TITLE: A NOTE ON THE DECAYS π₀ → νν̄

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A NOTE ON THE DECAYS $\pi^0 \rightarrow \nu \nu^-$

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ABSTRACT

Phenomenological aspects of $\pi^0 \rightarrow \nu \nu^-$ are considered. From the existing experimental information, we deduce an upper limit

$$\frac{\Gamma(\pi^0 \rightarrow \nu \nu^-)}{\Gamma(\pi^0 \rightarrow \text{all})} < 2.4 \times 10^{-5} \, (90\% \, \text{C.L.}).$$

Possible mechanisms which can give rise to this process are discussed. Branching ratios of the order of $10^{-7}$ for decays into the known neutrinos ($\nu_e, \nu_\mu$) are found possible. Rates comparable to the experimental limit for decays into final states involving additional neutrinos (massive or massless) cannot be ruled out.

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The decay of the neutral pion into two spin 1/2 neutral particles is an interesting one, since momentum and angular momentum conservation admit only final states in which the helicities of \( \nu \) and \( \nu' \) are the same. Consequently, \( \pi^0 + \bar{\nu} \bar{\nu} \) is forbidden if \( \nu \) is a massless Weyl neutrino. Hence, the observation of \( \pi^0 + \nu \nu' \) would imply either the existence of neutrino states of both helicities, or that lepton number is not conserved, or both.

The purpose of this note is to present an experimental upper limit for the branching ratio of \( \pi^0 \rightarrow \nu \nu' \), implied by the available data, and to consider the possible mechanisms and corresponding rates for this decay. We shall discuss here in detail only the case when \( \nu' = \bar{\nu} \), where \( \nu \) is any of the known, or yet to be discovered neutrinos (with mass \( m < \frac{m_{\pi}}{2} \)). As discussed above, more general final states may also be involved in \( \Sigma \Gamma(\pi^0 + \nu \nu') \). Also, since \( \pi^0 + \bar{\nu} \bar{\nu} \) is forbidden for a massless Weyl neutrino, we shall assume in the following that \( \nu \) is a four-component fermion.

The most general matrix element for \( \pi^0 \)-decay into a neutrino-antineutrino pair can be written as

\[
M(\pi^0 \rightarrow \nu \bar{\nu}) = c \gamma_5 \bar{\nu} + d \bar{\nu},
\]

where \( c \) and \( d \) are constants. The amplitude \( c \) is P and CP-conserving, while the \( d \)-term violates both P and CP. The decay rate is given by

\[
\Gamma = \frac{m_{\pi} \kappa}{8 \pi} \left( |c|^2 + \kappa^2 |d|^2 \right),
\]

where \( \kappa = (1 - 4m^2/m_{\pi}^2)^{1/2} \), \( m \) and \( m_{\pi} \) denote the neutrino and the pion mass, respectively.

An experimental search for \( \pi^0 \rightarrow \nu \nu' \) is difficult in view of the absence of a positive signature. A sensitive experiment must establish the presence of a \( \pi^0 \) and the absence of its electromagnetic decay products with a high
degree of certainty. A possible approach is to look for $\pi^0 + \nu\nu'$ following $K^+ \rightarrow \pi^+\pi^0$ decay. vs. The $K^+ \rightarrow \pi^+\pi^0$ decay mode, with a branching ratio of 21%, is a suitable source of tagged neutral pions. $K^+ \rightarrow \pi^+\pi^0$ is signaled by a $K^+$ decaying at rest into $\pi^+$ with a kinetic energy of 108 MeV.

Past experiments which searched for the process $K^+ \rightarrow \pi^+\nu\nu'$ specifically sought to exclude events with $\pi^+$ kinetic energies in the vicinity of 108 MeV, so as to eliminate the background caused by those $K^+ \rightarrow \pi^+\pi^0$ where the $\pi^0$ decay products were not detected. Two of the experiments completely eliminated $K^+ \rightarrow \pi^+\pi^0$ candidates by accurately measuring the $\pi^+$ momentum. Two other experiments determined the $\pi^+$ energy by measuring the $\pi^+$ range and restricted the kinetic energy to $60 \text{ MeV} < T_{\pi^+} < 105 \text{ MeV}$ and $117 \text{ MeV} < T_{\pi^+} < 127 \text{ MeV}$. However, the spread in the $\pi^+$ range due to straggling implies that the detection efficiency for 108 MeV $\pi^+$'s (and consequently for $K^+ \rightarrow \pi^+\pi^0$) was non-zero. In Ref. 5, a prescription is given for calculating an upper limit for $\Gamma(K^+ \rightarrow \pi^+\nu\nu')/\Gamma(K^+ \rightarrow \pi^+\pi^0)$ from the data of Refs. 4, 5 for any given $\pi^+$ spectrum. Using a delta function at 108 MeV for the spectrum, we find

$$\lim_{\nu\nu'} \frac{\Gamma(\pi^0 \rightarrow \nu\nu')}{\Gamma(\pi^0 \rightarrow \text{all})} \leq \frac{\Gamma(K^+ \rightarrow \pi^+\nu\nu')}{\Gamma(K^+ \rightarrow \pi^+\pi^0)}$$

(3)

and as a consequence

$$\lim_{\nu\nu'} \frac{\Gamma(\pi^0 \rightarrow \nu\nu')}{\Gamma(\pi^0 \rightarrow \text{all})} < 2.4 \times 10^{-5} \text{ (90\% C.L.)} .$$

(4)

This appears to be the best limit one can extract from the existing data.

To interpret this result, let us consider first the most general local nonderivative effective neutrino-quark interaction that could contribute to $\pi^0 + \nu\nu'$:

$$L = \frac{g}{\sqrt{2}} [g_{MM} \bar{\nu}_M \gamma^\nu \nu_M + (g_{PP} \bar{\nu}_P \gamma^\nu + g_{SP} \nu_P \gamma^\nu) P],$$

(5)

-3-
where $J^A_\mu = \frac{1}{2}(\bar{u}_\mu Y_{su} - \bar{d}_\mu Y_{sd})$, $J^P = \frac{1}{2}(\bar{u}_1 Y_{su} - \bar{d}_1 Y_{sd})$, and $g_{AA}$, $g_{PP}$, $g_{PS}$ are constants characterizing the strength of the corresponding terms relative to $2^{-3/2} G (G = 10^{-5} m_p^{-2})$.

The $\pi^0 + \nu \bar{\nu}$ amplitudes stemming from Eq. 5 are

$$c = c_A + c_P = 2 \frac{G_{m\pi}}{\sqrt{2}} g_{AA} \rho_A + \frac{G_{m\pi}}{\sqrt{2}} g_{PP} \rho_P,$$

$$(6)$$

$$d = \frac{G_{m\pi}}{\sqrt{2}} g_{SP} \rho_P,$$

$$(7)$$

where $\rho_A$ and $\rho_P$ are defined by

$$<0|J^A_\mu|\pi^0(p)> = m_{\pi} \rho_A \bar{\mu},$$

$$(8)$$

$$<0|J^P|\pi^0(p)> = -i m_{\pi} \rho_P.$$

$$(9)$$

The constant $\rho_A$ is related to the charged pion decay constant $f_\pi$ (defined by $<0|\bar{\pi}_\mu Y_{su}|\pi^+(p)> = f_\pi \bar{f}_\mu$) as $\rho_A = f_\pi / m_{\pi} \sqrt{2} \approx 0.7$. $\rho_P$ can be related to $\rho_A$ using the relation $\bar{q}\gamma^\mu Y_{su} = 2m_q \bar{q}_s Y_{su}$. One finds

$$\rho_P = \rho_A m_{\pi} / (m_u + m_d) \approx 8,$$

$$(10)$$

where we have used $m_u = 4.2$ MeV, $m_d = 7.5$ MeV for the quark masses.\(^6\)

Thus, we obtain

$$c \approx (2 \times 10^{-7}) g_{AA} m_{\pi} / m_{\pi} + (1.2 \times 10^{-6}) g_{PP},$$

$$(11)$$

$$d \approx (1.2 \times 10^{-6}) g_{SP}.$$

$$(12)$$

which gives (using $\Gamma(\pi^0 \rightarrow \nu \bar{\nu}) \approx 7.8$ eV) a branching ratio

$$B(\pi^0 \rightarrow \nu \bar{\nu}) \approx (9.4 \times 10^{-7}) \kappa \left[ (g_{PP} + 0.2 g_{AA} m_{\pi})^2 + \kappa^2 g_{SP}^2 \right].$$

$$(13)$$
Consequently, the experimental limit (4) constrains the strength of the effective interaction responsible for any possible $\nu\bar{\nu}$ channel to satisfy

$$[\kappa(g_{pp} + 0.2 g_{AA} m/m_\pi)^2 + \kappa^2 g_{SP}^2] \leq 5 .$$  \(14\)

Note that in the presence of $g_{pp}$ and/or $g_{SP}$, $B(\pi^0 + \nu\bar{\nu})$ would not vanish even for massless neutrinos. 7)

Could the strength of the interaction terms involved be as large as required to saturate this limit? For the axial-vector term, evidence from neutral-current neutrino experiments supports the value $g_{AA} = 1$ corresponding to the prediction of the Weinberg-Salam model. 8, 9) These experiments also indicate that (for the known neutrinos) 10) the $S, P$ couplings, if present, are weaker than the $V, A$ terms, obeying approximately the bounds 11, 12)

$$|g_{pp}|, |g_{SP}| \leq 0.3 .$$  \(15\)

With $g_{pp} = g_{SP} = 0.3$, $g_{AA} = 1$, the branching ratio would be

$$B(\pi^0 \rightarrow \nu\bar{\nu}) = 1.7 \times 10^{-7}$$  \(16\)

for $0 \leq m \leq 30$ MeV and then decreasing gradually with increasing $m$. For $\nu_1$ and other yet to be discovered neutrinos, values of $g_{pp}$, $g_{PS}$ exceeding the bounds (Eq. 15) cannot be, of course, ruled out.

In the framework of unified gauge theories of the weak and the electromagnetic interactions, $\pi^0 \rightarrow \nu\bar{\nu}$ could occur at the tree level via neutral gauge boson exchange (leading to the $g_{AA}$-term in Eq. 5) and also through the exchange of neutral Higgs mesons (giving rise to the $g_{pp}$ and $g_{SP}$ terms). In the simplest version of the Weinberg-Salam model, the neutrinos are taken to be massless left-handed fermions. As discussed earlier, $\pi^0 \rightarrow \nu\bar{\nu}$ would then be absolutely forbidden. To accommodate massive neutrinos, the simplest modification of the minimal model is to assign the right-handed components...
to SU(2) singlets with zero weak hyperchange. The right-handed neutrinos are then absent from the SU(2)$_{L}$ x U(1) gauge interactions.

The $\pi^0 \rightarrow \nu\bar{\nu}$ amplitude due to Z-exchange is given by the $c_A$ term in (Eq. 6) with $g_{AA} = 1$, leading to a branching ratio

$$B_Z(\pi^0 \rightarrow \nu\bar{\nu}) = 3 \times 10^{-8}(1 - 4m^2/m^2_\pi)^{1/2}(m/m_\pi^2).$$

The decay rate as a function of the neutrino mass has a broad maximum at $m = m_\pi/\sqrt{6} = 55$ MeV with a branching ratio

$$B_Z(\pi^0 \rightarrow \nu\bar{\nu}; m = 55 \text{ MeV}) = 2.8 \times 10^{-9}.$$ (17)

For $\nu_e$ and $\nu_\mu$, $B_Z(\pi^0 \rightarrow \nu\bar{\nu})$ is completely negligible. However, $\nu_\tau (m_{\nu_\tau} < 250 \text{ MeV})^{16}$ and any further possible neutrinos could lead to a decay rate comparable to (Eq. 18). The interpretation of a positive experimental result in terms of a neutrino mass would not, however, be unambiguous in view of possible other decay mechanisms. Even if the right-handed components had no gauge interactions, four-component neutrinos would, in general, be expected to couple also to Higgs mesons. In the simplest SU(2)$_{L}$ x U(1) model with only one Higgs doublet, the Higgs couplings are proportional to the mass of the fermion involved, and the Higgs mesons couple to scalar rather than pseudoscalar fermion densities, which would have no effect on $\pi^0 \rightarrow \nu\bar{\nu}$.

If, however, more than one Higgs doublet exists (which may even be necessary for various theoretical reasons) pseudoscalar couplings may also be present, contributing to $\pi^0 \rightarrow \nu\bar{\nu}$ decay. Also, in the presence of more doublets, the strength of the Higgs couplings may no longer be governed by the fermion mass involved, opening up the possibility of stronger couplings. Finally, the SU(2)$_{L}$ x U(1) model may be part of a theory based on a larger flavor group involving additional Higgs mesons, with a variety of possible couplings.
We shall parameterize the relevant Higgs-neutrino couplings as follows:

\[ I_H = f_\nu \frac{2}{\sqrt{G}} m \bar{\nu} i Y_s \nu H + f_\nu^* \frac{2}{\sqrt{G}} m \nu \bar{\nu} H \]

\[ + f_u \frac{2}{\sqrt{G}} m u \bar{u} i Y_s u H + f_d \frac{2}{\sqrt{G}} m d \bar{d} i Y_s d H \]

Thus,

\[ g_{pp} = 2 f_\nu m (f_u m_u - f_d m_d)/(m_{\pi}^2 - m_H^2) \]

and

\[ g_{Sp} = 2 f_\nu^* m (f_u m_u - f_d m_d)/(m_{\pi}^2 - m_H^2) \]

We shall consider two special cases for the parameters \( f \):

1. **Standard Higgs Couplings**: \( f_\nu = f_\nu^* = f_u = f_d = 1 \).

   In this case, \( g_{pp} = g_{Sp} = 2m(m_u - m_d)/(m_{\pi}^2 - m_H^2) \). These give, in general, negligible contributions to \( B(\pi^0 \rightarrow \nu \bar{\nu}) \). They are comparable to the axial vector contribution (with \( g_{AA} = 1 \)) only in the unlikely event when \( m_H^2 \approx m_{\pi}^2 \).

2. **Higgs Couplings Proportional to Some Large Fermion Masses Present in the Theory**: \(18\)

   Thus, \( f_\nu = M_L/m \), \( f_\nu^* = M'_L/m \), \( f_u = M_Q/m_u \), \( f_d = M'_Q/m_d \),

   and

   \[ g_{pp} = 2M_L(M_Q - M'_Q)/(m_{\pi}^2 - m_H^2) \], \( g_{Sp} = 2M'_L(M_Q - M'_Q)/(m_{\pi}^2 - m_H^2) \).

As seen from Eq. 13, the pseudoscalar contribution to \( \pi^0 \rightarrow \nu \bar{\nu} \) would exceed a contribution from the axial-vector coupling (with \( g_{AA} = 1 \)) for

\[ 2M_L(M_Q - M'_Q)/(m_{\pi}^2 - m_H^2) \approx 7 \times 10^{-7} \]. For not unreasonable values of masses, \( m_H \sim 10 \text{ GeV}, M_L \sim 2 \text{ GeV}, |M_Q - M'_Q| \sim 8 \text{ GeV} \) for example, the \( \pi^0 \rightarrow \nu \bar{\nu} \) branching ratio could be as large as the expected phenomenological upper bound (16) for the known neutrinos. For Higgs mesons coupled to new neutrinos,
larger values of $g_{pp}$, $g_{sp}$ (corresponding to larger fermion masses and/or a smaller Higgs mass) and thus, a larger branching ratio for $\pi^0 \rightarrow \nu \bar{\nu}$ cannot be excluded.

A new type of experiment could lead to a more sensitive search for $\pi^0 \rightarrow \nu \bar{\nu}$. The $K^+ \rightarrow \pi^+ \pi^0$ process would be established by accurately measuring the $\pi^+$ momentum in a magnetic spectrometer and by detecting the $\pi^+ \rightarrow \mu^+ \rightarrow e^+$ decay chain. NaI detectors surrounding the stopping target would detect the electromagnetic decay products of the $\pi^0$ with extremely high efficiency. Since the $\pi^0$ energy and direction would be known, the NaI inefficiency, as a function of energy, could be measured by comparing the number of events with two photons detected to the number with only one photon detected. In this way, it can be established if events with no photons detected are due to detection inefficiencies or due to $\pi^0 \rightarrow \nu \bar{\nu}$ decays. A sensitivity of below $10^{-7}$ appears feasible.

The reaction $\bar{p}p \rightarrow \pi^+ \pi^- \pi^0$ has also been suggested as a source of tagged $\pi^0$'s. This scheme is probably more complicated, requiring the momenta of the $\bar{p}$, $\pi^+$, and $\pi^-$ to be measured with high precision. In either case, it is probably the NaI inefficiency which would limit the attainable sensitivity.

To summarize, we have considered $\pi^0 \rightarrow \nu \bar{\nu}$ in the presence of contributions from both gauge bosons and Higgs mesons. Observation of a branching ratio larger than $2.8 \times 10^{-9}$ might imply the presence of Higgs contributions with couplings stronger than the standard ones. With not unreasonable values of the Higgs couplings, the $\pi^0 \rightarrow \nu \bar{\nu}$ branching ratio could be comparable to the experimental limit we have deduced from the existing data. It appears feasible to search for $\pi^0 \rightarrow \nu \bar{\nu}$ with improved sensitivity.
References

1. The process $\pi^0 \rightarrow \nu \bar{\nu}$ has been, to our knowledge, first considered by

2. This method was first suggested by B. Kayser, G. T. Garvey, E. Fischbach, and S. P. Rosen, Ref. 1.


10. Strictly speaking, the evidence refers only to $\nu_\mu$ since the low-energy experiments involving $\bar{\nu}_e$ are not sensitive to a pseudoscalar quark density.

12. This conclusion is not quite rigorous since the local approximation (5) would break down if the boson mediating the P,S interactions is not sufficiently heavy.


14. The right-handed neutrinos may still couple to new gauge bosons present in possible extensions of the SU(2)_L x U(1) model.


