ONE-DIMENSIONAL THERMONUCLEAR BURN COMPUTATIONS

FOR THE REVERSED-FIELD PINCH REACTOR (RFPR)*

by

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R. A. Nebel, G. H. Miley and R. W. Moses[†]

Fusion Studies Laboratory, Nuclear Engineering Program

University of Illinois, Urbana, Illinois 61801

MASTER

ABSTRACT

Conceptual fusion reactor designs of the Reversed-Field Pinch Reactor (RFPR) have been based on profile-averaged zero-dimensional (point) plasma models. The plasma response/performance that has been predicted by the point plasma model is re-examined by a comprehensive one-dimensional (radial) burn code (RFPBRN) that has been developed and parametrically evaluated for the RFPR. The RFPR plasma parameters have been optimized and effects of turbulent transport and stability have been studied.

I. INTRODUCTION

A comprehensive conceptual design of a Reversed Field Pinch (RFP) reactor was recently published.¹ The plasma physics model, used to estimate the thermonuclear burn dynamics for this study was based on a zero-dimensional or "point-properties" formulation. This "point" model averaged all crucial plasma properties (i.e., field, density, and temperature profiles) in accordance with the Bessel-function model^{2,3} for magnetic field profiles.

*Work supported by US DOE contracts EY-76-S-02-2218 and W-7405-ENG-36

[†]Los Alamos Scientific Laboratory

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In order to verify the accuracy of these studies and explore other phenomena, a more realistic one-dimensional (radial) plasma model has been developed⁴ and applied to the point design reported in Ref. 1. The results of this comparison and suggested changes inferred for future RFP designs are reported herein.

II. BACKGROUND

The RFP is a device similar to a tokamak in structure. It is toroidal and has confinement provided by toroidal (B_ϕ) and poloidal (B_θ) magnetic fields. The major difference is that B_ϕ and B_θ are the same order of magnitude and B_ϕ is reversed on the outside edge of the plasma (see Figure 1).¹

FIGURE 1

Comparison of magnetic and pressure profiles for a $dq/dr \neq 0$ stabilized RFP and a $q > 1$ stabilized tokamak.

The resulting high shear provides stability to high β ($\beta_0 \sim .5$)⁵ for pressure driven modes while a conducting wall stabilizes current driven modes. Unlike tokamaks which require $q > 1$ [$q = (r_p/R)(B_\phi/B_\theta)$] the RFP operates with $q < 1$ and hence has no aspect ratio constraint.

Since $B_\theta \sim B_\phi$, the RFP has large currents and the reactor can be ignited by ohmic heating alone. This feature, coupled with stability at high β and the lack of an aspect ratio constraint, potentially provides significant engineering advantages over tokamaks. However, since B_ϕ resistively decays, the RFP must operate in a pulsed mode. Still the resistive decay time is long enough at thermonuclear temperatures to allow pulse times on the order of tens of seconds so that, as shown later, favorable energy balances are possible.

Previous plasma studies of RFPR have been done using a point model. (A detailed description of this work is given in Reference 1). Transport coefficients were assumed to be classical, except for thermal conduction which was assumed to be anomalous ($1/200$ Bohm). This is consistent with the results seen on Zeta, where energy confinement was strongly anomalous even during the "quiescent" period.⁶

Results of the cost optimized point model RFPR design are summarized in Table I. These parameters are the base case values used for the one-dimensional simulations.

III. ONE-DIMENSIONAL RFPR THERMONUCLEAR BURN

The one-dimensional model developed here treats a three-species quasi-neutral gas of electrons, DT-ions and alpha particles. Radial field diffusion and plasma convection in this axisymmetric system are

TABLE I
SUMMARY DESCRIPTION OF RFPR DESIGN PARAMETERS

PARAMETER	VALUES
First-wall radius, r_w (m)	1.5
Major radius, R (m)	12.7
Toroidal plasma current, I_ϕ (MA)	20.0
Toroidal field at the coil, $B_{\phi c}$ (T)	2.0
Poloidal field at the coil, $B_{\theta c}$ (T)	2.0
Toroidal coil energy, $w_{B\phi}$ (GJ)	3.7
Poloidal coil energy, $w_{B\theta}$ (GJ)	11.0
Field rise time, τ_R (s)	0.1
Burn time, τ_B (s)	21.6
Cycle time, τ_C (s)	26.6
Average fuel burnup, f_B	0.5
Average plasma density, n ($1/m^3$)	$2.0(10)^{20}$
Average plasma temperature, $T_{i,e}$ (keV)	15.0
14.1-MeV neutron flux at first wall, I_w (MW/m^2)	2.7
Engineering Q-value, Q_E	5.8
Recirculating power fraction, $\epsilon = 1/Q_E$	0.17
Average blanket power density, P_{BLK} (MW/m^3)	4.7
Average system power density, P_{SYS} (MW/m^3)	0.9
Total thermal power, P_{TH} (MWt)	3000
Net electrical power, P_E (MWe)	750
Net plant efficiency, $\eta_p = \eta_{TH}(1-\epsilon)$	0.25

modeled under a magnetohydrostatic approximation, and thermal conduction for ions and electrons are computed separately. The plasma is heated ohmically, and radiation losses include bremsstrahlung and electron cyclotron. Alpha particle heating is modeled using an in-situ approximation with the power fractions going to ions and electrons determined by Ref. 7.

A. Radial Transport

The radial burn code uses an implicit Lagrangian meshing technique that has been developed specifically for modeling plasma processes quasi-statically. Inertial terms in the MHD equations are ignored, and plasma evolution is then determined by pressure balance, $\vec{J} \times \vec{B} = \nabla p$. This requires that the equilibrium profiles are stable, and this assumption is examined periodically by a separate algorithm (see Sec. IIIC). For systems that exhibit symmetry in θ and ϕ , the pressure balance constraint is expressed by the following integral equation:

$$p + B^2/2\mu_0 = (2/r^2) \int_0^r (p + B_\phi^2/2\mu_0) r' dr' \quad , \quad (1)$$

where p is the total plasma pressure, $B^2 = B_\phi^2 + B_\theta^2$, and mks units are consistently used except for temperature, $T(\text{keV})$.

Plasma parameters are computed as functions of time by a two-step method. First, the Lagrangian mesh is fixed in space for the computation of all diffusion, radiation, and heating processes. Next, the Lagrangian mesh is adjusted radially to satisfy the pressure balance constraint given by Eq. (1).

The first phase of this two-step process is described by the following expressions for ion density n_i , alpha-particle density n_α , energy of the j th species T_j ($j=e, i, \alpha$), and magnetic field.

$$\frac{\partial n_i}{\partial t} + \vec{\nabla} \cdot \vec{\Gamma}_i = S_i - 2f(1-f) \langle \sigma v \rangle n_i^2 \quad (2a)$$

$$\frac{\partial n_\alpha}{\partial t} + \vec{\nabla} \cdot \vec{\Gamma}_\alpha = f(1-f) \langle \sigma v \rangle n_i^2 f_\alpha \quad (2b)$$

$$\frac{3}{2} n_j \frac{dT_j}{dt} + p_j \vec{\nabla} \cdot \vec{\Gamma}_j = -\vec{\nabla} \cdot \vec{q}_j + Q_j \quad (2c)$$

$$\frac{\partial \vec{B}}{\partial t} = \vec{\nabla} \times \left[-\vec{v} \times \vec{B} + \frac{1}{en_e} \vec{R}_{TH} + n_\alpha J_\alpha \vec{e}_\parallel + n_i J_i \vec{e}_\parallel \right] \quad (2d)$$

The fluid velocity with respect to the Eulerian coordinates is $\vec{v}$, $\vec{\Gamma}_j = n_j \vec{v}_j$ is the particle flux, S_j is a refuel ion source term, f is the tritium fraction, and f_α (≤ 1.0) is an alpha-particle confinement efficiency based on finite orbit effects. The thermal heat flux is $\vec{q}_j$; Q_j is a heat source/sink including alpha heating, ohmic heating, ion-electron equilibration, bremsstrahlung and cyclotron

radiation; and R_{TH} is a thermal force. The parallel and perpendicular resistivities, $\eta_{\parallel}$ and $\eta_{\perp}$, are treated either classically or with anomalous effects included as described later (see Sec. III D).

B. Pressure Balance

When the transport (Eqs. (2a-d)) is evaluated for a given time step at each radial position, the new values of temperature, density and fields do not satisfy pressure balance (Eq. (1)). It is then necessary to adiabatically adjust the Lagrangian mesh so the new radial positions satisfy the pressure constraint. This procedure is subject to the appropriate boundary condition(s), e.g. maintaining a given plasma radius or toroidal current. Variable time steps are used to assure fast convergence, while energy balance is used to check the accuracy of the model.

C. Stability Model

Two methods are used to monitor the ideal MHD stability. Local modes are assumed to saturate and are monitored using a Suydam criterion.⁸ A Rayleigh-Ritz type energy principle is used to monitor global modes. Both methods for monitoring stability are derived from the ideal MHD equations using a cylindrical approximation.

In one-dimension the energy principle assumes the following form⁹

$$\delta W = (\pi/2) \int_0^{r_p} [f(d\xi/d\tau)^2 + g\xi^2] d\tau \quad (3)$$

where

$$f = \frac{r(krB_{\phi} + mB_{\theta})^2}{\mu_0 [(kr)^2 + m^2]} \quad (4a)$$

$$g = \frac{2(kr)^2}{(kr)^2 + m^2} \left(\frac{d\rho}{dr} \right) + \frac{[krB_{\phi} + mB_{\theta}]^2 [(kr)^2 + m^2 - 1]}{\mu_0 r [(kr)^2 + m^2]} \\ + \frac{2(kr)^2 [(kr)^2 B_{\phi}^2 - m^2 B_{\theta}^2]}{\mu_0 r [(kr)^2 + m^2]^2} \quad (4b)$$

The axial wave number is k , the mode number is m , and ξ is the radial displacement. The transform $\psi = \xi \sqrt{f}$ is employed in order to avoid the singularity in ξ when f approaches zero.⁵ Then Eq.(3) transforms to

$$\delta W = (\pi/2) \int_0^R \left\{ (d\psi/dr)^2 - (\psi/r) (df/dr) (d\psi/dr) + \right. \\ \left. [(df/dr)^2/4f^2 + g/r] \psi^2 \right\} dr \quad (5)$$

The parameter ψ is then expanded into a Fourier Series according to

$$\psi = \sum_{i=0}^n C_i \sin(\alpha_i r) \quad (6)$$

where $\alpha_i = (i+1)\pi/R$. The constant C_0 is normalized to unity and the remaining constants, $C_i (i=1, \dots, n)$, are computed to minimize the energy δW by requiring that $\partial \delta W / \partial C_i = 0$ for $i=1, \dots, n$. The resulting set of n inhomogeneous equations in n unknowns is inverted and solved for C_i . If the resulting value of δW is negative, the associated plasma

configuration is considered unstable. This check is applied to $m=0$, $k \neq 0$ and $m=1$, $-30/R < k < 30/R$, since the long wavelength modes are typically the most unstable.⁵

In general, $\delta W / \delta C_i = 0$ only provides a stationary solution rather than a minimum for the energy principle. A minimum requires that $\delta^2 W / \delta C_i^2 > 0$ also. However, if $\delta^2 W / \delta C_i^2 < 0$, it can be shown that the plasma is always unstable and the required answer is provided without inverting the matrix.

The Suydam criterion is included in the more general energy principle stated by Eq. (3). These Suydam modes, however, have eigenfunctions that are asymptotically singular at rational surfaces. Since the mesh size used by the one-dimensional computation is finite and determined by the previously described transport computation, eigenfunctions that are more localized than the mesh size will not be detected by the global energy-principle analysis. Consequently, the global stability analysis is able to find gross modes that may also occur simultaneously with Suydam modes (which are assumed to saturate). The global energy-principle analysis also quantifies the validity of the assumption that Suydam modes are localized in that the modes will be detected if the associated eigenfunctions extend over a "macroscopic" (i.e., mesh size) volume of the plasma.

D. Anomalous Transport

As will be shown, a large region of the plasma is found to be unstable to local Suydam modes for the plasma conditions selected for the point plasma operating parameters. The resulting instabilities are assumed to remain localized and to be manifested as enhanced transport.¹⁰ Specifically, the perpendicular resistivity, $\eta_\perp$, was modified relative to the classical value, η_{1c} , according to

$$\eta_\perp = \eta_{1c} + f_T f_s C B^2 \eta_B / n_e \quad (7)$$

where f_T is a triangular function that spreads the region of enhanced transport over mesh points (~ 10 Larmor radii) adjacent to the location of Suydam instability, $C = 10^{-4}$, and f_S is given by

$$f_S = \begin{cases} 0 & \text{if } \epsilon < 1 \\ (\epsilon-1)/\epsilon & \text{if } \epsilon > 1 \end{cases} \quad (8a)$$

$$\epsilon = - \frac{dp/dr}{(B_0^2/8\pi v_0)(dp/dr)^2} \quad (8b)$$

and

$$P = rB_\phi/B_0 = 1/v. \quad (8c)$$

Bohm-like transport scaling is used since it scales like to MHD turbulence.¹⁰ The triangular function, f_T , accounts for the width of a Suydam mode while f_S is a function varying from 0 to 1 which provides a measure in the degree of Suydam violation. The exact scaling of the perpendicular resistivity has been found to be relatively unimportant as long as it substantially exceeds (by 2 to 3 orders of magnitude) the classical values. The higher perpendicular resistivity increases radial transport of particles and energy, thereby reducing the pressure gradients that initiated the instability.

IV. RESULTS

Plasma performance was evaluated for four cases:

- Case A: The one-dimensional model was forced to follow exactly the radius trajectory predicted by the zero-dimensional model.
- Case B: The more physically correct assumption of constant radius and moderately programmed toroidal current was examined.
- Case C: With the plasma radius fixed, regions of Suydam instability were subjected to enhanced transport in order to model a flattening of the pressure profile (e.g., Eqs. (8)).
- Case D: Using Case B parameters, the magnetic field was varied to optimize Q_p .

For all cases the radial energy confinement time is set equal to 200 Bohm times and it is assumed that both the alpha-particle energy and mass are deposited at the place of birth. Table II summarizes the essential elements of the three simulation cases described above.

TABLE II
SUMMARY OF CONDITIONS ASSUMED FOR ONE-DIMENSIONAL SIMULATIONS

<u>Case</u>	<u>Radius Trajectory</u>	<u>Energy Transport</u>	<u>Burn time^(b) $\tau_R(s)$</u>
A (a)	moving	200	21.6
B	fixed	200	25
C	fixed	200 plus Suydam Turbulence ¹⁰	25
D	fixed	200	21.6

(a) closest simulation of zero-dimensional model

(b) quench time taken as 5 s

A. Discussion of Case A Results.

In order to match the point model as close as possible, the plasma radius was confined to a trajectory $r_p(t)$ given by

$$r_p(t) = r_p(t=0) / \sqrt{1 - R_\theta(t)}. \quad (9)$$

where the plasma radius r_p is defined by the toroidal field reversal point, and β_θ is the minimum poloidal beta. The initial temperature distribution was assumed uniform, and the initial density and toroidal field profiles within the plasma region were modeled by Bessel functions (consistent with the point model¹)

$$n(r,t=0) = n(r=0,t=0) J_0^2(0.94\alpha r) \quad (10)$$

$$B_\phi(r,t=0) = B_\phi(r=0,t=0) \quad (11)$$

where $\alpha = 2.405/r_p(0)$. The initial poloidal field profile is computed to satisfy pressure balance.

Figure 2 compares the averaged ion and electron temperatures obtained with the one-dimensional model for Case A with the predictions of the zero-dimensional model. Shown also is the time dependence of poloidal beta, β_θ , and the fractional burnup. The radial dependence of temperatures and fields at $t=12$ s into the burn is depicted in Fig. 3. Fig. 4 shows the density profile, the shear parameter(Eq. (8c)) , P and the Suydam parameter, f_s (Eq. (8a)) at 12 seconds. For the parameters shown, $f_s > 0$, everywhere so the plasma is unstable to local Suydam modes over its entire volume. Fig. 5 compares the neutron, transport, and radiation energy flux at the first wall.

The differences between the point and one-dimensional models illustrated in Figs. 2-5 can be attributed to the self-consistent profiles contained in the one-dimensional code. The point model assumes a uniform plasma temperature, while the one-dimensional code has radially varying temperatures which are higher and cause enhanced

FIGURE 2

Comparison of averaged parameters versus time as predicted by the Case A one-dimensional simulation and the zero-dimensional model.

FIGURE 3

Temperature and field profiles for the Case A simulation at 12.0 s into the burn phase.

FIGURE 4

Profiles of density, pitch parameter P , and Suydam parameter, f_s , for the Case A simulation at 12.0 s into the burn phase.

FIGURE 5

Comparison of first-wall neutron, thermal conduction, and radiation fluxes for the Case A one-dimensional simulation and the zero-dimensional model.

fusion reactivity (and alpha-particle heating) in the central plasma region. This results in a higher average temperature shortly after ignition (Fig. 2). The one-dimensional model burns out more rapidly, and has a lower value for f_B (Fig. 2). This lower burnup is principally caused by small burnup near the plasma edge where the plasma is cooler.

The point burn model must specify a magnetic field profile to assure pressure balance and does not account for resistive decay of the toroidal field B_ϕ . In forcing a radius trajectory, $r_p(t)$, the one-dimensional model forces the toroidal current, I_ϕ , to decay, as resistive losses consume B_ϕ field. This results in somewhat less ohmic heating, although this effect is small.

The radial profiles shown in Figs. 3 and 4 remain relatively unchanged throughout the burn period. Both the temperature and pressure show a maximum on the plasma centerline because of the assumed Bohm-like thermal conduction. However, since the bulk plasma is unstable to Suydam modes, enhanced transport occurs in this region. This is examined in Case C.

Table III summarizes performance parameters for both the zero-dimensional and Case A one-dimensional simulations. Of particular interest is the plasma Q-value, Q_p ,

$$Q_p = 1.25 \cdot W_N / (W_{OHM} + W_{pf}) \quad (12)$$

It is seen that for the conditions under which Case A was evaluated, a factor ~1.5 decrease in Q_p relative to the point model is predicted. This significant difference in Q_p illustrates the net influence of profile effects, with the reduction in Q_p a direct result of the lower fusion yield. However, as for any model, Q_p values are sensitive to modelistic assumptions. Some of the assumptions are examined further in the following cases.

TABLE III

COMPARISON OF ENERGY BALANCE FOR ZERO- AND ONE-DIMENSIONAL BURN MODELS

PARAMETER (MJ/m)	ZERO-D	ONE-D ^a CASE A
Burn time τ_B (s)	21.6	21.6
Initial plasma energy, W_{pi}	0.05	0.7
Final plasma energy, W_{pf}	2.5	1.9
Radiation energy, W_{RAD}	28.1	14.3
Ohmic heating energy, W_{OHM}	7.1	8.9
Plasma energy loss (conduction), W_{COND}	147.5	93.0
Plasma expansion energy, W_{pv}	0.7	1.1
Magnetic-field energy inside first wall at end of burn cycle, W_{Bf}	51.6	45.0
Fusion Neutron Energy, W_N (14.1 MeV)		
$Q_p = (\text{fusion energy})/(\text{ohmic} + \text{field energy})$	14.4	9.5

^aradius forced to follow zero-dimensional case

B. Comparison of Constant Radius Cases With and Without Enhanced Transport, Cases B and C.

The one-dimensional model was evaluated for a programmed rather than a constant toroidal current, using the formalism for Suydam turbulent transport represented by Eqs. (7) and (8). A comparison for the constant radius case with and without Suydam turbulence is given in Fig. 6, where the time dependence of average temperature, $\bar{\theta}_0$, and f_S is given. Typical radial profiles of temperatures and fields are depicted in Fig. 7. Like the global parameters in Fig. 6, Suydam turbulent transport has little effect on the temperature and field profiles. Fig. 8 gives the radial dependence of densities and the Suydam parameter, f_S . Key plasma parameters for Cases B and C are summarized in Table IV.

Even without Suydam turbulent transport a greater fraction of the plasma volume becomes Suydam stable (Fig. 8) for the constant-radius Case B. Suydam turbulence causes anomalous Bohm-like transport to occur in unstable regions (Eqs. (7) and (8)), flattening the density profiles in the center of the plasma. There is a similar leveling of the pressure profile, pushing the plasma towards the marginal Suydam stability limit.

Comparison of the Suydam stability parameter, f_S , for Cases B and C shows that enhanced transport changes the plasma from one of uniform Suydam instability to a pointwise, stochastic violation of the Suydam criterion. As an unstable plasma region is stabilized by anomalous transport, pressure gradients in adjacent regions are increased, causing these latter regions to become unstable. The net result is a randomized or statistical violation of the Suydam criterion in regions

TABLE IV

COMPARISON OF ENERGY BALANCE FOR NONTURBULENT AND TURBULENT TRANSPORT

PARAMETER (MJ/m)	ONE-D ^a CASE 8	ONE-D ^b CASE C
Burn time τ_B (s)	24.6	25.0
Initial plasma energy, W_{pi}	0.8	0.8
Final plasma energy, W_{pf}	3.0	3.1
Radiation energy, W_{RAD}	14.8	13.9
Ohmic heating energy, W_{OHM}	10.4	12.4
Plasma energy loss (conduction), W_{COND}	115.2	114.5
Plasma expansion energy, W_{pv}	0.0	0.0
Magnetic-field energy inside first wall at end of burn cycle, W_{Bf}	45.1	45.5
*Fusion Neutron Energy, W_N (14.1 MeV)		
* $Q_p = (\text{fusion energy})/(\text{ohmic} +$ field energy)	493.5 11.7	469.7 10.1

^afixed plasma radius, no Suydam turbulence

^bfixed plasma radius, Suydam turbulence

FIGURE 6

Comparison of Case B and Case C one-dimensional simulations showing the influence of Suydam turbulence on the average temperatures; poloidal beta, and burnup fraction. For f_B and β_{θ} , no difference was observed.

FIGURE 7

Typical temperature and field profiles at $t=10$ s for Cases B and C.

FIGURE 8

Comparison of Case B (no turbulence) and Case C (with turbulence) one-dimensional simulations in terms of density and f_s profiles that have evolved at ~ 10 s into the burn phase.

of weak shear. These results indicate that a major part of the plasma confinement is provided by the plasma edge regions.

The magnetic fields maintain a continuous topological structure despite the enhanced transport associated with the Case C model. Aberrations in both field and pitch parameter profiles are small compared to density perturbations (Figs. 7,8). This result is characteristic of anomalous transport scaling through only the perpendicular resistivity, $\eta_{\perp}$ (Eqs. (13) and (14)).

Despite the significant differences in transport models used in Cases B and C, the plasma Q_p is increased only 10-20% from the Case A comparison case. A factor of 1.3 then separates the predictions of the one-dimensional and zero-dimensional models.

C. Optimization of RFPR Design Point by One-Dimensional Model

The optimization of the RFPR design point by the one-dimensional model is the primary motivation for this study. Overall reactor system considerations are not within the scope of this work, and a comparison with the point model is used to gauge if the predictions of the one-dimensional model are favorable. Establishing a 14.1-MeV neutron wall loading near the reference design¹ (2.7 MW/m^2), while maintaining a similar power output, fixes the plasma line density at $1.5(10)^{21} \text{ m}^{-1}$.

Using Case D (with fixed radius) listed in Table V, the magnetic field levels were reduced to increase Q_p (see Fig. 9). If $B_{\phi 0}$ is reduced below $\sim 5.8\text{T}$, plasma ignition does not occur due to insufficient ohmic heating. As noted in Fig. 9, the poloidal beta increases from 0.39 to 0.51 as the magnetic fields are reduced, still remaining within stability limits as predicted by ideal MHD theory.

TABLE V

COMPARISON OF ENERGY BALANCE FOR ZERO- AND OPTIMIZED
ONE-DIMENSIONAL BURN MODELS

<u>PARAMETER (MJ/m)</u>	<u>ZERO-D</u>	<u>ONE-D^a CASE D (OPTIMIZED)</u>
Burn time τ_B (s)	21.6	22.0
Initial plasma energy, W_{pi}	0.05	0.99
Final plasma energy, W_{pf}	2.5	3.68
Radiation energy, W_{RAD}	28.1	19.3
Ohmic heating energy, W_{OHM}	7.1	8.93
Plasma energy loss (conduction), W_{COND}	147.5	146.7
Plasma expansion energy, W_{pv}	0.7	0.0
Magnetic-field energy inside first wall at end of burn cycle, W_{Bf}	51.6	30.6
Fusion Neutron Energy, W_N (14.1 MeV)	676.5	646.2
$Q_p = (\text{fusion energy})/(\text{ohmic} + \text{field energy})$	14.4	20.4

^afixed plasma radius

FIGURE 9

Dependence of plasma Q-values and associated maximum poloidal beta on the initial toroidal bias field.

The time dependence of the electron and ion temperatures, f_g and B_θ for the $Q_p = 20.4$ case shown in Fig. 9 are plotted in Fig. 10. The plasma power fluxes are also given in Fig. 10. The radial dependence of the magnetic fields is shown in Fig. 11 for $t=0, 10, 20$ s. At $t=0$ the plasma pressure is essentially zero, and the magnetic field profiles represent a pressureless plasma state. Expansion occurs as the plasma heats causing the magnetic fields near the center to decrease in order to maintain pressure balance. This behavior is illustrated by the $t=10$ s case in Fig. 11. Near the end of the burn ($t=20$ s) the plasma is again cool, and the fields relax toward the center. The effect of plasma expansion is also exhibited by the electron density profiles depicted in Fig. 11.

The effects of fuel burnup are illustrated in Fig. 11 by the decreasing ion density and corresponding increase in alpha-particle density as the burn proceeds. Regions of Suydam instability violation are shown in Fig. 11 as f_s . Only a small region of the plasma is initially unstable, but the region grows as the temperature is increased. The region of Suydam instability is reduced as the plasma temperature decreases near the end of the burn ($t=20$ s, Fig. 11), although relaxation of the profiles has also contributed to the degradation of the stability.

The existence of a turbulent Suydam-unstable central region is common to axisymmetric configurations (both the tokamaks and the RFP) and is not expected to limit severely the confinement properties because of the stable region between the turbulent plasma core and first wall. In fact, this study demonstrates that Suydam induced does not produce significantly different plasma performance from those

FIGURE 10

Time dependence of average plasma temperatures, beta, and DT fuel burnup fraction for the optimum design point. Also shown are the neutron, radiation and conduction powers expressed per unit first-wall area.

FIGURE 11

Radial profiles of the magnetic fields, particle densities, ion and electron temperatures, magnetic field pitch and the Suydam stability parameter (f_s) for the optimal design point at various times.

cases in which the existence of the instability was only monitored. The assumed anomalous energy confinement corresponding to $200\tau_{\text{GOHM}}$ dominates any Suydam induced losses.

V. CONCLUSIONS

Generally, the one-dimensional model behaves similar to the point model. Differences between the results of the two models are caused by profile effects.

One significant difference is that local ignition and radial burn propagation allow the one-dimensional model to ignite with lower currents. This has led to a new optimized RFPR operating with better performance (higher Q_p) than was exhibited by the point model. The lower burnup exhibited by the one-dimensional model (Case A, Table III) is also caused by profile effects. Lower plasma edge temperature leads to lower burnup in that region causing a smaller total fractional burnup.

Stability studies indicate that gross plasma stability can be maintained with proper field programming for the ideal $m=0$ and $m=1$ modes. Suydam instabilities occur over a substantial region of the plasma, but tend to self-stabilize when anomalous transport is included. This transport flattens the density profiles and causes the plasma to approach the marginally stable Suydam limit. This results in stochastic violation of Suydam's criterion in the inner plasma regions. However, this has little effect on gross plasma parameters.

Although the one-dimensional simulations give considerably more insight into RFP reactor behavior, a number of physics questions remain. Among these questions are the empirical scaling of energy and

particle transport; this will have to be resolved by experiment. Also, questions of startup and refueling for the RFP remain to be answered. These will be addressed by a future study, involving extension of the one-dimensional model used here.

Acknowledgments:

The advice and collaboration of Dr. R. L. Hagenson and Dr. R. A. Krakowski is gratefully acknowledged.

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