AN ASSESSMENT OF THE PERFORMANCE OF A CONCEPTUAL ACOUSTIC SURVEILLANCE SYSTEM

FOR ANOMALOUS EVENTS IN LMFBRs

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A method is developed for calculating the detectability of anomalous acoustic events. The example used is the sodium vapor bubble collapse in the subcooled regions of a Liquid Metal Fast Breeder Reactor (LMFBR). This method provides a range of estimates for detection and false alarm probabilities in acoustic surveillance systems for sodium boiling and voiding detection, as well as any other impulsive events such as loose-parts monitoring.

The signal excess at the receiver array from an impulsive source is computed by an extension of methods introduced by W. Carey. Assuming an exponential pulse form for the signal (or its envelope) the equivalent source level is determined from the energy flux spectral density for inclusion in the sonar equation. The signal excess (SE) is then given by the source level (SL) minus the noise level (NL) minus the transmission loss (TL) minus the detection threshold (DT) plus the receiving array gain (AG).

Numerical values are drawn from recent experiments at ANL and EBR-II. Signal excess values are computed to be in the range of 0 to 20 dB. The probability of false alarm associated with high probability of detection and computed signal excess is excessive for reactor instrumentation. This false alarm rate must be reduced by post processing, i.e., taking advantage of the impulse occurrence statistics and by cross correlation with neutronic noise.

Introduction

Previous work¹ introduced the concept of the sonar equation to the field of sodium boiling detection and presented a method for calculating the detectability of sodium vapor bubble collapse by passive acoustic means. By treating the bubble as a single-frequency monochromatic source of sound in a uniformly distributed field of random noise sources, a relationship was shown between the output of a square-law detector and the power spectral density of the input signal and noise which determines signal detectability.

The detection process was then shown to be expressible as an algebraic equation known as the sonar equation in which each term represents some aspect of the signal or noise generation, transmission or reception. The source of sound was represented by the estimated radiated intensity (power per unit area at an arbitrary distance of 1 meter) known as the source level.

Two points should be emphasized concerning the computation of signal detectability which relate to the present study. The first is that a single frequency steady source of sound was assumed¹ while the bubble collapse results in an impulsive sound transient. This does not negate the value of the previous work since in principle the transient can be synthesized by a series of delta function impulses with continuous power spectra. However, by modifying the treatment of R. Urlick², the source energy may be used to compute the "impulsive" source level.

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The second point has to do with the transmission loss term. Transmission loss results when the sound wavefront spreads with distance or the sound intensity is decreased by absorption or scattering in the sound-conducting medium. Further effects, such as dispersion, are present for the finite amplitude (shock) pulse which accompanies bubble collapse. Zero transmission loss was assumed in the work of reference 1. It should be noted that a non-zero value of transmission loss experienced by sound transmitted within a working nuclear reactor is not easy to estimate but is expected to be small for the frequency ranges of interest. Further work is underway to determine the actual transmission loss under these conditions.

The Energy Form of the Sonar Equation

According to Urlick² the active sonar equation in terms of energy is stated as:

$$\text{Echo Energy (E)} = \text{Noise Masking Energy, (NME)} \quad (1)$$

where the NME:

$$\text{Noise Masking Energy (NME)} = \text{Noise Intensity (NI)} \times \text{Echo Duration (T)}. \quad (2)$$

He then shows that the energy form reduces to the more familiar form of the sonar equation if the source level is defined as:

$$SL = 10 \log(E) - 10 \log(T), \quad (3)$$

where T is the echo duration and E is the energy flux density of a plane pressure wave of 1 dyne/cm² at 1 meter, i.e., the energy flux/unit area per second of signal duration.

When we consider the detection by passive "listening" of a sodium vapor bubble collapse¹, in an infinite sea of sodium, we are faced with a sharp transient as a signal in continuous background noise. Urlick has stated that for the passive sonar equation, the difference between the energy and intensity form is trivial since the same time integral occurs on both sides of the energy equation. This is undoubtedly true for continuous sources of sound but for transients it is not necessarily so. We will inquire further into this case.

Energy, Intensity and Integration Time

If we have a signal S(t) and its Fourier transform H(f), its energy SE is expressible as the integral:

$$SE = \int_{-\infty}^{+\infty} |S(t)|^2 dt \quad (4)$$

which, by Parseval's theorem, is:

$$SE = \int_{-\infty}^{+\infty} |H(f)|^2 df. \quad (5)$$

In terms of energy the sonar equation is stated:

$$\text{Transient Signal Energy} = \text{Noise Masking Energy} \quad (6)$$

$$\text{Noise Masking Energy} = \text{Noise Intensity} \times \text{Signal Duration}. \quad (7)$$

If we have a noise n(t) with its Fourier transform G(f), we have for total noise energy:

$$NME = \int_{-\infty}^{+\infty} |G(f)|^2 df = \int_{-\infty}^{+\infty} |n(t)|^2 dt. \quad (8)$$

However, not all of the noise energy masks the signal if the signal bandwidth is finite. Only that part of the noise intensity that lies within the signal intensity spectral band will mask the noise. If we know the signal spectrum (and thus the signal waveform) we may filter the noise to that bandwidth. While it is strictly true that a signal and its spectrum cannot both be of finite extent, respectively in time and frequency, it is approximately true for most signals³. Therefore, we will assume that Equation (8) expresses the noise masking energy NME when written:

$$NME = 2 \int_0^{f_0} |G(f)|^2 df \quad (9)$$

where f_0 is the effective maximum frequency of the signal power spectral density. If $n(t)$ is a noise pressure signal, the noise acoustic intensity is given by Morse⁴

$$N_I = \frac{1}{2T} \int_{-T}^{+T} n(t)u(t)dt \quad (10)$$

where $u(t)$ is the local particle velocity in the wave. For a plane wave:

$$u(t) = \frac{n(t)}{\rho c} \quad (11)$$

thus, Equation (10) becomes:

$$N_I = \frac{1}{2T\rho c} \int_{-T}^{+T} n^2(t)dt. \quad (12)$$

If we use Parseval's theorem in its approximate form, i.e., for T sufficiently long:

$$\lim_{T \rightarrow \infty} \frac{1}{\rho c} \int_{-T}^{+T} n^2(t)dt = \int_{-\infty}^{+\infty} \frac{|n(t)|}{\rho c} |^2 dt =$$

$$\int_{-\infty}^{+\infty} |G(f)|^2 df \approx 2 \int_0^{f_0} |G(f)|^2 df = NME \quad (13)$$

then we have the relationship between the noise acoustic intensity and the noise masking energy, by substitution into Equation (12), and letting $T = T_N$ for convenience:

$$N_I = \frac{NME}{2\rho c T_N} \quad (14)$$

Applying the same arguments to the signal, intensity, S_I , is given by:

$$S_I = \frac{1}{2T} \int_{-T}^{+T} s(t)u(t)dt \quad (15)$$

where $u(t)$ is the particle velocity of the signal wave. If the signal source is small compared with the observation range the wave may be considered to be plane for which:

$$u(t) = \frac{S(t)}{\rho c} \quad (16)$$

as with the noise. Thus:

$$S_I = \frac{1}{2T\rho c} \int_{-T}^{+T} s(t)^2 dt \quad (17)$$

which becomes, by the same arguments as before, letting $T = T_S$,

$$S_I = \frac{SE}{2\rho c T_S}. \quad (18)$$

The "intensity form" of the sonar equation holds for the case as the receiver integrates for only as long as the signal lasts. It is true, for the spherical source in the infinite sea, that the "intensity form" of the equations may be used to compute the detectability of the sodium vapor bubble collapse. This is because the signal energy E is a constant and the intensity I is thus:

$$I(r) = \frac{E}{A} = \frac{E}{4\pi r^2} \quad (19)$$

the r^2 factor in the divisor results in a spreading loss term when expressed in decibels. Thus, by detecting the signal power and integrating over signal duration the signal intensity at range is determined.

The Effects of Boundaries

The effect of the presence of boundaries (and inhomogeneities) is to extend the signal in time beyond that of the signal duration at the source. As mentioned before, the signal at the receiver is expressed by a convolution of the signal waveform (for linear signals), $S(t)$, and the impulse response of the structure. A simple example is that of a free surface. The impulse response of a semi-infinite medium with a free surface is composed of the initial impulse plus a time-delayed negative image. The time delay is equal to the path length difference between the direct and reflected paths from source to receiver divided by the propagation velocity C . The geometry is shown in Figure 1.

Figure 1. Geometry for Half-Space with Free Surface.

Mathematically, we have:

$$S(r_1) = [\delta(t - \frac{r_1}{c}) - \delta(t - \frac{r_2}{c})] * S(t) \quad (20)$$

where $\delta(t-a)$ is the Dirac delta function. The impulse in time is pictured in Figure 2.

Figure 2. Impulse Response of Half-Space with Point Source.

The signal "duration" at the receiver is now:

$$T_R = \frac{T_2 - T_1}{C} \quad (21)$$

From this simple example it is apparent that the signal duration T_R at the receiver depends upon the geometry of the boundary or boundaries, or inhomogeneities. Urlick² shows the influence of a boundary in the waveform connected with the transmission and echo of an explosive source of sound. This figure, taken from Urlick's Figure 1, is reproduced in Figure 3.

Figure 3. Pulse Waveforms for Active Sonar, After Urlick², Figure 1.

The "near source" curve is the pressure-time envelope of a shock wave at a sufficient distance from the source for the shock front to develop. The manner in which the shock wave steepens is illustrated in Cole⁵ and reproduced in Figure 4.

Figure 4. Wavefront Steepening into a Shockfront, After Cole⁵, Figure 2.1.

The velocity of the finite amplitude wave depends upon the amplitude so that point *b* catches up with point *a* at which time the velocity of the front approaches acoustic velocity. The signal as received, in the absence of boundaries, appears as the time inverse of this waveform; thus, the model of the exponentially decaying impulse in explosive sound transmission.

We thus arrive at a distinction between the source signal duration, T_S , and the received signal duration, T_R . It is evident that $T_R \geq T_S$. It is also apparent that the noise integration time, T_N , is equal to T_R .

In order to determine the proper value of T_R for sodium vapor bubble detection in LMFBRs, it is necessary to determine the transmission characteristics (impulse response, transmission loss, etc.) of the reactor geometry, either by modeling or in full scale tests with controlled impulsive sources.

Detectability of the Pulse Signal

It should be clear from the foregoing arguments that the sonar equations for possible detection of a pulse are the same expressed either in terms of intensity or energy if the integration times are the same on either side of the energy equality expression. However, by selective filtering, (a rudimentary matched filter), the noise energy can be reduced.

The previous section showed the expressions necessary to solve for the bandwidth from the signal energy at the source. We wish to apply the method to the detection of a sodium vapor bubble collapse such as might be experienced in the subcooled regions of a working liquid sodium cooled nuclear reactor (LMFBR). The first step is to determine the effective maximum frequency of the source signal power spectral density, f_0 . The signal envelope to be detected is assumed to be an exponential pulse of the form:

$$p(t) = P e^{-t/t_1} \quad (22)$$

where

t_1 = time to decay to $P e^{-1}$.

This pulse is sketched in Figure 5.

Figure 5. Exponential Pulse Waveform.

It has a plane wave energy density given by:

$$E = \frac{1}{\rho c} \int_0^\infty p^2(t) dt \quad (23)$$

where ρc is the characteristic acoustic impedance of the medium. If pressure is in dynes/cm², and ρc is in cgs units, E will be expressed in ergs/cm².

The energy flux spectral density is found by the magnitude squared Fourier transform of $p(t)$ and is given as⁶:

$$E_o(f) = \frac{2P_o^2}{\rho c} \frac{t_1^2}{1 + (2\pi t_1 f)^2} \quad (24)$$

The corner frequency of the spectrum occurs when the ratio $(E_o(f_o)/E_o(0))$ is equal to 0.5, from which $f_o = 1/2\pi t_1$.

Taking a value of t_1 from the pulse shown in Pepler⁸ of $t_1 = 200 \mu\text{sec}$, we have:

$$f_o = 1/2\pi t_1 = 7.960 \text{ kHz.} \quad (25)$$

Thus, we will use a lowpass filter at 8 kHz. The

success of this step depends upon the accuracy of estimation of t_1 .

To determine the source level, we need the energy flux density at 1 meter.

Recalling the previous section, the energy flux spectral density is of the equivalent form shown in Figure 6.

Figure 6. Equivalent Energy Flux Density Spectrum of Exponential Pulse.

Determining the value of $E_0(0)$ by using Equation (24), one finds that the energy can be approximated by:

$$E = \frac{2P_o^2}{\rho c} \left(\frac{f_o}{f_1^2} \right) \quad (26)$$

The source level is:

$$SL = 10 \log \left(\frac{E}{E_r} \cdot \frac{T_r}{T} \right) = 10 \log \left(\frac{P_o^2}{P_r^2} \right) + 10 \log \left(\frac{f_o}{f_1^2} \cdot \frac{T_r}{T} \right) \quad (27)$$

For P_o we use measured values of $P_o = 170$ to 190 dB re μ Pa which yield, from the above equation with $T = 125$ μ sec, $SL = 173$ to 193 dB re μ Pa at the fuel subassembly outlet into the reactor upper plenum. Assuming that transmission loss is due to the change in cross section between the fuel subassembly and the upper plenum, $TL \sim 22$ dB. The received signal level is then $SL - TL = 151$ to 171 dB. To determine the detection performance we assume a quadratic receiver¹⁰ working at a signal to noise ratio in the receiver band such that $DT = 10 \log (S/N)$. Examples of this performance are:

DT	P_{fa}	P_d
13	10^{-3}	.995
14	10^{-4}	.995
15	10^{-6}	.995

where the detection probability, P_d , is fixed and the false alarm probability, P_{fa} , varies with DT. For array gain we assume a practical number of sensors, (5) in the upper plenum for which $AG \sim 10 \log(5) = 7$ dB. The measured value of noise background in the receiver bandwidth is $NL = 151$ dB re μ Pa. Then substituting in the sonar equation:

$SE = 7$ to 27 dB -DT

i.e., we need a 7 to 27 dB detection threshold for zero signal excess. From the above table it is seen that the midpoint of this range, $DT = 13$ dB gives a $P_{fa} = .001$. Recalling that this probability is defined for a single pulse duration and that $T = 125$ μ sec, we compute a false alarm rate of 8 false alarms/sec. A satisfactory value for reactor instrumentation would be in the order of 1 false alarm/year which translates to a $P_{fa} = 10^{-17}$. Clearly the above computed performance is inadequate based on a single event. The false alarm probability must be reduced by post processing after energy detection in a manner similar to radar anti-clutter techniques. Since the process under consideration is a repeated occurrence of broadband, transient events further averaging will not reduce the false alarm rate. If the mean rate of occurrence of these events is much higher than the false alarm rate for a single event, then the detection of multiple pulses can be accomplished with a lower P_{fa} and a high detection probability. Furthermore, cross correlation of the acoustic signal with neutronic noise can further reduce this false alarm probability.

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