0.024 -0.034 ± 0.019	-0.365 ± 0.011	-0.010 ± 0.015
22) 885-937	0.093 ± 0.010 0.100 ± 0.010	0.013 ± 0.014 0.009 ± 0.013	0.097 ± 0.007	0.011 ± 0.010
23) 885-1384	a) -0.341 ± 0.015	a) 0.017 ± 0.018	-0.341 ± 0.015	0.017 ± 0.018
24) 937-447	0.115 ± 0.031 0.114 ± 0.020	0.055 ± 0.046 -0.015 ± 0.028	0.114 ± 0.017	0.026 ± 0.024

a) Due to the gain settings for the X- and Y-detectors this gate was unattainable.

(See Section II.3.1a and Figure II-2.).

agreement is another indication of the system's excellent performance and of the self-consistency of the normalization factors and solid-angle correction factors.

The E2/M1 mixing ratios listed in Table VIII were extracted directly from the measured correlations rather than indirectly from the fitted A_{kk} values. The ANGCOR and GGDC codes were used for these analyses. Many of the transitions were involved in several γ - γ directional correlation measurements. Those correlations involving a common transition were analyzed simultaneously to determine a "best" value of the mixing ratio for that transition.

Results for the mixing ratios are listed in Table VIII along with the results obtained by Krane⁷⁷ from $\gamma\gamma(0)$ measurements and by Johnston and Stone⁷⁸ and Wang *et al.*⁷⁹ from nuclear orientation measurements. The results for each mixed transition are discussed separately.

V.6.1. 446.8 - keV ($3^+ - 4^+$) transition

Figure V-12 shows the data and least-squares fits for the 447 - 937 keV and 447 - 885 keV correlations. The 447 - 658 keV correlation results showed poor agreement between the results obtained by gating on the 658-keV transition and those obtained by gating on the 447-keV transition. This is mainly due to the large Compton background in coincidence with the 658-keV transition which introduced large errors on the 447-keV peak area. The analysis of the 447 - 658 keV correlation data, shown in Fig. V-13a, yielded $\delta_{447} = -0.31 \pm 0.07$ or -4.83 ± 1.50 . The analysis of the 447 - 885 keV correlation also yielded two delta values, $\delta_{447} = -0.33 \pm 0.02$ or -4.30 ± 1.40 . This analysis is shown in Figure V-13b. Figure V-13c shows a single delta value of $\delta_{447} = -0.33 \pm 0.03$ for the analysis of the 447 - 937 keV correlation data. A simultaneous analysis of all three correlations yields, of $\delta_{447} = -0.33 \pm 0.05$.

Table VIII
The E2/M1 Mixing Ratios of the ^{110}Cd Gamma Transitions

Transition (keV)	Present Results	Krane and Steffen ⁷⁷	Johnston and Stone ⁷⁸	Wang <u>et. al.</u> ⁷⁹
446.8 $5^+ - 6^+$	-0.33 ± 0.05	-0.45 ± 0.20	--	-0.40 ± 0.06
620.3 $3^+ - 4^+$	-0.50 ± 0.08	-0.80 ± 0.50	--	--
677.7 $4^+ - 4^+$	-0.28 ± 0.05	-0.25 ± 0.20	-0.44 ± 0.05	-0.38 ± 0.03
687.0 $3^+ - 2^+$	-1.27 ± 0.38	$-1.10^{+0.8}_{-0.4}$	-1.80 ± 0.05	-1.65 ± 0.09
706.7 $5^+ - 4^+$	-1.10 ± 0.30	-1.00 ± 0.30	-0.58 ± 0.02	-1.42 ± 0.07
818.0 $2^+ - 2^+$	-1.44 ± 0.10	-1.20 ± 0.15	-1.36 ± 0.10	--
1384 $5^+ - 4^+$	-0.44 ± 0.02	-0.37 ± 0.03	-0.46 ± 0.01	-0.39 ± 0.02
1505 $3^+ - 2^+$	-1.24 ± 0.20	-0.55 ± 0.10	-1.26 ± 0.06	-1.09 ± 0.10

Figure V-12. Correlation data for 447-(937)-885 keV and 447-937 keV cascades in ^{110}Cd .

Figure V-13. χ^2 analysis of correlation data involving the 447-keV transition in ^{110}Cd . All three curves are results from the GGDC code.

This mixing ratio value indicates the presence of a $(90 \pm 3)\%$ admixture of M1 component in the 446.8-keV transition.

V.6.2. 620.2 - keV ($3^+ - 4^+$) transition.

The 620-keV transition was the weakest transition on which a directional correlation measurement was made. The intensity of the 620-keV transition is 2.5% relative to 100% for the 658-keV transition.⁸⁰ The statistics obtained on the 620-keV peak area in the 658-keV gate spectrum were $\pm 8\%$. The mixing ratio results from the five correlations measured which involved the 620-keV transition agreed within the limits of their errors. The 764 - (620) - 885 keV correlation is very weak, and consequently, was not used in the final analysis. The 764/658 keV correlation data also were not used in the final analysis, since the correlation is contributed to by three different cascades which involve five unknown mixing ratios. The 764/658 keV correlation data were used only as a check on the final results. The simultaneous analysis of the data from the three remaining correlations is shown in Figure V-14. The E2/M1 mixing ratio result is $\delta_{620} = -0.50 \pm 0.08$. This value indicates an $(80 \pm 5)\%$ M1 admixture in the 620 keV transition.

V.6.3. 677.6 - keV ($4^{1+} - 4^+$) transition

Four correlations were measured which involved the 678-keV transition. The A_{22} and A_{44} coefficients of the 678 - 885 keV and the 678 - (885) - 658 keV correlations should be equal, since the unobserved 885-keV transition in the latter correlation is pure E2. Table VII shows that the directional coefficients determined from these two correlations (entries 4 and 20) are in excellent agreement. The correlation data and the χ^2 analysis of these two correlations are given in Figures V-15 and V-16.

Figure V-14. Simultaneous χ^2 analysis of correlations involving the 620-keV transition in ^{110}Cd .

Figure V-15. Correlation data and χ^2 analysis for 678-885 keV cascade in ^{110}Cd .

Figure V-16. Correlation data and χ^2 analysis for 678-(885)-658 keV cascade in ^{110}Cd .

The 707 - (658) - 885 keV correlation involves mixing in both the 707-keV and the 678-keV transitions. The correlation data from this cascade were used to determine a mixing ratio value for the 707-keV transition, after having determined a mixing ratio value for the 678-keV transition. The latter was determined by simultaneously analyzing the data from the 678 - 885 keV and the 678 - (885) - 658 keV correlations. The results are shown in Figure V-17 which indicate a mixing ratio value of $\delta_{678} = -0.28 \pm 0.05$. This corresponds to a $(93 \pm 3)\%$ M1 radiation component in the transition.

The 707/658 keV correlation involves three cascades, one of which includes the 678-keV transition. This correlation was used only to check the final mixing ratio results.

V.6.4. 686.8 - keV ($3^+ - 2^+$) transition

Of the three correlations measured involving the 687-keV transition only the 764 - 687 keV correlation data gave meaningful results. The 687 - (818) - 658 keV and the 764 - (687) - 818 keV correlations have small asymmetries, consequently, the A_{22} and A_{44} coefficients are small. Therefore, even though the statistics obtained on the 687-keV and the 818-keV peak areas were $\pm 2.7\%$, the errors on the directional correlation coefficients were still relatively large. The mixing ratio results were ambiguous since the cascades involved mixing in both the 687-keV and 818-keV transitions. The mixing ratio value obtained from the 764 - 687 correlation data was $\delta_{687} = -1.27 \begin{smallmatrix} +0.80 \\ -0.42 \end{smallmatrix}$ which indicates a M1 admixture in the 687-keV transition of $(38 \begin{smallmatrix} +44 \\ -12 \end{smallmatrix})\%$.

V.6.5. 706.7 - keV ($5^+ - 4^+$) transition

The mixing ratio for the 707-keV transition was extracted from the 707 - (687) - 885 keV correlation data using a mixing ratio value

Figure V-17. Simultaneous χ^2 analysis of 678-(885)-658 keV and 678-885 keV correlation data.

$\delta = -0.28$ for the 678-keV transition. The correlation data and the results of the χ^2 analysis are shown in Figure V-18. The mixing ratio obtained for the 707-keV transition was -1.10 ± 0.30 . This represents a $(45 \pm 11)\%$ M1 admixture in the 707-keV transition.

V.6.6. 818.0 - keV ($2^{1+} - 2^+$) transition

The 818.0-keV and the 744 - (818) - 658 keV correlations were used to determine the mixing ratio for the 818-keV transition. The χ^2 results from the simultaneous analysis of the data from these two correlations are shown in Figure V-19. A mixing ratio of $\delta_{818} = -1.44 \pm 0.10$ ($33 \pm 3\%$ M1) was determined. This value is in good agreement with the results given by Krane⁷⁷ and by Johnston and Stone⁷⁸. It is also in good agreement with the value of -1.50 ± 0.9 $\left(\begin{smallmatrix} 31 & + & 43 \\ - & 11 & \end{smallmatrix} \right)\%$ M1 obtained from Coulomb excitation measurements by Milner et al.⁸¹

V.6.7. 1384.0 - keV ($5^+ - 4^+$) transition

The 1384 - (885) - 658 keV and 1384 - 885 keV correlations were measured to determine the mixing ratio of the 1384-keV transition. The A_{22} and A_{44} coefficients for these correlations should be equal. Table VII shows that the directional correlation coefficients of these two correlations are in good agreement (entries 11 and 23 in Table VII). Figure V-20 shows the data obtained for these two correlations. Simultaneous analysis of the data from these correlations yields a mixing ratio of $\delta_{1384} = -0.45 \pm 0.02$, $(83 \pm 1)\%$ M1. The results of this analysis are shown in Figure V-21.

V.6.8. 1505.0 keV ($3^+ - 2^+$) transition

Correlation data from both the 764 - 1505 keV and the 1505 - 658 keV cascades were measured to determine the multipole mixing of the

Figure V-18: Correlation data and χ^2 analysis of 707-885 keV cascade in ^{110}Cd

Figure V-19. Simultaneous χ^2 analysis of 818-658 keV and 744-(818)-658 keV correlation data.

Figure V-20. Correlation data for 1384-(885)-658 keV and 1385-885 keV cascades in ^{110}Cd .

Figure V-21. Simultaneous χ^2 analysis of 1384-(885)-658 keV and 1384-658 keV correlation data.

1505.0-keV transition. Analysis of the 764 - 1505 keV correlation data gave two mixing ratio values, $\delta_{1505} = -1.16 \pm 0.40$ and $\delta_{1505} = -0.54 \pm 0.20$. Analysis of the 1505 - 658 keV data gave a single mixing ratio value of $\delta_{1505} = -1.20 \pm 0.20$. Figure V-22 shows the results from the simultaneous analysis of the data from both correlations. This analysis yielded a mixing ratio value of $\delta_{1505} = -1.24 \pm 0.32$, $(39 \pm 10)\% \text{ Ml}$.

This mixing ratio value differs significantly from the mixing ratio value of $\delta_{1505} = -0.55 \pm 0.10$ quoted by Krane and Steffen.⁷⁷ Figure V-23 compares our 1505 - 658 keV correlation data with the correlation curve for $\delta = -0.55$. Krane does not give any correlation data for this cascade, so only the shapes of the two curves can be compared. There is a significant difference in the ratio of 90° data to 180° data. The correlation data gives a $90^\circ/180^\circ$ ratio of 2.76 ± 0.12 . The curve for $\delta_{1505} = -0.55$ indicates a $90^\circ/180^\circ$ ratio of 2.06. The 90° and 180° measurements were repeated three times, each under different conditions to determine whether there was a systematic or procedural error in our analysis of the 1505 - 658 keV correlation data. The source was changed and the 90° and 180° measurements were made with source-to-detector distances at 7.0 cm. The measurements were also made with source-to-detector distances at 5.0 cm and 10.0 cm. For all cases except the measurement with a source-to-detector distance of 5.0 cm the $90^\circ/180^\circ$ ratio overlapped the 2.76 ± 0.12 value. For the measurement at 5.0 cm the $90^\circ/180^\circ$ ratio was 818.0-keV transitions may have influenced the 1505 - 658 keV correlation data obtained by Krane who used a 5 cm source distance.

Calculations show that the 1505-keV sum peak would show a correlation with the 658-keV transition. The number of summed 687-keV and

Figure V-22. Simultaneous χ^2 analysis of 1505-658 keV and 764-1505 keV correlation data.

Figure V-23. Comparison of the 1505-658 keV correlation data with a theoretical correlation curve for a mixed E2/M1 transition having a mixing ratio of -0.55.

818-keV events in coincidence with the 658-keV transition would be greater at 180° than at 90° . This would result in a reduction of $90^\circ/180^\circ$ ratio for the 1505 - 658 keV correlation as observed by Krane. Summing would have been more prevalent in Krane's data, since his source-to-detector distances were 5 rather than 7 centimeters and apparently he did not use any pulse pileup discrimination.

The nuclear orientation results of Johnston and Stone⁷⁸ and Wang et al.⁷⁹ agree with our results. Johnston reported that their A_2 coefficient (see Eq. I-5) determined from the angular distribution of the 1505-keV transition gave two possible solutions for δ , either $\delta = -0.48 \pm 0.03$ or $\delta = -1.26 \pm 0.06$. Their value for the A_4 coefficient gives a solution of $\delta = -1.2 \pm 0.5$ suggesting the larger value of δ as the more likely solution. They chose to report both values, since the lower value was in agreement with the value reported by Krane and Steffen. Wang et al. initially reported a mixing ratio value of $\delta = -0.51 \pm 0.05$ obtained from nuclear orientation measurements.⁷⁹ However, after discussing our results with them, they reviewed their data and found that their data gave a better result of $\delta_{1505} = -1.09 \pm 0.10$.⁸² Thus we feel confident that the value of $\delta_{1505} = -1.24 \pm 0.30$ which we have obtained is correct.

V.7. Comments on the structure of ^{110}Cd

The nucleus ^{110}Cd seems to exhibit an ordering of excited states characteristic of a "good" vibrational nucleus. However, considerable evidence has accumulated, regarding the non-zero static quadrupole moment of the first excited state and the failure of the electromagnetic transition probabilities to conform to the vibrational model.⁸¹ This indicates that a strictly vibrational model may lead to incorrect predictions of nuclear properties in this region.

Model calculations for the even Cd nuclei by Gneuss and Greiner⁶⁸ indicate that this is the case. These calculations predict that, as neutron pairs are added beyond the $N = 50$ neutron-shell closure $\left(\begin{smallmatrix} 98 \\ 48 \end{smallmatrix} \text{Cd}_{50} \right)$, the minimum in the potential energy surface shifts towards more deformed positions until, near ^{110}Cd , a second minimum appears. If such a second minimum does occur, an additional quasi-rotational band might be expected to occur at the two-phonon energy. Meyer and Peker⁸³ have shown that a quasi-rotational band built on the known proton four-hole, two particle 0^+ level at 1473.07-keV level in ^{110}Cd can be associated with known levels of spin and parity $I^\pi = 0^+, 2^+, 4^+$, and 6^+ that have a moment of inertia greater than that of the ground-state band.

In view of these facts, the large M1 admixtures indicated by this and other work should not be surprising. It is interesting to note that all eight E2/M1 mixing ratios measured in ^{110}Cd have negative signs.

VI. Discussion and Conclusions

Advances in detectors and pulse processing electronics in recent years have significantly improved coincidence techniques. With the high resolution capabilities of Ge(Li) detectors comes the need for new multi-parameter analysis systems to process and store the large quantities of data available. Since relatively long count times are required to obtain adequate statistics, there is a practical advantage in a coincidence analyzer system which is capable of sorting and storing two-parameter data on-line. Such a system would also provide for rapid recovery of the sorted data for setting up and monitoring an experiment as well as for the subsequent detailed examination of the data.

A gamma-gamma coincidence pulse-height analysis system has been developed which records all observed coincidences on a moving-head disk. The system uses a PDP-8/E minicomputer to sort and store gamma-gamma coincidence events on-line. By coupling dynamic allocation of disk-track buffers with a compact arrangement of the more active portions of the data locations on the disk, the system is capable of processing 1,040 events/s. The system can store up to 262,144-coincidence events in any one of 1024×1024 data locations on the disk. An X- or Y-pulse-height spectrum in coincidence with up to four pulse-height windows can be arithmetically assembled from the data on the disk into the computer core for scope display within 2 to 20 seconds.

Three separate programs were developed for the operation of the PDP-8/E system. These programs provided for operation of the system as a coincidence analyzer, for assemblage of gated spectra from the sorted coincidence data on the disk, and for operation of the system as two 2048-channel pulse-height analyzers. The programs were written

extensively in machine language to provide rapid and efficient data acquisition and sorting, and rapid data recovery from the disk.

The coincidence electronics were designed to provide optimum timing and energy resolution. The goal was to develop coincidence electronics which would allow the experimenter to collect the maximum amount of data in a minimum amount of time, with minimum influences by Compton background and accidental coincidences and with as little intervention as possible by the experimenter. As the level of sophistication in the system electronics increases, more attention must be given to the logic of the electronic component interactions and to the time alignment of the signals.

By studying isotopes with well-known decay schemes, subtle flaws can be found in the logic of electronic modules which might not be uncovered if an unknown decay scheme is studied. For example, an error in the on-line subtraction of chance coincidence events was discovered because gamma-ray lines showed up in gates where they did not belong after a careful correction for Compton distribution contributions. The cause was finally traced to the loss of chance coincidence events due to the improper resetting of logic levels in the true/chance logic module.

The two 40 cc detectors used for this work gave energy resolutions of 2.3 keV FWHM. A timing resolution of 9.0 ns FWHM and 21 ns FWTM was obtained using ^{134}Ba gamma rays at singles rates of 10,000 counts/s. The significant parameter; however, when discussing time resolution, is the full-width at tenth maximum (FWTM). It is well-known that the FWTM is a function of the energy range being considered. In particular by discriminating against coincidences which involve low-energy events a significant improvement occurs in the FWTM. From our experience we found that in those cases where the coincidence study involves an isotope

with no significant gamma rays with energies less than 300 keV it is best to reject those coincidence events in which one or both are low-energy events. The result is a significant improvement in the timing spectrum, in the true-to-chance ratio, and in the coincidence spectra.

Directional correlation measurements were made for gamma-ray cascades in ^{134}Ba and ^{110}Cd . The correlation data were analyzed to determine directional correlation coefficients and multipole mixing ratios. These measurements were made to determine whether the coincidence pulse-height analysis system and the electronics were functioning properly. From our experience we recommend that every coincidence pulse-height analysis system under development be tested by performing a coincidence experiment on a well-known decay scheme before embarking on an unknown.

Excellent directional correlation results were obtained after all problems with the coincidence electronics and associated logic were discovered and eliminated. The directional correlation coefficients obtained from those correlations involving transitions with no multipole mixing were in excellent agreement with theoretical values. This reflects favorably on all phases of the correlation analysis including the normalization factors for source asymmetry and decay, the accidental and Compton distribution corrections, the peak area determinations and their associated errors, and the values used for the solid-angle correction factors. The good results obtained for these correlations adds confidence to the results obtained for other correlations where multipole mixing is an additional factor. For transitions with mixed M1 and E2 multipolarities the mixing ratio results were in excellent agreement with other published results. In addition, the data handling features of this system and the improved energy resolution, resolved an ambiguity in the 1505-keV

transition in ^{110}Cd and indicated a probable error in another measurement on the 563-keV transition in ^{134}Ba .

In summary, results from these measurements indicate that this megachannel, multiparameter pulse-height analysis system will allow rapid and accurate gamma-ray coincidence studies to be made. The analysis system and associated electronics with their capacity for handling reasonable data rates can have considerable impact on the study of short-lived fission products.

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