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FEBRUARY 1984

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PRINCETON UNIVERSITY
PRINCETON, NEW JERSEY

PREPARED FOR THE U.S. DEPARTMENT OF ENERGY,
UNDER CONTRACT DE-AC02-76-CHO-3073.

NONLINEAR EVOLUTION OF THE RESISTIVE INTERCHANGE MODE
IN THE CYLINDRICAL SPHEROMAK

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ABSTRACT

Results are presented of a study of various aspects of the single helicity nonlinear development of the resistive interchange mode in the cylindrical spheromak. A formulation of the helically symmetric resistive MHD equations that partially separates the ideal MHD characteristics is developed. Mode saturation can occur due to the quasilinear flattening of the pressure profile in the vicinity of the mode rational surface. However, this saturation process is defeated when the plasma overheats and in regions of the plasma where the shear is low. Finite fluid compression has significant, and optimistic, consequences on the long-term nonlinear behavior of this mode. For a tearing mode stable cylindrical spheromak equilibrium configuration with an axial beta value of 6%, complete overlap of the $m = 1$ islands occurs in about 3% of the resistive skin time for a magnetic Reynold's number of $S = 10^5$. For typical parameters of the S-1 device at Princeton, this time corresponds to nearly one-millisecond.

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I. INTRODUCTION

The spheromak is now being considered by the fusion community as a viable alternative to the tokamak for containing a hot plasma long enough to produce a sizable amount of energy from fusion reactions. Like the tokamak, the spheromak is an axisymmetric closed magnetic field line toroidal configuration. However, unlike the tokamak where the toroidal magnetic field is generated primarily by external poloidal currents, the toroidal magnetic field in the spheromak is generated by internal poloidal plasma currents so that the vacuum toroidal magnetic field is zero. These factors allow for a coil-blanket topology that does not link the center of the plasma, thereby simplifying reactor design, construction, and maintenance. Ideal MHD stability studies indicate stable engineering beta values that are several times larger than in the tokamak. The spheromak has very large ohmic heating power densities (comparable to the reversed field pinch) and also has the potential to control impurities since the plasma is located in a natural divertor system.

For these reasons, spheromaks and other compact toroids have received increased theoretical and experimental attention in the last few years. Three distinct methods of spheromak formation have been experimentally demonstrated¹⁻³ and stability has been observed for experimental times approaching 1000 Alfvén wave transit times. The general agreement with theory regarding both formation methods and stability with respect to large scale ideal MHD instabilities has been very good. However, until recently the energy content of the present small scale experiments has been insufficient to surpass the peaks in the radiation curves for oxygen and carbon impurities, and as a result the temperature in these experiments has been limited to a few tens of electron volts, and the lifetimes have been correspondingly short.

As experimental lifetimes are increased and the plasma collisionality parameter decreases in the present and the next generation of experiments, resistive instabilities are expected to dominate the dynamics of these devices. Theoretical understanding of these resistive instabilities is essential in interpreting experimental results as we pass through the transition region from small, relatively cold, collisional plasmas to larger, hot reactor relevant experiments.

A convenient model geometry to use for theoretical studies of resistive instabilities in the spheromak is the periodic cylindrical spheromak. This is a fictitious configuration in which the torus has been cut and straightened out to become a circular cylinder of radius a , length $2\pi R$, and with periodic boundary conditions. The usefulness of this model lies in the fact that it has a reduced parameter space and is more amenable to theoretical analysis. It was introduced in Ref. 4 where ideal MHD instabilities were examined and was then adopted in Ref. 5 for a study of resistive instabilities in the spheromak. These studies indicate that cylindrical spheromak equilibria can be constructed which are stable to both ideal MHD and resistive current driven tearing modes. However, due to the strongly unfavorable magnetic field line curvature, stable operating regimes disappear when resistive pressure driven (interchange) modes are considered.

Within the framework of the resistive MHD model, the resistive interchange mode is seen to pose considerable threat to confinement in the spheromak. In the present study, we concentrate on the nonlinear evolution of $m = 1$ resistive interchange modes with helical symmetry imposed. Our intention here is to examine the quasilinear equilibrium modifications due to the growing instability and the effects that this modification has on the growth rate and eigenfunctions. Documenting and understanding these two

processes will help to lay the groundwork, and to determine the usefulness of subsequent three-dimensional nonlinear calculations.

Since the resistive interchange mode is mostly fluid flow,⁶ we suspect that fluid compression will have important consequences on the long-term nonlinear behavior of this mode. Therefore, it becomes necessary to solve the full set of compressible resistive MHD equations and not make the simplifying assumption that the velocity is divergence-free.

In order to solve the full set of equations both accurately and efficiently we reformulate them in a manner which partially separates the description of the "fast" magnetoacoustic wave from the slower shear Alfvén wave. Since variations on the ideal MHD time scale are, in general, a superposition of the three ideal MHD waves, which differ widely in amplitude, frequency, and direction of propagation, any analytic partial separation of the ideal MHD characteristics will improve the accuracy and stability of the numerical solutions. This separation has made feasible the use of a time advancement scheme which treats the ideal MHD waves implicitly.

In Sec. 2 we review the periodic cylindrical spheromak model that was introduced in Ref. 4. An equilibrium configuration which is tearing mode stable but unstable to resistive pressure driven modes is described. Following this, in Sec. 3 we reformulate the 2D resistive MHD equations in the manner indicated above and then, in Sec. 4 describe the necessary boundary conditions.

Section 5 outlines the numerical procedure used to advance the resulting equations and boundary conditions in time. Spectral techniques are used in the periodic helical direction and finite difference techniques are used across the equilibrium flux surfaces and in time. We employ a partially implicit two-step time integration scheme, and zone packing about the mode

rational surface is utilized. Section 6 provides a demonstration that the ideal MHD characteristics are indeed separated in our formulation.

Finally, in Sec. 7 we study various aspects of the nonlinear development of the $m = 1$ resistive interchange mode in the cylindrical spheromak approximation. The effect of "superheating" the plasma is described, and the long-term consequences of finite fluid compression are calculated. An estimate of the time taken for a complete overlap of $m = 1$ islands to occur is then calculated. A summary of the physical results is given in Sec. 8.

II. THE CYLINDRICAL SPHEROMAK

The periodic cylindrical spheromak was demonstrated by Jardin⁴ to be a useful model for locating and understanding parameter regimes of ideal MHD stability in the spheromak. As long as the central safety factor q_0 is below unity, toroidicity was shown to have little effect on the ideal MHD internal stability boundaries given by the cylindrical model. This same model was adopted in Ref. 5 to study resistive instabilities in the spheromak. Toroidal effects on resistive modes are also expected to be small in the spheromak since the toroidal current density and temperature should peak near the magnetic axis where the flux surfaces are nearly circular.⁷ Also, since $q < 1$ everywhere, poloidal curvature will always dominate over toroidal curvature.

Using standard (r, θ, z) cylindrical coordinates, we define a periodicity length in the z -direction of $L = 2\pi R$, where R is the analog of the major radius in a toroidal device. The equilibrium equation in this geometry is

$$\frac{d}{dr} p + \frac{d}{dr} \frac{1}{2} B_z^2 + \frac{B_\theta}{r} \frac{d}{dr} r B_\theta = 0 \quad (1)$$

The pitch profile, $\mu(r) \equiv r B_z(r) / B_\theta(r) = R q(r)$, is taken to be of the form

$\mu(r) = \mu_0(1-r^2)$, where the plasma boundary is defined by $r = 1$. As in the previous works, the pressure is parameterized by introducing the parameter $0 < \alpha_s < 1$ such that $dp/dr = -\alpha_s r B_z^2 / 8(\mu'/\mu)^2$. Thus, when $\alpha_s = 0$, the equilibrium is force free and, when $\alpha_s = 1$, the pressure profile is everywhere marginally stable to the Suydam criterion. The equilibrium fields for such a system are given by

$$B_\theta(r) = \frac{r}{\mu_0} \exp\left\{ \int_0^r dr \left[\frac{(\alpha_s/8)r\mu'^2 - 2r - \mu\mu'}{\mu^2 + r^2} \right] \right\}, \quad (2a)$$

$$B_z(r) = \frac{11}{r} B_\theta(r) \quad (2b)$$

and

$$p(r) = \frac{1}{r} \int_0^r dr \left[\frac{\alpha_s}{8} (B_z \mu'/\mu)^2 \right] \quad (2c)$$

The equilibrium profiles are seen to depend only on two parameters, the pitch μ_0 (or the central safety factor Rq_0) and the pressure factor α_s .

Equilibrium magnetic field profiles for the ideal and tearing mode stable finite pressure configuration characterized by $\mu_0 = 0.8$ and $\alpha_s = 0.7$ are shown in Fig. 1. Note that the toroidal field decreases from unity at the magnetic axis to zero at the plasma edge. Also, unlike the tokamak, the poloidal and toroidal field components are of comparable magnitude. We use this configuration and the lower pressure configuration given by $\mu_0 = 0.8$ and $\alpha_s = 0.2$ to study the nonlinear development of the resistive interchange mode.

III. THE HELICALLY SYMMETRIC RESISTIVE MHD EQUATIONS

The MHD equations relate the electromagnetic fields $\vec{E}$ and $\vec{B}$ to the plasma ion fluid velocity $\vec{v}$, ion mass density ρ , and total fluid pressure p . Written in rationalized cgs units; the resistive MHD equations are

$$\frac{\partial}{\partial t} \rho + \nabla \cdot (\rho \vec{v}) = 0 \quad , \quad (3a)$$

$$\frac{\partial}{\partial t} \vec{B} + \nabla \times \vec{E} = 0 \quad , \quad (3b)$$

$$\rho \left(\frac{\partial}{\partial t} \vec{v} + \vec{v} \cdot \nabla \vec{v} \right) + \nabla p = \vec{J} \times \vec{B} \quad , \quad (3c)$$

$$\frac{\partial}{\partial t} p + \vec{v} \cdot \nabla p + \frac{5}{3} p \nabla \cdot \vec{v} = \frac{2}{3} \eta J^2 \quad , \quad (3d)$$

$$\vec{E} + \vec{v} \times \vec{B} = \eta \vec{J} \quad , \quad (3e)$$

$$\vec{J} = \nabla \times \vec{B} \quad , \quad (3f)$$

$$\nabla \cdot \vec{B} = 0 \quad . \quad (3g)$$

In the present analysis the plasma resistivity η is assumed to be constant in space and time. The reduction to two space dimensions is accomplished by introducing the periodic helical coordinate $\tau \equiv m\theta + kz$, and by constraining all solutions of Eq. (3) to depend only on r , τ , and time t . Here, $k = -n/R$ is the axial wavenumber and m and n are integers.

Equation (3g) and the assumed helical symmetry allows the magnetic field to be written in terms of a helical flux function $\chi(r, \tau, t)$ and a helical field function $g(r, \tau, t)$ as

$$\vec{B} = \frac{\hat{e}}{h} \times \nabla \chi + g \frac{\hat{e}}{h}, \quad (4)$$

where we have introduced the unit vectors $\hat{r} \equiv \nabla r$, $\hat{\tau} \equiv \nabla \tau / |\nabla \tau|$, $\hat{e} \equiv \hat{r} \times \hat{\tau}$, and the coordinate function $h(r) \equiv [m^2 + k^2 r^2]^{1/2}$. This form of the magnetic field has the advantage of being inherently divergence-free.

The current density is obtained from Eq. (3f) by taking the curl of Eq. (4), i.e.,

$$\vec{J} = \nabla g \times \frac{\hat{e}}{h} + (\Delta^* \chi - Mg) \frac{\hat{e}}{h}. \quad (5)$$

Here Δ^* is the helically symmetric equivalent to the Grad-Shafranov operator,

$$\Delta^* \equiv h^2 \nabla^2 + h^{-2} \nabla^2 = \frac{h^2}{r} \frac{\partial}{\partial r} \frac{r}{h^2} \frac{\partial}{\partial r} + \frac{h^2}{r^2} \frac{\partial^2}{\partial \tau^2}, \quad (6)$$

and $M \equiv 2mk/h^2$.

In order to partially decouple the ideal MHD characteristics we use a decomposition of the velocity field based on the velocity stream-function and potential method.^{8,9} The velocity is split into a divergence-free component and a curl-free component, i.e.,

$$\vec{v} = \frac{\hat{e}}{h} \times \nabla A + \omega \frac{\hat{e}}{h} + \nabla Q, \quad (7)$$

where $A(r, \tau, t)$ is the helical stream function, $\omega(r, \tau, t)$ is the helical velocity, and $Q(r, \tau, t)$ is the helical velocity potential. The compressible part of the velocity is contained entirely in the helical velocity potential Q . From Eq. (7) we obtain the curl of the velocity as

$$\nabla \times \vec{v} = \nabla \omega \times \frac{\hat{e}}{h} + (\Delta^* A - M\omega) \frac{\hat{e}}{h} , \quad (8a)$$

and the divergence of the velocity as

$$\nabla \cdot \vec{v} = \nabla^2 \phi . \quad (8b)$$

In this geometry the components of the field equation, Eq. (3b), become time advancement equations for the helical flux and the helical field

$$\frac{\partial}{\partial t} \chi + \vec{u} \cdot \nabla \chi = \eta (\Delta^* \chi - M g) + c(t) , \quad (9a)$$

$$\frac{\partial}{\partial t} (g - M\chi) + h^2 [\vec{u} \cdot \nabla \frac{g}{h^2} - \vec{B}_\perp \cdot \nabla \frac{\omega}{h^2}] + g \nabla^2 \Omega = \eta \Delta^* g - M c(t) . \quad (9b)$$

Here $\vec{u} \equiv (\hat{e}/h) \times \nabla A + \nabla \Omega$ and $\vec{B}_\perp \equiv (\hat{e}/h) \times \nabla \chi$ are the velocity and magnetic field components perpendicular to the symmetry direction $\hat{e}$, and $c(t)$ is an integration constant which depends only on time.

The incompressible shear Alfvén wave motion is mostly contained in the helical stream function A . This wave can be partially isolated by projecting the curl of the momentum equation, Eq. (3c), onto the symmetry direction to obtain the following time advancement equation for the vorticity, $\Delta^* A$:

$$h^{-2} \frac{\partial}{\partial t} \Delta^* A + \nabla \cdot \left[\frac{(\Delta^* A - M\omega)}{h^2} \vec{u} \right] - \frac{k^2}{h} \frac{\partial}{\partial \tau} (g^2 - \omega^2) = \vec{B}_\perp \cdot \nabla \left[\frac{\Delta^* \chi - M g}{h^2} \right] . \quad (9c)$$

The fast compressible magnetoacoustic wave motion is mostly contained in the helical velocity potential Ω . This wave can be partially isolated by taking the divergence of the momentum equation. The following time

advancement equation for the compressibility, $\nabla^2 Q$, results:

$$\frac{\partial}{\partial t} \nabla^2 Q + \nabla \cdot (\vec{v} \cdot \nabla \vec{v}) + (\nabla \times \vec{B})^2 + \nabla^2 p = \vec{B} \cdot \left\{ \frac{\hat{e}}{h} \times \nabla (\Delta^* \chi - Mg) + [(\Delta^* - M^2)g + M \Delta^* \chi] \frac{\hat{e}}{h} \right\}, \quad (9d)$$

where

$$\nabla \cdot (\vec{v} \cdot \nabla \vec{v}) = \frac{1}{r} \frac{\partial}{\partial r} r Q_r + \frac{h}{r} \frac{\partial}{\partial \tau} Q_\tau$$

and we have defined the quantities

$$Q_r \equiv u_r \frac{\partial}{\partial r} u_r + \frac{h}{r} u_\tau \frac{\partial}{\partial \tau} u_r - \frac{1}{h^2 r} (\mu u_\tau - k r \frac{\omega}{h})^2,$$

$$Q_\tau \equiv u_r \frac{\partial}{\partial r} u_\tau + \frac{h}{r} u_\tau \frac{\partial}{\partial \tau} u_\tau + \frac{\mu u_r}{h^2 r} (\mu u_\tau - 2 k r \frac{\omega}{h}),$$

$$u_r \equiv \vec{u} \cdot \hat{r} = -\frac{1}{r} \frac{\partial A}{\partial \tau} + \frac{\partial Q}{\partial r},$$

$$u_\tau \equiv \vec{u} \cdot \hat{\tau} = \frac{1}{h} \frac{\partial A}{\partial r} + \frac{h}{r} \frac{\partial Q}{\partial \tau}.$$

The time advancement equation for the helical velocity ω is obtained by projecting the momentum equation onto the symmetry direction $\hat{e}$,

$$\frac{\partial}{\partial t} \omega + \vec{u} \cdot \nabla \omega = \vec{B}_\perp \cdot \nabla g. \quad (9e)$$

Finally, the pressure equation becomes

$$\frac{\partial}{\partial t} p + \vec{u} \cdot \nabla p + \frac{5}{3} p \nabla^2 Q = \frac{2}{3} \eta (\nabla \times \vec{B})^2. \quad (9f)$$

In deriving Eqs. (9c-e) we have explicitly assumed that the plasma mass density ρ is constant, independent of both space and time. Formally, this is accomplished by setting $\rho = 1$ in the momentum equation. Doing this avoids having to calculate triple products in the momentum equation. Such calculations are cumbersome and extremely time consuming for a numerical method which calculates products using fully spectral techniques. Also, since ρ only enters into the calculation of the magnitude of the ideal MHD wave velocities, its fine scale fluctuations due to fluid convection and compression are unimportant on the time scale of the resistive instabilities that we are considering.

Equations (9a) through (9f) are the helically symmetric resistive MHD equations. They have several advantages over more primitive forms. First, the physically interesting incompressible resistive MHD equations are a subset of the full set of equations and are obtained by setting $\Omega = 0$ and dropping Eqs. (9d) and (9f). This fact can be exploited when developing and debugging a computer code to solve the full set of equations. Second, by taking the divergence and curl of the momentum equation we have removed, to lowest order in $\hat{B} \cdot \hat{r}/(r\hat{B} \cdot \hat{e}) \ll 1$, the shear Alfvén wave motion from the quantity Q and the fast magnetosonic wave motion from the quantity A . This partial separation of the ideal MHD characteristics has dramatic effects on the accuracy of the numerical solutions. It has made feasible the use of the implicit time advancement scheme described in Sec. 5.

IV. BOUNDARY CONDITIONS

Equations (9) are being applied to study the nonlinear development of radially localized internal instabilities in a circular cross-sectional plasma. It is the initial conditions and not the interaction of the plasma

with its external surroundings which provide the driving mechanism for these instabilities. Therefore, in order to minimize the effects of the plasma surroundings we choose simple boundary conditions, i.e., it is assumed that the plasma comes in contact with a rigid, conducting circular wall.

The momentum equation requires a knowledge of the normal velocity at the plasma/wall interface. For our model of the plasma surroundings the radial velocity at the plasma edge must vanish. This boundary condition can be satisfied by either of the following two conditions:

$$A|_{r=1} = \frac{\partial Q}{\partial r}|_{r=1} = 0, \quad (10a)$$

or

$$Q|_{r=1} = 0 \quad \text{and} \quad \frac{1}{r} \frac{\partial A}{\partial \tau}|_{r=1} = \frac{\partial Q}{\partial r}|_{r=1}. \quad (10b)$$

The reason that two conditions both satisfy the boundary condition on the radial velocity is that the velocity decomposition that we use, Eq. (7), possesses a gauge invariance, i.e., $\vec{v} \cdot \hat{r}$, $\vec{v} \cdot \hat{\tau}$, and $\vec{v} \cdot \hat{e}$ are invariant under the transformation

$$A \rightarrow A + r \frac{\partial}{\partial r} f(r, \tau, t),$$

$$Q \rightarrow Q + \frac{\partial}{\partial \tau} f(r, \tau, t),$$

provided f satisfies

$$\nabla^2 f = 0.$$

In practice we use Eq. (10a) so that the boundary conditions for the incompressible set of equations are decoupled from those of the full set of equations.

Magnetic boundary conditions require a knowledge of χ and g at the plasma edge. The helical flux at the wall is determined by the requirement that the perturbed radial magnetic field vanish there, i.e.,

$$\left. \frac{\partial}{\partial \tau} \chi \right|_{r=1} = 0 \quad . \quad (11a)$$

This must be the case since $\nabla \cdot \vec{B} = 0$ everywhere. The helical field at the wall is determined by the requirement that the radial current density vanish there, i.e.,

$$\left. \frac{\partial}{\partial \tau} g \right|_{r=1} = 0 \quad , \quad (11b)$$

this prevents any charge from accumulating at the plasma edge.

The helical velocity equation, Eq. (9e), can be expanded to obtain

$$\frac{\partial}{\partial \tau} \omega + u_r \frac{\partial \omega}{\partial r} + \frac{h}{r} u_\tau \frac{\partial \omega}{\partial \tau} = \frac{1}{r} \left[\frac{\partial \chi}{\partial r} \frac{\partial g}{\partial \tau} - \frac{\partial \chi}{\partial \tau} \frac{\partial g}{\partial r} \right] \quad .$$

Evaluating this equation at the plasma edge and applying Eqs. (10) and (11), we see that it can be satisfied trivially by the boundary condition

$$\omega|_{r=1} = 0 \quad . \quad (12)$$

Finally, we are free to adjust the pressure at the plasma edge within the constraint that the pressure gradient remains negative there. Doing this is

equivalent to adjusting the mechanical reaction force from the wall so that Eq. (10) is always satisfied. In practice we choose:

$$\left. \frac{\partial}{\partial \tau} p \right|_{r=1} = 0 \quad . \quad (13)$$

V. NUMERICAL METHODS

The helically symmetric resistive MHD equations are periodic in the helical coordinate $\tau \equiv m\theta + kz$. We Fourier decompose the equations in τ and then numerically integrate the resulting spectral equations using finite differences in the radial direction. The following Fourier expansion of a definite parity is used for the plasma quantities:

$$\gamma(r, \tau, t) = \gamma_0(r, t) + \sum_{l=1}^{\infty} \gamma_l(r, t) \cos(l\tau) \quad , \quad (14a)$$

$$g(r, \tau, t) = g_0(r, t) + \sum_{l=1}^{\infty} g_l(r, t) \cos(l\tau) \quad , \quad (14b)$$

$$A(r, \tau, t) = \sum_{l=1}^{\infty} A_l(r, t) \sin(l\tau) \quad , \quad (14c)$$

$$\omega(r, \tau, t) = \sum_{l=1}^{\infty} \omega_l(r, t) \sin(l\tau) \quad , \quad (14d)$$

$$\Omega(r, \tau, t) = \Omega_0(r, t) + \sum_{l=1}^{\infty} \Omega_l(r, t) \cos(l\tau) \quad , \quad (14e)$$

$$p(r, \tau, t) = p_0(r, t) + \sum_{l=1}^{\infty} p_l(r, t) \cos(l\tau) \quad . \quad (14f)$$

The plasma equations are Fourier decomposed in τ giving a set of time advancement equations for the Fourier components in Eqs. (14). Each of the many convolutions that result is divided into two distinct parts, the "linear"

part and the "nonlinear" part. The linear part of the convolution is made up of all terms which contain an equilibrium ($l = 0$) component of the magnetic field or pressure. For example, the "linear" part of the l^{th} Fourier component of the $g\nabla^2\Omega$ term in the helical field equation is

$$g_0 \left[\frac{1}{r} \frac{\partial}{\partial r} r \frac{\partial}{\partial r} a_l - \frac{l^2}{r^2} a_l \right] ,$$

for $l > 0$. All remaining terms in the convolution are denoted "nonlinear." The reason for splitting each convolution in this fashion will become apparent in the discussion that follows.

The spectral equations are truncated at a finite number of modes, $l = 0, 1, 2, \dots, L$, and the $6(L + 1)$ time advancement equations for $\chi_l, g_l, \dots$ are integrated in time using standard finite difference techniques. In order to resolve the singular behavior of the resistive MHD unstable eigenfunctions with as few radial grid points as possible, we utilize zone packing about the mode rational surface. The radial grid is constrained to be "mildly nonuniform" so that the standard central difference operators would be approximately second order accurate.⁵ Variable locations are staggered throughout the radial finite difference mesh so that radial derivatives would nearly always be taken over a single zone spacing.

In order to circumvent the CFL time step restrictions based on the ideal MHD wave velocities given by the equilibrium fields and pressure, we integrate the linear part of the convolutions in time using the implicit, generalized Crank-Nicholsen scheme.¹⁰ The resistive terms in the field equations are treated in the same fashion. Because the $l = 0$ fields and pressure are changing on a time scale which is much longer than the ideal MHD time scale, these quantities can be held constant over a single time step.

Numerical stability consideration of the generalized Crank-Nicholson scheme applied to simple hyperbolic or parabolic partial differential equations requires that the implicitness parameter has a value of $1 > \phi > 1/2$. Although Eqs. (9) represent a mixed parabolic and hyperbolic set of equations with multiple characteristics, it is plausible that the same numerical stability requirement is approximately true, especially since the characteristics are partially decoupled in the present formulation. This numerical stability requirement has been verified empirically in the computer code which solves the finite difference form of the linearized helically symmetric equations. To ensure numerical stability when integrating the full set of equations, we choose $\phi = 0.51$.

The remaining terms are nonlinear and are integrated in time using a simple two-step explicit scheme. Due to this explicit treatment of the nonlinear terms the following CFL time step restriction exists:

$$\Delta t_{\max} < \frac{\Delta r_s}{\delta B_r|_{r=r_s}}, \quad (15)$$

where Δr_s is the radial grid spacing at the singular surface and $\delta B_r|_{r=r_s}$ is the perturbed radial magnetic field at the singular surface. This restriction exists because we are treating explicitly the propagation of shear Alfvén waves parallel to the perturbed radial magnetic field across regions of the plasma where grid points are packed closely together. The finite difference equations for the time advancement scheme just described are given in Appendix A.

After discretizing the spectral equations in radius and time according to the methods discussed here, the result is a matrix tridiagonal system of equations which is inverted for each harmonic $0 < \ell < L$ at each half time

step. The inversion of such a system is a well-known algorithm¹⁰ and will not be discussed here.

VI. DEMONSTRATION OF IDEAL MHD TIME SCALE SEPARATION

With the particular choice of magnetic field and velocity representation given by Eqs. (4) and (7), and by taking the divergence and curl of the momentum equation, the ideal MHD wave solutions have been partially separated. The fast magnetosonic wave motion is thereby removed from the helical stream function $A(r, \tau, t)$ and the slower shear Alfvén wave motion is removed from the helical velocity potential $Q(r, \tau, t)$. In this section we demonstrate that this separation of the velocity into a slowly varying incompressible part and a rapidly varying compressible part has been successful.

In Fig. 2 we plot the time history of the $l = 1$ Fourier component of the radial velocity, along with the corresponding compressible and incompressible parts, at a radial location that is well removed from the mode rational surface. Here, the equilibrium is an ideally stable spheromax configuration. Time is normalized to the Alfvén time $t_A \equiv B_0 / (4\pi\rho_0)^{1/2}$, where B_0 is the toroidal field on axis and ρ_0 is the volume averaged ion mass density, so that the oscillations seen in this figure are occurring on the ideal MHD time scale. The slowly varying component of the radial velocity is the largest in amplitude and corresponds to the incompressible part of the eigenmode. The smaller amplitude high frequency component corresponds to the compressible part of the eigenmode. Examination of this figure shows that there is no high-frequency component in the incompressible part A_1 , and little or no low-frequency variation in the compressible part Q_1 .

VII. NONLINEAR EVOLUTION OF THE $m = 1$ RESISTIVE INTERCHANGE MODE

In this section we study various aspects of the nonlinear development of the resistive interchange mode in the cylindrical spheromak. Our model assumes helical symmetry so that the results presented here are a valid representation of the true physical situation, according to the resistive MHD model, only in the early stages of nonlinear development when the physics is dominated by modes of a single helicity. In the later stages of nonlinear development, modes with different helicities interact so that a fully three-dimensional calculation becomes necessary. Our intention here is to study the quasilinear modification of the equilibrium profile, caused by the exponentially growing perturbations, and the subsequent effect that the equilibrium modification has on the growth rate and eigenfunctions.

We consider a cylindrical spheromak equilibrium configuration in which the resistive interchange forces dominate the tearing forces. Such a configuration was determined in Ref. 5 to have the parabolic pitch profile $\mu(r) = 0.8(1-r^2)$. At zero pressure this configuration is stable to all internal ideal and resistive modes. Figure 1 shows the magnetic field profiles corresponding to the above pitch profile and a pressure gradient parameterized by $\alpha_s = 0.7$. The axial beta value for this configuration is approximately 6%.

Corresponding $m = 1$ mode rational surface positions, i.e., the radial locations r such that $m - n\mu(r)/R = 0$ for integer n , are shown in Fig. 3 for an aspect ratio of $R/a = 2$. Here we see that the mode rational surface positions are more densely packed toward the outer edge of the plasma so that adjacent $m = 1$ islands will overlap in this region while the resistive instabilities are still in their phase of linear growth. Therefore, we cannot effectively study the nonlinear development of resistive instabilities in the

outer region of the plasma by doing an analysis which assumes helical symmetry. Wall interactions, toroidicity, and rippling modes are also expected to be important factors near the plasma edge. We restrict our consideration here to modes which resonate in the hot, central region of the plasma where the majority of confinement is expected to take place.

In order to calculate the nonlinear development of these modes we first perturb the cylindrical equilibrium with the linear eigenfunctions. The helically symmetric equations along with the boundary conditions described in Sec. 4 are then advanced in time according to the numerical methods discussed in Sec. 5. All of the nonlinear calculations assume a magnetic Reynold's number of $S = 10^5$.

First, we consider the nonlinear development of the $n = 4$ mode, indicated in Fig. 3, with a pressure gradient characterized by $\alpha_s = 0.7$. Two physically different cases are examined, one with the full set of compressible helically symmetric equations, and one with the ohmic heating term absent from the pressure equation. Time histories of the island widths for these two cases are shown in Fig. 4.

In both cases, the island width evolution passes through three distinct phases. Initially, the nonlinear terms are small enough so that the growth rate is exponential. This is the linear phase. When the nonlinear terms become appreciable, the growth rate slows down and changes character from exponential to algebraic. This is the first stage of nonlinear development. Finally, the solution enters a second stage of nonlinear development in which the algebraic growth decreases.

Mode saturation for these two cases is caused by a quasilinear flattening of the pressure profile in the vicinity of the mode rational surface. This saturation process is illustrated in Fig. 5 where we plot the evolution of the

equilibrium pressure profiles. As the pressure gradient goes to zero at the singular surface, the driving mechanism for the resistive interchange instability is eliminated.

Further examination of Fig. 4 reveals that the nonlinear growth is much larger in the ohmically heated plasma. The reason for this is indicated in Fig. 5 where it is seen that the ohmic heating term causes the equilibrium pressure to increase dramatically in the central core of the plasma. As the central pressure increases, the magnitude of the pressure gradient at the singular surface tends to increase. In order to continually counteract this increase the magnetic field perturbations must continue to grow. The field perturbations can become large enough to excite some stable eigenmodes in the plasma. This is indicated by the oscillations on the tail of the island width evolution curve, seen in Fig. 4, for the case in which the ohmic heating term is included.

In practice, a laboratory plasma will not "superheat" to the extent indicated in Fig. 5, but the ohmic heating input power will be partially balanced by energy loss processes such as radiation and thermal conduction. Therefore, in the remaining discussions, these energy loss processes will be simulated by omitting the ohmic heating term in the pressure equation.

Next, we examine the effect that finite fluid compression has on the nonlinear development of the resistive interchange instability. This is easily accomplished in the present formulation since the incompressible equations are a subset of the full set of equations. The nonlinear development of the $m = 1$, $n = 4$ mode, for $\mu_0 = 0.8$ and $\alpha_s = 0.7$, has already been calculated with finite fluid compression (see Figs. 4 and 5). We recalculate this case, this time using the incompressible subset of equations.

Figure 6 presents a comparison of the island width evolution for both the

incompressible and compressible modes. The incompressible mode is seen to evolve through the same three phases of development as the compressible mode, i.e., an initial linear phase, and two consecutive phases of nonlinear algebraic growth. Also, the mode saturation mechanism is the same in both cases, and was seen in Fig. 5b. However, further examination of Fig. 6 shows that finite fluid compression has the effect of decreasing the algebraic growth rate of the resistive interchange instability during the phases of nonlinear development.

Figure 7 demonstrates that finite fluid compression can also have a significant effect on the resulting island structure. Here we see that a "double island" structure develops for the compressible mode. The reason for this difference is indicated in Fig. 8 where we plot the time histories of the amplitudes of the helical flux harmonics at the singular surface for both cases. In the incompressible mode the amplitudes of the $\ell > 1$ harmonics are all driven positive whereas in the compressible mode they are all driven negative.

Although the incompressible equations are more numerically tractable, the compressible equations are a more accurate representation of the real physical situation. Also, as was just demonstrated, finite fluid compression does indeed have a significant, and optimistic effect on the nonlinear development of the resistive interchange instability. For these reasons, all further two and three-dimensional nonlinear studies of this mode should retain fluid compression.

So far, we have seen that the magnetic field perturbations, resulting from the $m = 1, n = 4$ mode, never completely saturate nonlinearly, but slow down to an algebraic growth rate on the resistive skin time scale. Therefore, neighboring islands will eventually overlap at which time ergodic magnetic

field line structure will exist over most of the plasma cross section, and parallel electron transport will lead to a swift loss of particles along open field lines. To ascertain the amount of time before complete island overlap occurs, we calculate the nonlinear development of the three innermost $m = 1$ modes. These are the $n = 3, 4$, and 5 modes shown in Fig. 3. Their nonlinear evolution is calculated for $\alpha_s = 0.7$.

Time histories of the widths of these three neighboring $m = 1$ islands are shown in Fig. 9. Here we see that modes which resonate in regions of lower shear, toward the plasma core, grow faster nonlinearly. In fact, when the shear is small enough, the final stage of nonlinear growth occurs on a time scale much shorter than the resistive skin time. This is the case for the innermost, $n = 3$, mode which increases to more than 30% of the plasma minor radius in less than 3% of the resistive skin time.

The resulting magnetic island structure for these three cases are shown in Fig. 10. Again, in the region of lower shear, the behavior is quite different. The $n = 4$ and 5 modes display the "double island" behavior seen previously whereas the $n = 3$ mode leads to an island which has a single magnetic axis.

In Fig. 11 is plotted the radial positions of the $n = 3, 4$, and 5 island boundaries at the angular position, $\tau_{\max}$, where the island width is maximum. The value of $\tau_{\max}$ is the same in all three cases. Complete overlap of the $m = 1$ islands is seen to occur in just under 3% of the resistive skin time. Typical parameters of the experimental device S-1 at PPPL are $R \sim 40$ -50 cm, $B_0 \sim 3$ -4 k-gauss, $n_e \sim 10^{14} \text{ cm}^{-3}$, and $T_e \sim 100 \text{ eV}$. These values correspond to a magnetic Reynold's number of $S \sim 10^5$ and a resistive skin time of $\tau_R \sim 0.03$ sec. Therefore, for an axial beta value of 6%, it will be at least one millisecond before complete overlap of the $m = 1$ islands occurs in the S-1

device.

This overlap time can be extended by lowering the plasma beta. For instance, we have recalculated the nonlinear evolution of the $m = 1$, $n = 3$ and 4 modes, this time with a pressure parameter of $\alpha_s = 0.2$, corresponding to an axial beta value of 2%. The results from this calculation are shown in Fig. 12. The island overlap time for this case is more than double the previous case. Also, as the electron temperature in this device is increased the linear growth rate of the resistive interchange mode will decrease thereby increasing the amount of time before ergodic magnetic field line structure exists over the majority of the plasma minor radius.

A discussion of the accuracy and validity of the results presented in this section is given in Appendix B where some convergence checks are made along with a consistency check which uses the energy conservation theorem.

VII. SUMMARY AND DISCUSSIONS

Theoretical studies have indicated that resistive pressure driven modes pose a serious threat to confinement in the spheromak. Therefore, we were motivated to examine the nonlinear evolution of the resistive interchange mode in this device. Our studies have concentrated on the single helicity development of the $m = 1$ mode in the cylindrical spheromak approximation.

We found that the growth of this instability always leads to a quasilinear flattening of the equilibrium pressure profile in the vicinity of the mode rational surface, thereby eliminating the driving mechanism for this mode. In response to the equilibrium modification the mode growth rate can slow down from exponential on the resistive MHD time scale, $S^{1/3}t_A$, to algebraic on the resistive skin time scale. Here, S is the magnetic Reynold's number, $S > 10^5$, and t_A is the Alfvén time based on the axial field.

However, this saturation process is defeated if the plasma is allowed to overheat. In order to prevent the plasma from overheating in further studies, we simulated various energy loss processes by eliminating the ohmic heating term from the pressure equation.

We then found that in regions of the plasma where the shear is low, i.e., toward the magnetic axis, mode saturation does not occur in response to the pressure flattening. Modes growing in this region were seen to be very active nonlinearly, the final stage of algebraic growth occurs on a time scale much shorter than the resistive skin time.

In a tearing mode stable cylindrical spheromak equilibrium, with an axial beta value of approximately 6%, we found that complete overlap of $m = 1$ islands occurs in about 3% of the resistive skin time, for $S = 10^5$. For typical S-1 device parameters this time corresponds to one millisecond. This overlap time is longer in lower beta or higher temperature plasmas.

We also found that finite fluid compression does indeed have long-term consequences on the nonlinear development of the resistive interchange mode. It has the effect of substantially slowing down the growth of this mode throughout the phases of nonlinear development and was seen to sometimes have dramatic effects on the resulting island structure. Therefore, fluid compression should be retained in further 2D and 3D nonlinear studies.

Because the Reversed Field Pinch (RFP) is magnetically similar to the spheromak, the results obtained here should also apply to $m = 1$ modes which resonate inside of the field null in the RFP. In fact, such studies have been performed by Schnack, Killeen, and Gerwin¹¹ and by Hender and Robinson¹² in tearing mode stable RFP profiles. They also observe the quasilinear flattening of the pressure profile and the effects of overheating the plasma that has been seen here.

Finally, we found that by writing the magnetic field in a form that is inherently divergence-free, and by decomposing the velocity into an incompressible component, described by a helical stream function, and a compressible component, described by a helical velocity potential, we could partially separate the ideal MHD time scales by taking the divergence and curl of the momentum equation. This separation of the ideal MHD characteristics leads to accurate and stable numerical solutions. The particular representation of magnetic field and velocity variables is a generalization to compressible flow of the incompressible equations in Ref. 13 for studying the nonlinear development of the ideal internal kink mode in a cylinder.

ACKNOWLEDGMENTS

The authors wish to thank W. Park and D. A. Monticello for sharing their experience concerning nonlinear simulations of resistive instabilities in a cylinder. We also thank R. C. Grimm and M. Yamada for reviewing this work and for several enlightening discussions concerning both the formulation of the present problem and the physical results.

This work was supported by U. S. Department of Energy Contract No. DE-AC02-76-CHO-3073.

APPENDIX A

Here we give a schematic form of the finite difference equations which are a result of the numerical techniques described in Sec. V as applied to Eqs. (9). The spectral equations are written in the following matrix form for each Fourier harmonic $0 \leq l \leq L$:

$$S_l(r) \cdot \frac{\partial}{\partial t} Z_l(r,t) = [W_l(Z_l, r) + \eta R_l(r)] \cdot Z_l(r,t) + N_l(Z_l, r) \quad (16)$$

The solution vector $Z_l(r,t)$ is a function of the radius r and time t and has the seven components

$$Z_l = [\gamma_l \quad q_l \quad \phi_l \quad A_l \quad \omega_l \quad O_l \quad p_l]^T \quad (17)$$

where $\phi_l \equiv r \partial \gamma_l / \partial r$.

The 7×7 matrices S_l , W_l , and R_l are, in general, second order differential operators in r . The matrix S_l depends only on the radius and contains the coefficients of the terms involving derivatives with respect to time in Eqs. (9). All of the linear terms from the convolutions are contained in the $W_l \cdot Z_l$ term in Eq. (16) so that W_l is a functional of Z_0 . The W_l matrix contains all of the driving terms for the ideal MHD waves and varies on a time scale which is much longer than the oscillation periods of these waves. The resistive diffusion terms are contained in the $\eta R_l \cdot Z_l$ term in Eq. (16).

The remaining terms arise from the nonlinear part of the many convolutions and are contained in the N_l vector in Eq. (16). Calculating N_l is by far the most time consuming part of the numerical procedure. The

elements of S_l , W_l , R_l , and N_l are given in Ref. 14.

As described in Sec. V we use a two-step time advancement scheme which treats all linear terms implicitly and all nonlinear terms explicitly. Defining $F_l \equiv W_l + \eta R_l$ and letting the superscript "n" denote the level number, the time advancement scheme is

Step 1

$$S_l \cdot (Z_l^{n+1} - Z_l^n) = \frac{\Delta t}{2} \{ F_l^n \cdot [(1 - \phi) Z_l^n + \phi Z_l^{n+1}] + N_l^n \} , \quad (18a)$$

and

Step 2

$$S_l \cdot (Z_l^{n+2} - Z_l^n) = \Delta t \{ F_l^{n+1} \cdot [(1 - \phi) Z_l^n + \phi Z_l^{n+2}] + N_l^{n+1} \} , \quad (18b)$$

where Δt is the time step and $1/2 < \phi < 1$.

The operators S_l and F_l^n couple the values of Z_l^n at neighboring radial grid points so that Eq. (18a) is a matrix tridiagonal system of equations for Z_l^{n+1} at each grid point and similarly Eq. (18b) gives Z_l^{n+2} at each grid point. Eqs. (18) are inverted for each Fourier harmonic $0 \leq l \leq L$ at each half time step.

APPENDIX B

Exact statements concerning the accuracy of a numerical procedure as complicated as the one presented in this paper are not possible to make at present. In practice one designs a numerical scheme with general accuracy and stability considerations in mind and then checks the validity of the solutions by comparing with known solutions, doing convergence studies, and making consistency checks using conservation laws. In this appendix we verify that our method is indeed a convergent one and that the solutions presented in Sec. VI are reasonably accurate.

Convergence is demonstrated by calculating the linear growth rate and the island width evolution of a given mode for various values of the time step, Δt (normalized to t_A), and the number of grid points, J , using a uniform mesh. The number of Fourier modes ($L = 8$) is held constant in these simulations. The values of J and Δt are chosen such that the ratio between the grid spacing and time step is held constant, i.e., $J\Delta t = 32$. The particular mode considered is $m = 1$, $n = 4$, and the equilibrium configuration is $\mu_0 = 0.8$, $\alpha_s = 0.7$, and $R/a = 2$. This case was studied extensively in Sec. VI for the magnetic Reynold's number $S = 10^5$. For the purpose of doing convergence studies we choose $S = 10^4$ since the resistive layer width becomes large enough to be resolved on a uniform grid.

In Fig. 13 is plotted the linear growth rates calculated with $J = 64$, 128, 256, and 512 grid points, corresponding to $\Delta t = 1/2$, $1/4$, $1/8$, and $1/16$, respectively. For $J > 128$ the growth rate is converging linearly in J^{-1} to a growth rate $\gamma t_A \approx 8.044 \times 10^{-3}$. With $J = 64$ the growth rate has not yet begun to converge. The accuracy of the linear calculations was demonstrated in Ref. 5.

The island width evolution curves for $J = 64$, 128, and 256 grid points

are shown in Fig. 14. In the nonlinear regime the island width increases approximately 4% when J is increased from 64 to 128 and then again by approximately 2% when J is increased from 128 to 256 indicating that the nonlinear solutions are converging. Assuming the resistive layer width scales as $S^{-1/3}$ gives approximately 15 grid points inside of the resistive layer when $J = 256$ and $S = 10^4$. The simulations described in Sec. VI utilize grid packing to ensure that 50 or more grid points are inside the resistive layer. Therefore, we conclude that the nonlinear solutions presented in this paper are within 1% of their converged values. Note also from Fig. 14 that using too few grid points will give an overly optimistic estimate of the island overlap time.

Finally, we calculate how well energy is being conserved in the above simulations. The form of the resistive MHD equations described in this paper obey the following energy conservation theorem:

$$\frac{\partial}{\partial t} \int d^3x \left[\frac{1}{2} v^2 + \frac{1}{2} B^2 + \frac{3}{2} p \right] = - \int dS \cdot (\vec{E} \times \vec{B}) + \frac{1}{2} \int d^3x \vec{v} \cdot \nabla \cdot \vec{v}$$

where volume integrals are taken over the plasma volume and surface integrals are taken over the plasma/wall interface. The above equation represents the fact that the time rate of change of the sum of plasma kinetic, magnetic, and internal energies is balanced by the Poynting flux at the wall plus a small term proportional to the volume integral of $\vec{v}^2 \cdot \nabla \cdot \vec{v}$ which is present because we have held the plasma density ρ constant in space and time in the momentum equation.

integrating the above equation in time gives the relation

$$E_L(t) = E_L(0) + E_R(t) \quad ,$$

where

$$E_L(t) \equiv \int d^3x \left[\frac{1}{2} v^2 + \frac{1}{2} B^2 + \frac{3}{2} p \right] ,$$

and

$$E_R(t) \equiv \int_0^t dt \left[\frac{1}{2} \int d^3x v^2 \nabla \cdot \vec{v} - \int d\vec{S} \cdot (\vec{E} \times \vec{B}) \right] .$$

The degree to which energy is conserved is measured by calculating the error $\varepsilon(t)$ as a function of time where

$$\varepsilon(t) \equiv \left(\frac{E_L(t) - E_L(0) - E_R(t)}{E_L(0)} \right) .$$

Resulting error curves for the cases $J = 64$, 128 , and 256 are shown in Fig. 15. Clearly, the error is much smaller in the cases $J = 128$ and 256 than in the case $J = 64$. Also, for $t/t_R \lesssim 0.05$ the error for $J = 256$ is more than twice as small as when $J = 128$.

Eventually, the magnitude of the error curves for $J = 128$ and 256 become comparable. This behavior is most likely due to the fact that when the island width become larger than $\approx 35\%$ of the minor radius, as is the case here, the number of Fourier modes retained ($L = 8$) is no longer adequate to give detailed energy balance. However, for the cases studied in Sec. VI where $S = 10^5$, the island width is always less 30% of the minor radius by the time complete island overlap occurs so that energy is conserved reasonably well throughout the development of those modes.

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FIGURE CAPTIONS

- FIG. 1. Cylindrical spheromak equilibrium profiles for $\mu_0 = 0.8$ and $\alpha_s = 0.7$. Distance is normalized to the minor radius. Axial beta value is approximately 6%.
- FIG. 2. Ideal MHD time scale separation. (a) Incompressible part, $-A_1/r$, and compressible part, $\partial Q_1/\partial r$, of the first harmonic of the radial velocity. (b) Corresponding radial velocity. Time is normalized to the Alfvén time t_A , based on the axial magnetic field.
- FIG. 3. Mode rational surface positions for $m = 1$ in the cylindrical configuration characterized by $\mu_0 = 0.8$ and an aspect ratio of 2. Distance is normalized to the minor radius.
- FIG. 4. Time histories of the island width for the $m = 1$, $n = 4$ mode in the configuration $\mu_0 = 0.8$ and $\alpha_s = 0.7$. The island width is W , the minor radius is a , and the resistive skin time is t_R . The effects of ohmic heating are illustrated.
- FIG. 5. Evolution of the equilibrium pressure profiles for the two cases illustrated in Fig. 4.
- FIG. 6. Time histories of the island width for the $m = 1$, $n = 4$ mode in the configuration $\mu_0 = 0.8$ and $\alpha_s = 0.7$. The effects of finite fluid compression are illustrated.

FIG. 7. Effects of finite fluid compression on the island structure.

FIG. 8. Time histories of the amplitude of the first four helical flux harmonics at the singular surface for the two cases illustrated in Fig. 6.

FIG. 9. Time histories of the island widths for the $m = 1$, $n = 3, 4$, and 5 modes in the configuration $\mu_0 = 0.8$ and $\alpha_s = 0.7$.

FIG. 10. Resulting island structures for the modes illustrated in Fig. 9.

FIG. 11. Demonstration of complete overlap of $m = 1$ islands in the configuration $\mu_0 = 0.8$ and $\alpha_s = 0.7$.

FIG. 12. Demonstration of overlap of $m = 1$ islands in the configuration $\mu_0 = 0.8$ and $\alpha_s = 0.2$.

FIG. 13. Convergence study of the linear growth rate.

FIG. 14. Convergence study of the island width evolution.

FIG. 15. Truncation error calculated from the energy conservation theorem.

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Fig. 1

Fig. 2

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Fig. 3

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Fig. 4

Fig. 5

Fig. 6

83T0124

(a) INCOMPRESSIBLE

(b) COMPRESSIBLE

Fig. 7

Fig. 8

Fig. 9

#82T0256

Fig. 10

Fig. 11

Fig. 12

Fig. 13

Fig. 14

#83T0489

Fig. 15

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