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FINITE ELEMENT STRESS ANALYSIS
OF A COMPRESSION MOLD

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Prepared by C. E. Watterson, D/816

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SUMMARY

Theoretical and experimental investigations of a compression mold were conducted using finite element method (FEM) stress analysis and strain gage instrumentation. Static linear elastic and transient heat transfer stress analyses were performed using an axisymmetric model of the mold assembly. This first application of finite element techniques at Bendix Kansas City Division to a major production problem warranted an extensive experimental verification of the mathematical model by thermocouple and strain gage instrumentation of a production mold assembly. At a maximum production load of 6.67 MN (750 tons) and a temperature of 315°C, distortional stresses did not exceed 39 percent of the yield strength of the mold.

The structural integrity of this particular mold was verified by this analysis. Additional applications of these techniques are recommended for similar molds experiencing production problems. Implementation of more advanced computer codes for nonlinear, dynamic, and three-dimensional stress analyses may be necessary for production mold problems resulting from more complex failure mechanisms.

DISCUSSION

SCOPE AND PURPOSE

The compression mold under investigation is used to form hemispherical shells of highly abrasive materials at loads up to 4.4 MN (500 tons) and temperatures of 148°C. This effort was initiated to study the structural integrity of the mold by determining the maximum mechanical-thermal stress concentrations throughout the mold during the linear loading phase of the molding process. Two, recently acquired, finite element method (FEM) stress analysis computer codes, SASL¹ and WILSON², were to be used along with experimental strain gage measurements.

PRIOR WORK

Bendix Kansas City initiated a long term effort to provide a finite element stress analysis capability. This effort included a consulting contract with Doctor J. E. Akin of the University of Tennessee. The transient heat transfer and stress analysis codes WILSON and SASL are now operational at Bendix as a result of these efforts.

The WILSON code performs thermal analysis and heat transfer calculations. The output consists of temperatures at each nodal point of the finite element model; SASL then combines this input with the material properties and loading data and performs a stress analysis. Output information from SASL includes stress, strain, and displacement in each element as well as failure criteria. With the WILSON interface to SASL, fairly complex thermomechanical problems can be solved directly.

ACTIVITY

Background

Theoretical and experimental stress analyses were conducted on the compression mold. Some molds used to form filled parts were experiencing premature failure during production. After a certain number of parts were fabricated, the molding contours were beyond tolerance limits and had to be reworked at considerable expense. A prime candidate for investigation was general yielding of such mold inserts caused by mechanical-thermal stresses. This particular mold was chosen because it is similar to many Bendix molds and was readily accessible for experimentation.

Figure 1 is a view of the compression mold along with the computerized strain gage and thermocouple monitor used in this investigation. The mold has a load limit of 6.6 MN (750 tons) with controlled temperatures up to 315°C. Figure 2 is an internal view of the instrumented lower mold insert.

It follows from classical mechanics that the largest stress concentrations near the molding surfaces would occur during the linear portion of the stress-strain curve of the molding material. Any yielding of the material would tend to distribute the load by increasing the area of contact between the molding surfaces. A linear elastic stress code was, therefore, considered both suitable and sufficient for this analysis.

This mold study was conducted in two phases. The first phase combined a static linear elastic stress analysis, using the SASL computer code, with experimental strain gage and thermocouple measurements. The primary intentions of this phase were to develop a mathematical model of the mold, conduct a stress analysis on the model using realistic loads and constraints, and verify results of the computer model by experimental measurements on a production mold.

The available computer codes are restricted to two-dimensional or axisymmetric models. An axisymmetric model was employed for the compression mold, and the corresponding experimental verification of the model was limited to strains in the region of the molding surfaces. A successful verification of the model, for the parameters of interest, then allows the effects of load, temperature, and material changes to be determined through computer analysis. This method of structural redesign or analysis has proven to be cost effective compared to prototype trial and error design.

A second phase combined the linear SASL analysis with a transient heat conduction analysis, using the WILSON computer code. Maximum production loads and temperatures were assigned to this model in order to find regions of critical stress that could result in the out-of-tolerance conditions experienced during production with similar molds.

Finite Element Model

The two finite element computer codes, SASL and WILSON, are capable of modeling plane strain, plane stress, and axisymmetric solids. The compression mold can be reasonably approximated by an axisymmetric model. A two-dimensional cross section of the finite element model developed for this analysis is shown in Figure 3. Heating and cooling channels are indicated by shaded areas in the mesh. Mechanical loading of the model was achieved by constraining nodes along the top of the mold. The laminated preform material to be molded is shown between the upper and lower mold halves.

Figure 1. Compression Mold Instrumentation

Figure 2. Instrumented Lower Mold Insert

The heating and cooling channels in the production mold are not axisymmetric. However, this first approximation was considered adequate for the intent of this investigation. The concurrent experimental strain measurements were incorporated to verify the authenticity of this model.

Both upper and lower mold halves are constructed from Vega steel which has the following properties.

- Elastic Modulus, 200 GPa
- Shear Modulus, 76.8 GPa
- Yield Strength, 1.86 GPa
- Ultimate Strength, 2.06 GPa
- Poisson's Ratio, 0.3

Figure 3. Two-Dimensional Cross Section of Axisymmetric FEM Model

- Thermal Expansion, $11.8 \text{ m/m/}^{\circ}\text{C}$
- Thermal Conductivity, $46.7 \text{ W/m}\cdot^{\circ}\text{C}$
- Specific Heat, $416 \text{ J/kg}\cdot^{\circ}\text{C}$
- Density, 8030 kg/m^3

Static Linear Elastic Analysis

Before the introduction of finite element computer codes at Bendix, only rudimentary stress analysis of complex structures was possible. The SASL stress analysis code calculates stress, strain, and displacement for complex structures involving one or more materials. Theoretically, the accuracy of a finite element model is limited only by the size of the individual elements.

This study represents the first application of finite elements to a major production problem at Bendix. Therefore, an adequately complex experimental verification of the mathematical model was considered essential. The static linear elastic analysis reported in this section and the experimental measurements in the next section are intended to satisfy this requirement.

Since the experimental strain gage measurements were thermally compensated, no thermal stresses were calculated in the finite element analysis. Uniform pressurization of 6.894 MPa (1000 psi) was applied to the bottom mold surface (Figure 3). The top surface of the mold was constrained by horizontal rollers, eliminating movement of this surface parallel to the axis of symmetry.

The SASL failure criteria for a material is based on the von Mises distortional energy theory. An "effective stress," $\bar{\sigma}$, is calculated:

$$\bar{\sigma} = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2} \quad (1)$$

where σ_1 , σ_2 , and σ_3 are the principal stresses.

This von Mises stress, which basically predicts failure when it exceeds the yield strength of the material, can be readily inspected from the SASL output without a thorough analysis of the principal stresses.

A contour plot of the von Mises stress is shown in Figure 4. An enlargement of the region of interest is presented in Figure 5. The largest stress in the region identified by the arrow is 138 MPa (20,000 psi) or 7.4 percent of the yield strength of Vega steel. At the full 6.6 MN (750 ton) capacity of the mold, stress in this region would not exceed 25 percent of the yield strength, ignoring thermal effects. This result was anticipated, since thermally induced stresses--not mechanical loadings--were believed to be the most severe.

The SASL computer code computes strain both parallel and perpendicular to the axis of symmetry of the model. Experimental strain gages were positioned parallel to the hemispherical molding surface, and tensor transformations were required to compare finite element derived strains to experimental strains. Details of these transformations are included in the Appendix. Strain comparisons are presented in the experimental section.

Experimental Stress Analysis

An experimental stress analysis was initiated as an independent confirming effort to the finite element computer calculations described previously. The actual compression mold has a complex

Figure 4. Contour Plot of von Mises Stress

three-dimensional geometry, instead of the pure axisymmetric geometry of the SASL model. In addition, the mold is made of several parts which fit together with some tolerance. This nonbonded fit cannot be completely modeled with either SASL or WILSON. For these reasons, experimental data were needed to verify the theoretical model.

Even the smallest strain gage must be bonded to the surface of an elastic material. Provisions must be made for lead wires to the monitoring instrumentation and for thermocouples to measure the temperature at the strain gage location. These factors limit the choice of location for comparing theoretical and experimental results. The actual mold surface is the location of interest, but it cannot be used because the molding material would damage the instrumentation. The nearest protected location for bonding strain gages, the back-side of the bottom mold insert, was chosen. The five strain gage and thermocouple locations are shown by the X's in Figure 6. A right angle strain gage pair was centered at each location. Each pair had one gage along the mold curvature and another perpendicular to it. A pressure transducer was located in the hydraulic fluid supply line. Three deflectometers monitored mold closure.

Figure 5. Enlargement of von Mises Stress

After the mold insert was instrumented, the mold was reassembled and remounted on the hydraulic press. Two strain gages and one thermocouple were damaged in reassembly. Thermal compensation data were then accumulated over the temperature range of part manufacture; then, five parts were formed and experimental data were collected for all but the second. Two of the mold closure deflectometers were damaged on the first run, and repair was not complete until the final run. The five parts were not formed under computer control, so the actual manufacturing details were slightly different for each part.

The experimental data for the first part are shown in Figure 7. One minute elapsed between successive data readings. The hydraulic

Figure 6. Location of Strain Gages and Thermocouples

pressure for the duration of the mold closure and precure of the part is shown. There was a slight drop in pressure as the precure progressed, and a slight decay in pressure occurred during all tests. A plot shows the temperatures for each thermocouple on the same time scale as for hydraulic pressure. TC2, located at the pole, lagged slightly during heat up for all tests. The mold was always cooled to the preheat temperature before it was opened. A plot shows the data for strain gages 1, 2, 3, and 4 which were along the curvature of the mold. Large strain transients occurred during the rapid temperature changes of heat up and cool down. Similar data for strain gages 6, 7, 8, and 10 are shown in Figure 7 for the tangential strains. Gages 5 and 9 and thermocouple 5 were lost in mold reassembly.

No data were recorded for test 2. Figure 8 shows pressure, temperature, strain along the curvature, and tangential strain for test 3. This test was of longer duration than test 1. Figure 9 shows similar data for test 4. This was of still longer duration than tests 1 or 3, and again large strain transients

Figure 7. Pressure, Strains, and Temperature Characteristics for Test 1

Figure 8. Pressure, Strains, and Temperature Characteristics for Test 3

Figure 9. Pressure, Strains, and Temperature Characteristics for Test 4

occurred during heat up and cool down. Figure 10 shows the data for test 5. The pressure was applied five minutes before heating started, so the mold was fully closed before the temperature was increased above the preheat temperature. This test duration was about equal to test 4 and also had large strain transients during heat up and cool down.

Table 1 shows several specific differences of the four tests made. The maximum initial pressure in the hydraulic fluid line varied 2.5 percent between tests and decreased 34 to 251 kPa (5 to 37 psi) during the tests which lasted from 31 to 81 minutes at pressure. The time to fully close the mold and form the part from the preform biscuit ranged from 3 to 9 minutes as measured by the one undamaged deflectometer for tests 1, 3, and 4 and by three deflectometers for test 5. The mold preheat temperature was 69°C for tests 1 and 4 and 75°C for tests 3 and 5.

Comparison Between Theoretical and Experimental Methods

The finite element model was designed to calculate the maximum mold stresses and strains just before the biscuit of molding material yielded. The corresponding experimental data are the strains for the first reading after full pressure is applied and the deflectometers show that closure has started. These data are shown in Table 2 for each strain gage for each test. There were differences between strains for gages 1 and 5, and 2 and 6. These had identical orientations at symmetrical locations relative to the pole of the mold, so they might be expected to show equal strains. The differences are probably related to any difference in tolerance of fit of the mold insert, the actual spiral geometry of the heating fluid channels, and the few seconds difference in time between readings for these strain gages. If both gages 3 and 4 were truly at the pole, they would be expected to show identical strains. However, the center of this right angle pair was at the pole, so each gage element was slightly off center. In addition, gage 4 was read 5 seconds later than gage 3.

The strain transformation equations of the Appendix were used to transform the SASL finite element computed strains from cylindrical coordinates to the spherical coordinate geometry of the experimental measurements. Table 3 shows the comparative data. Both the large and small mesh finite element results are exhibited. The small mesh was used for the thermal analysis. Both the average value and range of the experimental results for all four tests from Table 3 are included. Two important observations should be made. First, the large and small mesh results agree adequately for 0 degree and 20 degree locations, which is the area of interest, and only differ significantly for 45 degree and 72 degree locations which are well removed from the preform biscuit of material. Second, theory and experiment agree adequately for most locations. Only the 0 degree tangential strain, the 45 degree curvature strain for the large mesh, and the 72 degree tangential strain are outside the range of the corresponding experimental data.

Figure 10. Pressure, Strains, and Temperature Characteristics for Test 5

Table 1. Specific Differences Between Mold Test Conditions

Test Conditions	Test			
	1	3	4	5
Initial Pressure (MPa)	644	635	651	649
Pressure Decrease (MPa)	0.18	0.03	0.28	0.24
Test Duration (Minutes)	31	65	81	80
Time to Close Mold (Minutes)	5	3	9	4,3,3*
Mold Preheat Temperature (°C)	69	75	69	75

*Only one deflectometer was operative for tests 1, 3, and 4. All three at 120-degree increments around the mold were operative for test 5.

The small mesh tangential strain at 72 degrees may not be sufficiently accurate because of larger element size. The 20 degree experimental data suffer from too long a time interval of one minute between data points and thus are probably too low.

With these observations in mind, the comparison data of Table 3 show the FEM model to be adequate. Since the strains agree adequately at the strain gage locations, the theoretical predictions can be believed at other locations as well. The maximum predicted stresses at the mold surface and the thermal stresses during heating are thus believed to be accurate.

Transient Heat Transfer Analysis

Thermally generated stresses in the compression mold were considered to be key factors in any out-of-tolerance conditions that might be experienced during production. Any general yielding near the molding surface would occur during the linear loading phase. A transient thermal stress analysis was conducted at maximum capacities of the mold.

Table 2. Summary of Experimental Strain Data

Gage Number	Gage Location (Degrees)	Strain			
		Test 1 ($\mu\text{m/m}$)	Test 3 ($\mu\text{m/m}$)	Test 4 ($\mu\text{m/m}$)	Test 5 ($\mu\text{m/m}$)
1	-20*	191	267	225	106
2	-20**	269	206	290	144
3	0*	98	119	83	173
4	0**	182	219	146	218
5	20*	151	52	147	178
6	20**	105	10	93	146
7	45*	83	50	70	166
8***	45**				
9***	72*				
10	72**	14	215	82	39

*Curvature gage orientation

**Tangential gage orientation

***Gage or lead wires damaged during mold reassembly

The WILSON heat transfer code, with the model presented in Figure 11, established temperature gradients throughout the mold for input to the linear SASL stress code. An initial temperature of 232°C was assigned to the immediate area of the heating channels. All other points in the mold were initially at -17°C, a restriction of the WILSON code. Outer boundaries of the mold were thermally insulated from the environment.

The WILSON code solves the classical transient heat conduction problem. Initial temperatures, heat sources, insulation boundaries, and convection coefficients are assigned to the model. Through iterative calculations, thermal gradients throughout the model are determined as the system approaches thermal equilibrium.

To determine what time (thermal gradient profile) will generate the highest thermal stresses, several cycles of the WILSON-SASL stress analysis are required. The compression mold generated the highest thermal stresses during the first one-half-second of operation. Contour plots of temperature are presented in Figure 12. The lower molding surface increases in temperature faster than the upper molding surface. The actual temperature difference, however, is less than 38°C and most likely does not affect the quality of the molded hemisphere.

Table 3. Comparison Between Theoretical and Experimental Strains

	Theoretical Strains				
	Large Mesh		Small Mesh		Experimental Strain Range ($\mu\text{m/m}$)***
Location (Degrees)	Element ($\mu\text{m/m}$)	Strain ($\mu\text{m/m}$)	Element ($\mu\text{m/m}$)	Strain ($\mu\text{m/m}$)	
0*	749	143	155	80	83-118-173
0**	749	88	155	65	146-191-219
20*	758	233	159	294	52-165-267
20**	758	146	159	115	10-158-290
45*	770	34	165	140	50-92-166
45**	770	-6	165	-53	
72*	989	24	171	78	
72**	989	-13	171	-57	14-88-215

*Curvature gage orientation

**Tangential gage orientation

***Middle number is average range

Stress concentrations for these five thermal profiles were obtained by successive SASL computer analysis. The thermal gradients illustrated in Figure 12 produced the highest effective stress levels, as shown in Figure 13. The arrow points to the region of highest effective stress. The stress in that region is plotted as a function of time in Figure 14. The maximum stress of 751 MPa (109,000 psi), occurring at 0.02 second, represents 39 percent of the yield strength of Vega steel. The thermal model, therefore, predicts that the mold will not fail even at maximum loads and temperatures.

ACCOMPLISHMENTS

Thermal heat transfer and linear elastic stress analyses have verified the structural integrity of a compression mold.

Successful application of axisymmetric modeling techniques, computer finite element codes (WILSON and SASL), and experimental verification of theoretical results provide Bendix with a powerful new tool for mold design and evaluation.

Figure 11. Wilson Temperature Distribution Model

A general survey of molds encountering production problems should be evaluated for finite element treatment. Mold design modifications as suggested by finite element stress analysis could result in significant cost savings by decreasing the number of mold reworks required for a production run.

For those mold problems with more complex failure mechanisms, more sophisticated strain gage experimentation and more advanced computer codes are recommended.

Integration of existing industrial nonlinear and three dimensional stress analysis and heat transfer codes should proceed as rapidly as possible for maximum benefit to the manufacturing complex.

0.01 SECONDS

0.02 SECONDS

0.03 SECONDS

0.04 SECONDS

Figure 12. Temperature Distribution After Indicated Times

A -17°C
 B 27°C
 C 71°C
 D 115°C
 E 160°C
 F 204°C

Figure 12 Continued. Temperature Distribution After Indicated Times

Figure 13. von Mises Stress at Maximum Loads and Temperatures

Figure 14. von Mises Stress as a Function of Time

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¹M. L. Callabresi and S. T. Heidelberg, SASL - A Finite Element Code for the Static Analysis of Axisymmetric and Plane Solids Subjected to Axisymmetric and Plane Loadings. Sandia Livermore: SCL-ER-70061, December 1972 (Available From NTIS).

²R. E. Nickell and E. L. Wilson, "Application of the Finite Element to Heat Conduction Analysis," Nuclear Engineering and Design, 4, North-Holland Publishing House, Amsterdam, 1966, pp 276-286.

Appendix

TRANSFORMATION OF STRAINS FROM CYLINDRICAL TO SPHERICAL COORDINATE SYSTEMS

This appendix gives the equations needed to transform the strains calculated by SASL for an axisymmetric solid using cylindrical coordinates to the spherical coordinate system of the experimental strain measurement on the insert of the compression mold. This transformation was necessary to compare the strains from the SASL model with those of the actual experiment and verify the validity of the SASL model. The SASL model could then be used for thermal analysis as well. Stresses at other locations could also be believed with confidence.

Stresses and strains are symmetric contravariant tensors of the second rank. Tensors in one coordinate system are linked to their counterpart in another coordinate system by a coordinate transformation. Let A^{ij} be a contravariant tensor of the second rank in the original coordinate system, X^i . The transformed tensor, $\bar{A}^{mn}$, in the transformed coordinate system, $\bar{X}^m$, is then given by Equation A-1.

$$\bar{A}^{mn} = \frac{\partial \bar{X}^m}{\partial X^i} \frac{\partial \bar{X}^n}{\partial X^j} A^{ij} \quad (A-1)$$

The cylindrical and spherical coordinate systems are shown in Figure A-1. The coordinate relations are given by Equations A-2 to A-4. The symbols $\bar{r}$, $\bar{\phi}$, $\bar{\theta}$ on the left hand side of these equations refer to the transformed spherical coordinate system, and the R , ϕ , Z on the right refer to the original cylindrical coordinate system of SASL. The usual relation to the X , Y , Z rectangular Cartesian coordinates is given in Figure A-1.

$$\bar{r} = \sqrt{R^2 + Z^2} \quad (A-2)$$

$$\bar{\phi} = \phi \quad (A-3)$$

$$\bar{\theta} = \cos^{-1} (Z / \sqrt{R^2 + Z^2}) \quad (A-4)$$

The A-1 transformation can be made for each strain component in turn by using the appropriate derivatives of Equations A-2 to A-4. The results are collected in Equations A-5 to A-10. The strain terms on the left hand sides are in the spherical coordinate system of the experiment. The strain components on the right are from the cylindrical coordinate system of SASL. The angle θ is measured

$$X = R \sin \varphi$$

$$Y = R \cos \varphi$$

$$Z = Z$$

(a) CYLINDRICAL COORDINATE SYSTEM

$$X = r \sin \theta \sin \varphi$$

$$Y = r \sin \theta \cos \varphi$$

$$Z = r \cos \theta$$

(b) SPHERICAL COORDINATE SYSTEM

Figure A-1. Coordinates Systems for Transformation of Strains From SASL to Compare With Punch Experiment

from the positive Z axis, so it is just the complement of the angle to each strain gage measured relative to the pole of the mold.

$$\bar{\epsilon}^{rr} = \epsilon^{RR} \sin^2 \theta + 2\epsilon^{RZ} \sin \theta \cos \theta + \epsilon^{ZZ} \cos^2 \theta \quad (A-5)$$

$$\bar{\epsilon}^{\phi\phi} = \epsilon^{\phi\phi} \quad (A-6)$$

$$\bar{\epsilon}^{\theta\theta} = \epsilon^{RR} \cos^2 \theta - 2\epsilon^{RZ} \sin \theta \cos \theta + \epsilon^{ZZ} \sin^2 \theta \quad (A-7)$$

$$\bar{\epsilon}^{r\phi} = \epsilon^{R\phi} \sin \theta + \epsilon^{Z\phi} \cos \theta \quad (A-8)$$

$$\bar{\epsilon}^{r\theta} = (\epsilon^{ZZ} - \epsilon^{RR}) \sin \theta \cos \theta + \epsilon^{RZ} (\cos^2 \theta - \sin^2 \theta) \quad (A-9)$$

$$\bar{\epsilon}^{\phi\theta} = \epsilon^{R\phi} \cos \theta - \epsilon^{Z\phi} \sin \theta \quad (A-10)$$

The strain gages of the experiment measured $\bar{\epsilon}^{\phi\phi}$ and $\bar{\epsilon}^{\theta\theta}$, so only those terms can be compared. In addition, SASL uses engineering strains, so $2\epsilon^{RZ} = \delta^{RZ}$. Using angles as measured relative to the pole of the mold, equations A-6 and A-7 become A-11 and A-12 and were used for comparison in that form.

$$\bar{\epsilon}^{\phi\phi} = \epsilon^{\phi\phi} \quad (A-11)$$

$$\bar{\epsilon}^{\theta\theta} = \epsilon^{RR} \cos^2 \theta + \delta^{RZ} \sin \theta \cos \theta + \epsilon^{ZZ} \sin^2 \theta \quad (A-12)$$

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MECHANICAL: Stress Analysis

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