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SHAFT78, A TWO-PHASE MULTIDIMENSIONAL COMPUTER PROGRAM FOR GEOTHERMAL RESERVOIR SIMULATION

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DISCLAIMER

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PREFACE

Large computer programs tend to exhibit characteristics not unlike a personality. When a single person is responsible for the entire program development, the computer program's personality (i.e., style, responses, etc.) reflect that of the programmer. In the case of SHAFT78, the development has not been due to one person, but many. As a result, the program has both desirable and undesirable features due to the many people who have worked to make it a usable tool.

The program employs the solution technique pioneered by A. Edwards and used in the TRUMP heat-flow program.[1] The TRUMP algorithm was adapted for use in various different porous flow calculations by Edwards,[2] Lasseter,[3] Sorey,[4] Lippmann,[5] and Narasimhan.[6,7] Narasimhan and Witherspoon[8] have presented the basic method and applications in several publications, and are responsible for the name used to describe the basic numerical method—i.e., integrated finite difference (IFD) method.[8] Lasseter[3] constructed the first version of SHAFT which has subsequently been extensively modified.

We pointed out above that there have been many people associated with this program during its development. But the support and encouragement of P. A. Witherspoon is the acknowledged reason for its existence and its current state of development. Finally the encouragement and support of our colleagues in Italy and Iceland should be acknowledged. The support provided by R. Cataldi and G. Manetti, and the technical contributions during application of the program to reservoir simulation by G. Neri, V. Jonsson, and C. Ruffili are gratefully appreciated.

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GENERAL APPROACH

1. INTRODUCTION

The computer program SHAFT78 was developed to compute two-phase flow phenomena in geothermal reservoirs. The program solves transient initial-value problems with prescribed boundary-conditions in up to three space dimensions. The solution method is an explicit-implicit IFD[8] approach which does not distinugish between 1, 2, or 3-D coordinate systems and allows a flexible choice of the shape of the discrete grid elements. The mass-and-energy equations are formulated in conservative form. The stability and convergence of the algorithm is controlled by an automatic choice of time steps - partially controlled by the user.

Although the program has been developed for use in simulating production and injection in geothermal reservoirs, there are other two-phase problems for which it is either immediately applicable, or for which it can be modified to be applicable. Since the equation of state is a tabular array, fluids other than water can be used with no fundamental modification. However, the pressure difference[9] between the wetting and non-wetting phases is neglected. The relative permeabilities, [9-10] for the wetting and non-wetting phases are available as analytical approximations, or in tabular form and can be specified for any fluid.

All fluid parameters, such as viscosity, heat capacity, heat conductivity, etc., can be specified as functions of temperature and pressure, and all parameters can vary with position. The program can handle up to seven different anisotropic rocks, with all rock parameters assumed to be independent of position, temperature, and pressure.

The solution algorithm is based upon statements of mass and energy conservation in both the rock and the two-phase fluid. The porous medium is assumed to have sufficiently small pores so that the thermal equilibration between rock and fluid is instantaneous. For most geothermal reservoir

problems where the time scale is in years and reservoir dimensions are often several kilometers, this approximation appears to be acceptable.

The solution variables are fluid density and fluid internal energy. These two intensive variables completely define the pressure, temperature, and phase of the (in general) two-phase mixture.[11] However, the initial conditions can be specified in terms of:

(1) temperature and pressure (for single-phase fluid)
(2) temperature and steam saturation (for two-phase fluid), or
(3) fluid density and internal energy, from which the remaining quantities are calculated.

The program CYCN, which solves the coupled energy and density equations, uses a technique designed to make the calculations both accurate and efficient. The energy equation and flow equation (mass conservation) are advanced in time according to separate time-step controls.[12] This was done since the flow calculation often requires smaller discrete time steps for convergence and stability than the energy calculation - the reason being that energy often changes at a smaller rate than density. Hence, the calculations proceed by first determining the allowable energy time step and then solving the energy equation. This is followed, in general, by a few flow (density) cycles which always end at the time calculated by the energy equation. The number of flow cycles for a given energy cycle can be partially controlled by the user. During each flow cycle the linearly interpolated value of internal energy is used to improve the acccuracy of the flow calculation. Either after the flow step or after each flow cycle a correction is made to ensure that the rock and fluid are in thermal equilibrium (same temperature). A two-step iteration can be carried out to ensure high accuracy for the rock/fluid equilibration.

Two grid generators are available for SHAFT78, and in addition the grid can be specified directly as input to CYCN by the user, if so desired. The grid generators provide input to CYCN for large problems, and since the

generators calculate areas, volumes, and distances needed for the solution procedure the generator input is quite simple compared to the direct input of these values by the user. The grid generator programs are called SHAME and OGRE.[13] The SHAME program is used when generating relatively regular grids in two dimensions. The OGRE program is used for large-scale simulation where the elements are irregular and one-, two-, or three-dimensional. The OGRE grid generator allows accurate discretization of geological features such as uneven bedding, distorted layers, etc. The grid generated by OGRE ensures that all elements are closed polyhedra with perpendicular flow areas between elements.

A plotting program is available for use with the grid generators and with CYCN. The OGRE program provides three-dimensional plots allowing several rotated views of the grid to evaluate complicated grid geometries. The SHAME program generates data which is used by CYCN and its plotting program SHAPE. The latter plotting routine can only be used when the SHAME program is used to generate the grid.

Plotting of the transient flowrate for all elements in which sources are present is also possible, and the same plotting program is used for comparison of the transient history matching plots with the production values.

In order to generate the tabular equation of state, a pre-processor program is available called PROPER, which uses input data for the fluid properties in terms of pressure, temperature, and phase. The program PROPER produces a table of ten fluid parameters as functions of internal energy and density, using a special interpolation scheme. Most input data for PROPER are generated by a program called WATER using the analytical formulas for the thermodynamic properties of water as provided by the International Formulation Committee (IFC Tables, 1967).[14] The program PROPER only needs to be run once over the range of energies and densities of interest, since the table can be used repeatedly. If a table with different size, different accuracy, or a table of some other fluid is desired, then the program PROPER

must be rerun with the appropriate input. The table interpolation is four-pointed everywhere except near the saturation curve where special care has been taken to ensure that interpolation does not occur across the phase change. The interpolation scheme ensures that good accuracy can be obtained in all regions. The program CYCN uses the same special interpolation methods that are used in generating the table during the preprocessing calculations.

The programs discussed above have all been used extensively, and have been applied to solve a variety of problems.[15-18] The mesh generators and plotting routines are free of obvious bugs. The program SHAFT78 has been tested by comparison with analytical solutions,[15] and by comparison with numerical calculations available in published literature.[16] Although extensive tests and reservoir simulation calculations have been carried out with the program, and although the basic equations and the physical model is believed free of errors, it is difficult to guarantee that a program as large and complicated as SHAFT78 is free of all errors. This is especially true since different people have worked on the program. However, we believe the program is calculating the phenomena of vaporization, condensation, and two-phase porous flow correctly within the limits of the physical model to be presented below.

2. THE GENERAL APPROACH

The microscopic structure of porous rock is highly heterogeneous. The channels through which the fluids move are tortuous and have (in general) non-regular shapes. In addition, the porous rock in a geological setting generally has many structural variations, and fractures of widely varying aperture and extent. The fluids, in general, move through the fractures more rapidly than through the microscopic pores. The macroscopic single-phase fluid flow through small pores has been found to be proportional to the pressure gradient (Darcy's Law).[19,20] Heterogeneous rock can usually be approximated by macroscopic rock and fluid parameters. When the flow rates are large,[21] or if the fracture velocities are very large relative

to the microscopic pore velocities,[23] the relationship between fluid flow-rate and macroscopic pressure gradient becomes non-linear. When the physical dispersion of fluid velocities in the porous channels is considered, another non-linear term appears.[24] In the case of two phases - one wetting and the other non-wetting - the relationship between flow rate and pressure gradient can be specified in terms of a function of wetting or non-wetting volumetric saturation.[25]

It is possible to apply statistical averaging techniques to derive transport equations in terms of volumetrically averaged rock and fluid properties.[26,27,28] This approach is entirely analogous to the microscopic averaging in fluid mechanics, where the effects of molecular motions within the fluid can be analyzed.[29] In both cases diffusion terms appear. In the case of microscopic fluid averaging the diffusion is molecular within an element of volume of fluid. In the case of macroscopic averaging over the pore volumes, the velocity dispersion term is due to fluid mixing in the channels.[30] In both cases, the relationship between the microscopic and the macroscopic fluxes is through a macroscopic coefficient. In the molecular case it is the diffusion coefficient, and in the macroscopic case it is the coefficient of dispersivity.

In any case, macroscopic equations can be derived using statistical averaging which have exactly the same form as the point equations (differential or integral equations obtained assuming a macroscopic representative elementary volume).[28] Although there are pedagogical considerations in taking a particular approach, the transport equations (obviously) have the same macroscopic form for the same set of initial assumptions. This is true also with regard to the method for deriving the governing equations. The same equations are obtained when the differential laws are integrated as when the differential-integral forms are derived directly when the same assumptions are made. The choice of presenting the equations in one form or another is primarily a matter of style and preference. We shall start from the macroscopic (point) differential equations, integrate them over a

volume which will have special significance for discretization, and then discuss the particular numerical solution procedure incorporated in the algorithm of SHAFT78.

The choice of variables is another question which is partly subjective. There is more than one possible choice of intensive thermodynamic variable pairs from which all other thermodynamic information can be derived.[31] Internal energy and specific density are two such variables. When the equation of state (EOS) is known in terms of energy and density the EOS gives pressure, temperature, and vapor saturation. This completely specifies the thermodynamic state in terms of macroscopic quantities we can measure. It is also possible to use triplets of variables such as temperature, pressure, and steam saturation.[32] Of these three variables only two are independent, namely, temperature and pressure in the one-phase region, and temperature and saturation in the two-phase region. Therefore, the numerical treatment becomes somewhat awkward for problems involving phase transitions.

In the subsequent development of the equations we will discuss them only in terms of internal energy and density as the basis for the program SHAFT78.

DEVELOPMENT OF EQUATIONS

1. DENSITY EQUATION

Consider the equation for the transient change in density, usually referred to as mass conservation or as the continuity equation.[33] For a two-phase fluid we have the total fluid density, ρ, such that:

$$\frac{\partial \phi \rho}{\partial t} = \frac{\partial \phi S \rho_v}{\partial t} + \frac{\partial \phi (1-S) \rho_\ell}{\partial t} \tag{1}$$

$$= - \nabla \cdot \bar{F} + q_\rho$$

where the symbols are defined by

ρ_α	=	density (mass per volume) of phase α
ϕ	=	porosity (V_p/V)
V	=	volume element of rock/fluid mixture
V_p	=	pore volume in V
S	=	vapor saturation (V_v/V_p)
V_v	=	vapor volume in V
t	=	time
$\bar{F}$	=	mass flux vector
q_ρ	=	external sources (mass rate/volume negative source corresponds to mass being withdrawn)

The mass flux of the fluid is the sum of the fluxes of the vapor and liquid.

$$\bar{F} = \bar{F}_v + \bar{F}_\ell \tag{2}$$

Darcy's Law[20] gives the form of the flux for each phase as

$$\bar{F}_\alpha = -K'_\alpha (\nabla p - \rho_\alpha \bar{g}) \tag{3}$$

The coefficients (conductances) are given by

$$K'_\alpha = \rho_\alpha \frac{k\, k_\alpha}{\mu_\alpha} \tag{4}$$

where

k = absolute permeability
k_α = relative permeability of phase α
μ_α = fluid viscosity of phase α
p = pressure
$\bar{g}$ = gravitational acceleration vector

Capillary pressure $p_c = p_\ell - p_v$ is presently neglected.[9] Integration of equation (1) over an arbitrary volume τ having the total closed surface σ gives

$$\int_\tau \left\{\frac{\partial \phi \rho}{\partial t}\right\} dV = \oint_J \left\{\bar{F} \cdot \bar{n}\right\} da + \int_\tau q_\rho dV \tag{5}$$

Here n is the inward normal for surface element da. The volume τ is taken to be an arbitrary non-reentrant, closed, N-sided polyhedron in three-space. Volume averages are given by

$$\rho = \frac{\int_\tau \rho' dV}{\int dV} = \frac{1}{\tau} \int_\tau \rho' dV \tag{6}$$

with an analogous definition adopted for surface integrals. The surface integral can be divided into N integrals, and the average flux through the area A_m is then $F_m = \oint \bar{F} \cdot \bar{n} da / A_m$. Then

$$\frac{\partial \phi \rho}{\partial t} = \sum_{m=1}^{N} \frac{F_m A_m}{\tau} + q \tag{7}$$

The summation, as implied by the definition of F_m (although not explicitly noted in eq. (7)) must be taken over a closed surface whose fluxes, F_m, are determined by a gradient in pressure that is perpendicular to the polygonal flow area, A_m.

The time differential can be expanded by noting that

$$\frac{\partial \phi\rho}{\partial t} = \phi \frac{\partial \rho}{\partial t} + \rho \left[\left(\frac{\partial \phi}{\partial \rho} \right)_u \frac{\partial \rho}{\partial t} + \left(\frac{\partial \phi}{\partial u} \right)_\rho \frac{\partial u}{\partial t} \right] \tag{8}$$

2. ENERGY EQUATION

Consider the equation for the transient change in internal energy, usually referred to as the energy conservation equation.[34] For a two-phase fluid in porous rock we have for the total time change in internal energy (per unit volume of the rock/fluid mixture)

$$\frac{\partial(\text{energy/volume})}{\partial t} = \frac{\partial(u\rho\phi + u_s\rho_s(1-\phi))}{\partial t}$$

$$= -\nabla \cdot \bar{G} + \left(\frac{\bar{F}_v}{\rho_v} + \frac{\bar{F}_\ell}{\rho_\ell} \right) \cdot \nabla_p + Q_u \tag{9}$$

where the symbols are defined by

- u = specific internal energy of the fluid
- u_s = specific internal energy of the solid
- ρ_s = specific density of the solid
- $\bar{G}$ = energy flux
- Q_u = external sources (internal energy rate/volume, negative source corresponds to heat being withdrawn)

The energy flux of the fluid is the sum of the conductive heat flux in the rock and fluid, plus the sum of the enthalpy fluxes of the vapor and liquid. The flux is given by

$$\overline{G} = -K\nabla T + \overline{F}_v h_v + \overline{F}_\ell h_\ell \tag{10}$$

where

- K = coefficient of heat conductivity for the fluid saturated rock[35]
- T = temperature
- $\overline{F}_\alpha$ = mass flux of phase α
- h_α = specific enthalpy of phase α

In evaluating the left-hand side of eq. (9), we impose the assumption that rock and fluid are in thermal equilibrium at all times, i.e.,

$$\frac{\partial u_s}{\partial t} = \left(\frac{\partial u_s}{\partial u}\right)_\rho \frac{\partial u}{\partial t} + \left(\frac{\partial u_s}{\partial \rho}\right)_u \frac{\partial \rho}{\partial t} \tag{11}$$

Thus the change in rock energy depends upon both changes in fluid energy and density. This introduces a very important coupling between energy and density equations (see p. 17). Neglecting changes in porosity and rock density, we obtain

$$\frac{\partial(\text{energy/volume})}{\partial t} = \frac{\partial u}{\partial t}\left[\phi\rho + (1-\phi)\rho_s\left\{\left(\frac{\partial u_s}{\partial u}\right)_\rho + \left(\frac{\partial u_s}{\partial \rho}\right)_u \frac{\partial\rho/\partial t}{\partial u/\partial t}\right\}\right] + u\frac{\partial\phi\rho}{\partial t} \tag{12}$$

Substituting for $\partial\phi\rho/\partial t$ from eq. (1) and neglecting the $\frac{\overline{F}}{\rho}\cdot\nabla p$ terms the energy equation (9) becomes

$$\frac{\partial u}{\partial t}\left[\phi\rho + (1-\phi)\rho_s\left\{\left(\frac{\partial u_s}{\partial u}\right)_\rho + \left(\frac{\partial u_s}{\partial \rho}\right)_u \frac{\partial\rho/\partial t}{\partial u/\partial t}\right\}\right] \tag{13}$$

$$= -\nabla\cdot\bar{G} + u\nabla\cdot\bar{F} + (Q_u - uq_\rho)$$

In analogy to section 1 we integrate equation (13) over some volume τ to obtain an equation for volume and surface averages, respectively:

$$\tau\frac{\partial u}{\partial t} = \frac{\sum_{m=1}^{N} A_m(\bar{G}_m - u\bar{F}_m) + \tau(Q - Q')}{\phi\rho + (1-\phi)\rho_s\left\{\left(\frac{\partial u_s}{\partial u}\right)_\rho + \left(\frac{\partial u_s}{\partial \rho}\right)_u \frac{\partial\rho/\partial t}{\partial u/\partial t}\right\}} \tag{14}$$

where Q is the volume averaged energy source term and Q' is the volume average of the product uq_ρ, where q_ρ is the mass source term (see eq. (6)).

3. THE SOURCE TERMS

In reservoir simulation the external sources corresponding to the terms q and Q in eq. (7) and eq. (14) are almost exclusively line sources penetrating (or partially penetrating) one or more of the simulation volume elements.[37] Conceptually, this is no problem, but modeling these sources is difficult for a line source penetrating a volume τ at an arbitrary orientation. At present, SHAFT78 has no special provisions for modeling the boundary conditions at a source. Sources are assumed to be uniformly distributed throughout the volume element in which they reside. In order to obtain good approximations for the large gradients of variables near a source, volume elements must be chosen in a special way. The symmetry of the source has to be taken into account, and volume elements must be "small" in the direction of rapid variations.

An improved technique, which can handle sources accurately without the expense of introducing many additional volume elements, is currently being investigated.

4. DIFFERENCE FORM OF THE EQUATIONS

Consider an arbitrary polyhedron in three-space as shown in Figure 1. Assume that all of the polygonal faces of the polyhedron relate to other polyhedra with one polygonal face in common. For any pair of polyhedra (n,m) we have an interface A_{nm} and a total flux through the interface labeled F_{nm}. The distance from the element location n to the interface with area A_{nm} is d_n, etc. In this general description of finite areas, volumes, and distances between volume elements, no distinction needs to be made between one-, two-, or three-dimensional figures. In the discussions, however, the two-dimensional case is easiest to pictorially describe and will be used in subsequent examples, as shown in Figure 2.

The discussion above has described the spatial discretization which is used in SHAFT78. The time discretization is an explicit-implicit scheme which has been described elsewhere.[2,8] It is basically an approach which solves an implicit equation by iteration for the finite difference in time, but uses an explicit equation to "get started." The integrated finite difference forms of the density and energy equations (eq. (7) and eq. (11)) are, for a volume element V_n

$$\Delta\rho_n = \Delta t \left[\frac{\frac{1}{V_n} \sum_{m=1}^{N} \overline{F_{nm}} A_{nm} + \bar{q}_n}{\phi} \right] \tag{15}$$

$$\Delta u_n = \Delta t \frac{\frac{1}{V_n} \sum_{m=1}^{N} A_{nm} \left(\overline{G_{nm} - u_n F_{nm}}\right) + \left(\overline{Q_n - Q'_n}\right)}{\phi\rho + (1-\phi)\rho_s \left\{ \left(\frac{\partial u_s}{\partial u}\right)_\rho + \left(\frac{\partial u_s}{\partial \rho}\right)_u \frac{\partial\rho/\partial t}{\partial u/\partial t} \right\}} \tag{16}$$

where the bars indicate averages over the time interval Δt.

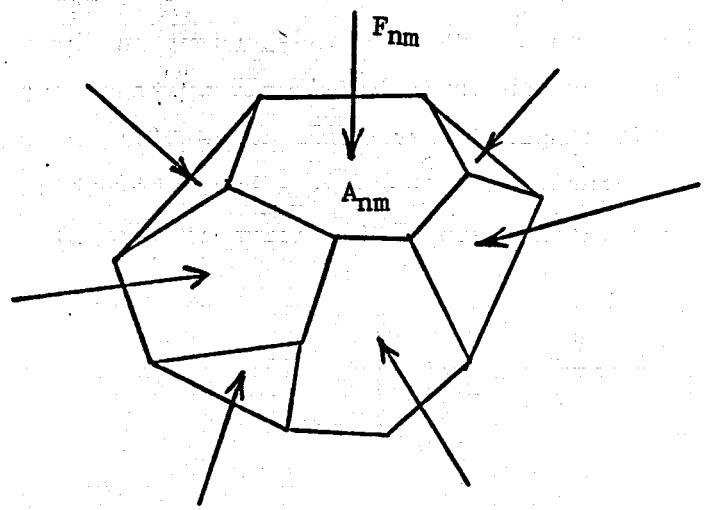

Figure 1. A three-dimensional polyhedron with a polygonal face of area A_{nm} and flux through that area F_{nm} from an adjoining polyhedral volume.

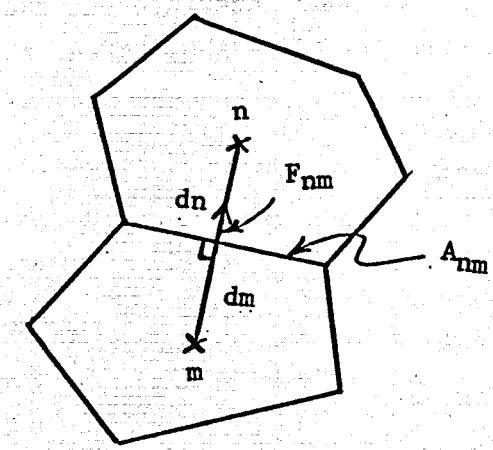

Figure 2. Two-dimensional polygons with interface area A_{nm} and flux F_{nm} at the interface between volume elements n and m.

In eq. (2) the total mass flux was noted to have a vapor component and a liquid component. The two mass flows are independent in the sense that the Darcy equation for each flux can be independently written, but the two phases are coupled through their respective relative permeabilities which are both dependent on the vapor saturation. The fluxes have the finite difference forms for phase α in element n given by (positive if into n)

$$F^{\alpha}_{nm} = K'^{\alpha}_{nm} \left[\frac{p_m - p_n}{d_n + d_m} - \rho^{\alpha}_{nm} \gamma_{nm} g \right] \tag{17}$$

and

$$G_{nm} = K_{nm} \left(\frac{T_m - T_n}{d_n + d_m} \right) + F^{v}_{nm} h^{v}_{nm} + F^{\ell}_{nm} h^{\ell}_{nm} \tag{18}$$

where γ_{nm} is the direction cosine for gravity.

To obtain the interface quantities we assume that permeability can change discontinuously at the element interface but that the fluid parameters must be continuous across the interface. Then K'^{α}_{nm} has the form

$$K'^{\alpha}_{nm} = k^{eff}_{nm} \rho^{\alpha}_{nm} \left(\frac{k^{\alpha}}{\mu^{\alpha}} \right)_{nm} \tag{19}$$

$$k^{eff}_{nm} = \frac{d_n + d_m}{\frac{d_n}{k_n} + \frac{d_m}{k_m}} \tag{20}$$

and ρ^{α}_{nm} is linearly interpolated between elements n and m. The interface mobilities $(k/\mu)_{nm}$ can be either spatially interpolated, or 100% upstream weighted. The latter is necessary for problems with phase fronts. For the thermal conductivity the coefficient is given by

$$K_{nm} = \frac{d_n + d_m}{\frac{d_n}{K_n} + \frac{d_m}{K_m}} \tag{21}$$

where K_n is defined in equation (10).

The specific enthalpy h_{nm} can be upstream weighted (see p. 17) or computed from linearly interpolated internal energy and density. The pressure at the interface is calculated from flux continuity. When $d_n = d_m$ the linearly interpolated interface values are the mean of the values at element m and element n. Note that harmonic weighting of all variables would give incorrect results in two-phase flow problems.

5. BOUNDARY CONDITIONS

In reservoir simulation there are several types of boundary conditions which are sometimes required to simulate a given production history, or geological feature. These boundary conditions are distinct from the models used for external sources. For example, a reservoir pay-section is sometimes bounded by impermeable layers. In this case the boundary condition is a no-flow in the impermeable direction. In SHAFT78 this condition can be imposed by defining flow elements for the pay-section which have no connections in the impermeable direction. In a geothermal reservoir the impermeable boundary in the above example may allow appreciable heat conduction, in which case elements can be used for the impermeable materials and can be given a non-zero coefficient of heat conductivity with zero permeability. In general, all necessary boundary conditions can be satisfied by assigning the appropriate conditions to the connections and elements used in SHAFT78. Obviously, in some cases this can be an inefficient use of computer storage, since the Dirichlet or Neuman conditions[38] could be defined in the IFD solution procedure very simply by supplying the gradient or element value at the boundary as input data. Implementation of this is planned as a future modification.

6. SOLUTION PROCEDURE

The explicit-implicit scheme uses the iteration technique described in ref. 40. Basically it consists of expanding the time average in eq. (15) as

$$\bar{F}_{nm} = F_{nm}\,(t + \theta\Delta t) \tag{22}$$

$$\approx F_{nm}(t) + \theta\Delta t \frac{\partial F_{nm}}{\partial t}$$

$$\approx F_{nm}(t) + \theta \left[\left(\frac{\partial F_{nm}}{\partial \rho_n}\right) \Delta\rho_n + \left(\frac{\partial F_{nm}}{\partial \rho_m}\right) \Delta\rho_m + \left(\frac{\partial F_{nm}}{\partial u_n}\right) \Delta u_n + \left(\frac{\partial F_{nm}}{\partial u_m}\right) \Delta u_m\right]$$

where θ is a parameter between 0 and 1. From eqs. (15) and (22),

$$\Delta\rho_n = (\Delta\rho_n)_{exp} + \theta(\Delta\rho_n)_{imp} \tag{23}$$

The explicit part of $\Delta\rho_n$ can be computed from quantities known at the beginning of a time step, t^k. The implicit part depends upon the changes occurring at the time $t^{k+1} = t^k + \Delta t$.

The solution algorithm computes $(\Delta\rho_n)_{exp}$ first, and then uses the value as zero-order approximation in an iterative scheme to compute the full expression eq. (23). For stability we must have $\theta > 1/2$. SHAFT78 uses a default initial value of $\theta = .57$, which can be modified (increased) automatically using a technique described in ref. 2. The user can prescribe other values for θ. Similar considerations apply to the mixed explicit-implicit solution of eq. (16).

Two additional parameters are used in the solution procedure. First, an upstream weighting parameter, λ, is used to provide values for u_{nm} according to [39]

$$u_{nm} = \lambda u_{up} + (1-\lambda)\, u_{down} \tag{24}$$

and

$$\Delta u_{nm} = \Delta u_{up} \tag{25}$$

where "up" and "down" refers to the variable values at element n or m, depending upon which is upstream or downstream, respectively. The default value is $\lambda = 0.667$. This upstream weighting is only applied to the energy equation. For the case where a phase which is present upstream is not present downstream we take $\lambda = 1$.

The second parameter referred to is the acceleration parameter introduced by Evans[40] to accelerate the convergence of the iteration procedure.

7. THE EQUILIBRATION BETWEEN ROCK AND FLUID

The energy and density equations are coupled in two distinct ways, namely across elements and within elements. The coupling across elements arises because pressure gradients, which in turn drive mass and energy flow, depend upon both fluid energy and density. Therefore, a change in fluid density modifies the flow of energy (and vice versa). This coupling usually is not very strong. It is treated in SHAFT78 (as in earlier versions of SHAFT) by solving first the energy equation, then the density equation, and restricting the maximum permissible changes in important variables (such as pressure, temperature, and vapor saturation) to a small percentage (1% default).

The coupling between the energy and density equations within elements arises from the presence of the rock matrix. Its importance was not sufficiently realized in the older work on SHAFT, and SHAFT78 contains a much improved treatment.

The cause of the coupling within elements is due to the dependence of fluid temperature on density. During a density step the fluid temperature changes. Subsequently, energy has to flow between rock and fluid in order to maintain local thermal equilibrium. In eq. (16) the coupling is represented by the term

$$R = \frac{\partial\rho/\partial t}{\partial u/\partial t} \tag{26}$$

which gives the ratio of density to energy change. This term is needed as input for computing the energy equation, but we know it only after the energy and density equations are solved in a completely self-consistent way. We are presently investigating whether in some cases the coupling between energy and density equations can be so strong that an interative solution technique is needed. SHAFT78 uses a simpler and faster approach, which has been found to yield satisfactory accuracy for a variety of test problems. Namely, we make a good guess for R, and correct for inaccuracies after the energy and density equations have been solved.

Generally speaking, we take as our estimate for R the value computed in the last time-step in each volume element. Such an extrapolation is not applicable in all cases. Usually the terms referring to the rock in the denominator of eq. (16) are much larger than the fluid term $\phi\rho$. Occasionally, however the estimated value of R is such that the two large rock terms approximately cancel each other. This yields a small denominator with a large relative numerical inaccuracy. Physically, this occurs in situations where the rock energy changes very little, i.e., the process is estimated to be nearly isothermal. SHAFT78 checks whether a substantial cancellation between the rock terms occurs, and if so it replaces the extrapolated value of R with the isothermal limit

$$R_{iso} = - \frac{(\partial u_s/\partial u)_\rho}{(\partial u_s/\partial \rho)_u} \tag{27}$$

corresponding to $\partial u_s/\partial t = 0$ (cf. eq. (11)). The isothermal limit for R is also used for the zeroth time-step ($t = 10^{-12}$ sec), where no values are available to be extrapolated from.

After the energy step, then, rock and fluid are equilibrated at a temperature $T_i = T(u+\Delta u, \rho+R\cdot\Delta u)$. However, the subsequent density step will usually yield a density change $\Delta\rho \neq R\Delta u$, leaving the fluid at the somewhat different temperature $T_f = T(u+\Delta u, \rho+\Delta\rho)$. To correct for this we perform an equilibration of rock and fluid temperatures within each element after a density step, which leaves both at the same temperature $T(u+\Delta u', \rho+\Delta\rho)$.

Noting that the equilibration must not change the energy per volume in each element, we have

$$\frac{\text{energy}}{\text{volume}} = \phi(\rho+\Delta\rho)(u+\Delta u) + (1-\phi)\rho_s\, u_s(u+\Delta u, \rho+R\Delta u)$$

(before equilibration)

$$= \phi(\rho+\Delta\rho)(u+\Delta u') + (1-\phi)\rho_s\, u_s(u+\Delta u', \rho+\Delta\rho)$$

(after equilibration) (28)

Expanding $u_s(u,\rho)$ to first order we find

$$\Delta u' = \Delta u + \frac{(1-\phi)\rho_s\,(\partial u_s/\partial\rho)_u\,(R\Delta u-\Delta\rho)}{\phi(\rho+\Delta\rho) + (1-\phi)\rho_s\,(\partial u_s/\partial u)_\rho} \tag{29}$$

The user can choose to perform a rock/fluid equilibration just once at the end of a density step, or after each density sub-cycle. While the former option requires less computing time the latter option is more accurate and is strongly recommended.

8. THE EQUATION OF STATE

The basic concept of SHAFT78 involves setting up and solving coupled equations for the flow of mass and energy directly in terms of fluid specific internal energy u and density ρ. An advantage of this formulation is that the same two variables (u, ρ) can be used to describe the state of water even if phase transitions occur. In order to implement this concept, the material properties - in particular water temperature T, pressure p, and vapor saturation S - must be computed as functions of u and ρ (see Fig. 3). We achieve this by tabulating T, p, and S over a rectangular grid $(u_n, \rho_m)(n=1, \ldots, N; m=1, \ldots, M)$. Parameters for the desired values of (u, ρ) are then "looked up" through bivariate interpolation.

The tabulation essentially involves a straightforward inversion of steam tables which give u and ρ as functions of (T, p, S). The inversion is done by interpolation in two steps (Fig. 4). The following problems, however, arise:

(i) Interpolation must not be made across the saturation line, at which T, p, and S change slope.

(ii) Due to the minimal compressibility of liquid water, the entire liquid region is very much condensed in the (u, ρ) diagram. How can it then be tabulated?

Point (i) is solved in the following manner:

(a) Table construction: First a check is made to see whether the saturation line intersects a given interpolation interval. If so, one of the two end points is discarded and replaced with this intersection.

(b) Table "look-up": For bivariate interpolation at least three reference points are needed. When table points (u_n, ρ_m) are chosen at random, situations are encountered near the saturation line where fewer than

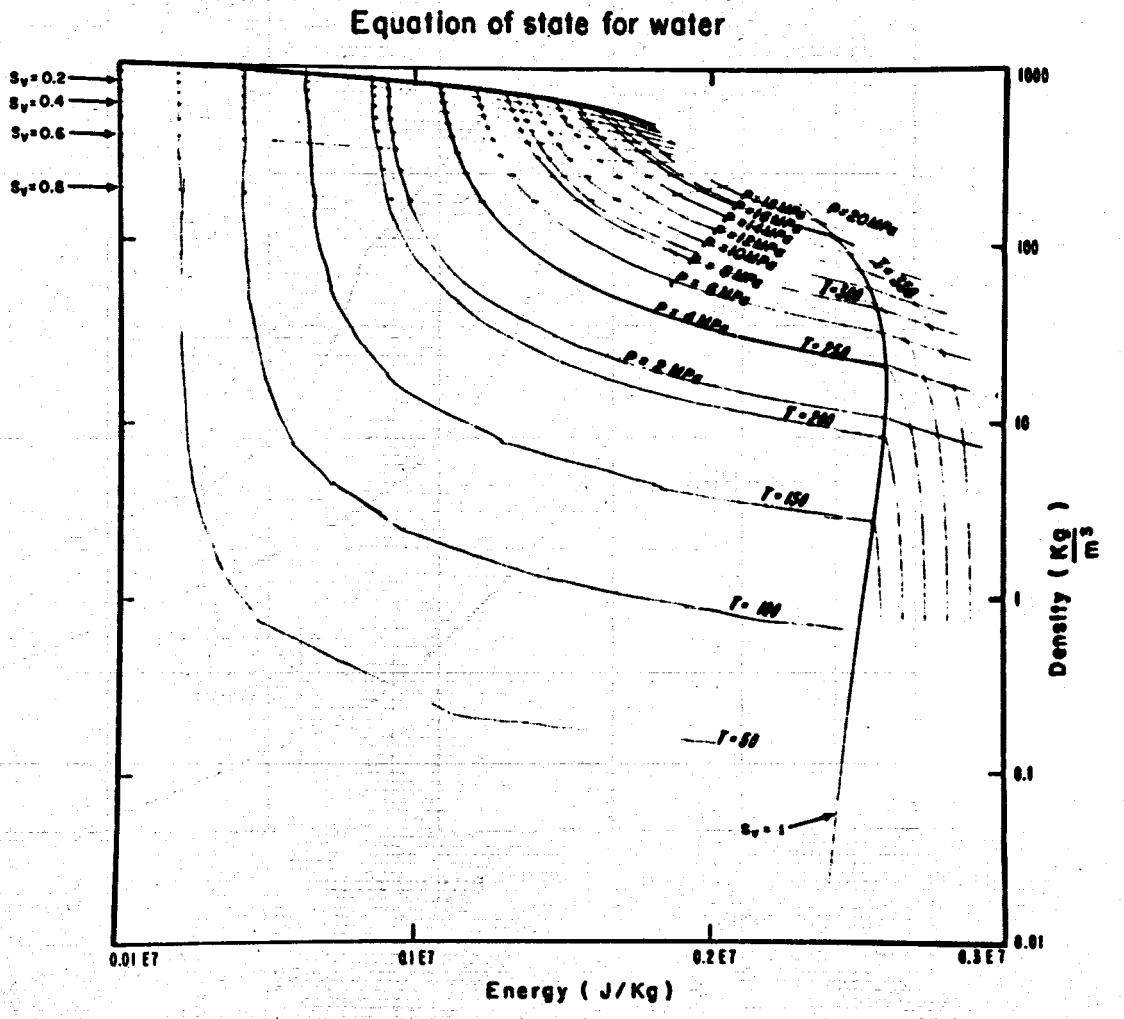

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Figure 3. The saturation curve of water in terms of internal energy and density. A few isobars and isotherms are also shown.

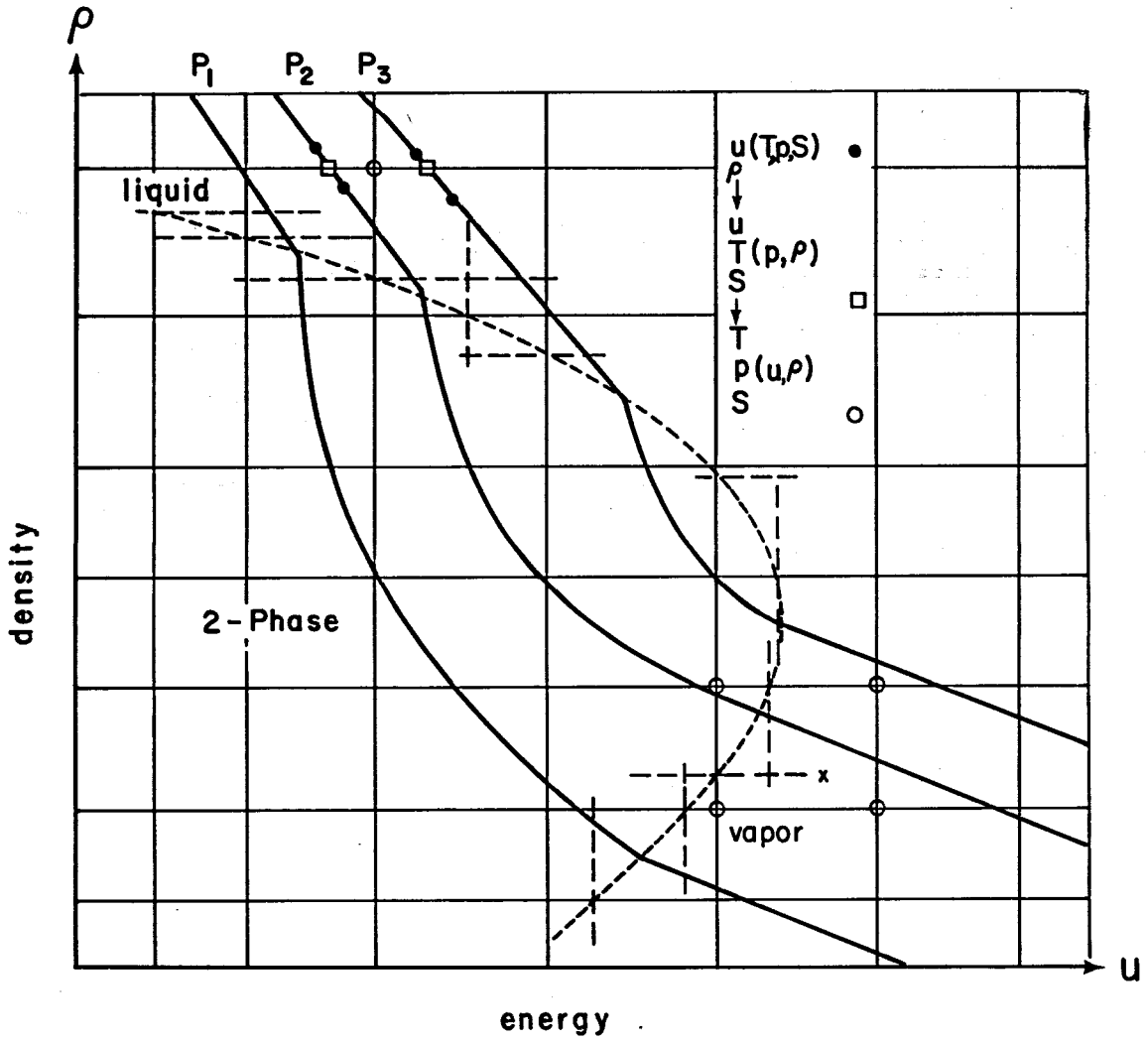

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Figure 4. A pictorial description of the numerical conversion of the equation of state of water to the u, ρ form.

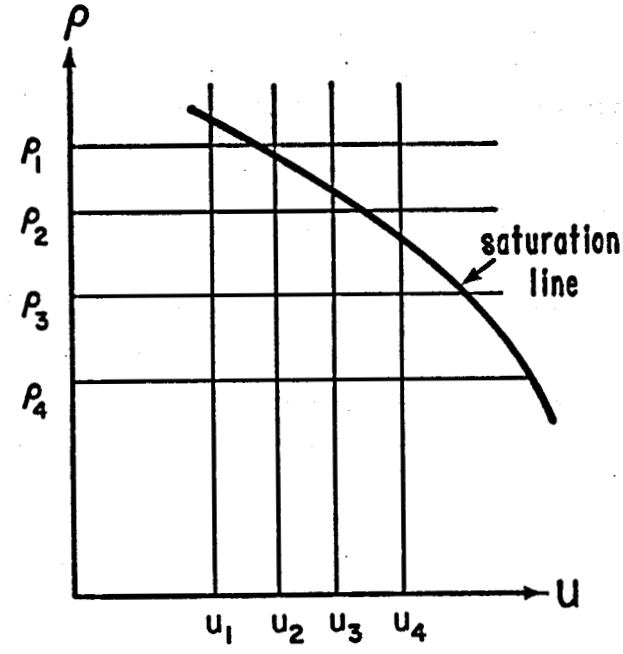
three points of a quadrangle lie on either side of the saturation line (Fig. 5(a)). In order to avoid this situation, all the intersections of basis energies u_n and densities ρ_m with the saturation line are actually tabulated (Fig. 5(b)).

Point (ii) is solved through "triangulation" of the liquid region (Fig. 6). A series of energy values, u_n and density values ρ_m is determined such that the entire liquid region between the saturation line and the highest isobar as provided by the input data is covered with triangles. By tabulating (T, p, S) at the corners of the triangles, an interpolation can be made for any point (u, ρ) within the liquid region.

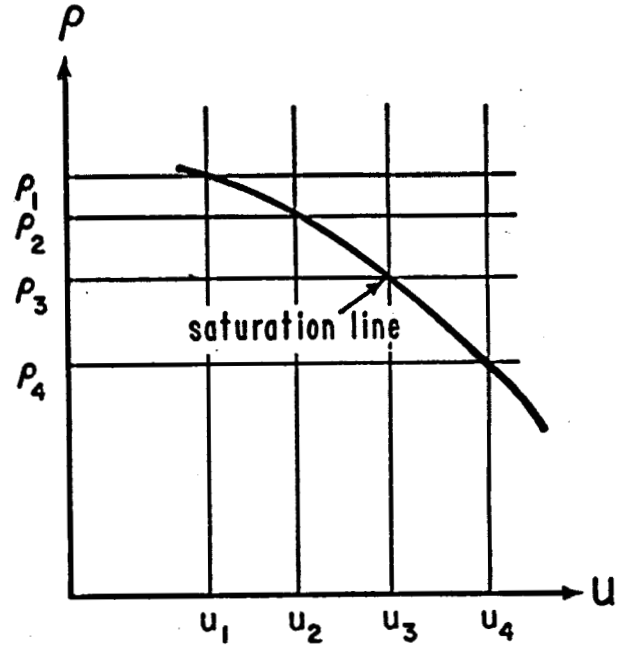
The question remains as to the accuracy which can be obtained for the interpolated values of T, p, and S as functions of (u, ρ). As an example a table was constructed using 55 energy and 59 density values which covers most of the equation of state between temperatures of 100-350°C, pressures of 1-220 bars, and which extends throughout the liquid, two-phase, and vapor regions. With input data provided from the Keenan/Keyes steam table,[41], the following accuracy was obtained:

$\Delta T/T$	<.02%	for most of two-phase region
	<2%	for most of one-phase regions
	≤1%	for small densities (ρ<2kg/m^3)
$\Delta p/p$	<1%	for most of two-phase and vapor regions
	<5%	for small pressures (p < 5 bars)
	≈4-6%	in liquid region
$\Delta S/S$	<.01%	everywhere except for very small S

The large pressure errors in the liquid region are due mainly to the minimal compressibility of liquid water, since very small inaccuracies in specific volume $v = 1/\rho$ as given by steam tables translate into large pressure errors. For example, using the Keenan/Keyes table and assuming an accuracy of ±1 in the seventh digit, we have in a typical case



(a)



(b)

XBL-791-7302

Figure 5. The two types of cases to deal with in the table lookup using bivariate interpolation.

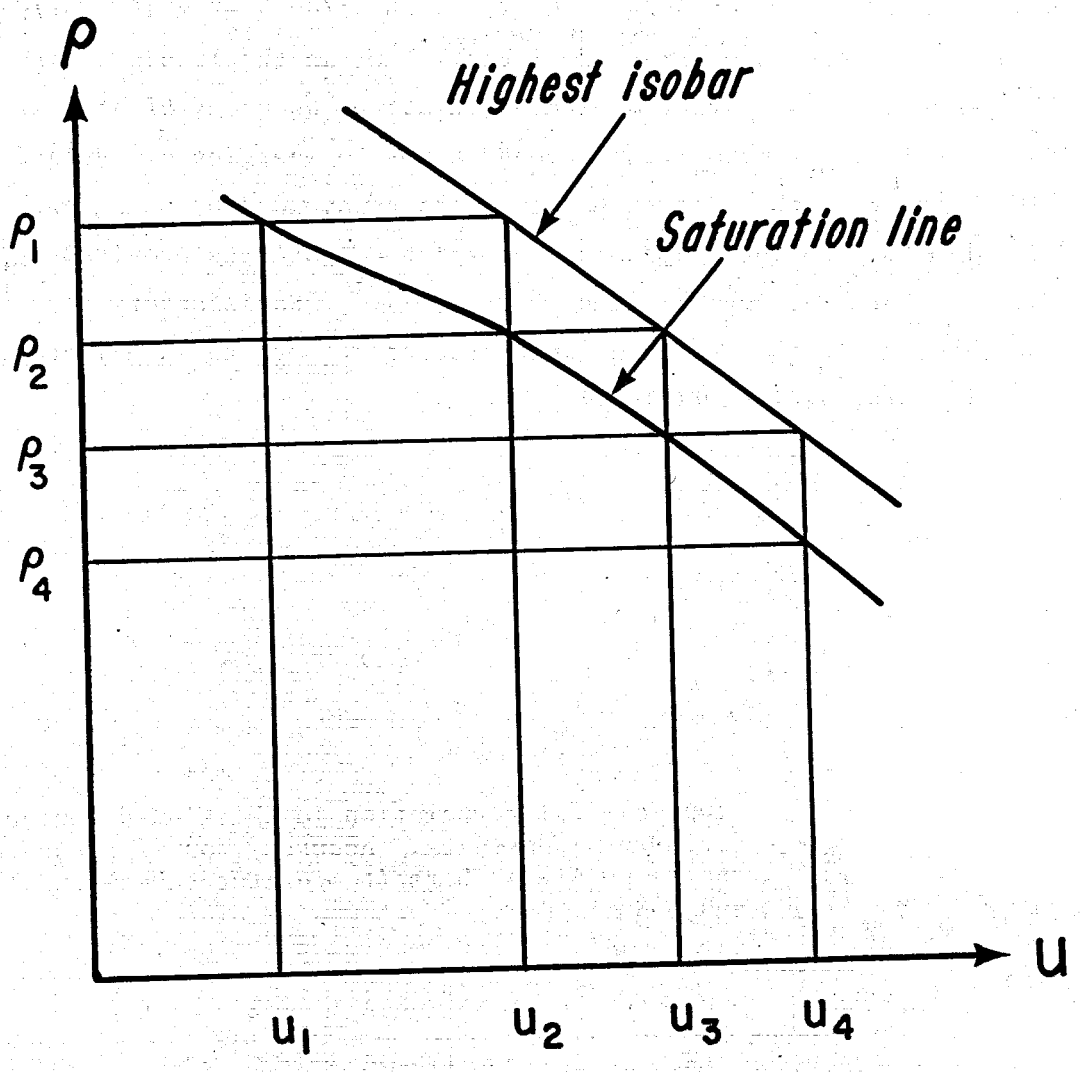


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Figure 6. The method of triangulation used in SHAFT78 for the tabulation of the liquid region.

$\Delta v = \pm 1 \times 10^{-7} m^3/kg$ or $\Delta v/v = 10^{-2}\%$. With $dp/dv \approx -2 \times 10^7$ bar/$\frac{m^3}{kg}$ this results in a pressure error of ±2 bar. Thus in the liquid region, we require reference point densities ρ with a (relative) accuracy of at least six digits. Somewhat less stringent requirements apply to energies u. Relative errors with respect to the reference points can be avoided by using the analytical formulas for the thermodynamic properties of water as provided by the International Formulation Committee (IFC, 1967).[14] Satisfactory results are then obtained, while the remaining inaccuracies reflect the non-linearity of the function $p(u, \rho)$ (see Table 1).

Table 1. Accuracy of interpolation in the liquid region for different input data. Accuracy for 12 pressures in the range 41-142 bars (temperatures between 217°C and 338°C).

	A	B	C
$\overline{\Delta P}$ (bar)	3.48	1.29	.62
ΔT (°C)	.1	.01	.01

(A) Input data from Keenan/Keyes and Vukalovitch tables.

(B) Input data from 1967 ASME - Steam Table Equation

(C) As (B) with double triangulation of liquid region.

HOW TO USE SHAFT78

1. GENERAL PROCEDURES

If the program has not been run before, the equation of state must first be generated. The equation of state file (or tape) only needs to be generated once. The user begins by running the program WATER. WATER computes the 1967-IFC-steam table equations.

The user specifies a lowest and a highest temperature (T_l, T_h) and a number of temperatures N_T. The equation of state for water is then tabulated for N_T evenly spaced temperatures in the interval (T_l, T_h). The user may also specify an explicit set of temperature values which he wishes to tabulate.

What was said for temperatures also holds for pressure, except that the N_p pressures in the interval (P_l, P_h) are spaced such that subsequent values differ by the same ratio rather than by the same absolute difference.

WATER will then compute a number of isotherms. On each isotherm it tabulates the values of energy and density for all pressures, including the saturation pressure P_s for that temperature (P_s is automatically computed in WATER).

After the WATER calculation has been completed, the user runs PROPER, which generates a table of fluid properties suitable for the program CYCN. CYCN will then perform an actual reservoir simulation.

PROPER tabulates, for a set of energies u and densities ρ, the following ten parameters:

Parameter	Meaning	
1	Temperature in ^oC	
2	Pressure in N/m^2	
3	Vapor saturation	
4	Heat conductivity in J/m sec^oC	
5	Rel. permeability/viscosity (for liquid)	in m^2/N sec
6	Rel. permeability/viscosity (for steam)	in m^2/N sec
7	Density in kg/m^3 (for liquid)	
8	Density in kg/m^3 (for steam)	
9	Specific internal energy in J/kg (for liquid)	
10	Specific internal energy in J/kg (for steam)	

The input to PROPER consists of a tape written by WATER and data cards. Some of them specify fluid data (viscosity, heat conductivity, relative permeability); others specify the tabulation to be performed by PROPER. PROPER operates in different modes, which the user specifies through the data cards. These options are extensively documented by the COMMENT cards in the MAIN program of PROPER. Some of the more important options include:

(i) Input/output tapes can be formatted or binary.

(ii) Different units can be employed (with SI-units as default).

(iii) Relative permeabilities can or cannot be included into p_5, p_6. In the latter case they have to be handled in CYCN (see below).

(iv) A set of energy values can be computed so as to "triangulate" the entire liquid region. Triangulation may be performed up to six times for increased accuracy of resulting table.

(v) Saturation densities or energies can be computed and appended to whatever list of densities the user supplies.

If the user intends to study problems which involve subcooled water it is essential that (iv) and (v) be executed. In order to decide upon starting energies and the number of triangulation steps it is convenient to have advance knowledge of the energy values that the triangulation will produce.

This can be determined with the little utility program LIQPO, which performs up to 60 triangulation steps beginning with an initial energy value specified by the user (the initial energy value, however, must not be smaller than the smallest saturation energy provided from the WATER-run).

After a fluid table has been generated by PROPER, the user may perform reservoir simulations with CYCN.

CYCN can be used to check the accuracy with which interpolations in the fluid table are made. The user specifies the initial conditions for elements in terms of (T,p) (one-phase) or (T,S) (two-phase). CYCN then determines internal energies and densities for these initial conditions and, upon completion of the zero-cycle, will find values of (T, p, S) from the fluid table by interpolating energies and densities. If the difference from the initially supplied values is greater than the user can tolerate, the user must supply a more accurate fluid table by using more closely spaced u, ρ-values.

CYCN operates in different modes, which the user selects by means of data cards. The more important choices include:

- Fluid table is read either as binary or formatted.
- Relative permeabilities are/are not included in fluid table.
- Rock/fluid equilibration can be made just once after a density step or after each density subcycle.

In what follows, we give some general rules that the user should observe in running CYCN.

- Begin a new problem with rather stringent accuracy requirements (e.g., use default values for EVARY, DVARY, PCT, RATEQ: see p. 50); run only a small number of cycles, 10-20 say, and generate full printout for each cycle.

- Determine whether the time steps are controlled by energy changes or by density changes, i.e., is $\Delta u_{max} \approx$ EVARY or is $\Delta \rho_{max} \approx$ DVARY? If the maximum changes are considerably smaller than both EVARY and DVARY, this indicates that time steps are controlled by implicit corrections (cf. eq. (23)).

- Set EVARY and DVARY such that the number of density cycles per energy cycle does not get too large ($\lesssim 5$ is a good value).

- In problems with phase transitions always use the option MOP(13) = 2 (see p. 50); this ensures that rock and fluid are equilibrated after each density subcycle.

- Compare the energy changes Δu from the energy equation with those after the density equation, which contain corrections from the rock/fluid-equilibration. If the latter differ by more than a few thousand Joules/kg from the former reduce the energy time steps.

- Problems involving one-phase liquid water need very tight controls on density steps, because of the large pressure response of liquid water upon small density changes. A DVARY of .1 would permit pressure changes of approximately 1 bar per time step.

- Run longer simulations in segments and restart the problem repeatedly. The optimum balance between accuracy and efficiency of the calculation may change during the simulation. E.g., a system which initially only has liquid water may become two-phase during the simulation, requiring much less stringent accuracy controls.

SUMMARY

The reservoir simulator SHAFT78 is made up of several working programs which can be used to study both simple idealized problems providing insight into the physical phenomena of two-phase flow, and to simulate the full-scale behavior of real producing geothermal reservoirs. All of the two-phase phenomena associated with evaporation, condensation, production, and injection can be realized. In Table 2 the equations in SHAFT78 have been gathered together and are presented in the differential-integral form appropriate for the IFD solution method. In Table 3 the functional dependence of the material parameters is presented.

Table 2. A summary of the basic equations used in SHAFT78 for two-phase flow in porous media.

$$\frac{\partial \phi \rho}{\partial t} = \frac{\int_\tau \frac{\partial \phi' \rho'}{\partial t} dv}{\tau}$$

$$F_m = \frac{\int_{F_m} \bar{F} \cdot \bar{n} \, da}{A_m}$$

$$\frac{\partial \phi \rho}{\partial t} = \sum_{m=1}^{N} \frac{F_m A_m}{\tau} + q$$

$$\rho = S\rho_v + (1-S)\rho_\ell$$

$$\bar{F} = \bar{F}_v + \bar{F}_\ell$$

$$\bar{F}_v = -\frac{\rho_v k \, k_v}{\mu_v} (\nabla p - \rho_v \bar{g})$$

$$\bar{F}_\ell = -\frac{\rho_\ell k \, k_\ell}{\mu_\ell} (\nabla p - \rho_\ell \bar{g})$$

$$G_m = -K_m \nabla T_m + F_{vm} h_{vm} + F_{\ell m} h_{\ell m}$$

$$\phi \rho \frac{\partial u}{\partial t} + \rho_s \frac{\partial (1-\phi) u_s}{\partial t} = \sum_{m=1}^{N} \frac{(G_m - u F_m) A_m}{\tau} + (Q - \tilde{Q})$$

$$q = \frac{\int_\tau q_\rho' \, dv}{\tau}, \quad \tilde{Q} = \frac{\int_\tau u q_\rho' \, dv}{\tau}$$

Table 3. A summary of the functional dependence of the parameters used in the SHAFT78 model.

$\bar{r} = \bar{r}(x, y, z)$

$\rho = \rho(\bar{r}, t)$

$u = u(\bar{r}, t)$

$S = S(\rho, u)$ (Equation of State)

$p = p(\rho, u)$

$T = T(\rho, u)$

$h = h(\rho, u)$

$\phi = \phi(\bar{r})$

$k = k(\bar{r})$

$K = K(\bar{r})$

$C_s = C_s(\bar{r})$

$\rho_s = \rho_s(\bar{r})$

$k_v = k_v(S)$, $k_\ell = k_\ell(S)$

$\mu_v = \mu_v(\rho, u)$, $\mu_\ell = \mu_\ell(\rho, u)$

$q = q(\bar{r}, t)$, $Q = Q(\bar{r}, t)$

Although the program has been developed by several people in the past, the current version is quite new and the authors take full responsibility for the content. No stability or error analysis is presented here, although some studies were carried out by Lasseter on a very early version of the program.[3,12] However, in all cases we have studied, the solution procedure converges quickly and no unstable oscillations have been observed. No formal study of the numerical dispersion associated with the solution method has been made, although this type of study will also be carried out in the future.

Good judgment is required to choose (define) problems that are physically reasonable, and to provide correct input in the formats described below. The sample problems below provide some examples of how the various parameters for time-step and accuracy control may be chosen. Otherwose, the program and the various pre- and post-processors provide tools for two-phase studies which are relatively free of "hidden" assumptions and restrictions.

We could write another section on improvements planned for future versions of SHAFT78. However, this is not the proper place to discuss programming changes, theoretical studies, or why these future developments are necessary. However, it is of importance to alert potential users that this version of the simulator is still under development, and new versions will be made available periodically.

In Figure 7 the generalized block diagram of the program SHAFT78 and its associated pre- and post-processor programs is summarized. In Figure 8 a generalized flow diagram of the program CYCN is presented.

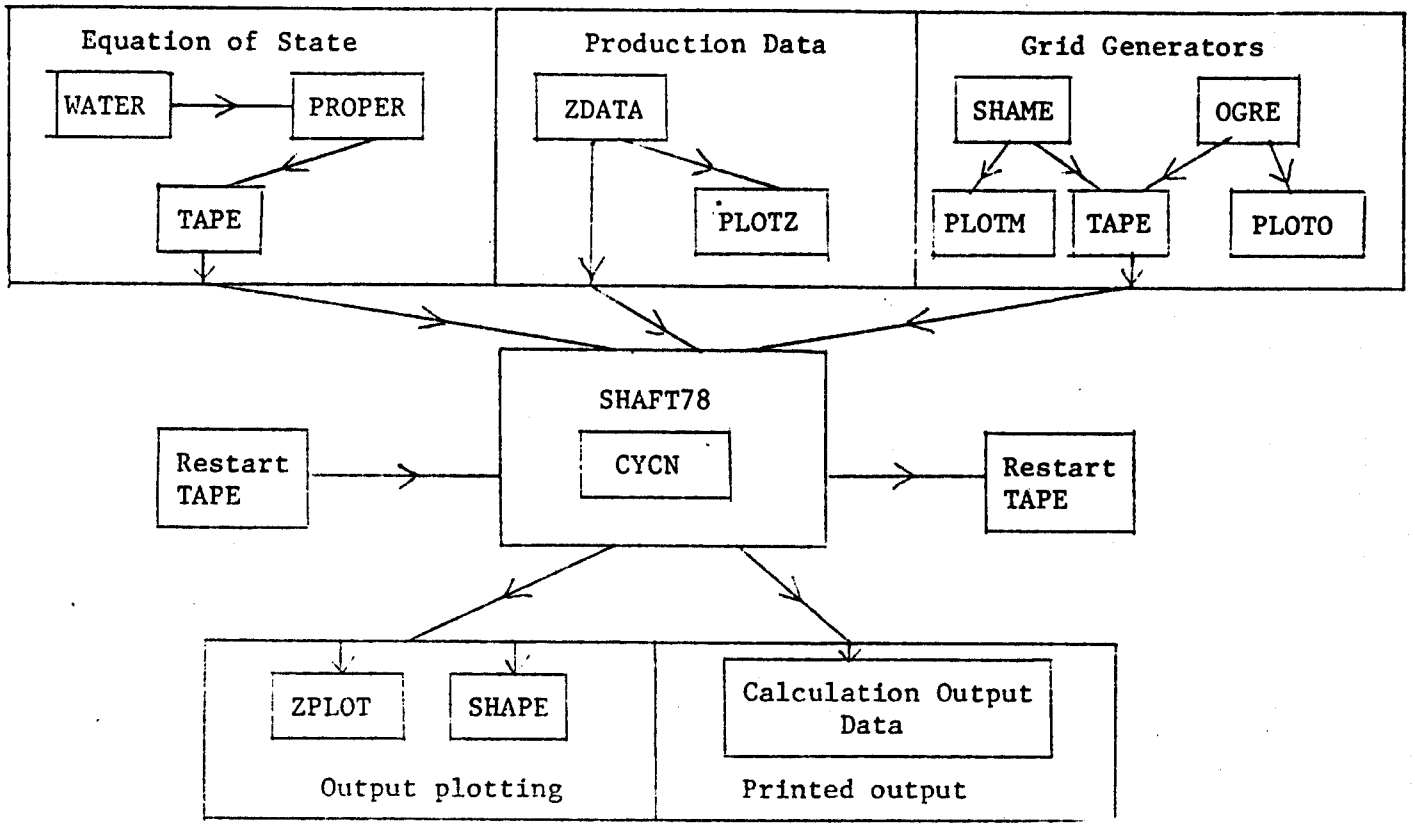


Figure 7. A general block diagram of the computer program SHAFT78 and the supporting programs used for input and output functions.

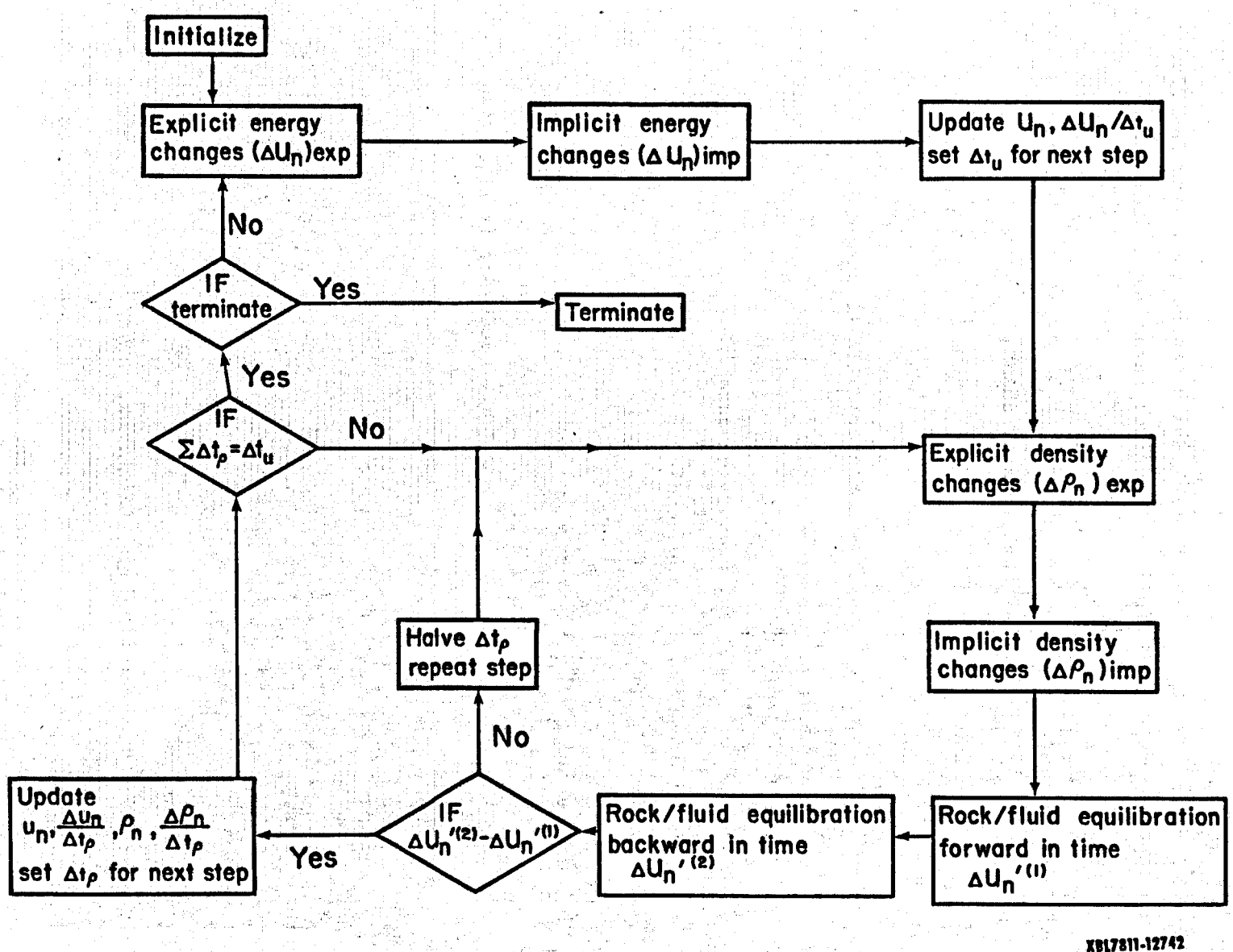


Figure 8. Flow diagram of CYCN.

2. THE INPUT FOR SHAME

SHAME is a 2-D mesh generating program which can be used with SHAFT78. To use the mesh generator, the following control cards should be used:

```
                    < JOB CARD >
                    SFL,70000.
                    FETCHPS, CHRISPUBL, SHAME, SHAME.
                    FTN4, I=SHAME
                    LGO

                    EOR

                       TITLE CARD (79-character title to output)
                    *MESH
                    < General parameter card >                    See next page
                    Region parameter card
                    Region corner card
1 set per region    Side radii and proportion card                See next page
                    I side proportion card*
                    J side proportion card*
                    INTER

1 per connection     Interregion connection crd                   See next page
                  .
                  .
                  .                              *only necessary if EL, EJ =0
                  Blank card
                  ENDME
                  *SPLIT

                  EOR


                  EOF
```

General Parameter Card

Containing the following: LABEL, NR, NSYMN

Format:	(A5, 15, 5X, A5)
LABEL	5-character word
NR	Number of regions constituting the mesh
NSYM	Should be CYLIN if cylindrical mesh. If anything else, rectangular symmetry is assumed.

Region Parameter Card

Contains the following: IDENT, MAT, FLOW, IMAX, JMAX, RINT

Format:	(A1, 4X, A5, 5X, A5, 2X, 3, 2X, I3, E10.3)
IDENT	1-character region code number
MAT	Name of material in which current region is imbedded
FLOW	Fluid flow index - if equal to NOFLO this and all subsequent regions are considered to be no-fluid-flow regions. If equal to anything else, the region is considered normal.
IMAX	Number of nodes along sides 1 and 3
JMAX	Number of nodes along sides 2 and 4
RINT	Uniform heat transfer resistance at interfaces between elements (msec ^{o}C/J)

Region Corners Coordinates Card

Format: (8E10.3)
(XC(N), YC(N), N=1,4)

where XC and YC denote the coordinates along the horizontal and vertical directions, respectively, of the four region corners listed counter-clockwise, starting at the corner defined by sides number 4 and 1.

Side Radii and Proportion Indices Card

Format:	(6E10.3) (RS(N), N=1,4), EI, EJ
RS(N)	=0.0 if side number N is straight, otherwise equal to the radius of the arc of circle, positive if convex toward the inside of the region, negative otherwise.
EI	Proportion indices defining how the elements are distributed along the two I-sides of the region. If zero, a set of IMAX-1 values is to be fed in, each being proportional to the length of the corresponding interval along both sides. If positive, the lengths of the intervals are a geometric series with ratio EI. If negative, the lengths of the intervals are in a geometric series with ratio -EI, beginning with the far end of the first I-side.
EJ	Proportion indices for the J-sides.

I-side Proportion Cards (only if EI is zero)

A set of IMAX-1 values proportional to the lengths of the intervals along the sides is entered with Format (8E10.3).

J-side Proportion Cards (only if EJ is zero)

A set of IMAX-1 values proportional to the lengths of the intervals along the sides is entered with Format (8E10.3).

Interregion Connection Parameter Card

Format:	(2(A2, 2X, I1, 5X), I5, 5X, 3E10.3)
	IDR1, NS1, IDR2, NS2, ISO, DEL1, DEL2, RINT
IDR1	Code number of one of the two connected regions
NS1	Number of the connected side
IDR2	
NS2	Same as above, for the other of the two connected regions.
ISO	Anisotropy direction index to be assigned to every elementary connection if the indices as defined in both regions differ.
DEL1	If both are zero, the connection distances are determined
DEL2	by the geometry of the connection. If any one is nonzero, the corresponding value is uniformly assigned to the distance from the center of every side-element in the corresponding region to the side commonto both regions.
RINT	Uniform heat transfer resistance (msec ^{o}C/J) along the connection.

Card #5 is repeated for each of the interregion connections.

3. THE INPUT FORMAT FOR OGRE

The program OGRE* requires, as input, the x, y, z coordinates of the discrete element locations being used to specify a particular reservoir's geological system. From the list of element locations, the program finds the midpoints of lines joining adjacent elements. At each midpoint the program constructs a perpendicular plane. The intersections of the planes in the three-space define an irregular (in general) n-sided polyhedron around each element center. In two-dimensions the program produces a unique "tiling" which has polygons with all faces perpendicular to the lines joining adjacent elements. The areas between adjoining elements and the volume of each element are calculated. The end result, in general, is a three-dimensional grid of n-sided polyhedra for which the element locations, the connecting (flow) areas, and the element volumes are all known. Since the grids are finite the program must have information about the boundary of

* OGRE was developed by Oleh Weres of LBL.

the grid.This is supplied as a set of "dummy" elements which are used only to limit the extent of the grid and are not intended for use in the reservoir simulation.

The program currently allows definition of up to 800 elements and 4000 interface areas. Each element can be assigned an arbitrary material identifier. The program uses only small core memory on the CDC 7600.

Input for OGRE

Card 1 (8A10)

IPRINT	=	80 column title (label) card

Card 2 (615)

NNR	=	the number of elements in the grid
NNT	=	the total number of elements including dummy elements
MRANK	=	the total number of elements to be ranked
MRMAX	=	approximate number of neighbors to be tested for connection
MRMAXS	=	approximate number of neighbors to be tested for connection within a single material group
NGIVEN	=	number of connections supplied as input

Default values for Card 2

MRANK	=	150
MRMAX	=	36
MRMAXS	=	11
NGIVEN	=	0

The program uses these values if the corresponding field is blank.

Card 3 (4E10.3)

EPSBC	
EPSYZ	Parameters for various geometric tests.
EPSIN	Set by default. Leave this card blank.
EPSI	

Card 4 (4E10.3)

THETA PHI PSI	Internal rotation angles. Set by default to nonzero values.
ANGC	If ANGC ≠ 0, the above variables may be set equal to zero.

For default leave this card blank.

Card 5 to NGIVEN + 5 (A5, 5X, A5)

Identifiers of pairs of elements whose connections are to be included in the calculation (above and beyond those which the program finds itself). Not present when NGIVEN = 0. These pairs of specified elements allow the user to insert connections as desired.

Card NGIVEN + 5 (A5, 5X, 3E20.12, 5X, A5)

.	
.	
.	
NGIVEN + NNR + 5	Identifier, x, y, and z location coordinates and material identifier
.	
.	
.	
NGIVEN + NNT + 5	For each of NNR real elements, followed by same for each of NNT-NNR dummy elements.

The large number of significant figures is essential if input cards have been prepared by another program, but they are usually unnecessary otherwise.

The purpose for having a large number of significant digits is to keep I/O associated truncation from causing "exactly" (i.e., to within computational roundoff error) parallel planes to be "almost parallel" and thereby wreaking havoc with the calculation. Manually prepared input data is not subject to this and may be provided in any E20.n or F20.m format desired.

Discussion of OGRE Printed Output

EPSBC, etc., are threshold values used to test whether or not various quantities are equal to zero. These quantities arise in tests to determine the identity of points, the parallelness of lines, the verticality of planes, etc. A threshold value is necessary because of roundoff error accumulation. The default values are appropriate for a problem with typical element separation on the order of 100 meters with all I/O and calculations in meters. Much smaller grid dimensions might require smaller test values and vice-versa.

The three angles THETA, PHI, PSI define an internal coordinate system which is actually employed in the calculation. The choice of default values is arbitrary, although the values employed probably should not be much smaller than the default values. The default values are used when zeros or blank fields are inputted, and the value of ANGC is also zero. Setting ANGC not equal to zero will allow zero values as input for the three angles, should this be desired.

The conversion to the rotated internal coordinates is done at the time of input, and all coordinates ae converted to the external coordinate system just prior to output. Thus, values in the internal coordinate system would only be seen in the context of an error message and source code controlled dump.

The purpose of this coordinate change is to eliminate exactly vertical planes and certain similar special elements from the mesh being calculated because exactly vertical planes would cause non-physical numerical divergences in certain of the algebraic manipulations.

The values of EPSBC, etc., and the rotation angles are best left alone unless trouble develops. Trouble is evidenced by the appearance of descriptive error messages ("TROUBLE-A THREE ENDED LINE," etc.) along with appropriate internal data values.

An oddball vertical plane or something similar caused by an unfortunate choice for the rotation angles will cause one or a few error-messages to be generated. A poor choice for one or more of the threshold test values will usually cause 100 (the maximum number allowed) error messages to be generated. Either case can be dealt with by changing the appropriate values. In the former case, a completely arbitrary change should suffice.

There have been no program-generated error messages since the code was debugged and the present default values adopted, but the possibility remains. The code is designed to "skip over" any difficulty that it recognizes during execution and complete the rest of the mesh the best it can. The list of inputted elements is self-explanatory.

The following page is generated during the actual calculation, which is performed element by element. K is the internal identifier of each element. It is simply the ordinal position within the input deck of elements. MMR is the total number of neighboring elements which the program identified as possible connected neighbors. (Neighbors externally supplied under the NGIVEN option and not redundant to those identified by the program are included above and beyond MMR). NMAT is the number of material designation groups to which the MMR neighbors belong. NMS and IMS are the number of neighbors falling into the first material group identified, and the corresponding material identifier. In this example, all elements belong to the material (blank). If more than one material group is present, the (NMS, IMS) field is repeated as many times as necessary. In all cases, MMR $\leq$ MRMAX and NMS $\leq$ MRMAXS. This is the significance of these input parameters. When more than one material group is present, usually MMR $\geq$ NMS.

Total Number of Geometric Trouble Flags is the number of times the program noted difficulties and acted upon them. Each instance results in the printing of an error message up to a maximum of 100 messages.

In Output Node List, IMAT is the material identifier for each element. NC is the number of neighboring elements that it is connected to, and a list of the connected elements follows.

The Total Number of Connections (NCT within the program) is the total number of connections given or identified, including those between real elements and dummy elements. The Number of Interelement Connections (NCF) does not include those between elements and dummy elements.

IFACE is the internal numerical identifier for each connection generated and IA, IB are the numerical identifiers for the connected elements. NODEA and NODEB are the element names. DELT is the perpendicular (i.e., shortest) distance from either of the two element centers to the plane interface between them. (The two distances are always equal. Note that the line between the two nodes need not actually pass through the polygonal interface between them). The values A through D are the coefficients for the equation of the interface plane written in the form

$$Ax + By + Cz = D \tag{30}$$

User Defined File Outputs

There are three user defined output files required. TAPE4 is written by the main program, OGRE, and TAPE5 and TAPE6 are written by the subroutine ISECT. TAPE4 is the SHAFT78 compatible output file. The first record gives the number of elements twice (NNR) followed by the total number of connections. The next NNR records give the name, material identifier, and volume for each of the elements. Note that OGRE uses the material identifier as a computation control input variable (more on this below). In general, this will not be the physical material identifier used by SHAFT78. Therefore, IMAT might have to be changed between running OGRE and running SHAFT78. The last NCT records describe the connections.

The values NA and NB are the elements which are connected. ISI is 1, 2, or 3 depending on which one of A, B, and C has the greatest absolute value. ZR is C of the printed output. It is required by SHAFT78 to handle

the effect of gravity upon flow. HTR is the interface heat transfer resistance used by SHAFT78. OGRE sets HTR = 0. If another value is desired, it must be supplied by OGRE and SHAFT78.

TAPE5 is the plotter output file which contains a list of all connections calculated. There is one record for each connection, for a total of NCT records. Each record contains the following:

NAMEA, NAMEB, N+1, (X, Y, Z) repeated N+1 times.

NAMEA and NAMEB are the names of the two connected nodes. N + 1 is the number of corners (edges) which the convex plane polygon which represents that connection has, plus one. Each triplet (X, Y, Z) is the position in space of one of those corners. The coordinate triplets are arranged in proper order around the polygon, and the first one is repeated at the end, thereby closing the polygon.

TAPE5 is input for the program PLOTO which generates the actual plotter commands. It is not a plotter command file itself.

The records written on TAPE6 are a subset of those on TAPE 5. That is, only connections between two real elements are included. Those which connect a real element to a dummy element are not included. There are a total of NCF records in TAPE6, each of which is identical to one of the records in TAPE5.

TAPE5 contains the information needed to create a complete picture of the grid. Making such a picture after each run is essential when dealing with any but the simplest problem. This plot is done with half weight lines because all of the line segments are retraced two or three times as a consequence of representing shared edges.

TAPE6 allows the internal flow connections within the grid to be visualized. This is also needed to properly check out a grid. These plots are even more useful as pairs of stereo plots rotated by a small angle

relative to each other. Line them up in front of you, cross your eyes, and see the grid pop out in 3D.

4. THE INPUT FORMAT FOR WATER

The units employed are ^{o}C for temperatures, N/m^2 for pressures.

Card 1 — specifies the temperatures for the tabulation
FORMAT (2I5, 2E10.4)

IT	=	number of evenly spaced temperatures
ITT	=	number of explicitly provided temperatures
T(1)	=	first of evenly spaced temperatures
T(IT)	=	last of evenly spaced temperatures

Card 2 — gives additional temperature values (optional, only if ITT $\neq$ 0.
FORMAT (8E10.4)

T(IT+1)
T(IT+2)
.
.
.
T(IT+ITT)

} explicitly provides temperature values

(more than one card if ITT > 8).

Card 3 — specifies the pressure values for the tabulation.
FORMAT (2I5, 2E10.4)

IP	=	number of pressures with constant ratio P_{i+1}/P_i
IPP	=	number of explicitly provided pressures
PP(1)	=	first of IP pressures
PP(IP)	=	last of IP pressures

Card 4 — gives additional pressure values (optional, only if IPP $\neq$ 0).
FORMAT (8E10.4)

PP(IP+1)
PP(IP+2)
.
.
.
PP(IP+IPP)

} explicitly provided pressures values

(more than one card if IPP > 8)

WATER generates printed output and two files.

TAPE9 contains "special data," i.e., data for the highest isobar and for the saturation line.

TAPE8 contains "isotherms," i.e., for each temperature value, water energy, density, and vapor saturation are tabulated as a function of pressures. The saturation pressure appears twice, with function values referring to liquid water and steam, respectively.

Both files are written in formatted mode. The input file needed by PROPER is obtained by executing

COPY, TAPE9/RB, IFXF, TAPE8/RB, NEW.

i.e., first TAPE9 and then TAPE8 has to be copied, with the end-of-file mark after TAPE9 being deleted.

5. THE INPUT FORMAT FOR PROPER

Card 1 holds various options

FORMAT (26I3)

IUNIT determines the units used in PROPER

- 0: input and output are in metric units (energies in Joules, pressures in N/m^2); use this option in conjunction with WATER.
- 1: energies are input in kcal, pressures in ata (= kgf/cm^2); output is in metric units.
- 2: energies are input in Joules, pressures are input in bars; output is in metric units.

ICHOIC(1) set equal to 2

ICHOIC(2)

0: generation of (u,ρ) - tables is made for input values of u and ρ.

1: saturation densities for input energies are appended to list of input densities (imperative if problems are studied which involve phase transitions).

ICHOIC(3) set equal to 0

ICHOIC(4)

0: tabulates only T, p, S as functions of u and ρ.

1: reads data for relative permeability, viscosity, and thermal conductivity; then proceeds to generate a full (u,ρ) table of all 10 fluid properties.

ICHOIC(5)

0: equation of state (EOS) - input is as isobars (for each pressure a list of T, S, u, ρ).

1: EOS - input is as isotherms (for each temperature a list of p, S, u, ρ); this option is needed in conjunction with WATER.

ICHOIC(6)

0: tapes are read/written in binary mode.

1: tapes are read/written in formatted mode (to be used with WATER).

ICHOIC(7)

0: relative permeabilities are included in parameters P_5 and p_6.

1: relative permeabilities are not included in parameters p_5 and p_6.

Card 2 controls triangulation of liquid region of EOS. FORMAT (6(I3, E10.4))

MAX(1) number of energy values to be generated by triangulation of the liquid region of the EOS.

ESTART(1) starting energy for triangulation process (must not be smaller than the lowest saturation energy provided from input data).

MAX(2), ESTART(2) } as before, for performing a second triangulation to increase accuracy

.
.
.

MAX(6), ESTART(6) } up to six triangulations can be performed

The total number of energy values must not exceed 99. Triangulation is necessary for problems involving (subcooled) liquid water.

Card 3 reads number of explicitly provided energy and density values.
FORMAT (26I3)

NE number of energies
ND number of densities

Card 4 reads NE energy values, 8 per card
FORMAT (8E10.4)

Card 5 reads ND density values, 8 per card
FORMAT (8E10.4)

Card 6
FORMAT (I5)

NR number of points for table of relative permeability (not to exceed 33)

Card 7 reads NR values of vapor saturation in ascending order, 8 per card.
FORMAT (8E10.4)

Card 8 reads NR values of liquid relative permeability corresponding to the above vapor satuations, 8 per card.
FORMAT (iE10.4)

Card 9 reads NR values of vapor relative permeability corresponding to the above vapor saturations, 8 per card.
FORMAT (8E10.4)

Card 10 begins the viscosity data.
FORMAT (E10.4, I5, A5)

PATM — pressure in units of technical atmospheres (ata = kgf/cm^2).

NTT — number of temperature (and viscosity) - values for this pressure (not to exceed 50).

JEND — ENDbb: PATM is the last (highest) pressure for which viscosity data are read.
blank: continue to read viscosity data.

Card 11 — reads NTT temperature values, 8 per card. The temperatures must appear in ascending order, with the saturation temperature corresponding to PATM appearing twice.
FORMAT (8E10.4)

Card 12 — reads NTT viscosity values, 8 per card, corresponding to PATM and the NTT temperatures read in card 11. For the saturation temperature, need the viscosity of saturated liquid first, then that of saturated steam.
FORMAT (8E10.4)

Unless JEND was equal to END, PROPER will continue reading viscosity data, going through the cycle CARD 10 - CARD 11 - CARD 12. Up to 25 isobars can be read. If JEND was equal to END, PROPER proceeds to read data for thermal conductivity of water.

Card 13 — begins the thermal conductivity data. These data are read exactly like the viscosity above:

PATM
NTT
JEND

Card 14 — reads NTT temperature values.

Card 15 — reads NTT thermal conductivity values.

Then proceeds to read another Card 13, etc., unless JEND is equal to END, in which case the input deck is closed.

In addition to the cards, PROPER needs as input a file generated by WATER, to be supplied to PROPER as TAPE8.

PROPER generates printed output and a file TAPE11, which contains a table of fluid properties suitable as input for CYCN.

6. THE INPUT FORMAT FOR CYCN

During the development of SHAFT78, a number of options have been created for running CYCN in different modes. The following description includes those options which are useful for applications. An overview of the input format for CYCN is given in Figure 9.

The input deck for CYCN begins with one or two "starred" cards.

Card 1 *(TITLE, optional)

The card has a star in column 1, followed by a 79-character title. The title is printed as a header of every page of output and can be used to identify a problem.

Card 2 *CYCLE (to be punched into columns one to six)

This card initiates the actual simulation.

Card 3 determines the input mode for the table of fluid properties, as generated by PROPER.

FORMAT (I2, 2I4, 7E10.4)

MODE
0: fluid properties are read from binary tape
1: fluid properties are read from formatted tape.

IDIM number of energy or density values in the fluid table, whichever is larger. If left blank, the default value is 81. IDIM must not exceed 99.

IREL
0: relative permeabilities are obtained from the fluid table.

≠0: relative permeabilities are computed in CYCN according to one of the following choices:

1: Corey's equation as given in ref. (42), with REL(1) = S_{wc}, REL(2) = S_{gc}.

2: Relative permeabilities are interpolated from a table of values, specified in array REL2 in subroutine PARAM. At present, REL2 contains data as given in ref. (43).

REL(I) parameters to be used in computing relative permeabilities (at present only REL(1) and REL(2) are operative).

Card 4 Rock properties.
FORMAT (A5, 5X, 7E10.4)

MAT	five character material identifier
DM	specific density (in kg/m^3)
POR	porosity
PER(1) PER(2) PER(3)	absolute permeabilities in the direction specified by the index ISOT (see card 8.1) (in m^2)
CM	heat conductivity (in J/m sec^oC)
CH	specific heat (in J/kg^oC)

Repeat card 4 for each different rock material.

At present, up to seven different materials can be defined.

Card 5 blank card indicates end of rock property data.

Card 6 has two options, to be punched in columns one through five.

"START": begin a new problem.

"RESTA": restart a problem, in which case CYCN needs a file RESTA to be described below.

The next five cards describe parameters which are needed in the calculation.

Card 6.1 FORMAT (2I2, 3I4, 14I1, 5(5X, A5)

NOEOF specifies the type of calculation to be performed.

-1: solve density equation only.

0: solve both energy and density equations (standard option).

1: solve energy equation only.

KDATA specifies what is to be printed.

0: print only the dependent variables and some parameters for each element.

1: print in addition fluxes and transductances.

2: full printout.

MCYC — maximum number of cycles to be performed.

MSEC — maximum duration, in machine seconds, of the simulation.

MCYPR — printout will occur for every multiple of MCYPR cycles.

MOP(I), I=1, 14 — allows choice of various options. Default options prevail for MOP(I) = 0. MOP(1) through MOP(8) generate additional printout in various subroutines, if set $\neq$ 0.

MOP(1) — INPUT

MOP(2) — PARAM (the larger MOP(2) is the more printout will be generated).

MOP(3) — THICK

MOP(4) — TRIP

MOP(5) — GETIT

MOP(6) — FLICK

MOP(7) — FLIP

MOP(8) — SHALK (do not set MOP(8) > 4)

MOP(9) through MOP(14) modify the way calculations are being performed. Use only those options that are discussed here!

MOP(9) — determines the composition of the source fluid. The relative amounts of liquid and vapor are determined:

0: according to relative mobilities in the source element.

1: source fluid has the same (u, ρ) as the producing element.

2: only vapor is produced.

3. only liquid is produced.

MOP(10) — set equal to 0.

MOP(11) — determines mobilities at interfaces.

0: mobilities are spatially interpolated between adjacent elements.

≠ 0: mobilities are upstream weighted, namely

1: upstream weighting with WUP (default is.667).

2: 100% upstream weighting

For problems with a phase front MOP(11) = 2 is imperative!

MOP(12) set equal to 2.

MOP(13) 0: perform rock/fluid equilibration just once after density step.

2: perform rock/fluid equilibration after every density-subcycle (strongly recommended!).

MOP(14) 0: default

7: update derivatives for rock/fluid equilibration (this option to be used only with MOP(13) = 0).

For problems in which elements will change phase MOP(13) = 2, MOP(14) = 0 is required. This option should be used as standard in almost all cases.

PRINT set equal to "STEP".

ENUM set equal to the name of one element, to obtain a short printout every cycle.

MATID set equal to the name of one of the materials (rocks), to obtain volume-, mass-, energy-, and heat-balances for (part of) the system.

PUNCH if set equal to "PUNCH" energies and densities are punched for each element after normal completion of the run (KWIT ≤ 4). The cards are compatible with the initial condition-deck "INCON" (see below); they provide an alternative for restarting a problem. If left blank no cards will be punched.

FLOW if set equal to "FLOWb" mass fluxes are punched under the format of deck "INFLOW" (see below), provided PUNCH was set equal to "PUNCH". Otherwise no additional punching will occur.

Card 6.2 time parameters
FORMAT (8E10.3)

TSTART, TIMAX, TIMEP, DELTEN, DELTEX, DELTFN, DELTFX, SCALE

TSTART : starting time of the calculation in seconds.

TIMAX : time in seconds at which calculation should stop. If zero or blank, simulation proceeds until calculations stop according to some other criterion.

TIMEP : time interval (sec.) between printouts.

DELTEN : minimum energy time step (sec.); default = 10^{-12}

DELTEX : maximum energy time step (sec.); default = 10^{+12}

DELTFN : minimum density time step (sec.); default = 10^{-12}

DELTFX : maximum density time step (sec.); default = 10^{+12}

SCALE : scale factor used to change the size of the mesh to be fed in (default = 1.0).

Card 6.3 variation parameters
FORMAT (8E10.3)

EVARY, DVARY, ERRE, ERRF, FORP, FORFP, WUP, PCT

EVARY : optimum change (J/kg) in the fluid specific internal energy for every element over any energy step. Set equal to 1000 J/kg if zero or blank.

DVARY : optimum change (kg/m^3) in the fluid density for every element over any density step. Set equal to 1 kg/m^3 if zero or blank.

ERRE : convergence criterion (J/kg) for the iterative scheme solving the energy balance equation. Set equal to 10^{-4} EVARY if zero or blank.

ERRF : convergence criterion (kg/m^3) for the iterative scheme solving the mass balance equation. Set equal to 10^{-4} DVARY if zero or blank.

FORP : implicit-explicit weighting factor in the energy balance. Computed every energy step if zero or left blank.

FORF : implicit-explicit weighting factor in the density balance. Computed every density step if zero or left blank.

WUP : upstream weighting factor used in evaluating the minimum value of the explicit energy convection term of the heat balance. Set equal to .667 if zero omitted.

PCT : maximum relative change over a time step in any variable and some parameters. Set equal to .01 if zero or left blank.

Card 6.4 default initial conditions
FORMAT (6E10.3)

EONE, DONE, FONE, GF, RATEQ, CUT

EONE : fluid specific internal energy (J/kg) to be used as initial value at any element for which no other value is specified in deck "INCON" (#10; initial conditions). IF EONE $\leq$ 1000, EONE is taken to be temperature TX (in ^{o}C) rather than energy.

DONE : initial fluid density (kg/m^3) for every element for which no other value is specified in deck "INCON" (#10; initial conditions). If EONE $\leq$ 1000, meaning temperature, DONE is taken to be pressure PX (N/m^2, for DONE > 1) in the one-phase case; it is taken to be volumetric vapor saturation SX (for DONE $\leq$ 1) in the two-phase case.

FONE : initial fluid mass flow rate (kg/sec) across every connection for which no other value is specified in deck "INFLO" (#11; initial mass flow rate).

GF : magnitude (m/sec^2) of the gravitational acceleration vector. Blank or zero gives "no gravity" calculation.

RATEQ : accuracy criterion (J/kg) for rock/fluid-equilibration; set equal to 50 J/kg if left zero or blank.

CUT : determines when the isothermal limit will be taken in eq. (6.2); if the denominator in eq. (6.2) is less than CUT * $\phi\rho$, it is replaced with $\phi\rho$. Default value is CUT = .5.

If the problem is restarted using restart-file ("RESTA" in Card 6) the cards 7 through 11 are omitted.

Card 7 | ELEME

This card precedes the element information, described below (columns 1 to 5).

Card 7.1

FORMAT (A5, 2I5, A5, E10.3)
EL1, NSEQ, NADD, AMA, VOLX

EL1 : code number of the first element in the sequence.

NSEQ : number of additional elements.

NADD : increment between the code numbers of two successive elements.

AMA : name of material of which the sequence of elements is composed.

VOLX : volume (m^3) of each element.

The card | NOFLO may be inserted among the deck of cards 7.1 to indicate that the following elements are outside the flow domain. Conversely, card | FLOW indicates that the following elements do belong to the flow domain.

Card 7.2 blank card

This card placed after all elements in the mesh have been specified by way of card 7.1, indicates the end of deck 7.

Card 8 | CONNE

This card introduces some connection information; connections having the same characteristics are listed as a sequence on a single card by using appropriate values of NSEQ, NAD1, and NAD2 (see below).

Card 8.1

FORMAT (2A5, 4I5, 5E10.3)

EL1, EL2, NSEQ, NAD1, NAD2, ISOT, D1, D2, AREAX, BETAX, RINTX

EL1
EL2 : code numbers of the two elements linked by the first connection in the sequence.

NSEQ : number of additional connections in the sequence.

NAD1
NAD2 : increments between code numbers of corresponding elements in two successive connections.

ISOT : set equal to 1,2 or 3; specifies absolute permeability to be PER(ISOT) for the materials in elements EL1 and EL2, where PER is read in through card #4. This allows assignment of different permeabilities in e.g. the horizontal and vertical direction.

D1
D2 : (connection) distances (m) from center of each of any pair of elements to their common interface.

AREAX : (connection) area (m^2) of each common interface.

BETAX : (direction) cosine of the gravitational acceleration and the normal to the common interface, oriented from the first to the second element in each connected pair. No effect if GF in card 6.4 is zero or blank.

RINTX : heat transfer resistance ($msec^oC/J$).

Card 8.2 blank card

This card, placed after all connections have been listed by way of cards 8.1, indicates the end of deck 8.

The information provided by decks 7 and 8 supplements but does not supersede completely the information that may have been provided through tape "MESH" from program SHAME.

Card 9 GENER

Introduces heat and mass generation data; any element may contain several heat/mass sources or sinks. Sources or sinks having the same characteristics can be listed as a sequence on a single card.

Card 9.1

FORMAT (2A5, 4I5, 5X, A4, 1X, 2E10.3)

EL1, S1, NSEQ, NADD, NADS, LTAB, TYPE, GX, EX

EL1 : code number of the first element in the sequence.

S1 : code number of the first source in the sequence.

NSEQ : number of subsequent elements.

NADD : increment between code numbers of two successive elements.

NADS : increment between code numbers of two successive sources.

LTAB : number of points in table of generation rate versus time (must not exceed 300). Set 0 or 1 for constant generation rate.

TYPE : MASS - mass source
HEAT - heat source
Default (blank) is MASS.

GX : constant generation rate, positive for injection, negative for production; GX is mass (kg/sec) for a MASS source, energy (J/sec) for a HEAT source.

EX : fixed specific enthalpy (J/kg) of the fluid for mass injection (GX > 0).

Card 9.2 table of generation rates versus time (LTAB > 1 only).

FORMAT (4E14.7)

Read all generation times

(F1(J), J = 1, LTAB)

Read all generation rates.

(F2(J), J = 1, LTAB)

F1 : time (sec)

F2 : generation rate (J/sec or kg/sec)

Card 9.3 table of specific enthalpy (J/kg) of injected fluid (for LTAB > 1 and F2 > 0 only).

FORMAT (8E10.3)
Read all generation specific enthalpies.

(F6(J), J = 1, LTAB)

F6 : specific enthalpy (J/kg).

See MOP(9) (Card 6.1) for specification of fluid energy for production.

Repeat cards 9.1 through 9.3 (if appropriate), for all source elements.

Card 9.4 blank card

This card indicates the end of deck 9.

Card 10 INCON

This card introduces the initial conditions for specific elements. This data overrides the initial conditions specified for all elements on card 6.4 by EONE and DONE.

Card 10.1

FORMAT (A5, 2I5, 5X, 2E10.4)

EL1, NSEQ, NADD, EX, DX

EL1 : code number of the first element in the sequence.

NSEQ : number of additional elements.

NADD : increment between code numbers of two successive elements.

EX : fluid internal energy (J/kg)

DX : fluid density (kg/m^3).

This card is repeated as many times as necessary to prescribe initial conditions other than the default values specified in card 6.4. (EX, DX) can be (T,p) or (T,S), as explained in connection with card 6.4.

Card 10.2 blank card
This card indicates the end of deck 10.

Card 11 INFLO

This card introduces the initial mass flow rate, which may be useful whenever NOEOF equals 1; connections having the same characteristics are listed as a sequence on a unique card.

Card 11.1

FORMAT (2A5, 3I5, 5X, E10.3)

EL1, EL2, NSEQ, NAD1, NAD2, FX

EL1, EL2 : code numbers of the two elements linked by the first connection in the sequence.

NSEQ : number of additional connections.

NAD1, NAD2 : increments between code numbers of corresponding elements in two successive connections.

FX : initial fluid mass flow rate (kg/sec)

This card is repeated as many times as necessary to prescribe initial mass flow values other than the default value specified in card 6.4.

Card 11.2 blank card indicates the end of deck 11.

Card 12 ENDCY

This card closes the data deck for CYCN (columns 1 to 5).

If the program SHAPE is to be run, follow this data deck for CYCN with the control cards as shown in the section below. If the CYCN data ends the data deck the following card begins the calculation.

Card 13 | *SPLIT (columns 1 to 6).

This card indicates the end of the data deck, and will transfer control to the computer system. An example of how to execute a data deck on the BKY system is shown in a later section. Figure 9 shows the standard form for SHAFT78 CYCN input data.

7. THE INPUT FORMAT FOR SHAPE

PLOTS

The plotting of the mesh, the variables u and ρ and some parameters is initiated by the following card:

Card 1 | *PLOTS

followed by:

Card(s) 2 region identifiers and border plot flags

LABEL, (F1(I), F2(I), I=1,7)

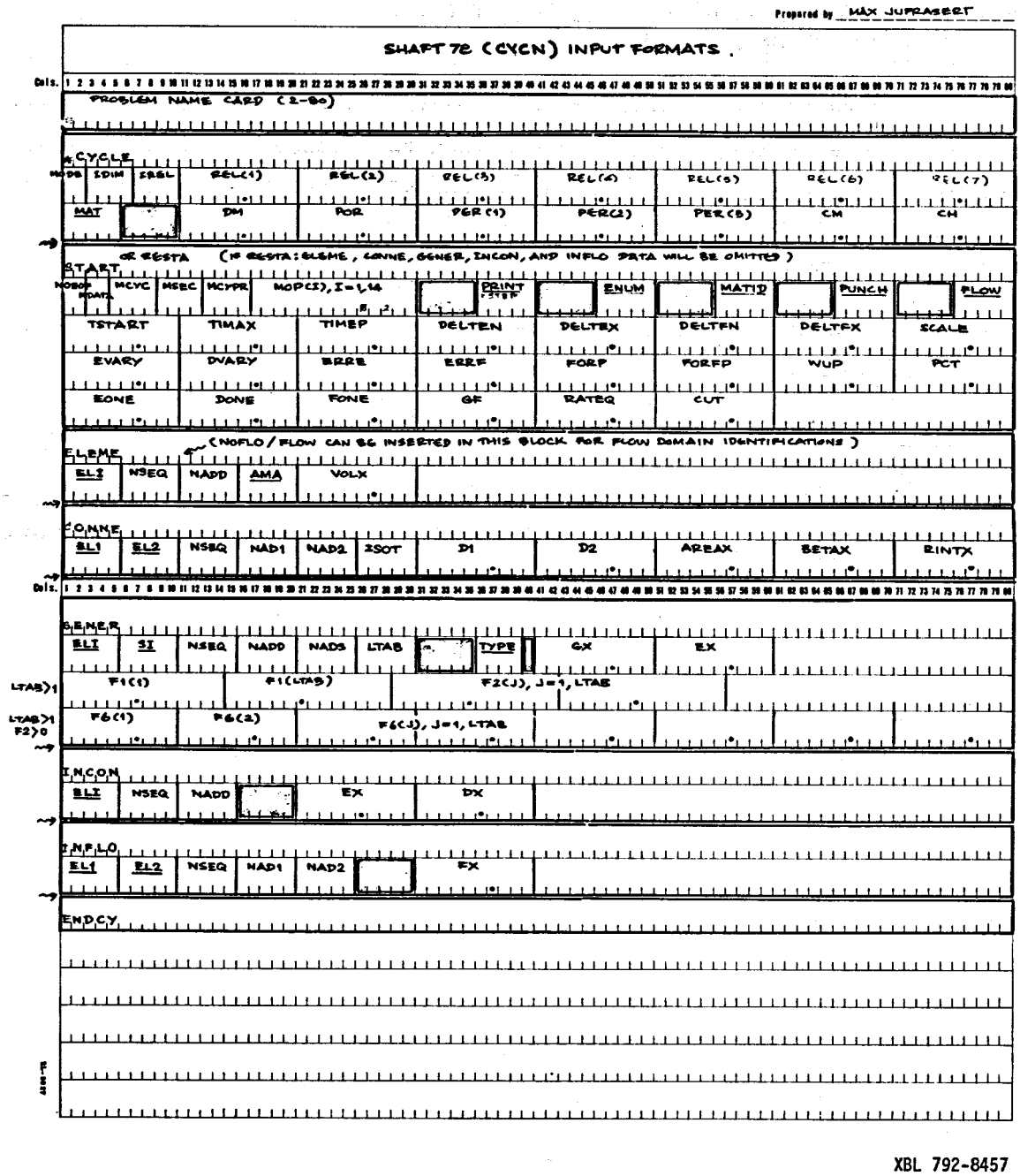
FORMAT (A5, 5X, 7 (A1, F5.0, 4x))

LABEL = REGIO

F1 : identifier of any region to be included in the plots.

F2 : border plot flag for the corresponding region. This is a one- to four-digit floating point number; each digit is between 1 and 4 and indicates that the corresponding side of the region is to be drawn. If no border plot flag is specified, all four sides of the region are drawn; therefore, if no side is to be drawn, F2 must be assigned the value 0.

Additional cards may be inserted, containing (F1(I), F2(I), I=1,7) with Format (10X, 7(A2, 3X, F5.0).



XBL 792-8457

Figure 9. Standard form for SHAFT78 CYCN input data.

Card 3 paper coordinates

LABEL, PXDF, PYDF

Format (A5, 5X, 2E10.3)

LABEL = COORP

PXDF, PYDF : horizontal and vertical space (inches) available for plotting the grid. Neither should exceed two-thirds of the total space available.

: The X and Y paper coordinates PXMIN and PYMIN of the lower left-hand corner of the plotting space available for the grid are then automatically set to half PXDF or PYDF, whichever is smaller.

The character height is automatically set to one-eighth of the foregoing minimum.

If card 3 is omitted, the default value 6, is assigned to PXDF and PYDF.

Card 4 space coordinates

LABEL, XMN, XMX, YMN, YMX

FORMAT (A5, 5X, 4E1Ø.3)

LABEL = COORS

XMN : minimum abscissa of the plot.

XMX : maximum abscissa of the plot.

YMN : minimum ordinate of the plot

YMX : maximum ordinate of the plot.

The paper coordinates are automatically adjusted so that the X and Y scales are the same and the grid is as large as can fit within the paper coordinates specified.

If card 4 is omitted, space coordinates are automatically adjusted to accommodate all the regions generated by SHAME.

Card 5 plot types

LABEL, (F1(I), I=1, 11)

FORMAT (A5, 5X, 11A2)

LABEL = TYPES

F1 : right-adjusted, two-character identifiers specifying which plots are to be made:

E: internal energy (J/kg)
D: density (kg/m^3)
T: temperature (^oC)
P: pressure (N/m^2)
S: vapor saturation
HF: heat flux (J/m^2 sec)
LF: liquid (mass) flux (kg/m^2.sec)
VF: vapor (mass) flux (kg/m^2.sec)
LV: liquid seepage velocity (m/sec)
VV: vapor seepage velocity (m/sec)
MF: (fluid) mass flux (kg/m^2.sec)

If card 5 is omitted, all eleven plots are made.

Card 6 maximum vector length and units

LABEL, VMAX, TIMU, DIMU

FORMAT (A5, 5X, E10.3, 2A10)

LABEL : MISCE

VMAX : maximum allowed vector length (m) in space coordinates. The default value is one-tenth the length of the plot along the X or the Y direction, whichever is less. This value is reset whenever space coordinates (card 4) are reset.

TIMU : time and length unit names to appear on the plots;

DIMU : default names are "SECONDS" and "METERS", respectively.

Card(s) 7 scale multipliers

LABEL (F1(I), F2(I), I=1,7)

FORMAT (A5, 5X, 7(A2, E8.2))

LABEL : SCALE

F1 : right-adjusted, two-character identifier of the parameter to be multiplied by a scale factor before plotting. F1 may be any of the identifiers mentioned with deck 5.

F2 : scale factor

Additional cards may be inserted, containing (F1(I), F2(I), I=1,7) with FORMAT (10X, 7(A2, E8.2)).

If deck 7 is omitted, the parameters are plotted with no scaling.

Card 8 contour levels

LABEL, TYPE, BOLD, XLAB, CMIN, CDEL

FORMAT (A5, 5X, A2, 8X, 4E10.3)

LABEL : CONLV

TYPE : right-adjusted, two-character identifier of the parameter for which contour levels are to be drawn: may be E (energy), D (density), T (temperature), P (pressure), or S (saturation).

BOLD : increment between bold contours.

XLAB : increment between labeled contours.

CMIN : minimum value to be contoured; this value applies after scaling.

CDEL : increment between contours.

Additional cards may be inserted, containing TYPE, BOLD, XLAB, CMIN, CDEL with Format (10X, A2, 8X, 4E10.3).

Card 9 CRID

This card initiates the plotting of the mesh.

Card(s) 10 plot times

LABEL, (F1(I), I=1,7)

FORMAT (A5, 5X, 7E1Ø.3)

LABEL : TIMES

F1 : time to be plotted: the corresponding information is plotted if F1 is within 4 percents of any time stored on tape "SAVE" (#6).
If F1 is negative all times stored on tape "SAVE" are plotted.

Additional cards may be inserted, containing (F1(I), I=1,7) with FORMAT (1ØX, 7E1Ø.3).

Card 11 ENDPL

This card closes the PLOTS data deck.

8. THE INPUT FORMATS FOR PLOTZ

PLOTZ is a package of utility programs which allow the user to edit and plot certain output variables from CYCN calculations. Four programs are described below. Two programs are for data manipulation and two are for graphics output. These programs all reside on PSS library DUCK and are accessed by the standard PSS LIBCOPY command. Figure 10 shows the program structure for the utility package called PLOTZ.

FILTER : Extracts Time/Temperature and Time/Pressure data for a given set of elements.

The input structure is:

Number of Elements (I5)
Element Names (10A5)
Tape 15
Blank Card

The output is:

Tape 1	Time/temperature
Tape 2	Time/pressure

An example of FILTER output plots is given in Figure 11.

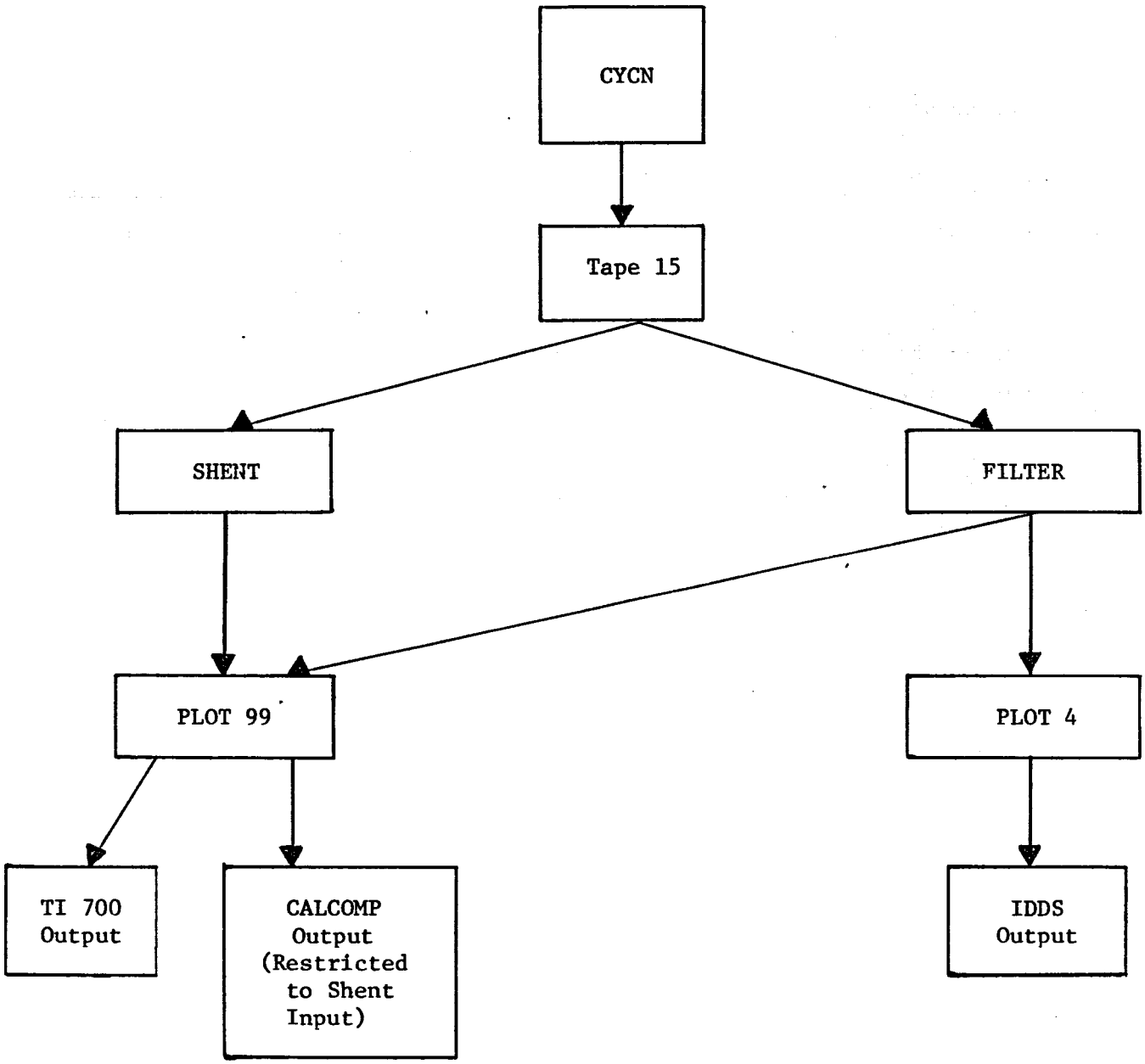


Figure 10. Generalized block diagram for data manipulation and plotting from CYCN output.

SHENT : calculates the total specific enthalpy for specified elements as $H = (\rho u + p)\phi + C_s\rho_s T(1-\phi)$ and produces a file for input to Plot 99 which will have up to 6 different frames (Enthalpy vs. position, Energy vs. position, Density vs. position, Temperature vs. position, Pressure vs. position, Phase vs. position).

The input structure is:

Number of Spatial Points (I5)
x position of Spatial Points (10E5.1)
Maximum Cycle number, Rock spec. ht. , Rock density (I5, 3E10.4), Porosity
IOPT (=1 suppresses tilting to produce graphic compatible output)
TAPE15

An example of SHENT output plots is given in Figure 11.

PLOT 99 : (SHENT Output) produces a CALCOMP Plot (IOPT (7)=0) or a TI700 formatted teletype plot (IOPT(7)=1) of H,E,D,T,P, Phase vs position from SHENT output for different times.

CALCOMPT Output has the time for each curve printed next to its maximum value. TI700 output plots a different symbol (0-9, A-Z) for each time, then a list of symbols and corresponding times after each plot.

IOPT is a string of 7 ones or zeros. If IOPT (I) ≠ for any I from 1 to 6 the variable number I of the set (H,T,P, Phase, E,D) is plotted.

Input (SHENT Output)

SHENT	(A5)
Total Number of Different Times	(I5)
IOPT	(I7)
TAPE 1 from SHENT	

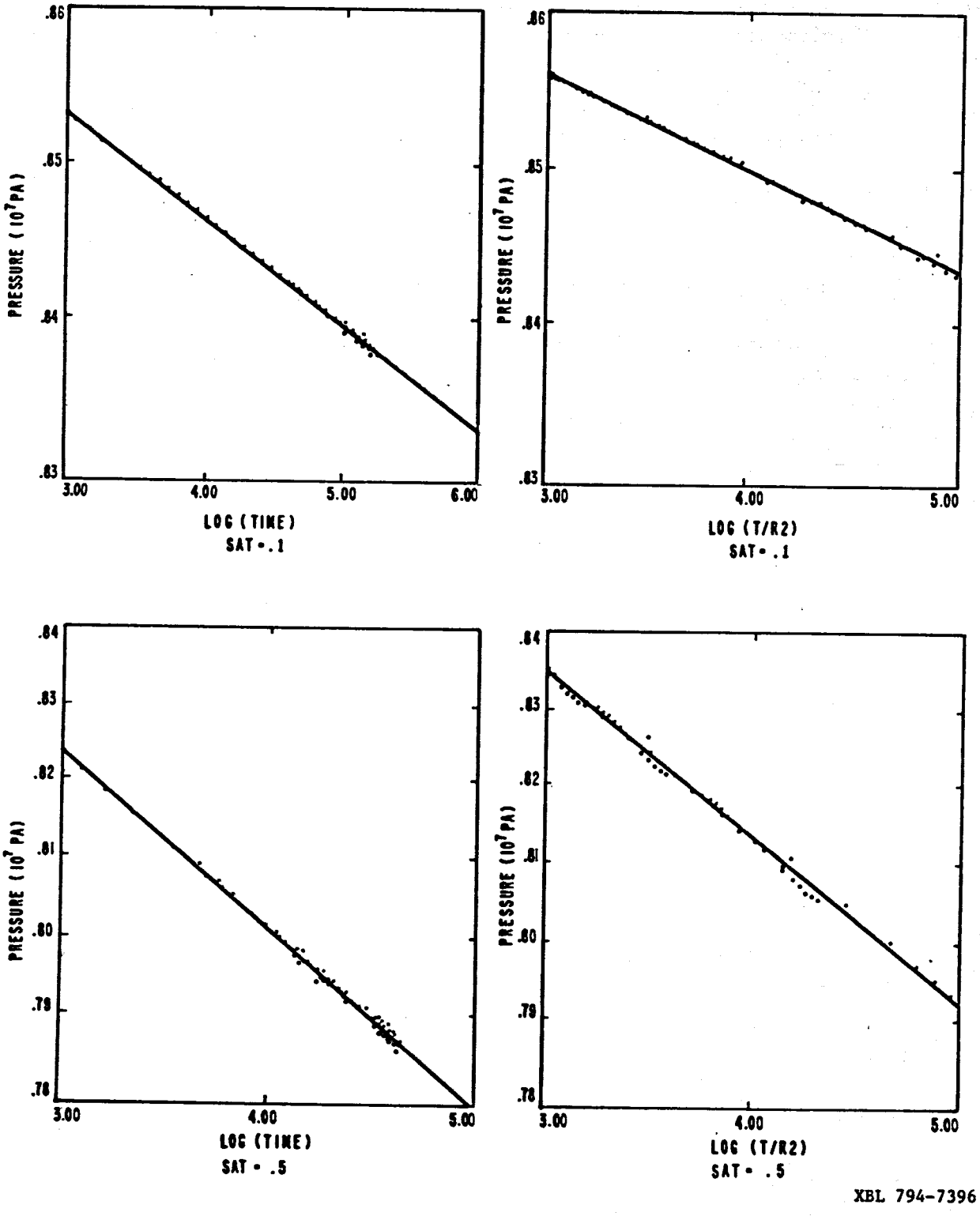


Figure 11. Examples of plots of pressure versus log time from FILTER (drawdown of wells in two-phase reservoir).

PLOT 99 : (FILTER Output) produces a plot of temperature or pressure vs log time for a given element (hence, in FILTER, the number of elements = 1).

Input (FILTER Output)

```
FILTER                                                    (AS)
Number of elements (=1)
IOPT (= 1000001 for TI700, = 1000000 for CALCOMP)
XMIN, XMAX (2F5.2)
YMIN, YMAX (2E10.4)
```

PLOT 4 : Produces an IDDS (Device Independent) Plot of temperature vs. time or pressure vs. time for a given set of elements (1 plot per element) uses filter output from PSS (Tape 1 or Tape 2 from FILTER) in the control deck.

An example of PLOT99 output is shown in Figure 12.

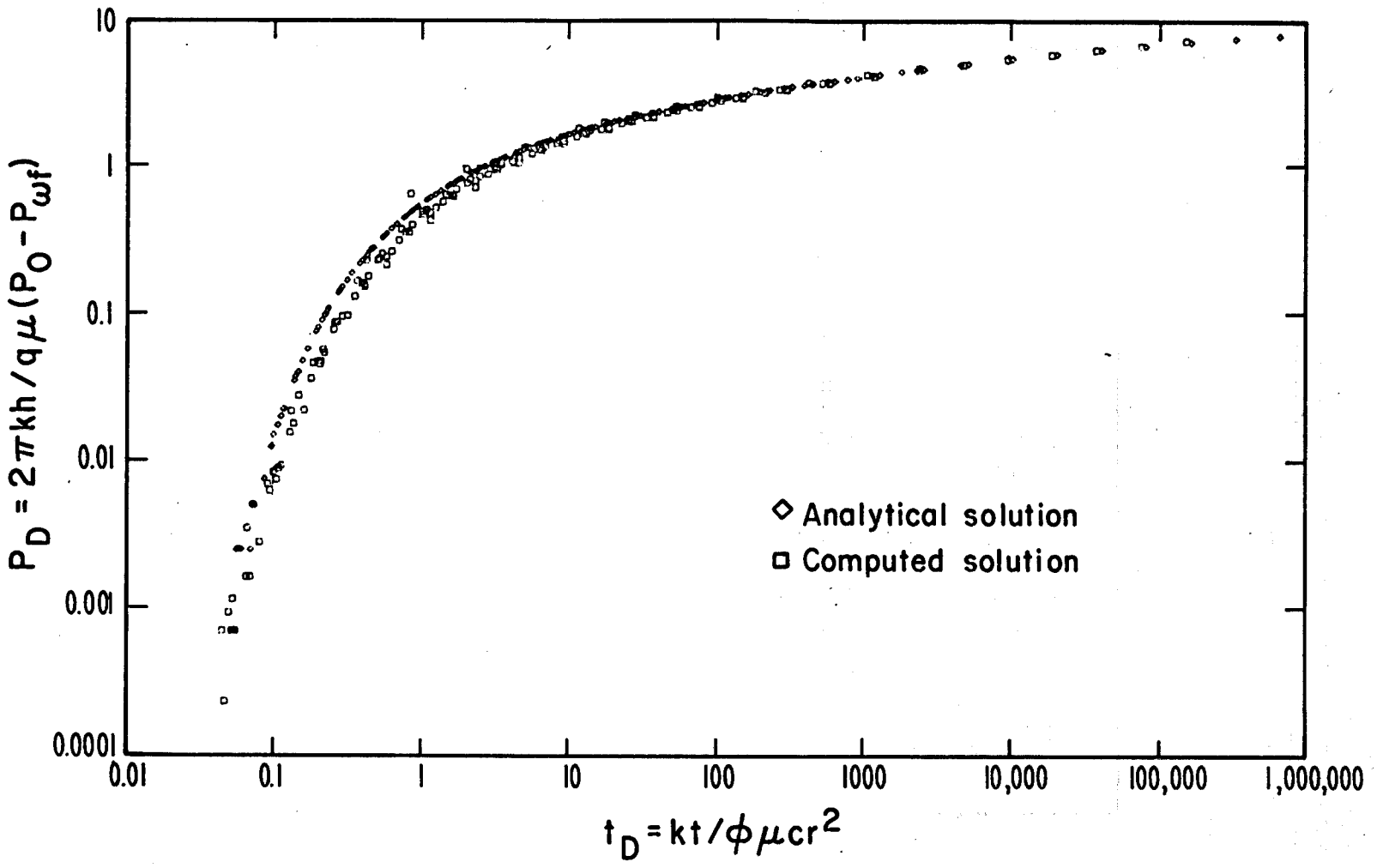


Figure 12. An example of a plot dimensionless pressure change versus dimensionless log time from SHENT.

9. THE OUTPUT OF SHAFT78

Most of the output of SHAFT78 is self-explanatory. In the following, we explain in some detail the output of CYCN.

For every cycle there is a short printout for the element ENUM as specified in card 6.1 of the input deck of CYCN.

For energy cycles we have:

KCYC : number of energy step.

TIME : time at end of energy step.

DELTE : length of time step.

ENERGY : fluid energy of element ENUM after step.

DE : change of fluid energy in ENUM.

TEMPERATURE
PRESSURE
PHASE : values of parameters (vapor saturation printed with a minus sign for two-phase elements) before current energy step.

RAT : estimated ratio of (maximum error in implicit energy change)/(explicit energy change).

NUTS : number of iterations in solving for implicit energy changes.

For density cycles we have:

KCYCF : number of density subcycle.

BEGIN TIME : time at which calculation begins.

DELTF : time step for density subcycle.

DENSITY : fluid density of element ENUM after subcycle.

DD : change of fluid density in ENUM during subcycle; for the last density subcycle belonging to an energy step DD is the cumulated sum of the density changes in all subcycles.

DEF : net mass flow into ENUM.

PRESSURE, PHASE : parameters after subcycle.

RATF : estimated ratio of (maximum error in implicit density change)/(explicit density change).

NUTSF : number of iterations in solving for implicit density changes.

The full ENERGY CYCLE OUTPUT DATA are:

KCYC : number of energy steps.

KWIT : termination parameter

NUTS : number of iterations in solving for implicit energy changes

NUTX : maximum of all prevoius NUTS.

NUTSUM : sum over all previous NUTS.

MAX.DE : absolute value of maximum energy change.

SLIMIN, SLIMAX, DESLMX : parameters in implicit iteration.

RATG : as RAT in "short printout."

RATGS : RATG of previous energy step.

TOTAL TIME : calculation time.

CUM. HEAT GEN. : cumulative heat exchange with sources/sinks.

CUM. HEAT GAIN : cumulative change of total heat content of reservoir, computed from interface heat fluxes.

AV. GAIN RATE : (CUM. HEAT GAIN)/(TOTAL TIME).

TIME STEP : energy time step

HEAT GEN. : heat exchange with sources/sinks.

HEAT GAIN : change of total heat content of reservoir, computed from interface heat fluxes.

GAIN RATE : (HEAT GAIN)/(TIME STEP).

WGHT. FACTOR : implicit weight factor (see eq. (23), p. 16)

ELEM. : element name.

INDEX : element number.

E : (fluid) energy.

DE : energy change.

DDE : rate of energy change.

T : temperature
P : pressure
} before E-step

PHZX : volumetric vapor saturation

DD/DE : estimated ratio of density to energy change (cf. eq. (26), p. 18)

DTDE : $(\partial T/\partial u)_\rho$

DD : estimated density change, i.e., DE * (DD/DE).

TOTAL DE : total energy change of element.

FLOH : rate of heat flow across interface.

TRAN1
TRAN2
ZIP
SLIM
CAN
} : parameters for implicit iteration

H : total cumulative energy change of element.

DF : net inflow of enthalpy

DVF : convected volume.

DMF : convected mass.

DTDD : $(\partial T/\partial \rho)_u$

DES/DE : $(\partial u_s/\partial u) = (1-\phi)\rho_s c_s (\partial T/\partial u)_\rho$.

DES/DD : $(\partial u_s/\partial \rho)_u = (1-\phi)\rho_s c_s (\partial T/\partial \rho)_u$.

The meaning of most DENSITY CYCLE OUTPUT DATA corresponds to that of the analogous energy step quantities. Exceptions are:

DE : change of fluid energy including rock/fluid equilibration.

DPDD : $(\partial P/\partial \rho)_u$.

B : liquid mobility
= (absolute permeability) * (relative permeability)/viscosity

VB : vapor mobility.

FLOF : explicit rate of fluid flow across interface.

FLOL : explicit rate of liquid flow across interface.

FLOV : explicit rate of vapor flow across interface.

VELL : velocity of explicit liquid flow.

VELV : velocity of explicit vapor flow.

TRAN1, TRAN2, ZIP, SLIM, CAD : parameters for implicit iteration.

HF : cumulative change of fluid mass.

FF : cumulative inflow of fluid mass.

DHF : change of fluid mass.

DFF : net inflow of fluid mass.

SOME SAMPLE PROBLEMS

WATER

Table 4 gives an example of an input deck to be used for running WATER (WATERG is a compiled version of the FORTRAN program WATER). The results are stored in library DIABLO, subset ISOT1, in a form suitable for use with PROPER. Table 5 shows part of the printout generated by WATER; it is the data for isotherm no. 71 at T = 350°C.

Table 4. Input deck for running WATER.

```
KPWA,07,03,471504,PRUESS
*INPUT 66008 15.52.33  07 JUL 78  VIA KPGCCCB
*HCLUOUT
LIBCOPY,DIABLC,LGO,WATERG.
REWIND,LGO.
LINK,X,PP=(LC=77777).
COPY,TAPE9/RB,1FXF,TAPE8/RB,NEW.
LIST,NEW.
LIBRITE,DIABLO,NEW/RB,ISCT1,143.
EXIT.
DUMP,0.
FIN.
REWIND,INPUT.
COPYSBF,INPUT,OUTPUT.



   80    1  5.      400.
238.0
   60    9      10.E5    220.E5
      1.E5       2.E5       3.E5       4.E5       5.E5       6.E5       7.E5       8.E5
      9.E5
```

Table 5. Printout from WATER.

71 350.0000 574.3432 113.6479 1643154.2689 2422157.1688

INDE	PRES	DENS	ENERGY	SATUR
1	220C0000.0	611.2202	16011c9.8	0.00000000
2	2C877C79.9	605.2551	1608033.6	0.00000000
3	19811475.8	599.0368	1615165.2	0.00000000
4	18800261.9	592.4947	1622631.3	0.00000000
5	17840662.3	585.5285	1630540.3	0.00000000
6	16930042.4	577.9879	1639055.4	0.00000000
7	16535124.1	574.3432	1643154.3	0.00000000
8	16535124.1	113.6479	2422157.2	1.00000000
9	16065902.2	103.6332	2460011.1	1.00000000
10	15245865.3	90.5386	2509966.7	1.00000000
11	14467652.4	80.7738	2548575.0	1.00000000
12	13729235.0	72.9599	2580497.0	1.00000000
13	13028469.9	66.4661	2607648.2	1.00000000
14	12363473.0	60.9320	2631139.3	1.00000000
15	11732418.9	56.1284	2651723.5	1.00000000
16	11133574.8	51.9001	2669948.1	1.00000000
17	10565296.9	48.1374	2686224.4	1.00000000
18	10026024.9	44.7599	2700868.4	1.00000000
19	9514278.3	41.7069	2714125.7	1.00000000
20	9028652.2	38.9313	2726190.6	1.00000000
21	8567813.4	36.3960	2737218.1	1.00000000
22	8130496.6	34.0710	2747334.0	1.00000000
23	7715501.2	31.9316	2756641.5	1.00000000
24	7321688.0	29.9572	2765226.7	1.00000000
25	6947975.7	28.1307	2773162.2	1.00000000
26	6593338.4	26.4372	2780510.4	1.00000000
27	6256802.4	24.8639	2787324.8	1.00000000
28	5937443.8	23.3999	2793652.7	1.00000000
29	5634385.8	22.0353	2799535.6	1.00000000
30	5346796.5	20.7618	2805010.5	1.00000000
31	5073686.3	19.5717	2810110.4	1.00000000
32	4814905.9	18.4582	2814865.4	1.00000000
33	4569144.3	17.4155	2819302.2	1.00000000
34	4335926.9	16.4380	2823445.5	1.00000000
35	4114613.3	15.5209	2827317.6	1.00000000
36	3904595.9	14.6597	2830938.8	1.00000000
37	3705298.2	13.8505	2834327.7	1.00000000
38	3516173.1	13.0897	2837501.3	1.00000000
39	3336701.2	12.3738	2840475.3	1.00000000
40	3166389.9	11.6999	2843264.0	1.00000000
41	3004771.6	11.0652	2845880.5	1.00000000
42	2851402.6	10.4670	2848336.8	1.00000000
43	2705661.8	9.9031	2850644.2	1.00000000
44	2567749.7	9.3713	2852812.9	1.00000000
45	2436687.1	8.8694	2854852.2	1.00000000
46	2312214.1	8.3958	2856771.0	1.00000000
47	2194289.4	7.9486	2858577.0	1.00000000
48	2082286.9	7.5263	2860277.9	1.00000000
49	1976005.0	7.1272	2861880.5	1.00000000
50	1875146.1	6.7502	2863390.9	1.00000000
51	1779435.3	6.3938	2864815.2	1.00000000
52	1688609.7	6.0568	2866158.8	1.00000000
53	1602419.9	5.7382	2867426.7	1.00000000
54	1520629.5	5.4368	2868623.6	1.00000000
55	1443013.8	5.1517	2869753.7	1.00000000
56	1369359.8	4.8820	2870821.3	1.00000000
57	1299465.1	4.6267	2871829.9	1.00000000
58	1233136.1	4.3851	2872783.2	1.00000000
59	1170156.5	4.1564	2873684.4	1.00000000
60	1110467.5	3.9398	2874536.5	1.00000000
61	1053787.2	3.7348	2875342.5	1.00000000
62	1000000.0	3.5407	2876104.9	1.00000000
63	9000000.0	3.1807	2877516.3	1.00000000
64	8000000.0	2.8221	2878820.0	1.00000000
65	7000000.0	2.4648	2880316.2	1.00000000
66	6000000.0	2.1089	2881705.3	1.00000000
67	5000000.0	1.7542	2883087.5	1.00000000
68	4000000.0	1.4008	2884463.1	1.00000000
69	3000000.0	1.0487	2885832.4	1.00000000
70	2000000.0	.6979	2887195.7	1.00000000
71	1000000.0	.3483	2888553.4	1.00000000

PROPER

Table 6 gives an example of an input deck used for running PROPER. This particular deck generates a table of fluid properties called FLUTAB3, which covers a temperature range from 5°C to 400°C and a pressure range from 10^{-2} bars to 220 bars. FLUTAB3 was used for most of the simulations carried out so far. Tables 7 and 8 show part of the printout generated by PROPER.

Table 6. Input deck for running PROPER.

```
KPP,07,120.471504,PROCESS
*INPUT 66CCE 13.32.36  25 SEP 78  VIA KPCCDC0
*HOLDOUT
FETCHCS,TAPE3=PSS3ACKUP/DIAB_C/ISOT1,11106.
REWIND,TAPE3.
LIBCOPY,DIAB_O,LGO,PROPERG.
REWIND,LGO.
LINK,X,FL,L=0,PP=(LC=77777).
LIBRITE,DIAB_O,TAPE11/RB,FLUT163,143.
EXIT.
DUMP,0.
FIN.
REWIND,INPUT.
COPYSBF,INPUT,OUTPUT.
```

Table 6 (continued)

```
 0  2  1  6  1  1  1  1
 44 21100.     24 120000.
 26  9
50300.      60000.      1884260.    2093400.    2236320.    2362000.    2487410.    2471212.
2512080.    2533014.    2553948.    2574882.    2591629.2   2601256.8   2603352.2   2602525.
2602530.    2687694.    2679552.    2721420.    2763286.    2805156.    2847024.    2895000.
2925300.    2955000.
130.        260.        340.        380.        1005.51414      220.        300.        420.
1010.46626
  11
0.0         0.1         0.2         0.3         0.4         0.5         0.6         0.7
0.8         0.9         1.0
1.0         0.7         0.465       0.285       0.16        0.085       0.03        0.01
0.005       0.001       0.0
0.0         0.01        0.045       0.1         0.17        0.255       0.38        0.63
0.91        0.99        1.0
 .7000E-02   43
      0.00        1.62        1.62       10.00       20.00       30.00       40.00       50.00
     60.00       70.00       80.00       90.00      100.00      110.00      120.00      130.00
    140.00      150.00      160.00      170.00      180.00      190.00      200.00      210.00
    220.00      230.00      240.00      250.00      260.00      270.00      280.00      290.00
    300.00      310.00      320.00      330.00      340.00      350.00      360.00      370.00
    380.00      390.00      400.00
 .1753E-02 .1665E-02 .3105E-05 .8446E-05 .8853E-05 .9260E-05 .9667E-05 .1007E-04
 .1043E-04 .1089E-04 .1131E-04 .1171E-04 .1211E-04 .1252E-04 .1292E-04 .1333E-04
 .1374E-04 .1414E-04 .1455E-04 .1496E-04 .1537E-04 .1577E-04 .1618E-04 .1659E-04
 .1699E-04 .1740E-04 .1781E-04 .1821E-04 .1862E-04 .1903E-04 .1944E-04 .1984E-04
 .2025E-04 .2066E-04 .2106E-04 .2147E-04 .2188E-04 .2229E-04 .2269E-04 .2310E-04
 .2351E-04 .2391E-04 .2432E-04
 .2294E-01   43
      0.00       10.00       19.38       19.38       20.00       30.00       40.00       50.00
     60.00       70.00       80.00       90.00      100.00      110.00      120.00      130.00
    140.00      150.00      160.00      170.00      180.00      190.00      200.00      210.00
    220.00      230.00      240.00      250.00      260.00      270.00      280.00      290.00
    300.00      310.00      320.00      330.00      340.00      350.00      360.00      370.00
    380.00      390.00      400.00
 .1753E-02 .1300E-02 .1017E-02 .6826E-05 .9351E-05 .9253E-05 .9665E-05 .1007E-04
 .1043E-04 .1089E-04 .1129E-04 .1170E-04 .1211E-04 .1252E-04 .1292E-04 .1333E-04
 .1374E-04 .1414E-04 .1455E-04 .1496E-04 .1537E-04 .1577E-04 .1618E-04 .1659E-04
 .1699E-04 .1740E-04 .1781E-04 .1821E-04 .1862E-04 .1903E-04 .1944E-04 .1984E-04
 .2025E-04 .2066E-04 .2106E-04 .2147E-04 .2188E-04 .2229E-04 .2269E-04 .2310E-04
 .2351E-04 .2391E-04 .2432E-04
 .7241E-01   43
      0.00       10.00       20.00       39.27       39.27       40.00       50.00
     60.00       70.00       80.00       90.00      100.00      110.00      120.00      130.00
    140.00      150.00      160.00      170.00      180.00      190.00      200.00      210.00
    220.00      230.00      240.00      250.00      260.00      270.00      280.00      290.00
    300.00      310.00      320.00      330.00      340.00      350.00      360.00      370.00
    380.00      390.00      400.00
 .1753E-02 .1300E-02 .1002E-02 .7972E-05 .9636E-05 .9630E-05 .9660E-05 .1007E-04
 .1048E-04 .1088E-04 .1129E-04 .1170E-04 .1210E-04 .1251E-04 .1292E-04 .1333E-04
 .1373E-04 .1414E-04 .1455E-04 .1496E-04 .1536E-04 .1577E-04 .1618E-04 .1659E-04
 .1699E-04 .1740E-04 .1781E-04 .1821E-04 .1862E-04 .1903E-04 .1944E-04 .1984E-04
 .2025E-04 .2066E-04 .2106E-04 .2147E-04 .2188E-04 .2229E-04 .2269E-04 .2310E-04
 .2351E-04 .2391E-04 .2432E-04
 .1955E+00   43
      0.00       10.00       20.00       39.27       40.00       50.00       59.15       59.15
     60.00       70.00       80.00       90.00      100.00      110.00      120.00      130.00
    140.00      150.00      160.00      170.00      180.00      190.00      200.00      210.00
    220.00      230.00      240.00      250.00      260.00      270.00      280.00      290.00
    300.00      310.00      320.00      330.00      340.00      350.00      360.00      370.00
```

Table 6 (continued)

```
   380.00    390.00    400.00
.1753E-02 .1300E-02 .1002E-02 .7972E-03 .6514E-03 .5442E-03 .4692E-03 .1043E-04
.1046E-04 .1087E-04 .1128E-04 .1168E-04 .1210E-04 .1250E-04 .1291E-04 .1332E-04
.1373E-04 .1414E-04 .1454E-04 .1495E-04 .1536E-04 .1577E-04 .1617E-04 .1658E-04
.1699E-04 .1740E-04 .1780E-04 .1821E-04 .1862E-04 .1903E-04 .1943E-04 .1984E-04
.2025E-04 .2066E-04 .2106E-04 .2147E-04 .2188E-04 .2229E-04 .2269E-04 .2310E-04
.2351E-04 .2392E-04 .2432E-04
.4647E+00   43
     0.00      10.00     20.00     30.00     40.00     50.00     60.00     70.00
    79.04      79.04     80.00     90.00    100.00    110.00    120.00    130.00
   140.00     150.00    160.00    170.00    180.00    190.00    200.00    210.00
   220.00     230.00    240.00    250.00    260.00    270.00    280.00    290.00
   300.00     310.00    320.00    330.00    340.00    350.00    360.00    370.00
   380.00     390.00    400.00
.1753E-02 .1300E-02 .1002E-02 .7972E-03 .6514E-03 .5442E-03 .4631E-03 .4004E-03
.3553E-03 .1122E-04 .1126E-04 .1167E-04 .1209E-04 .1249E-04 .1289E-04 .1330E-04
.1371E-04 .1412E-04 .1453E-04 .1494E-04 .1535E-04 .1576E-04 .1617E-04 .1657E-04
.1698E-04 .1739E-04 .1780E-04 .1821E-04 .1862E-04 .1902E-04 .1943E-04 .1984E-04
.2025E-04 .2066E-04 .2106E-04 .2147E-04 .2188E-04 .2229E-04 .2270E-04 .2310E-04
.2351E-04 .2392E-04 .2433E-04
.1007E+01   43
     0.00      10.00     20.00     30.00     40.00     50.00     60.00     70.00
    80.00      90.00     99.29     99.29    100.00    110.00    120.00    130.00
   140.00     150.00    160.00    170.00    180.00    190.00    200.00    210.00
   220.00     230.00    240.00    250.00    260.00    270.00    280.00    290.00
   300.00     310.00    320.00    330.00    340.00    350.00    360.00    370.00
   380.00     390.00    400.00
.1753E-02 .1300E-02 .1002E-02 .7972E-03 .6514E-03 .5442E-03 .4631E-03 .4004E-03
.3510E-03 .3113E-03 .2811E-03 .1231E-04 .1204E-04 .1245E-04 .1286E-04 .1327E-04
.1368E-04 .1410E-04 .1451E-04 .1492E-04 .1533E-04 .1574E-04 .1615E-04 .1656E-04
.1697E-04 .1739E-04 .1779E-04 .1820E-04 .1861E-04 .1902E-04 .1943E-04 .1984E-04
.2025E-04 .2066E-04 .2107E-04 .2147E-04 .2188E-04 .2229E-04 .2270E-04 .2311E-04
.2352E-04 .2393E-04 .2434E-04
.1974E+01   43
     0.00      10.00     20.00     30.00     40.00     50.00     60.00     70.00
    80.00      90.00    100.00    110.00    119.21    119.21    120.00    130.00
   140.00     150.00    160.00    170.00    180.00    190.00    200.00    210.00
   220.00     230.00    240.00    250.00    260.00    270.00    280.00    290.00
   300.00     310.00    320.00    330.00    340.00    350.00    360.00    370.00
   380.00     390.00    400.00
.1753E-02 .1299E-02 .1002E-02 .7972E-03 .6514E-03 .5442E-03 .4631E-03 .4005E-03
.3510E-03 .3113E-03 .2790E-03 .2523E-03 .2316E-03 .1277E-04 .1288E-04 .1322E-04
.1363E-04 .1405E-04 .1446E-04 .1488E-04 .1529E-04 .1571E-04 .1612E-04 .1653E-04
.1695E-04 .1736E-04 .1777E-04 .1819E-04 .1860E-04 .1901E-04 .1942E-04 .1983E-04
.2024E-04 .2065E-04 .2107E-04 .2149E-04 .2189E-04 .2230E-04 .2271E-04 .2312E-04
.2353E-04 .2394E-04 .2435E-04
.3454E+01   43
     0.00      10.00     20.00     30.00     40.00     50.00     60.00     70.00
    80.00      90.00    100.00    110.00    120.00    130.00    137.74    137.74
   140.00     150.00    160.00    170.00    180.00    190.00    200.00    210.00
   220.00     230.00    240.00    250.00    260.00    270.00    280.00    290.00
   300.00     310.00    320.00    330.00    340.00    350.00    360.00    370.00
   380.00     390.00    400.00
.1753E-02 .1299E-02 .1002E-02 .7972E-03 .6514E-03 .5442E-03 .4631E-03 .4005E-03
.3510E-03 .3114E-03 .2786E-03 .2523E-03 .2308E-03 .2111E-03 .1944E-03 .1345E-04
.1355E-04 .1397E-04 .1439E-04 .1481E-04 .1523E-04 .1565E-04 .1607E-04 .1649E-04
.1691E-04 .1733E-04 .1774E-04 .1816E-04 .1858E-04 .1899E-04 .1941E-04 .1982E-04
.2024E-04 .2065E-04 .2107E-04 .2149E-04 .2190E-04 .2231E-04 .2272E-04 .2314E-04
.2355E-04 .2396E-04 .2437E-04
.5922E+01   43
     0.00      10.00     20.00     30.00     40.00     50.00     60.00     70.00
    90.00      90.00    100.00    110.00    120.00    130.00    140.00    150.00
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Table 6 (continued)

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   157.58    157.58    160.00    170.00    180.00    190.00    200.00    210.00
   220.00    230.00    240.00    250.00    260.00    270.00    280.00    290.00
   300.00    310.00    320.00    330.00    340.00    350.00    360.00    370.00
   380.00    390.00    400.00
.1753E-02 .1239E-02 .1002E-02 .7972E-03 .6515E-03 .5442E-03 .4632E-03 .4005E-03
.3511E-03 .3114E-03 .2791E-03 .2524E-03 .2300E-03 .2111E-03 .1950E-03 .1811E-03
.1718E-03 .1417E-04 .1427E-04 .1471E-04 .1514E-04 .1557E-04 .1599E-04 .1642E-04
.1685E-04 .1727E-04 .1771E-04 .1812E-04 .1854E-04 .1897E-04 .1939E-04 .1981E-04
.2023E-04 .2065E-04 .2107E-04 .2149E-04 .2191E-04 .2233E-04 .2275E-04 .2316E-04
.2358E-04 .2400E-04 .2441E-04
.3869E+01   43
     0.00     10.00     20.00     30.00     40.00     50.00     60.00     70.00
    80.00     90.00    100.00    110.00    120.00    130.00    140.00    150.00
   160.00    170.00    178.49    178.49    180.00    190.00    200.00    210.00
   220.00    230.00    240.00    250.00    260.00    270.00    280.00    290.00
   300.00    310.00    320.00    330.00    340.00    350.00    360.00    370.00
   380.00    390.00    400.00
.1752E-02 .1239E-02 .1002E-02 .7972E-03 .6515E-03 .5443E-03 .4632E-03 .4006E-03
.3512E-03 .3115E-03 .2792E-03 .2524E-03 .2301E-03 .2112E-03 .1951E-03 .1812E-03
.1691E-03 .1586E-03 .1506E-03 .1490E-04 .1497E-04 .1542E-04 .1596E-04 .1630E-04
.1674E-04 .1718E-04 .1762E-04 .1806E-04 .1849E-04 .1892E-04 .1936E-04 .1979E-04
.2022E-04 .2065E-04 .2107E-04 .2150E-04 .2193E-04 .2235E-04 .2278E-04 .2321E-04
.2363E-04 .2406E-04 .2448E-04
.1480E+02   43
     0.00     10.00     20.00     30.00     40.00     50.00     60.00     70.00
    80.00     90.00    100.00    110.00    120.00    130.00    140.00    150.00
   160.00    170.00    180.00    190.00    196.77    196.77    200.00    210.00
   220.00    230.00    240.00    250.00    260.00    270.00    280.00    290.00
   300.00    310.00    320.00    330.00    340.00    350.00    360.00    370.00
   380.00    390.00    400.00
.1752E-02 .1239E-02 .1002E-02 .7972E-03 .6515E-03 .5443E-03 .4633E-03 .4007E-03
.3512E-03 .3116E-03 .2792E-03 .2525E-03 .2302E-03 .2113E-03 .1952E-03 .1813E-03
.1692E-03 .1587E-03 .1494E-03 .1412E-03 .1361E-03 .1554E-04 .1569E-04 .1615E-04
.1661E-04 .1707E-04 .1752E-04 .1797E-04 .1842E-04 .1887E-04 .1931E-04 .1976E-04
.2020E-04 .2064E-04 .2108E-04 .2152E-04 .2196E-04 .2239E-04 .2283E-04 .2326E-04
.2370E-04 .2413E-04 .2456E-04
.1974E+02   43
     0.00     10.00     20.00     30.00     40.00     50.00     60.00     70.00
    80.00     90.00    100.00    110.00    120.00    130.00    140.00    150.00
   160.00    170.00    180.00    190.00    200.00    210.00    210.77    210.77
   220.00    230.00    240.00    250.00    260.00    270.00    280.00    290.00
   300.00    310.00    320.00    330.00    340.00    350.00    360.00    370.00
   380.00    390.00    400.00
.1752E-02 .1239E-02 .1002E-02 .7972E-03 .6515E-03 .5444E-03 .4634E-03 .4007E-03
.3513E-03 .3117E-03 .2793E-03 .2526E-03 .2303E-03 .2114E-03 .1953E-03 .1814E-03
.1694E-03 .1588E-03 .1495E-03 .1413E-03 .1339E-03 .1273E-03 .1268E-03 .1602E-04
.1647E-04 .1694E-04 .1742E-04 .1789E-04 .1835E-04 .1881E-04 .1927E-04 .1973E-04
.2018E-04 .2064E-04 .2110E-04 .2154E-04 .2198E-04 .2243E-04 .2287E-04 .2332E-04
.2376E-04 .2420E-04 .2464E-04
.2961E+02   43
     0.00     10.00     20.00     30.00     40.00     50.00     60.00     70.00
    80.00     90.00    100.00    110.00    120.00    130.00    140.00    150.00
   160.00    170.00    180.00    190.00    200.00    210.00    220.00    230.00
   232.10    232.10    240.00    250.00    260.00    270.00    280.00    290.00
   300.00    310.00    320.00    330.00    340.00    350.00    360.00    370.00
   380.00    390.00    400.00
.1751E-02 .1239E-02 .1001E-02 .7972E-03 .6516E-03 .5445E-03 .4635E-03 .4009E-03
.3515E-03 .3119E-03 .2795E-03 .2528E-03 .2305E-03 .2116E-03 .1955E-03 .1816E-03
.1696E-03 .1590E-03 .1497E-03 .1415E-03 .1341E-03 .1275E-03 .1216E-03 .1162E-03
.1151E-03 .1678E-04 .1719E-04 .1769E-04 .1819E-04 .1869E-04 .1918E-04 .1966E-04
.2015E-04 .2062E-04 .2110E-04 .2157E-04 .2204E-04 .2251E-04 .2297E-04 .2343E-04
.2389E-04 .2435E-04 .2481E-04
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Table 6 (continued)

.4935E+02	43						
0.00	10.00	20.00	30.00	40.00	50.00	60.00	70.00
80.00	90.00	100.00	110.00	120.00	130.00	140.00	150.00
160.00	170.00	180.00	190.00	200.00	210.00	220.00	230.00
240.00	250.00	260.00	261.96	261.96	270.00	280.00	290.00
300.00	310.00	320.00	330.00	340.00	350.00	360.00	370.00
380.00	390.00	400.00					
.1750E-02	.1298E-02	.1001E-02	.7972E-03	.6517E-03	.5447E-03	.4638E-03	.4012E-03
.3518E-03	.3122E-03	.2799E-03	.2532E-03	.2309E-03	.2121E-03	.1959E-03	.1821E-03
.1700E-03	.1595E-03	.1502E-03	.1419E-03	.1346E-03	.1280E-03	.1221E-03	.1167E-03
.1117E-03	.1072E-03	.1031E-03	.1023E-03	.1794E-04	.1840E-04	.1897E-04	.1952E-04
.2006E-04	.2060E-04	.2112E-04	.2164E-04	.2216E-04	.2267E-04	.2317E-04	.2366E-04
.2417E-04	.2467E-04	.2516E-04					
.6909E+02	43						
0.00	10.00	20.00	30.00	40.00	50.00	60.00	70.00
80.00	90.00	100.00	110.00	120.00	130.00	140.00	150.00
160.00	170.00	180.00	190.00	200.00	210.00	220.00	230.00
240.00	250.00	260.00	270.00	280.00	283.69	283.69	290.00
300.00	310.00	320.00	330.00	340.00	350.00	360.00	370.00
380.00	390.00	400.00					
.1749E-02	.1298E-02	.1001E-02	.7971E-03	.6518E-03	.5449E-03	.4646E-03	.4015E-03
.3522E-03	.3126E-03	.2803E-03	.2536E-03	.2313E-03	.2125E-03	.1964E-03	.1825E-03
.1705E-03	.1599E-03	.1506E-03	.1424E-03	.1351E-03	.1285E-03	.1225E-03	.1171E-03
.1122E-03	.1077E-03	.1036E-03	.9972E-04	.9615E-04	.9499E-04	.1894E-04	.1934E-04
.1996E-04	.2057E-04	.2115E-04	.2173E-04	.2229E-04	.2285E-04	.2340E-04	.2394E-04
.2447E-04	.2501E-04	.2553E-04					
.9869E+02	43						
0.00	10.00	20.00	30.00	40.00	50.00	60.00	70.00
80.00	90.00	100.00	110.00	120.00	130.00	140.00	150.00
160.00	170.00	180.00	190.00	200.00	210.00	220.00	230.00
240.00	250.00	260.00	270.00	280.00	290.00	300.00	308.67
308.67	310.00	320.00	330.00	340.00	350.00	360.00	370.00
380.00	390.00	400.00					
.1747E-02	.1297E-02	.1001E-02	.7971E-03	.6520E-03	.5452E-03	.4644E-03	.4020E-03
.3527E-03	.3131E-03	.2809E-03	.2542E-03	.2319E-03	.2131E-03	.1970E-03	.1831E-03
.1711E-03	.1606E-03	.1513E-03	.1431E-03	.1357E-03	.1292E-03	.1232E-03	.1178E-03
.1129E-03	.1084E-03	.1043E-03	.1004E-03	.9687E-04	.9353E-04	.9040E-04	.8782E-04
.2041E-04	.2050E-04	.2121E-04	.2169E-04	.2254E-04	.2317E-04	.2378E-04	.2439E-04
.2498E-04	.2557E-04	.2614E-04					
.1234E+03	43						
0.00	10.00	20.00	30.00	40.00	50.00	60.00	70.00
80.00	90.00	100.00	110.00	120.00	130.00	140.00	150.00
160.00	170.00	180.00	190.00	200.00	210.00	220.00	230.00
240.00	250.00	260.00	270.00	280.00	290.00	300.00	310.00
320.00	325.40	325.40	330.00	340.00	350.00	360.00	370.00
380.00	390.00	400.00					
.1746E-02	.1296E-02	.1000E-02	.7970E-03	.6521E-03	.5454E-03	.4648E-03	.4024E-03
.3531E-03	.3136E-03	.2814E-03	.2547E-03	.2325E-03	.2136E-03	.1975E-03	.1837E-03
.1717E-03	.1611E-03	.1519E-03	.1436E-03	.1363E-03	.1297E-03	.1238E-03	.1184E-03
.1135E-03	.1091E-03	.1049E-03	.1011E-03	.9747E-04	.9414E-04	.9101E-04	.8805E-04
.8524E-04	.8378E-04	.2172E-04	.2207E-04	.2280E-04	.2350E-04	.2418E-04	.2483E-04
.2547E-04	.2610E-04	.2672E-04					
.1480E+03	43						
0.00	10.00	20.00	30.00	40.00	50.00	60.00	70.00
80.00	90.00	100.00	110.00	120.00	130.00	140.00	150.00
160.00	170.00	180.00	190.00	200.00	210.00	220.00	230.00
240.00	250.00	260.00	270.00	280.00	290.00	300.00	310.00
320.00	330.00	339.61	339.61	340.00	350.00	360.00	370.00
380.00	390.00	400.00					
.1745E-02	.1295E-02	.9995E-03	.7970E-03	.6522E-03	.5457E-03	.4651E-03	.4027E-03
.3536E-03	.3141E-03	.2818E-03	.2552E-03	.2330E-03	.2141E-03	.1981E-03	.1842E-03
.1722E-03	.1617E-03	.1524E-03	.1442E-03	.1369E-03	.1303E-03	.1244E-03	.1190E-03

Table 6 (continued)

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.1141E-03 .1096E-03 .1055E-03 .1016E-03 .9808E-04 .9475E-04 .9162E-04 .8867E-04
.8586E-04 .8316E-04 .8170E-04 .2316E-04 .2315E-04 .2395E-04 .2468E-04 .2538E-04
.2606E-04 .2672E-04 .2737E-04
.1727E+03   43
     0.00      10.00      20.00      30.00      40.00      50.00      60.00      70.00
    80.00      90.00     100.00     110.00     120.00     130.00     140.00     150.00
   160.00     170.00     180.00     190.00     200.00     210.00     220.00     230.00
   240.00     250.00     260.00     270.00     280.00     290.00     300.00     310.00
   320.00     330.00     340.00     350.00     352.07     352.07     360.00     370.00
   380.00     390.00     400.00
.1743E-02 .1295E-02 .9996E-03 .7973E-03 .6524E-03 .5459E-03 .4654E-03 .4031E-03
.3540E-03 .3145E-03 .2823E-03 .2557E-03 .2335E-03 .2147E-03 .1986E-03 .1848E-03
.1728E-03 .1622E-03 .1533E-03 .1449E-03 .1375E-03 .1303E-03 .1250E-03 .1196E-03
.1147E-03 .1102E-03 .1061E-03 .1022E-03 .9868E-04 .9536E-04 .9223E-04 .8926E-04
.8648E-04 .8381E-04 .8123E-04 .7875E-04 .7824E-04 .2480E-04 .2539E-04 .2610E-04
.2680E-04 .2746E-04 .2815E-04
.1974E+03   43
     0.00      10.00      20.00      30.00      40.00      50.00      60.00      70.00
    80.00      90.00     100.00     110.00     120.00     130.00     140.00     150.00
   160.00     170.00     180.00     190.00     200.00     210.00     220.00     230.00
   240.00     250.00     260.00     270.00     280.00     290.00     300.00     310.00
   320.00     330.00     340.00     350.00     360.00     363.10     363.10     370.00
   380.00     390.00     400.00
.1742E-02 .1294E-02 .9993E-03 .7969E-03 .6525E-03 .5462E-03 .4657E-03 .4035E-03
.3544E-03 .3156E-03 .2828E-03 .2562E-03 .2340E-03 .2152E-03 .1991E-03 .1853E-03
.1733E-03 .1628E-03 .1535E-03 .1453E-03 .1380E-03 .1315E-03 .1255E-03 .1202E-03
.1153E-03 .1108E-03 .1067E-03 .1029E-03 .9929E-04 .9597E-04 .9285E-04 .8990E-04
.8710E-04 .8443E-04 .8186E-04 .7933E-04 .7696E-04 .7622E-04 .2670E-04 .2714E-04
.2779E-04 .2844E-04 .2910E-04
.2244E+03   43END
     0.00      10.00      20.00      30.00      40.00      50.00      60.00      70.00
    80.00      90.00     100.00     110.00     120.00     130.00     140.00     150.00
   160.00     170.00     180.00     190.00     200.00     210.00     220.00     230.00
   240.00     250.00     260.00     270.00     280.00     290.00     300.00     310.00
   320.00     330.00     340.00     350.00     360.00     370.00     373.82     373.82
   380.00     390.00     400.00
.1740E-02 .1293E-02 .9990E-03 .7969E-03 .6527E-03 .5464E-03 .4661E-03 .4039E-03
.3549E-03 .3155E-03 .2833E-03 .2569E-03 .2346E-03 .2158E-03 .1997E-03 .1859E-03
.1739E-03 .1634E-03 .1542E-03 .1460E-03 .1386E-03 .1321E-03 .1262E-03 .1208E-03
.1159E-03 .1114E-03 .1073E-03 .1035E-03 .9995E-04 .9663E-04 .9352E-04 .9056E-04
.8773E-04 .8512E-04 .8255E-04 .8007E-04 .7766E-04 .7529E-04 .7439E-04 .2915E-04
.2940E-04 .2989E-04 .3044E-04
.00539088    13
0.         0.         50.        100.       150.       200.       250.       300.
350.       400.
.569       .0176      .0212      .0248      .0287      .0332      .0382      .0434
.0490      .0549
.0229378     11
0.         20.        40.        50.        100.       150.       200.       250.
300.       350.       400.
.569       .602       .0188      .0212      .0248      .0287      .0332      .0382
.0434      .0490      .0549
.072412      11
0.         40.        48.        50.        100.       150.       200.       250.
300.       351.       400.
.569       .630       .0201      .0212      .0248      .0287      .0332      .0382
.0434      .0490      .0549
.195545      11
0.         50.        60.        80.        100.       150.       200.       250.
300.       350.       400.
.569       .643       .653       .0216      .0248      .0287      .0332      .0382
.0434      .0490      .0549
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Table 6 (continued)

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.464737      11
0.           50.          80.          80.          100.         150.         200.         250.
300.         350.         400.
.569         .643         .669         .0231        .0248        .0297        .0332        .0382
.0434        .0490        .0549
1.01972      11
0.           50.          99.63        99.63        100.         150.         200.         250.
300.         350.         400.
.569         .643         .650         .0246        .0248        .0287        .0332        .0382
.0434        .0490        .0549
5.09858      11
0.           50.          100.         150.         151.86       151.86       200.         250.
300.         350.         400.
.569         .644         .681         .687         .687         .0318        .0333        .0386
.0438        .0494        .0553
10.19716     11
0.           50.          100.         150.         179.91       179.91       200.         250.
300.         350.         400.
.570         .644         .681         .687         .677         .0335        .0351        .0393
.0444        .0499        .0557
25.49291     11
0.           50.          100.         150.         200.         223.99       223.99       250.
300.         350.         400.
.571         .645         .682         .688         .665         .638         .0425        .0429
.0465        .0516        .0572
50.98581     11
0.           50.          100.         150.         200.         250.         263.99       263.99
300.         350.         400.
.573         .647         .684         .690         .668         .618         .590         .0543
.0525        .0554        .0602
76.47872     11
0.           50.          100.         150.         200.         250.         290.59       290.59
300.         350.         400.
.575         .649         .686         .691         .670         .622         .549         .0664
.0637        .0608        .0639
101.9716     11
0.           50.          100.         150.         200.         250.         300.         311.86
311.86       350.         400.
.577         .651         .686         .693         .672         .625         .545         .512
.0800        .0688        .0686
152.9574     11
0.           50.          100.         150.         200.         250.         300.         342.24
342.24       350.         400.
.581         .655         .691         .696         .676         .633         .559         .459
.120         .104         .0822
203.9432     11
0.           50.          100.         150.         200.         250.         300.         350.
365.81       365.81       400.
.585         .659         .695         .700         .681         .639         .571         .454
.422         .162         .107
224.4        11END
0.           50.0         100.0        150.0        200.0        250.0        300.0        350.0
373.80       373.80       400.0
.596         .660         .696         .701         .682         .642         .576         .463
.385         .138         .124
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Table 7. Printout from PROPER (i).

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NE =  70
 .211000E+05  .500000E+05  .800000E+05  .181935E+06  .261054E+06  .326477E+06  .385056E+06  .439527E+06  .491191E+06  .540971E+06
 .589259E+06  .636412E+06  .682755E+06  .728442E+06  .773558E+06  .818205E+06  .862462E+06  .906337E+06  .950024E+06  .993408E+06
 .103653E+07  .107935E+07  .112166E+07  .116400E+07  .120596E+07  .124739E+07  .128827E+07  .132861E+07  .136809E+07  .140677E+07
 .144447E+07  .148094E+07  .151621E+07  .155017E+07  .158268E+07  .161362E+07  .164301E+07  .167033E+07  .169584E+07  .171974E+07
 .174099E+07  .176054E+07  .177994E+07  .179920E+07  .182062E+07  .183444E+07  .186406E+07  .209340E+07  .223632E+07  .236200E+07
 .240741E+07  .247021E+07  .251208E+07  .253301E+07  .255395E+07  .257488E+07  .259163E+07  .260126E+07  .260253E+07  .260253E+07
 .260335E+07  .263768E+07  .267955E+07  .272142E+07  .276329E+07  .280516E+07  .284702E+07  .289500E+07  .292500E+07  .295500E+07
NE =  94
 .211000E+05  .500000E+05  .800000E+05  .120000E+06  .181935E+06  .217770E+06  .261054E+06  .289752E+06  .326477E+06  .351300E+06
 .385056E+06  .408409E+06  .439527E+06  .461594E+06  .491191E+06  .512360E+06  .540971E+06  .561481E+06  .589259E+06  .609273E+06
 .636412E+06  .656037E+06  .682755E+06  .702044E+06  .728442E+06  .747461E+06  .773558E+06  .792381E+06  .818205E+06  .836877E+06
 .862462E+06  .881063E+06  .906387E+06  .924825E+06  .950124E+06  .968360E+06  .993408E+06  .101162E+07  .103653E+07  .105457E+07
 .107935E+07  .109725E+07  .112156E+07  .113966E+07  .116408E+07  .118177E+07  .120598E+07  .122347E+07  .124739E+07  .126465E+07
 .128827E+07  .132361E+07  .136809E+07  .140677E+07  .144447E+07  .148094E+07  .151621E+07  .155017E+07  .158264E+07  .161362E+07
 .164301E+07  .167033E+07  .169584E+07  .171974E+07  .174099E+07  .176054E+07  .177994E+07  .179920E+07  .182062E+07  .183444E+07
 .186406E+07  .209340E+07  .223632E+07  .236200E+07  .240741E+07  .247021E+07  .251208E+07  .253301E+07  .255395E+07  .257488E+07
 .259163E+07  .260126E+07  .260253E+07  .260253E+07  .260335E+07  .263768E+07  .267955E+07  .272142E+07  .276329E+07  .280516E+07
 .284702E+07  .289500E+07  .292500E+07  .295500E+07
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Table 7 (continued)

INTERSECTION OF BASIS ENERGIES WITH SATURATION LINE				. . . WITH HIGHEST ISOBAR	
ENERGY	DENSITY	PRESSURE	TEMPERATURE	DENSITY	TEMPERATURE
.211000E+05	.999969E+03	.873429E+03	.502244E+01	1010.46626E1275	5.0502907707858
	-.777700E+77	-.777700E+77	-.777700E+77		
.500000E+05	.999525E+03	.140931E+04	.119114E+02	1009.6152637748	12.083759331231
	-.777700E+77	-.777700E+77	-.777700E+77		
.800000E+05	.999442E+03	.221935E+04	.190776E+02	1008.1960956277	19.382056097307
	-.777700E+77	-.777700E+77	-.777700E+77		
.120000E+06	.998091E+03	.395020E+04	.286458E+02	1005.5141416698	29.185953566466
	-.777700E+77	-.777700E+77	-.777700E+77		
.181935E+06	.998046E+03	.599540E+04	.434671E+02	999.96891724979	44.145874670316
	-.777700E+77	-.777700E+77	-.777700E+77		
.217770E+06	.997866E+03	.137241E+05	.523390E+02	996.09071642305	52.638712034052
	-.777700E+77	-.777700E+77	-.777700E+77		
.261054E+06	.998187E+03	.223477E+05	.623855E+02	990.84789838947	63.328889534767
	-.777700E+77	-.777700E+77	-.777700E+77		
.289752E+06	.998089E+03	.302252E+05	.692387E+02	987.06603251540	70.277758942674
	-.777700E+77	-.777700E+77	-.777700E+77		
.326477E+06	.998867E+03	.433347E+05	.779996E+02	981.87015319363	79.160453671245
	-.777700E+77	-.777700E+77	-.777700E+77		
.351800E+06	.996952E+03	.557846E+05	.840336E+02	978.08891467096	85.278830022056
	-.777700E+77	-.777700E+77	-.777700E+77		
.385056E+06	.996373E+03	.757218E+05	.919467E+02	972.86654308062	93.303678761547
	-.777700E+77	-.777700E+77	-.777700E+77		
.408409E+06	.995901E+03	.925687E+05	.974950E+02	969.05252064634	98.931790730547
	-.777700E+77	-.777700E+77	-.777700E+77		
.439327E+06	.994551E+03	.122321E+06	.104877E+03	963.77297266713	106.42031336329
	-.777700E+77	-.777700E+77	-.777700E+77		
.461594E+06	.950612E+03	.143789E+06	.110101E+03	959.98136737264	111.72233654028
	-.777700E+77	-.777700E+77	-.777700E+77		
.491191E+06	.945148E+03	.191404E+06	.117093E+03	954.55090652460	118.82156218011
	-.777700E+77	-.777700E+77	-.777700E+77		
.512360E+06	.941131E+03	.212531E+06	.122094E+03	950.61243509632	123.83974895066
	-.777700E+77	-.777700E+77	-.777700E+77		
.540971E+06	.935553E+03	.261113E+06	.128814E+03	945.14889372571	130.72621294688
	-.777700E+77	-.777700E+77	-.777700E+77		
.561481E+06	.931443E+03	.301276E+06	.133626E+03	941.13098404686	135.61632161744
	-.777700E+77	-.777700E+77	-.777700E+77		
.589259E+06	.925759E+03	.362753E+06	.140127E+03	935.55336152255	142.22328563263
	-.777700E+77	-.777700E+77	-.777700E+77		
.609273E+06	.921558E+03	.413421E+06	.144797E+03	931.44752147094	146.97288234565
	-.777700E+77	-.777700E+77	-.777700E+77		
.636412E+06	.915728E+03	.490438E+06	.151110E+03	925.75907758624	153.39658381664
	-.777700E+77	-.777700E+77	-.777700E+77		
.656937E+06	.911429E+03	.553213E+06	.155661E+03	921.55751333997	158.02896537854
	-.777700E+77	-.777700E+77	-.777700E+77		
.692755E+06	.903444E+03	.643429E+06	.161836E+03	915.72755563861	164.31851813447
	-.777700E+77	-.777700E+77	-.777700E+77		
.782049E+06	.901841E+03	.724132E+06	.166298E+03	911.42685360716	168.84574883993
	-.777700E+77	-.777700E+77	-.777700E+77		
.723442E+06	.894394E+03	.838971E+06	.172338E+03	905.44387244819	175.02153821928
	-.777700E+77	-.777700E+77	-.777700E+77		
.747461E+06	.893359E+03	.923626E+06	.176686E+03	901.04119378019	179.45527366809
	-.777700E+77	-.777700E+77	-.777700E+77		
.773558E+06	.884075E+03	.106613E+07	.182630E+03	894.89441388537	185.51875093739
	-.777700E+77	-.777700E+77	-.777700E+77		
.792391E+06	.879442E+03	.117339E+07	.186699E+03	890.38341471537	189.87692842628
	-.777700E+77	-.777700E+77	-.777700E+77		
.816205E+06	.872973E+03	.133356E+07	.192731E+03	884.07476215317	195.63053256429
	-.777700E+77	-.777700E+77	-.777700E+77		

Table 8. Printout from PROPER (ii).

28	792381.	186.600	.116851E+07	.09513	.60865	.693641E+04	.658590E+05	879.77001	5.95845	791061.	2586065.	
29	816205.	192.421	.132467E+07	.09156	.60830	.716591E+04	.649063E+05	873.31720	6.73761	816831.	2584374.	
30	836877.	196.614	.144913E+07	.08655	.60810	.734085E+04	.643727E+05	868.56868	7.34665	835473.	2591560.	
31	862462.	202.334	.163397E+07	.07955	.60791	.755265E+04	.635697E+05	861.93564	8.25138	861032.	2594245.	
32	881008.	206.464	.177802E+07	.07431	.60784	.771246E+04	.630025E+05	857.05026	8.95716	879572.	2595977.	
33	906387.	212.090	.199062E+07	.06669	.60783	.793174E+04	.622392E+05	850.23316	10.00086	904964.	2598024.	
34	924828.	216.162	.215567E+07	.06132	.60789	.807477E+04	.616823E+05	845.19481	10.81337	923431.	2599292.	
35	950024.	221.693	.239861E+07	.05344	.60807	.827774E+04	.609403E+05	838.17725	12.01007	948692.	2600675.	
36	968360.	225.713	.258522E+07	.04751	.60777	.843133E+04	.604145E+05	832.98496	12.93830	967097.	2601469.	
37	993403.	231.161	.285931E+07	.03909	.60654	.864941E+04	.597138E+05	825.74869	14.30500	992277.	2602175.	
38	1011620.	235.109	.306936E+07	.03277	.60569	.878657E+04	.591824E+05	820.40290	15.35766	1010612.	2602473.	
39	1036530.	240.478	.337656E+07	.02375	.60465	.896792E+04	.584608E+05	812.93238	16.91146	1035740.	2602463.	
40	1054567.	244.346	.361252E+07	.01697	.60398	.910329E+04	.579517E+05	807.42201	18.11053	1053970.	2602194.	
41	1079350.	249.635	.395387E+07	.00729	.60316	.929513E+04	.572698E+05	799.70961	19.86307	1079074.	2601460.	
42	1097245.	253.430	.421566E+07	0.00000	.60267	.943783E+04	.567903E+05	794.01830	21.22219	1097245.	2600607.	
43	1121857.	261.254	.144375E+03	0.00000	.61544	.952455E+04	0.	794.01830	0.00000	1121857.	0.	
44	1139659.	266.970	.220[illegible]E+03	0.00000	.61959	.955567E+04	0.	794.01830	0.00000	1139659.	0.	

ND = 55 DENSITY = 799.62463

NE	ENERGY	T	P	Z	K	MOB(L)	MOB(V)	DEN(L)	DEN(V)	ENE(L)	ENE(V)
1	21100.	5.021	.373361E+03	.20035	.46521	.648975E+03	.121314E+06	999.96896	.00681	21096.	2382462.
2	50000.	11.910	.143916E+04	.20300	.47459	.775459E+03	.117333E+06	999.52543	.01071	49994.	2391946.
3	80000.	19.075	.221957E+04	.19913	.48463	.972653E+03	.113461E+06	998.44269	.01646	79990.	2401773.
4	120000.	28.642	.334939E+04	.19724	.49679	.117503E+04	.108691E+06	996.09180	.02838	119964.	2414836.
5	181935.	43.459	.690199E+04	.19300	.51555	.161221E+04	.102056E+06	990.85110	.06105	181902.	2434853.
6	217770.	52.027	.137162E+05	.18992	.52554	.185951E+04	.985865E+05	987.87152	.09169	217721.	2446264.
7	261054.	62.367	.223258E+05	.18564	.53736	.221840E+04	.947168E+05	981.87958	.14491	260980.	2459826.
8	289752.	69.218	.301971E+05	.18251	.54392	.242979E+04	.923378E+05	978.10176	.19237	289657.	2468665.
9	326477.	77.964	.437799E+05	.17814	.55258	.276674E+04	.894659E+05	972.88570	.27243	326347.	2479751.
10	351800.	83.996	.557055E+05	.17432	.55793	.296619E+04	.876161E+05	969.07664	.34142	351641.	2487247.
11	385056.	91.898	.755818E+05	.17043	.56463	.324505E+04	.855106E+05	963.80636	.45424	384852.	2496068.
12	408409.	97.438	.927168E+05	.16716	.56946	.347399E+04	.837654E+05	959.94193	.54935	408168.	2503469.
13	439527.	104.807	.120048E+06	.16247	.57425	.374010E+04	.818280E+05	954.60212	.70046	439232.	2512060.
14	461594.	110.021	.143377E+06	.15903	.57720	.393048E+04	.805178E+05	950.67442	.82708	461256.	2517986.
15	491191.	116.999	.188844E+06	.15420	.58133	.421776E+04	.788291E+05	945.22377	1.02761	490768.	2525693.
16	512360.	121.978	.211815E+06	.15062	.58431	.441226E+04	.776906E+05	941.21601	1.19154	511907.	2531052.
17	540971.	128.691	.260174E+06	.14561	.58849	.465955E+04	.762316E+05	935.65674	1.44476	540446.	2538068.
18	561481.	133.490	.300107E+06	.14188	.59158	.485403E+04	.752217E+05	931.56432	1.65172	560901.	2542923.
19	589259.	139.973	.361120E+06	.13667	.59586	.511755E+04	.739041E+05	925.89683	1.96526	588600.	2549281.
20	609275.	144.630	.411509E+06	.13277	.59904	.528644E+04	.729926E+05	921.70811	2.22185	608356.	2553669.
21	636412.	150.924	.488442E+06	.12731	.60346	.555331E+04	.717955E+05	915.90210	2.61065	635613.	2559372.
22	656037.	155.463	.550244E+06	.12325	.60515	.572610E+04	.709564E+05	911.61930	2.92103	655179.	2563325.
23	682755.	161.619	.644845E+06	.11751	.60701	.596286E+04	.698763E+05	905.65713	3.39307	681815.	2568429.
24	702048.	166.051	.719951E+06	.11325	.60841	.612702E+04	.691288E+05	901.27171	3.76605	701051.	2571939.
25	728442.	172.092	.834041E+06	.10722	.61044	.636591E+04	.681355E+05	895.14688	4.33002	727370.	2576458.
26	747461.	176.430	.923974E+06	.10275	.61197	.654929E+04	.674396E+05	890.69269	4.77306	746338.	2579527.
27	773558.	182.366	.105962E+07	.09648	.61395	.677829E+04	.665043E+05	884.36420	5.43935	772372.	2583448.
28	792381.	186.622	.116608E+07	.09169	.61273	.693723E+04	.658557E+05	879.74643	5.96122	791156.	2586077.
29	816205.	192.445	.132535E+07	.08500	.61237	.716689E+04	.649832E+05	873.29036	6.74097	816937.	2589387.
30	836677.	196.639	.144993E+07	.08001	.61217	.734195E+04	.643690E+05	868.53913	7.35058	835588.	2591572.
31	862462.	202.363	.163493E+07	.07295	.61188	.755373E+04	.635657E+05	861.90624	8.25610	861159.	2594257.
32	881008.	206.495	.177914E+07	.06766	.61190	.771367E+04	.629984E+05	857.01366	8.96264	879709.	2595989.
33	906387.	212.124	.199195E+07	.06017	.61169	.793290E+04	.622346E+05	850.19204	10.00738	905116.	2598035.
34	924828.	216.197	.215720E+07	.05455	.61134	.807605E+04	.616775E+05	845.14991	10.82060	923554.	2599301.
35	950024.	221.737	.239981E+07	.04659	.61213	.827920E+04	.609357E+05	838.12708	12.01895	948871.	2600688.
36	968360.	225.751	.258726E+07	.04061	.61181	.843294E+04	.604092E+05	832.93037	12.94849	967268.	2601475.
37	993403.	231.206	.286169E+07	.03211	.61057	.865125E+04	.597081E+05	825.68795	14.31695	992446.	2602178.
38	1011620.	235.157	.307203E+07	.02573	.60971	.878815E+04	.591760E+05	820.33727	15.37111	1010833.	2602471.
39	1036530.	240.536	.338[illegible]E+07	.01662	.60867	.896969E+04	.584543E+05	812.85933	16.92736	1035981.	2602459.
40	1054567.	244.401	.361583E+07	.00977	.60799	.910523E+04	.579446E+05	807.34459	18.12738	1054226.	2602190.

Table 8 (continued)

41	1079350.	249.693	.395765E+07	0.00000	.60716	.329728E+04	.572624E+05	799.62463	19.88249	1079350.	2601451.
42	1097245.	255.351	.114561E+08	0.00000	.61941	.937678E+04	0.	799.62463	0.00000	1097245.	0.
43	1121857.	263.216	.220000E+08	0.00000	.62455	.942715E+04	0.	799.62463	0.00000	1121857.	0.

ND = 56 DENSITY = 807.24166

NE	ENERGY	T	P	S	K	MOB(L)	MOB(V)	DEN(L)	DEN(V)	ENE(L)	ENE(V)
1	21100.	5.022	.673364E+03	.19273	.46947	.648976E+03	.121314E+06	999.96896	.00691	21096.	2382462.
2	50000.	11.910	.140917E+04	.19238	.47893	.775468E+03	.117333E+06	999.92542	.01071	49994.	2391940.
3	80000.	19.075	.221958E+04	.19150	.48907	.972656E+03	.113461E+06	998.44267	.01646	79991.	2401773.
4	120000.	28.642	.394542E+04	.18960	.50134	.117504E+04	.108691E+06	996.09175	.02038	119985.	2414836.
5	181935.	43.460	.890207E+04	.18532	.52027	.161222E+04	.102855E+06	990.85094	.06106	181982.	2434854.
6	217770.	52.023	.137166E+05	.18220	.53035	.185953E+04	.985863E+05	987.07124	.09169	217724.	2446264.
7	261854.	62.368	.223307E+05	.17789	.54227	.221842E+04	.947165E+05	981.87910	.14492	260964.	2459827.
8	299752.	69.217	.301936E+05	.17472	.54869	.242983E+04	.923374E+05	978.10189	.19238	289662.	2468666.
9	326477.	77.970	.437828E+05	.17031	.55763	.276681E+04	.894654E+05	972.88469	.27245	326354.	2479753.
10	351680.	83.998	.557098E+05	.16706	.56302	.296626E+04	.876155E+05	969.87533	.34144	351649.	2487249.
11	385056.	91.961	.755655E+05	.16252	.56978	.324515E+04	.853098E+05	963.80451	.45428	384863.	2496871.
12	408409.	97.441	.927277E+05	.15916	.57466	.347414E+04	.837645E+05	959.93964	.55001	408182.	2503472.
13	439527.	104.611	.120064E+06	.15446	.57348	.374024E+04	.818269E+05	954.59515	.70055	439249.	2512065.
14	461594.	110.026	.143401E+06	.15100	.58248	.393067E+04	.805167E+05	950.67075	.82722	461276.	2517992.
15	491191.	117.804	.180878E+06	.14613	.58658	.421881E+04	.788278E+05	945.21914	1.02779	490812.	2525695.
16	512360.	121.384	.211860E+06	.14252	.59962	.441249E+04	.776691E+05	941.21255	1.19178	511935.	2531055.
17	540971.	128.699	.260235E+06	.13745	.59384	.465986E+04	.762259E+05	935.65804	1.44507	540480.	2538076.
18	561441.	133.459	.300165E+06	.13368	.59695	.485441E+04	.752136E+05	931.55655	1.65212	560939.	2542532.
19	589259.	139.984	.361223E+06	.12842	.60127	.511793E+04	.739028E+05	925.88749	1.96579	588645.	2549292.
20	609273.	144.641	.411637E+06	.12448	.60447	.528689E+04	.729903E+05	921.65740	2.22251	608678.	2553680.
21	636412.	150.938	.488627E+06	.11896	.60893	.553387E+04	.717929E+05	915.88916	2.61158	635673.	2559384.
22	656037.	155.478	.550471E+06	.11485	.61063	.572677E+04	.709536E+05	911.60464	2.92217	655244.	2563338.
23	682755.	161.637	.645134E+06	.10905	.61251	.596348E+04	.698733E+05	905.63596	3.39450	681891.	2568443.
24	702048.	166.070	.726360E+06	.10474	.61393	.612775E+04	.691257E+05	901.25245	3.76778	701134.	2571954.
25	728442.	172.114	.834477E+06	.09885	.61598	.636680E+04	.681322E+05	895.12458	4.33217	727465.	2576474.
26	747461.	176.454	.924455E+06	.09412	.61753	.655032E+04	.674358E+05	890.62785	4.77562	746442.	2579543.
27	773556.	182.387	.106026E+07	.08771	.61861	.677925E+04	.665052E+05	884.33569	5.44249	772489.	2583464.
28	792381.	186.651	.116664E+07	.08294	.61828	.693833E+04	.658514E+05	879.71488	5.96493	791263.	2586094.
29	818205.	192.477	.132627E+07	.07617	.61791	.716819E+04	.649785E+05	873.25445	6.74547	817079.	2589484.
30	836877.	196.674	.145101E+07	.07113	.61778	.734342E+04	.643640E+05	868.49559	7.35584	835741.	2591589.
31	862462.	202.401	.163622E+07	.06398	.61750	.755517E+04	.635605E+05	861.86155	8.26242	861330.	2594274.
32	881886.	206.535	.178063E+07	.05862	.61742	.771528E+04	.629929E+05	856.96467	8.96938	879893.	2596805.
33	906387.	212.169	.199373E+07	.05104	.61741	.793444E+04	.622284E+05	850.13730	10.01610	905318.	2598049.
34	924822.	216.245	.215924E+07	.04536	.61746	.807776E+04	.616710E+05	845.08983	10.83067	923811.	2599314.
35	950024.	221.789	.240221E+07	.03729	.61765	.828116E+04	.609269E+05	838.05992	12.03804	949110.	2600698.
36	968360.	225.906	.259000E+07	.03123	.61731	.843510E+04	.604020E+05	832.45738	12.96213	967543.	2601483.
37	993408.	231.266	.286428E+07	.02262	.61604	.865372E+04	.597804E+05	825.60663	14.33294	992765.	2602183.
38	1011620.	235.220	.307560E+07	.01616	.61517	.879025E+04	.591673E+05	820.24941	15.38911	1011131.	2602473.
39	1036530.	240.565	.338426E+07	.00693	.61412	.897206E+04	.584449E+05	812.76163	16.94862	1036385.	2602454.
40	1054567.	244.473	.362028E+07	0.00000	.61342	.910779E+04	.579352E+05	807.24166	18.14977	1054567.	2602185.
41	1079350.	252.285	.141966E+08	0.00000	.62833	.920943E+04	0.	807.24166	0.00000	1079350.	0.
42	1097245.	257.981	.228000E+08	0.00000	.63146	.924844E+04	0.	807.24166	0.00000	1097245.	0.

ND = 57 DENSITY = 812.69341

NE	ENERGY	T	P	S	K	MOB(L)	MOB(V)	DEN(L)	DEN(V)	ENE(L)	ENE(V)
1	21100.	5.022	.673366E+03	.19728	.47252	.648976E+03	.121314E+06	999.96896	.00681	21096.	2382462.
2	50000.	11.910	.140918E+04	.19692	.48284	.775461E+03	.117333E+06	999.92541	.01071	49994.	2391940.
3	80000.	19.076	.221959E+04	.19604	.49225	.972659E+03	.113461E+06	998.44265	.01646	79991.	2401773.
4	120000.	28.642	.394545E+04	.19412	.50459	.117504E+04	.108691E+06	996.09171	.02038	119985.	2414836.
5	181935.	43.460	.890219E+04	.17981	.52364	.161222E+04	.102855E+06	990.85083	.06106	181934.	2434854.
6	217770.	52.023	.137169E+05	.17668	.53379	.185955E+04	.985861E+05	987.07185	.09169	217726.	2446265.
7	261854.	62.368	.223313E+05	.17253	.54579	.221844E+04	.947163E+05	981.87877	.14492	260966.	2459828.

CYCN

Garg's Problem

A rather severe test of the accuracy of SHAFT78 is provided by a problem which was recently studied by Garg.[44] The reservoir is a sector of a cylinder with finite height and a very large radius. Fluid is withdrawn at the center at a constant rate. A complete specification of the problem is given in Garg's paper, ref. (44).

The reservoir is initially filled with liquid subcooled water. As production proceeds, water begins to boil and several elements cross the saturation line.

A satisfactory computation can be obtained using the default values for time step controls. Table 9 shows the input deck used. The calculation terminated after 460 time steps, because machine time reached 60 seconds. Table 10 shows part of the printout after 460 cycles, and Figure 13 compares the results of the SHAFT78 computation with those obtained by Garg. A more complete discussion of the problem is given in refs. 15 and 16.

Table 9. Input deck for running Garg's problem with CYCN.

```
KPGA,nN,400,300000,467301,PRUEBB
$HOLDOUT
LGOCOPY,DIABLO,LGO,CYCNG.
FETCHGO,TABLE=P88BACKUP/DIABLO/FLUTABS,ST009.
REWIND,LGO.
$FL,170000.
LGNK,Y,LMO,PP=(PL=777777).
$KIT.
DUMP,0.
PGN.
REWIND,INPUT.
COPYSBF,INPUT,OUTPUT.
```

Table 9 (continued)

```
*TITLE
*CYCLES
  1  99    1        .30        .05
RFLOW      2650.      .20            1.E+14                      5.25      1000.

START
 0 2 600  60  2000000000102220     STEP      F  0      RFLOW

   300.00          9.E6                                50.
ELEMENT INFORMATION
F   0          RFLOW       4.90
F   1          RFLOW      16.73
F   2          RFLOW      28.54
F   3          RFLOW      38.36
F   4          RFLOW      44.18
F   5          RFLOW      58.00
F   6          RFLOW      63.81
F   7          RFLOW      73.63
F   8          RFLOW      83.45
F   9          RFLOW      93.27
F  10          RFLOW     103.08
F  11          RFLOW     136.66
F  12          RFLOW     182.65
F  13          RFLOW     246.05
F  14          RFLOW     334.03
F  15          RFLOW     456.42
F  16          RFLOW     628.01
F  17          RFLOW     869.11
F  18          RFLOW    1209.42
F  19          RFLOW    1690.92
F  20          RFLOW    2373.73
F  21          RFLOW    3346.06
F  22          RFLOW    4730.18
F  23          RFLOW    6706.26
F  24          RFLOW    9531.08
F  25          RFLOW   13573.65
F  26          RFLOW   19365.24
F  27          RFLOW   27667.17
F  28          RFLOW   39578.97
F  29          RFLOW   56680.72
F  30          RFLOW   81244.78
F  31          RFLOW  116539.35
F  32          RFLOW  167274.56
F  33          RFLOW  240224.09
F  34          RFLOW  344144.78
F  35          RFLOW  496069.8
F  36          RFLOW  713221.9
F  37          RFLOW 1025682.
F  38          RFLOW 1475372.
F  39          RFLOW 2122505.
F  40          RFLOW 3058196.
F  41          RFLOW 4395241.
F  42          RFLOW 6325806.
F  43          RFLOW 8663520.
F  44          RFLOW 13348165.
F  45          RFLOW 18867637.
F  46          RFLOW 27162454.
F  47          RFLOW 39105615.
F  48          RFLOW 56301977.
F  49          RFLOW 81062899.
```

Table 9 (continued)

```
CONNE
FI   0F   1              1     .5       .5        9.82
FI   1F   2              1     .5       .5       19.63
FI   2F   3              1     .5       .5       29.45
FI   3F   4              1     .5       .5       39.27
FI   4F   5              1     .5       .5       49.09
FI   5F   6              1     .5       .5       58.90
FI   6F   7              1     .5       .5       68.72
FI   7F   8              1     .5       .5       78.54
FI   8F   9              1     .5       .5       88.36
FI   9F  10              1     .5       .5       98.17
FI  10F  11              1     .5       .6      107.99
FI  11F  12              1     .6       .72     119.77
FI  12F  13              1     .72      .864    133.91
FI  13F  14              1     .864    1.037    150.87
FI  14F  15              1    1.037    1.244    171.24
FI  15F  16              1    1.244    1.493    195.66
FI  16F  17              1    1.493    1.792    224.98
FI  17F  18              1    1.792    2.150    260.15
FI  18F  19              1    2.150    2.580    302.37
FI  19F  20              1    2.580    3.096    353.03
FI  20F  21              1    3.096    3.715    413.81
FI  21F  22              1    3.715    4.458    486.76
FI  22F  23              1    4.458    5.350    574.29
FI  23F  24              1    5.350    6.420    679.33
FI  24F  25              1    6.420    7.703    805.38
FI  25F  26              1    7.703    9.244    956.33
FI  26F  27              1    9.244   11.093   1138.15
FI  27F  28              1   11.093   13.312   1355.96
FI  28F  29              1   13.312   15.974   1617.33
FI  29F  30              1   15.974   19.169   1930.98
FI  30F  31              1   19.169   23.003   2307.36
FI  31F  32              1   23.003   27.603   2759.02
FI  32F  33              1   27.603   33.124   3301.00
FI  33F  34              1   33.124   39.748   3951.38
FI  34F  35              1   39.748   47.698   4731.84
FI  35F  36              1   47.698   57.238   5668.39
FI  36F  37              1   57.238   68.686   6792.25
FI  37F  38              1   68.686   82.223   8140.9
FI  38F  39              1   82.223   98.907   9759.2
FI  39F  40              1   98.907  118.688  11701.3
FI  40F  41              1  118.688  142.226  14031.7
FI  41F  42              1  142.226  170.911  16828.2
FI  42F  43              1  170.911  205.093  20184.1
FI  43F  44              1  205.093  246.112  24112.9
FI  44F  45              1  246.112  295.334  29043.4
FI  45F  46              1  295.334  354.401  34842.1
FI  46F  47              1  354.401  425.281  41801.0
FI  47F  48              1  425.281  510.337  50151.3
FI  48F  49              1  510.337  622.405  60171.8

GENERATION
FI   0                   1     MASS    =.21875

ENDCYCLES
@SPLIT
```

Table 10. Printout for Garg's problem with CYCN.

```
*TITLE

FLOW DOMAIN DATA                 ELAPSED TIME SINCE START OF SIMULATION =  .84537E+04    =  .26806E-03 YEARS
                                                  SINCE START OF RUN    =  .84537E+04
                                                  SINCE LAST PRINT-OUT  =  .57031E+03

VOLUME, MASS, ENERGY AND HEAT BALANCES

      FLUID    VOLUME  IN PLACE           =  .53094E+08
      LIQUID   VOLUME  IN PLACE           =  .53094E+08
      VAPOR    VOLUME  IN PLACE           =  .13581E+01

      FLUID     MASS   IN PLACE           =  .37864E+11
      LIQUID    MASS   IN PLACE           =  .37864E+11
      VAPOR     MASS   IN PLACE           =  .57912E+02

      MARGINAL  MASS PRODUCTION           =  .12469E+03
      MARGINAL  MASS PRODUCTION RATE      =  .21863E+00
      CUMULATED MASS PRODUCTION           =  .18484E+04     FLUID MASS FRACTION PRODUCED  =  .48817E-07
      AVERAGE   MASS PRODUCTION RATE      =  .21865E+00

      SOLID  INTERNAL ENERGY              =  .16884E+18     SOLID  HEAT CONTENT             =  .16884E+18
      FLUID  INTERNAL ENERGY              =  .50432E+17     FLUID  HEAT CONTENT             =  .50912E+17
      LIQUID INTERNAL ENERGY              =  .50432E+17     LIQUID HEAT CONTENT             =  .50912E+17
      VAPOR  INTERNAL ENERGY              =  .14889E+09     VAPOR  HEAT CONTENT             =  .15977E+09

      MARG.   SOLID  INT. ENERGY LOSS =  .58056E+08     MARG.   SOLID  HEAT LOSS        =  .58056E+08
      MARG.   FLUID  INT. ENERGY LOSS =  .12159E+09     MARG.   FLUID  HEAT LOSS        =  .15257E+09
      MARG.   RESER. INT. ENERGY LOSS =  .17965E+09     MARG.   RESER. HEAT LOSS        =  .21063E+09
      MARG.   RESER. ENERGY LOSS RATE =  .31500E+06     MARG.   RESER. HEAT LOSS RATE   =  .36933E+06

      CUMUL.  SOLID  INT. ENERGY LOSS =  .79713E+09     CUMUL.  SOLID  HEAT LOSS        =  .79713E+09
      CUMUL.  FLUID  INT. ENERGY LOSS =  .17703E+10     CUMUL.  FLUID  HEAT LOSS        =  .22443E+10
      CUMUL.  RESER. INT. ENERGY LOSS =  .25674E+10     CUMUL.  RESER. HEAT LOSS        =  .30414E+10
      AVERAGE RESER. ENERGY LOSS RATE =  .30371E+06     AVERAGE RESER. HEAT LOSS RATE   =  .35977E+06

MASS AVERAGES

      FLUID  SPECIFIC INTERNAL ENERGY =  .13319E+07     FLUID  SPECIFIC ENTHALPY        =  .13446E+07
      LIQUID SPECIFIC INTERNAL ENERGY =  .13319E+07     LIQUID SPECIFIC ENTHALPY        =  .13446E+07
      VAPOR  SPECIFIC INTERNAL ENERGY =  .25709E+07     VAPOR  SPECIFIC ENTHALPY        =  .27588E+07

VOLUME AVERAGES

      FLUID  DENSITY                      =  .71315E+03
      LIQUID DENSITY                      =  .71315E+03
      VAPOR  DENSITY                      =  .42642E+02

      LIQUID TEMPERATURE                  =  .30000E+03
      VAPOR  TEMPERATURE                  =  .29490E+03

      LIQUID PRESSURE ( N/M2 )            =  .90343E+07
      LIQUID PRESSURE ( ATM. )            =  .92124E+02
      VAPOR  PRESSURE ( N/M2 )            =  .80116E+07
      VAPOR  PRESSURE ( ATM. )            =  .81695E+02

      P/Z             ( N/M2 )            =  .11196E+08
      P/Z             ( ATM. )            =  .11417E+03
      P/Z             ( PSIA )            =  .16238E+04
      Z                                   =  .71559E+00
```

Table 10 (continued)

*TITLE

DENSITY CYCLE OUTPUT DATA — 460 DCYCLES

KCYC	KCYCF	KCYCFC	KWIT	NUTSF	NUTXF	NUTSUMF	MAX. CD	SLIMIN	SLIMAX	DCSLMX	RATGF	RATGSF
460	2	1089	4	4	23	3006	.4869452+00	.1541002+01	.6817785+04	.1946162+03	.7849745-05	.7820113-05

TOTAL TIME	CUM. MASS GEN.	CUM. GAIN	AV. GAIN RATE	TIME STEP	MASS GEN.	MASS GAIN	GAIN RATE	MGMT. FACTOR
.8453702+04	-.1849255+04	-.1849255+04	-.2187505+00	.1527695+02	-.6683635+01	-.6683635+01	-.2187505+00	.5700006+00

CLLM.	INDEX	D	DC	DDD	T	P	PHZ	DE	DPDD	B	VB
F	1	.2614475+03	-.9760075+00	-.3194605-01	.2906835+03	.7526495+07	.6790155+00	.4195545+03	.1205085+050.		.5167875+05
F	2	.5857335+03	-.2212295+00	-.7240675-02	.2987585+03	.8454645+07	.1924155+00	-.6097735+01	.3958045+04	.2710435+04	.2229675+04
F	3	.6952655+03	-.3557835+00	-.1164455-01	.2998385+03	.8580875+07	.2563955-01	.3320465+02	.2936445+04	.9426205+04	.6049995+01
F	4	.7123305+03	-.5034385-02	-.1647715-04	.2999505+03	.8627575+070.		.4897445+00	.8225615+06	.1107515+050.	
F	5	.7124005+03	-.4828475-03	-.1580325-04	.2999555+03	.8662705+070.		.4577795+00	.8225615+06	.1107415+050.	
F	6	.7124575+03	-.4754625-03	-.1556195-04	.2999585+03	.8690745+070.		.4519065+00	.8225615+06	.1107335+050.	
F	7	.7125045+03	-.4683835-03	-.1532985-04	.2999625+03	.8714085+070.		.4448845+00	.8225615+06	.1107265+050.	
F	8	.7125445+03	-.4642105-03	-.1519335-04	.2999645+03	.8734035+070.		.4417495+00	.8225615+06	.1107215+050.	
F	9	.7125795+03	-.4594935-03	-.1503895-04	.2999665+03	.8751465+070.		.4369575+00	.8225615+06	.1107165+050.	
F	10	.7126105+03	-.4563925-03	-.1493745-04	.2999685+03	.8766925+070.		.4343535+00	.8225615+06	.1107125+050.	
F	11	.7126385+03	-.4530905-03	-.1482935-04	.2999705+03	.8780795+070.		.4310035+00	.8225615+06	.1107085+050.	
F	12	.7126665+03	-.4505005-03	-.1474455-04	.2999725+03	.8794635+070.		.4287135+00	.8225615+06	.1107045+050.	
F	13	.7126965+03	-.4476225-03	-.1465035-04	.2999745+03	.8809545+070.		.4259755+00	.8225615+06	.1107005+050.	
F	14	.7127295+03	-.4449025-03	-.1456135-04	.2999765+03	.8825465+070.		.4234385+00	.8225615+06	.1106955+050.	
F	15	.7127615+03	-.4420585-03	-.1446955-04	.2999785+03	.8842315+070.		.4207915+00	.8225615+06	.1106915+050.	
F	16	.7127975+03	-.4391735-03	-.1437385-04	.2999815+03	.8859965+070.		.4180295+00	.8225615+06	.1106865+050.	
F	17	.7128345+03	-.4357215-03	-.1426115-04	.2999835+03	.8878265+070.		.4147665+00	.8225615+06	.1106805+050.	
F	18	.7128725+03	-.4310895-03	-.1410925-04	.2999865+03	.8897035+070.		.4103575+00	.8225615+06	.1106755+050.	
F	19	.7129105+03	-.4240725-03	-.1387955-04	.2999885+03	.8916045+070.		.4035885+00	.8225615+06	.1106705+050.	
F	20	.7129485+03	-.4129505-03	-.1351555-04	.2999915+03	.8935005+070.		.3930995+00	.8225615+06	.1106655+050.	
F	21	.7129855+03	-.3954575-03	-.1294305-04	.2999935+03	.8953565+070.		.3764485+00	.8225615+06	.1106595+050.	
F	22	.7130215+03	-.3690265-03	-.1207805-04	.2999955+03	.8971275+070.		.3512875+00	.8225615+06	.1106545+050.	
F	23	.7130545+03	-.3313465-03	-.1084475-04	.2999975+03	.8987635+070.		.3154175+00	.8225615+06	.1106505+050.	
F	24	.7130835+03	-.2814285-03	-.9210915-05	.2999995+03	.9002045+070.		.2678975+00	.8225615+06	.1106465+050.	
F	25	.7131075+03	-.2213075-03	-.7233385-05	.3000015+03	.9013955+070.		.2103805+00	.8225615+06	.1106435+050.	
F	26	.7131255+03	-.1555835-03	-.5092135-05	.3000025+03	.9022945+070.		.1481025+00	.8225615+06	.1106405+050.	
F	27	.7131375+03	-.9399205-04	-.3076295-05	.3000035+03	.9028925+070.		.8947255-01	.8225615+06	.1106385+050.	
F	28	.7131445+03	-.4588305-04	-.1501715-05	.3000035+03	.9032255+070.		.4367695-01	.8225615+06	.1106375+050.	
F	29	.7131475+03	-.1677855-04	-.5491475-06	.3000045+03	.9033715+070.		.1597195-01	.8225615+06	.1106375+050.	
F	30	.7131485+03	-.4257915-05	-.1393585-06	.3000045+03	.9034175+070.		.4053255-02	.8225615+06	.1106375+050.	
F	31	.7131485+03	-.7069085-06	-.2313665-07	.3000045+03	.9034275+070.		.6729365-03	.8225615+06	.1106375+050.	
F	32	.7131485+03	-.7416385-07	-.2427985-08	.3000045+02	.9034295+070.		.7061955-04	.8225615+06	.1106375+050.	
F	33	.7131485+03	-.4850825-08	-.1587645-09	.3000045+03	.9034295+070.		.4617815-05	.8225615+06	.1106375+050.	
F	34	.7131485+03	-.1967625-08	-.6439875-11	.3000045+03	.9034295+070.		.1873125-06	.8225615+06	.1106375+050.	
F	35	.7131485+03	-.4707265-11	-.1540695-12	.3000045+03	.9034295+070.		.4481245-08	.8225615+06	.1106375+050.	
F	36	.7131485+03	-.3972785-12	-.1300265-13	.3000045+03	.9034295+070.		.3781845-09	.8225615+06	.1106375+050.	
F	37	.7131485+03	-.2613915-13	-.8555815-15	.3000045+03	.9034295+070.		.2487875-10	.8225615+06	.1106375+050.	
F	38	.7131485+03	-.8147475-15	-.2666675-16	.3000045+03	.9034295+070.		.7754255-12	.8225615+06	.1106375+050.	
F	39	.7131485+03	-.7572115-15	-.2477645-16	.3000045+03	.9034295+070.		.7206235-12	.8225615+06	.1106375+050.	
F	40	.7131485+03	-.6734535-15	-.2204165-16	.3000045+03	.9034295+070.		.6409365-12	.8225615+06	.1106375+050.	
F	41	.7131485+03	-.5437775-15	-.1779745-16	.3000045+03	.9034295+070.		.5177535-12	.8225615+06	.1106375+050.	
F	42	.7131485+03	-.5384475-16	-.1762305-17	.3000045+03	.9034295+070.		.5114405-13	.8225615+06	.1106375+050.	
F	43	.7131485+03	-.7252845-16	-.2373805-17	.3000045+03	.9034295+070.		.6900385-13	.8225615+06	.1106375+050.	
F	44	.7131485+03	-.5169795-16	-.1364745-17	.3000045+03	.9034295+070.		.3968395-13	.8225615+06	.1106375+050.	
F	45	.7131485+03	-.3887595-16	-.1272385-17	.3000045+03	.9034295+070.		.3699775-13	.8225615+06	.1106375+050.	

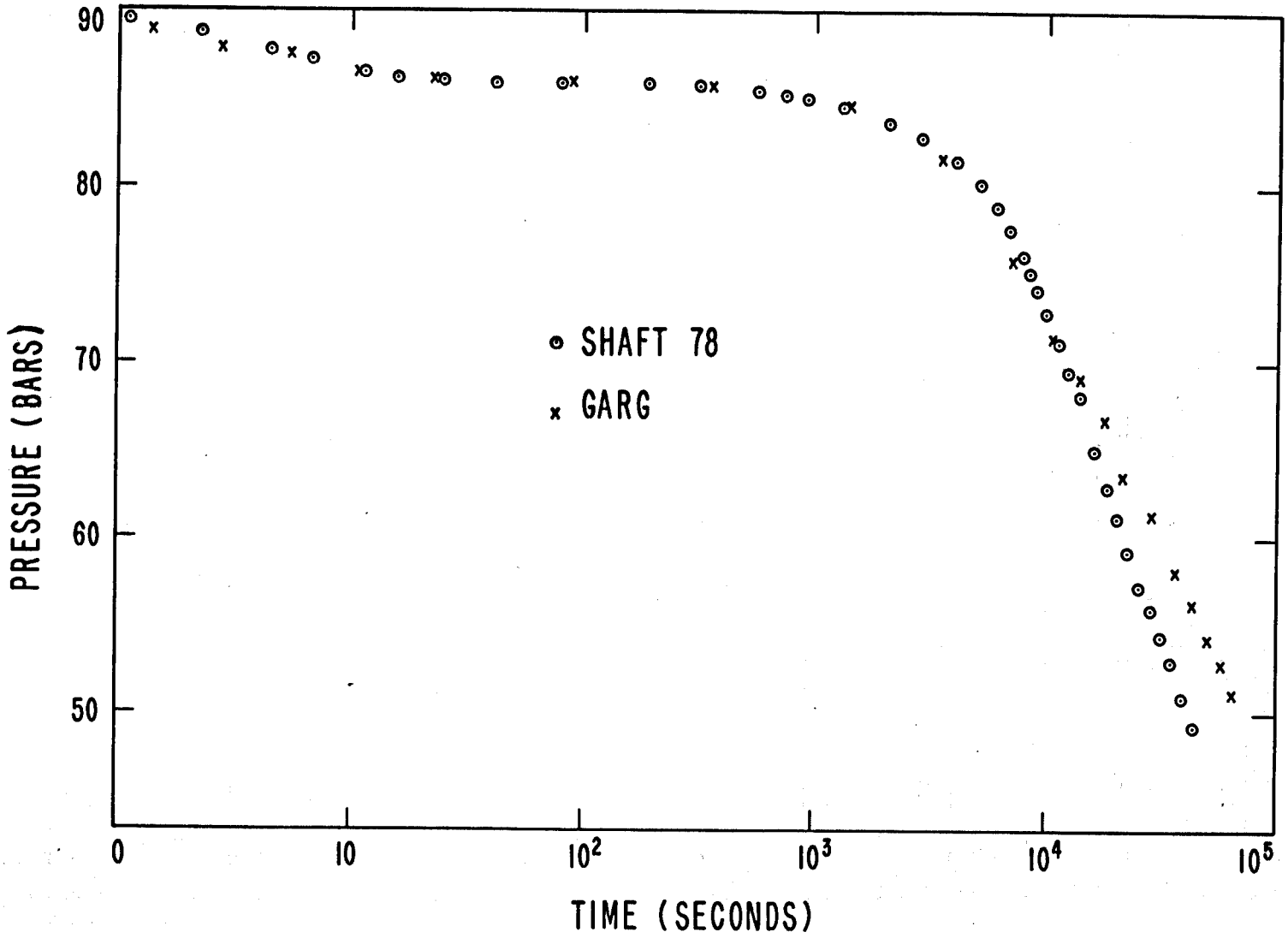

Figure 13. Comparison of SHAFT78 results for pressure decline in wellblock (circles) with Garg's calculations (crosses).

XBL 7811-2168

Toronyi's Problem

This problem involves production from an areal two-dimensional reservoir. It was initially investigated by Toronyi and Farouq-Ali, and their calculation was subsequently verified by other investigators.[42,45] Table 11 shows the input deck used for running the first 40 time steps. Note that this problem requires different relative permeabilities, which are provided in the fluid table "FLUTOR." Table 12 shows part of the printout after 40 time steps. The input deck provides for storing the file SAVE, which is then used for restarting the problem. Table 13 shows the input deck for restarting the calculation. Table 14 gives part of the printout generated after the total time reached the desired value of 6.76512×10^6 seconds. Table 15 compares the results for liquid water saturations as computed by SHAFT78 with those obtained by Toronyi.

Table 11. Input deck for running Toronyi's problem with CYCN.

```
TOR,07,63,170608.466712,PRUESS
*INPUT 66008 17.19.47  15 NOV 78  VIA KP08002
LIBCOPY,DIAB,0,LGO,CYCNG.
REWIND,LGO.
FETCHGS,TABLE=PSSBACKUP/DIABLO/FLUTOR,11186.
LINK,X,PP=(PL=777771.
LIBRITE,DIABLO,SAVE/RB,TO40,143.
EXIT.
DUMP,B.
FIN.
REWIND,INPUT.
COPYSBF,INPUT,OUTPUT.
```

```
*TITLE
*CYCLES
TORON      2562.9536 3.85       9.4619E-13                    1.728558  1010.5930
START PARAMETERS
    2  40 640  1000000000882220      STEP      F  16      TORON              COO
          6765120.
500.                                                  1.          1.
                                                25.       .1
ELEME
F   1   35    1TORON2831684.66
CONNE
F   1F   2    4    1    1    1 152.4      152.4      9290.304
F   7F   8    4    1    1    1 152.4      152.4      9290.304
F  13F  14    4    1    1    1 152.4      152.4      9290.304
F  19F  20    4    1    1    1 152.4      152.4      9290.304
F  25F  26    4    1    1    1 152.4      152.4      9290.304
F  31F  32    4    1    1    1 152.4      152.4      9290.304
F   1F   7    5    1    1    1 15.24      15.24      92903.04
F   7F  13    5    1    1    1 15.24      15.24      92903.04
F  13F  19    5    1    1    1 15.24      15.24      92903.04
F  19F  25    5    1    1    1 15.24      15.24      92903.04
F  25F  31    5    1    1    1 15.24      15.24      92903.04
GENER
F  16                              MASS -25.1994
INCON
F   1   35    1     256.8889   0.8
ENDCYCLES
*SPLIT
```

Table 12. Printout for Toronyi's problem from CYCN after 40 time steps.

*TITLE

DENSITY CYCLE OUTPUT DATA 40 CYCLES

KCYC	KCYCF	KCYCFC	KWIT	NLTSF	NUTXF	NUTSUMF	MAX. DD	SLIMIN	SLIMAX	DDSLMX	RATGF	RATGSF
40	1	80	3	16	55	922	.232689E+00	.192871E+04	.416192E+04	.875537E+03	.132315E-02	.118035E-02

TOTAL TIME	CUM. MASS GEN.	CUM. GAIN	AV. GAIN RATE	TIME STEP	MASS GEN.	MASS GAIN	GAIN RATE	WGHT. FACTOR
.506832E+05	-.127719E+08	-.127719E+08	-.251594E+02	.115847E+05	-.291927E+06	-.291927E+06	-.251594E+02	.100000E+01

ELEM.	INDEX	D	DD	DDD	T	P	PHI	DE	DPDD	E	VB
F	1	1 .175751E+03	-.409776E-03	-.353716E-07	.256777E+03	.445848E+07	.799946E+00	.446649E+00	.128849E+05	.661251E+01	.384275E+05
F	2	2 .175641E+03	-.569241E-02	-.491373E-06	.256775E+03	.445834E+07	.800008E+00	.601945E+01	.128896E+05	.659722E+01	.384438E+05
F	3	3 .174227E+03	-.523120E-01	-.451561E-05	.256734E+03	.445591E+07	.801912E+00	.545945E+02	.129498E+05	.639588E+01	.386537E+05
F	4	4 .163780E+03	-.227476E+00	-.196359E-04	.256346E+03	.443034E+07	.815437E+00	.238257E+03	.133996E+05	.492157E+01	.402114E+05
F	5	5 .174227E+03	-.523312E-01	-.451727E-05	.256734E+03	.445591E+07	.801912E+00	.546160E+02	.129498E+05	.639962E+01	.386538E+05
F	6	6 .175639E+03	-.608362E-02	-.525091E-06	.256775E+03	.445833E+07	.800096E+00	.644511E+01	.128899E+05	.659634E+01	.384447E+05
F	7	7 .175751E+03	-.409449E-03	-.353439E-07	.256777E+03	.445848E+07	.799946E+00	.446038E+00	.128849E+05	.661252E+01	.384275E+05
F	8	8 .175641E+03	-.566353E-02	-.488581E-06	.256775E+03	.445834E+07	.800087E+00	.603656E+01	.128896E+05	.659720E+01	.384437E+05
F	9	9 .174236E+03	-.526741E-01	-.454687E-05	.256734E+03	.445590E+07	.801909E+00	.540754E+02	.129497E+05	.639599E+01	.386534E+05
F	10	10 .163480E+03	-.221024E+00	-.190790E-04	.256325E+03	.442887E+07	.815826E+00	.246604E+03	.134101E+05	.467848E+01	.402569E+05
F	11	11 .174229E+03	-.528931E-01	-.456851E-05	.256734E+03	.445590E+07	.801909E+00	.540968E+02	.129497E+05	.639995E+01	.386534E+05
F	12	12 .175635E+03	-.605405E-02	-.522590E-06	.256775E+03	.445833E+07	.800093E+00	.645980E+01	.128899E+05	.659641E+01	.384447E+05
F	13	13 .175751E+03	-.407082E-03	-.351397E-07	.256777E+03	.445848E+07	.799946E+00	.445139E+00	.128849E+05	.661252E+01	.384275E+05
F	14	14 .175642E+03	-.569393E-02	-.491504E-06	.256775E+03	.445834E+07	.800087E+00	.598481E+01	.128896E+05	.659737E+01	.384436E+05
F	15	15 .174238E+03	-.519713E-01	-.448620E-05	.256734E+03	.445593E+07	.801898E+00	.549019E+02	.129493E+05	.640115E+01	.386522E+05
F	16	16 .163085E+03	-.231401E+00	-.199747E-04	.256288E+03	.442561E+07	.816339E+00	.233344E+03	.134194E+05	.482147E+01	.403184E+05
F	17	17 .174238E+03	-.519913E-01	-.448784E-05	.256734E+03	.445593E+07	.801898E+00	.549232E+02	.129493E+05	.640111E+01	.386522E+05
F	18	18 .175636E+03	-.608212E-02	-.525013E-06	.256775E+03	.445833E+07	.800095E+00	.640432E+01	.128899E+05	.659649E+01	.384446E+05
F	19	19 .175751E+03	-.408636E-03	-.352739E-07	.256777E+03	.445848E+07	.799946E+00	.442590E+00	.128849E+05	.661252E+01	.384275E+05
F	20	20 .175643E+03	-.564006E-02	-.487631E-06	.256775E+03	.445834E+07	.800086E+00	.603463E+01	.128895E+05	.659741E+01	.384435E+05
F	21	21 .174257E+03	-.527422E-01	-.455275E-05	.256735E+03	.445598E+07	.801874E+00	.539141E+02	.129486E+05	.640377E+01	.386494E+05
F	22	22 .163898E+03	-.220693E+00	-.190469E-04	.256346E+03	.443032E+07	.815283E+00	.249687E+03	.133923E+05	.493837E+01	.401941E+05
F	23	23 .174256E+03	-.527612E-01	-.455438E-05	.256735E+03	.445598E+07	.801874E+00	.539354E+02	.129486E+05	.640374E+01	.386494E+05
F	24	24 .175636E+03	-.603890E-02	-.521282E-06	.256775E+03	.445834E+07	.800094E+00	.645654E+01	.128898E+05	.659659E+01	.384445E+05
F	25	25 .175751E+03	-.407406E-03	-.351676E-07	.256777E+03	.445848E+07	.799946E+00	.445068E+00	.128849E+05	.661253E+01	.384275E+05
F	26	26 .175643E+03	-.568616E-02	-.490834E-06	.256775E+03	.445834E+07	.800085E+00	.599104E+01	.128895E+05	.659753E+01	.384435E+05
F	27	27 .174276E+03	-.520714E-01	-.449484E-05	.256736E+03	.445605E+07	.801848E+00	.547686E+02	.129479E+05	.640656E+01	.386464E+05
F	28	28 .164600E+03	-.232689E+00	-.200859E-04	.256387E+03	.443320E+07	.814372E+00	.236449E+03	.133652E+05	.503884E+01	.400882E+05
F	29	29 .174276E+03	-.520104E-01	-.449648E-05	.256736E+03	.445605E+07	.801849E+00	.547899E+02	.129479E+05	.640653E+01	.386464E+05
F	30	30 .175637E+03	-.607464E-02	-.524367E-06	.256775E+03	.445834E+07	.800093E+00	.641540E+01	.128898E+05	.659667E+01	.384444E+05
F	31	31 .175751E+03	-.409208E-03	-.353294E-07	.256777E+03	.445848E+07	.799946E+00	.445127E+00	.128849E+05	.661252E+01	.384275E+05
F	32	32 .175643E+03	-.567017E-02	-.489453E-06	.256775E+03	.445834E+07	.800085E+00	.602376E+01	.128895E+05	.659756E+01	.384434E+05
F	33	33 .174288E+03	-.524756E-01	-.452968E-05	.256737E+03	.445609E+07	.801834E+00	.542972E+02	.129474E+05	.640817E+01	.386447E+05
F	34	34 .164983E+03	-.227490E+00	-.196371E-04	.256410E+03	.443477E+07	.813875E+00	.244729E+03	.133497E+05	.509364E+01	.400383E+05
F	35	35 .174288E+03	-.524941E-01	-.453133E-05	.256737E+03	.445609E+07	.801834E+00	.543187E+02	.129474E+05	.640813E+01	.386447E+05
F	36	36 .175637E+03	-.606039E-02	-.523137E-06	.256775E+03	.445834E+07	.800093E+00	.644794E+01	.128898E+05	.659665E+01	.384443E+05

Table 12 (continued)

*TITLE

INTERNAL CONNECTIONS DATA KCYC = 40 - KCYCF = 1 - TIME = .58683E+06

ELEM1		ELEM2		INDEX	FLOH	FLOF	FLOL	FLOV	VELL	VELV	TRAN1	TRAN2
F	1	F	2	1	-.882086E+04	-.340477E-02	-.204237E-04	-.338435E-02	-.278676E-11	-.161943E-07	-.336808E+00	.336934E+00
F	2	F	3	2	-.158345E+06	-.611194E-01	-.365793E-03	-.607536E-01	-.499085E-10	-.290789E-06	-.336827E+00	.336838E+00
F	3	F	4	3	-.170211E+07	-.656917E+00	-.381176E-02	-.653106E+00	-.519871E-09	-.313579E-05	-.337600E+00	.349836E+00
F	4	F	5	4	.170213E+07	.656914E+00	.381172E-02	.653102E+00	.519865E-09	.313577E-05	-.349936E+00	.337601E+00
F	5	F	6	5	.158007E+06	.609888E-01	.364957E-03	.606236E-01	.497945E-10	.290168E-06	-.338445E+00	.336843E+00
F	7	F	8	6	-.880333E+04	-.339803E-02	-.203831E-04	-.337761E-02	-.278115E-11	-.161620E-07	-.336808E+00	.336933E+00
F	8	F	9	7	-.158003E+06	-.609878E-01	-.365009E-03	-.606228E-01	-.498015E-10	-.290163E-06	-.336827E+00	.336832E+00
F	9	F	10	8	-.181352E+07	-.699914E+00	-.406239E-02	-.695851E+00	-.554041E-09	-.334166E-05	-.337426E+00	.358015E+00
F	10	F	11	9	.181351E+07	.699911E+00	.406235E-02	.695848E+00	.554035E-09	.334164E-05	-.350015E+00	.337426E+00
F	11	F	12	10	.157669E+06	.608581E-01	.364180E-03	.604940E-01	.496884E-10	.289546E-06	-.338441E+00	.336842E+00
F	13	F	14	11	-.868813E+04	-.335354E-02	-.201164E-04	-.333343E-02	-.274477E-11	-.159506E-07	-.336808E+00	.336933E+00
F	14	F	15	12	-.157740E+06	-.608858E-01	-.364404E-03	-.605214E-01	-.497190E-10	-.289677E-06	-.336826E+00	.336842E+00
F	15	F	16	13	-.201456E+07	-.777511E+00	-.451537E-02	-.772595E+00	-.615796E-09	-.371341E-05	-.337088E+00	.349939E+00
F	16	F	17	14	.201456E+07	.777509E+00	.451533E-02	.772592E+00	.615789E-09	.371339E-05	-.349935E+00	.337088E+00
F	17	F	18	15	.157406E+06	.607566E-01	.363578E-03	.603930E-01	.496063E-10	.289063E-06	-.338434E+00	.336841E+00
F	19	F	20	16	-.861685E+04	-.332603E-02	-.199514E-04	-.330613E-02	-.272225E-11	-.158197E-07	-.336809E+00	.336932E+00
F	20	F	21	17	-.152862E+06	-.590028E-01	-.353136E-03	-.586496E-01	-.481817E-10	-.280716E-06	-.336832E+00	.336840E+00
F	21	F	22	18	-.172313E+07	-.665047E+00	-.386196E-02	-.661186E+00	-.526710E-09	-.317464E-05	-.337510E+00	.349639E+00
F	22	F	23	19	.172317E+07	.665044E+00	.386192E-02	.661182E+00	.526710E-09	.317462E-05	-.349639E+00	.337510E+00
F	23	F	24	20	.152532E+06	.588756E-01	.352324E-03	.585232E-01	.480708E-10	.280111E-06	-.338431E+00	.336847E+00
F	25	F	26	21	-.847093E+04	-.326969E-02	-.196135E-04	-.325006E-02	-.267614E-11	-.155518E-07	-.336809E+00	.336932E+00
F	26	F	27	22	-.149243E+06	-.576060E-01	-.344779E-03	-.572613E-01	-.470414E-10	-.274069E-06	-.336837E+00	.336896E+00
F	27	F	28	23	-.151268E+07	-.583812E+00	-.339048E-02	-.580422E+00	-.462434E-09	-.278582E-05	-.337769E+00	.349092E+00
F	28	F	29	24	.151268E+07	.583809E+00	.339044E-02	.580418E+00	.462429E-09	.278580E-05	-.349092E+00	.337789E+00
F	29	F	30	25	.148915E+06	.574794E-01	.343971E-03	.571354E-01	.469313E-10	.273467E-06	-.338403E+00	.336852E+00
F	31	F	32	26	-.842475E+04	-.325188E-02	-.195066E-04	-.323237E-02	-.266156E-11	-.154671E-07	-.336809E+00	.336932E+00
F	32	F	33	27	-.146236E+06	-.564453E-01	-.337831E-03	-.561075E-01	-.460935E-10	-.268545E-06	-.336841E+00	.336886E+00
F	33	F	34	28	-.142024E+07	-.548131E+00	-.318363E-02	-.544947E+00	-.434230E-09	-.261511E-05	-.337913E+00	.348826E+00
F	34	F	35	29	.142023E+07	.548127E+00	.318359E-02	.544944E+00	.434225E-09	.261510E-05	-.348826E+00	.337913E+00
F	35	F	36	30	.145911E+06	.563197E-01	.337031E-03	.559827E-01	.459843E-10	.267948E-06	-.338393E+00	.336856E+00
F	1	F	7	31	.251548E+03	.970874E-04	.582375E-06	.965050E-04	.794619E-14	.461775E-10	-.336821E+02	.336821E+02
F	2	F	8	32	.200820E+04	.775276E-03	.463862E-05	.770637E-03	.632913E-13	.368759E-09	-.337064E+02	.337064E+02
F	3	F	9	33	.362635E+05	.139931E-01	.809409E-04	.139121E-01	.110431E-11	.666058E-08	-.340156E+02	.340154E+02
F	4	F	10	34	-.115403E+06	-.444975E+01	-.192983E-01	-.443046E+01	-.263102E-09	-.213376E-05	-.363369E+02	.363369E+02
F	5	F	11	35	.362661E+05	.139940E-01	.809460E-04	.139131E-01	.110438E-11	.666104E-08	-.340156E+02	.340155E+02
F	6	F	12	36	.225844E+04	.871862E-03	.521576E-05	.866646E-03	.711659E-13	.414701E-09	-.337076E+02	.337078E+02
F	7	F	13	37	.149197E+03	.576019E-04	.345523E-06	.572564E-04	.471447E-14	.273971E-10	-.336821E+02	.336821E+02
F	8	F	14	38	.116702E+05	.450430E-02	.269505E-04	.447735E-02	.367723E-12	.214246E-08	-.337063E+02	.337063E+02
F	9	F	15	39	.382449E+05	.147640E-01	.854165E-04	.146786E-01	.116537E-11	.702751E-08	-.340141E+02	.340146E+02
F	10	F	16	40	-.208809E+08	-.805159E+01	-.345906E-01	-.801700E+01	-.471561E-09	-.386314E-05	-.363716E+02	.363706E+02
F	11	F	17	41	.382584E+05	.147692E-01	.854461E-04	.146837E-01	.116578E-11	.703000E-08	-.340142E+02	.340146E+02
F	12	F	18	42	.118072E+05	.455718E-02	.272630E-04	.452592E-02	.371587E-12	.216762E-08	-.337077E+02	.337077E+02
F	13	F	19	43	.539558E+03	.208249E-03	.124917E-05	.207000E-03	.170443E-13	.990491E-10	-.336821E+02	.336821E+02
F	14	F	20	44	.767081E+04	.296112E-02	.177175E-04	.294340E-02	.241745E-12	.140845E-08	-.337061E+02	.337062E+02
F	15	F	21	45	.498028E+06	.192203E+00	.111253E-02	.191091E+00	.151788E-10	.914859E-07	-.340111E+02	.340116E+02
F	16	F	22	46	.307707E+08	.118655E+02	.516646E-01	.118138E+02	.704337E-09	.569177E-05	-.363161E+02	.362649E+02
F	17	F	23	47	.498034E+06	.192206E+00	.111254E-02	.191093E+00	.151789E-10	.914870E-07	-.340112E+02	.340117E+02
F	18	F	24	48	.820663E+04	.316792E-02	.183521E-04	.314896E-02	.258390E-12	.150682E-08	-.337075E+02	.337076E+02
F	19	F	25	49	.182123E+03	.703114E-04	.421761E-06	.698896E-04	.575469E-14	.334421E-10	-.336821E+02	.336821E+02
F	20	F	26	50	.147832E+05	.570589E-02	.341410E-04	.567175E-02	.465833E-12	.271400E-08	-.337060E+02	.337060E+02
F	21	F	27	51	.378492E+06	.146079E+00	.845953E-03	.145233E+00	.115418E-10	.695304E-07	-.340069E+02	.340060E+02
F	22	F	28	52	.221887E+08	.855595E+01	.380819E-01	.851787E+01	.519219E-09	.410321E-05	-.361935E+02	.361576E+02
F	23	F	29	53	.378506E+06	.146085E+00	.845980E-03	.145239E+00	.115421E-10	.695330E-07	-.340070E+02	.340081E+02

Table 13. Input deck for restarting Toronyi's problem with CYCN.

```
TOR,07,400,170000,466712,PRUESS
LIBCOPY,DIABLO,LGO,CYCNG.
REWIND,LGO.
FETCHGS,TABLE=PSSBACKUP/DIABLO/FLUTOR,11106.
LIBCOPY,DIABLO,RESTA,TO40.
LINK,X,PP=(PL=77777).
EXIT.
DUMP,0.
FIN.
REWIND,INPUT.
COPYSBF,INPUT,OUTPUT.
```

```
*TITLE
*CYCLES
TORON     2562.9536 0.05       9.86619E-13                       1.728558  1010.5930
RESTART
  2 460 600   500000000002220      STEP        F  16     TORON                  000
          6765120.
500.                                              1.        1.
                                    25.           .1
ENDCYCLES
*SPLIT
```

Table 14. Printout for Toronyi's problem from CYCN after 19% of reservoir fluid has been produced.

```
*TITLE
 FLOW DOMAIN DATA                ELAPSED TIME SINCE START OF SIMULATION =  .67651E+07     =  .21452E+00 YEARS
                                                 SINCE START OF RUN     =  .62583E+07
                                                 SINCE LAST PRINT-OUT   =  .27354E+06

 VOLUME, MASS, ENERGY AND HEAT BALANCES

      FLUID   VOLUME  IN PLACE      =  .50970E+07
      LIQUID  VOLUME  IN PLACE      =  .79804E+06
      VAPOR   VOLUME  IN PLACE      =  .42990E+07

      FLUID   MASS    IN PLACE      =  .72525E+09
      LIQUID  MASS    IN PLACE      =  .63088E+09
      VAPOR   MASS    IN PLACE      =  .94841E+08

      MARGINAL  MASS PRODUCTION     =  .68979E+07
      MARGINAL  MASS PRODUCTION RATE=  .25217E+02
      CUMULATED MASS PRODUCTION     =  .17059E+09      FLUID MASS FRACTION PRODUCED  =  .19043E+00
      AVERAGE   MASS PRODUCTION RATE=  .25216E+02

      SOLID  INTERNAL ENERGY        =  .64120E+17      SOLID  HEAT CONTENT           =  .64120E+17
      FLUID  INTERNAL ENERGY        =  .96589E+15      FLUID  HEAT CONTENT           =  .96819E+15
      LIQUID INTERNAL ENERGY        =  .69912E+15      LIQUID HEAT CONTENT           =  .70262E+15
      VAPOR  INTERNAL ENERGY        =  .26658E+15      VAPOR  HEAT CONTENT           =  .26539E+15

      MARG.  SOLID  INT. ENERGY LOSS=  .11764E+14      MARG.  SOLID  HEAT LOSS       =  .11764E+14
      MARG.  FLUID  INT. ENERGY LOSS=  .75500E+13      MARG.  FLUID  HEAT LOSS       =  .75456E+13
      MARG.  RESER. INT. ENERGY LOSS=  .19314E+14      MARG.  RESER. HEAT LOSS       =  .19329E+14
      MARG.  RESER. ENERGY LOSS RATE=  .70606E+08      MARG.  RESER. HEAT LOSS RATE  =  .70663E+08

      CUMUL. SOLID  INT. ENERGY LOSS=  .28909E+15      CUMUL. SOLID  HEAT LOSS       =  .28909E+15
      CUMUL. FLUID  INT. ENERGY LOSS=  .18833E+15      CUMUL. FLUID  HEAT LOSS       =  .18874E+15
      CUMUL. RESER. INT. ENERGY LOSS=  .47741E+15      CUMUL. RESER. HEAT LOSS       =  .47783E+15
      AVERAGE RESER. ENERGY LOSS RATE=  .70569E+08     AVERAGE RESER. HEAT LOSS RATE =  .70631E+08

 MASS AVERAGES

      FLUID  SPECIFIC INTERNAL ENERGY =  .13042E+07    FLUID  SPECIFIC ENTHALPY      =  .13350E+07
      LIQUID SPECIFIC INTERNAL ENERGY =  .11082E+07    LIQUID SPECIFIC ENTHALPY      =  .11137E+07
      VAPOR  SPECIFIC INTERNAL ENERGY =  .25999E+07    VAPOR  SPECIFIC ENTHALPY      =  .27983E+07

 VOLUME AVERAGES

      FLUID  DENSITY                  =  .14229E+03
      LIQUID DENSITY                  =  .79054E+03
      VAPOR  DENSITY                  =  .22061E+02

      LIQUID TEMPERATURE              =  .25569E+03
      VAPOR  TEMPERATURE              =  .25561E+03

      LIQUID PRESSURE ( N/M2 )        =  .43811E+07
      LIQUID PRESSURE ( ATM. )        =  .44675E+02
      VAPOR  PRESSURE ( N/M2 )        =  .43754E+07
      VAPOR  PRESSURE ( ATM. )        =  .44617E+02

      P/Z             ( N/M2 )        =  .53916E+07
      P/Z             ( ATM. )        =  .54979E+02
      P/Z             ( PSIA )        =  .78200E+03
      Z                               =  .81152E+00
```

Table 14 (continued)

*TITLE

DENSITY CYCLE OUTPUT DATA 367 DCYCLES

KCYC	KCYCF	KCYCFC	KWIT	NUITSF	NUTXF	NUITSUMF	MAX. DD	SLIMIN	SLIMAX	DDSLMX	RATGF	RATGSF
367	3	882	1	12	54	8510	.778005E-01	.977057E+03	.381428E+04	.245906E+03	.528574E-02	.528414E-02

TOTAL TIME	CUM. MASS GEN.	CUM. GAIN	AV. GAIN RATE	TIME STEP	MASS GEN.	MASS GAIN	GAIN RATE	WGHT. FACTOR
.676512E+07	-.170477E+09	-.170477E+09	-.251994E+02	.129979E+05	-.873440E+06	-.873440E+06	-.251994E+02	.100000E+01

ELEM.	INDEX	C	DD	DDD	T	P	PHZ	DE	DPDD	R	VB
F	1	1 .162348E+03	-.142670E+00	-.411614E-05	.256328E+03	.442922E+07	.817299E+00	.154288E+03	.134850E+05	.471724E+01	.404220E+05
F	2	2 .154201E+03	-.133195E+00	-.394173E-05	.256048E+03	.440773E+07	.827916E+00	.148837E+03	.138774E+05	.354205E+01	.416331E+05
F	3	3 .138993E+03	-.161225E+00	-.465147E-05	.255531E+03	.436739E+07	.847815E+00	.260992E+03	.174494E+05	.159314E+01	.439318E+05
F	4	4 .115579E+03	-.199883E+00	-.576678E-05	.254621E+03	.430599E+07	.878041E+00	.495630E+03	.230375E+05	.547753E+00	.477124E+05
F	5	5 .135678E+03	-.199087E+00	-.574294E-05	.255416E+03	.436002E+07	.852073E+00	.327310E+03	.177223E+05	.142609E+01	.444630E+05
F	6	6 .146733E+03	-.189378E+00	-.546371E-05	.255800E+03	.438708E+07	.837703E+00	.221027E+03	.142430E+05	.244798E+01	.427415E+05
F	7	7 .162348E+03	-.142676E+00	-.411632E-05	.256328E+03	.442922E+07	.817292E+00	.154289E+03	.134846E+05	.471805E+01	.404211E+05
F	8	8 .154191E+03	-.133092E+00	-.383981E-05	.256048E+03	.440773E+07	.827930E+00	.148654E+03	.138780E+05	.354056E+01	.416346E+05
F	9	9 .138957E+03	-.161338E+00	-.465473E-05	.255531E+03	.436740E+07	.847848E+00	.262168E+03	.174518E+05	.159184E+01	.439359E+05
F	10	10 .115328E+03	-.199993E+00	-.576996E-05	.254599E+03	.430464E+07	.878359E+00	.497387E+03	.230473E+05	.541916E+00	.477545E+05
F	11	11 .135621E+03	-.199028E+00	-.574211E-05	.255416E+03	.436002E+07	.852146E+00	.327430E+03	.177278E+05	.142323E+01	.444715E+05
F	12	12 .146705E+03	-.189478E+00	-.546660E-05	.255800E+03	.438709E+07	.837741E+00	.221886E+03	.142447E+05	.244381E+01	.427456E+05
F	13	13 .154235E+03	-.142678E+00	-.411636E-05	.256328E+03	.442922E+07	.817293E+00	.154289E+03	.134847E+05	.471790E+01	.404213E+05
F	14	14 .154213E+03	-.133124E+00	-.384072E-05	.256048E+03	.440773E+07	.827901E+00	.148845E+03	.138766E+05	.354376E+01	.416314E+05
F	15	15 .139051E+03	-.161199E+00	-.465071E-05	.255532E+03	.436740E+07	.847727E+00	.260130E+03	.174431E+05	.159659E+01	.439210E+05
F	16	16 .115109E+03	-.201021E+00	-.579960E-05	.254559E+03	.430194E+07	.878632E+00	.501440E+03	.230828E+05	.536864E+00	.477929E+05
F	17	17 .135769E+03	-.199248E+00	-.574846E-05	.255417E+03	.436004E+07	.851955E+00	.327357E+03	.177136E+05	.143068E+01	.444486E+05
F	18	18 .146764E+03	-.189237E+00	-.545962E-05	.255800E+03	.438708E+07	.837663E+00	.220262E+03	.142411E+05	.245253E+01	.427370E+05
F	19	19 .162355E+03	-.142680E+00	-.411642E-05	.256328E+03	.442922E+07	.817290E+00	.154292E+03	.134846E+05	.471825E+01	.404209E+05
F	20	20 .154194E+03	-.133072E+00	-.383922E-05	.256048E+03	.440773E+07	.827926E+00	.148597E+03	.138779E+05	.354094E+01	.416342E+05
F	21	21 .138919E+03	-.161477E+00	-.465874E-05	.255532E+03	.436747E+07	.847899E+00	.262744E+03	.174558E+05	.158987E+01	.439419E+05
F	22	22 .115698E+03	-.203651E+00	-.587550E-05	.254621E+03	.430590E+07	.877888E+00	.504593E+03	.230183E+05	.550542E+00	.476932E+05
F	23	23 .135557E+03	-.199141E+00	-.574738E-05	.255417E+03	.436009E+07	.852216E+00	.327838E+03	.177334E+05	.142049E+01	.444804E+05
F	24	24 .146688E+03	-.189524E+00	-.546790E-05	.255800E+03	.438709E+07	.837763E+00	.222189E+03	.142457E+05	.244133E+01	.427480E+05
F	25	25 .162352E+03	-.142675E+00	-.411629E-05	.256328E+03	.442922E+07	.817294E+00	.154288E+03	.134847E+05	.471760E+01	.404214E+05
F	26	26 .154208E+03	-.133139E+00	-.384116E-05	.256048E+03	.440774E+07	.827908E+00	.148845E+03	.138770E+05	.354298E+01	.416322E+05
F	27	27 .139108E+03	-.161492E+00	-.465915E-05	.255534E+03	.436752E+07	.847654E+00	.260770E+03	.174384E+05	.159949E+01	.439118E+05
F	28	28 .116251E+03	-.206532E+00	-.595860E-05	.254666E+03	.430852E+07	.877186E+00	.508604E+03	.229508E+05	.563409E+00	.476009E+05
F	29	29 .135807E+03	-.199501E+00	-.575876E-05	.255418E+03	.436015E+07	.851906E+00	.327686E+03	.177106E+05	.143260E+01	.444425E+05
F	30	30 .146774E+03	-.189319E+00	-.546200E-05	.255800E+03	.438709E+07	.837649E+00	.220496E+03	.142405E+05	.245403E+01	.427355E+05
F	31	31 .162349E+03	-.142671E+00	-.411616E-05	.256328E+03	.442922E+07	.817299E+00	.154289E+03	.134847E+05	.471734E+01	.404219E+05
F	32	32 .154198E+03	-.133156E+00	-.384166E-05	.256048E+03	.440774E+07	.827921E+00	.148783E+03	.138776E+05	.354156E+01	.416336E+05
F	33	33 .139033E+03	-.161632E+00	-.466322E-05	.255534E+03	.436755E+07	.847790E+00	.262168E+03	.174484E+05	.159413E+01	.439285E+05
F	34	34 .116586E+03	-.201457E+00	-.600590E-05	.254689E+03	.430983E+07	.876760E+00	.510655E+03	.229082E+05	.571222E+00	.475452E+05
F	35	35 .135685E+03	-.199450E+00	-.574427E-05	.255419E+03	.436018E+07	.852064E+00	.328005E+03	.177225E+05	.142644E+01	.444617E+05
F	36	36 .146707E+03	-.189479E+00	-.546681E-05	.255800E+03	.438709E+07	.837738E+00	.221588E+03	.142446E+05	.244409E+01	.427453E+05

Table 15. Percentage of liquid water saturations for Toronyi's problem.*

1 18.3 18.3	2 17.2 17.3	3 15.2 15.2	4 12.2 11.8	5 14.8 14.9	6 16.2 16.4
7 18.3 18.3	8 17.2 17.3	9 15.2 15.2	10 12.2 11.7	11 14.8 14.9	12 16.2 16.4
13 18.3 18.3	14 17.2 17.3	15 15.2 15.2	16 12.1 11.8	17 14.8 14.9	18 16.2 16.4
19 18.3 18.3	20 17.2 17.3	21 15.2 15.2	22 12.2 11.8	23 14.8 14.9	24 16.2 16.4
25 18.3 18.3	26 17.2 17.3	27 15.2 15.2	28 12.3 12.0	29 14.8 14.9	30 16.2 16.4
31 18.3 18.3	32 17.2 17.3	33 15.2 15.2	34 12.3 12.0	35 14.8 14.9	36 16.2 16.4

*The lower numbers are Toronyi's results, whereas the upper ones were computed from SHAFT78.

Depletion of Two-Phase Reservoirs

Table 16 shows the input deck used for running the first 25 time steps of a problem which was recently investigated by Brigham and Morrow.[46] Table 17 shows part of the printout obtained from this run. More extensive results and discussions are given in ref. 16.

A similar depletion problem involving a reservoir with a uniform initial water saturation was investigated with the input deck shown in Table 18. Table 19 shows some results; more extensive calculations for this problem are presented and discussed in ref. 16.

Table 16. Input deck for running Brigham and Morrow's problem with CYCN (depletion of two-phase reservoirs).

```
84F,7,100,17COCC.466712,PRUESS.
*INPUT 66008 14.46.14  10 NOV 78  VIA KP00009
*HOLDOUT
LIBCOPY,DIABLO,LGO,CYCNG.
REWIND,LGO.
FETCHCS,TABLE=PSSBACKUP/DIABLC/FLUTAB3,11106.
LINK,X,PP=(PL=9999991.
LIBRITE,DIABLO,SAVE/PB,F25,143.
EXIT.
DUMP,J.
FIN.
REWIND,INPUT.
COPYS3F,INPUT,OUTPUT.



*   BRIGHAM/MORROW FINE MESH 2202
*CYCLES
 1   99    1   .70
RONP1      2000.      .10              1.E-13                    0.0          1232.

START
 0 2  25 600  2500000000202220      STEP       F  11      RONP1      00000      000
0.                  0.                          1.50E+7
05000.      05.0                                                              .05
                                                100.

ELEMENT INFORMATION
F    1     2     1RONP1      1.068
F    4    19     1RONP1      .2558
F   24     1     1RONP1      1.068

CONNE
F    1F    2     1    1     1    1    50.00     50.00     1000000. 0.
F    3F    4                      1    50.00     12.50     1000000. 0.
F    4F    5    18    1     1    1    12.50     12.50     1000000. 0.
F   23F   24                      1    12.50     50.00     1000000. 0.
F   24F   25                      1    50.00     50.00     1000000. 0.

GENER
F    1G01        0     0     0    1     MASS     -  50.0

INCON
F    1    10     1          252.      41.00E+5
F   12    13     1          252.       1.00E-3

ENDCYCLES
*SPLIT
```

Table 17. Printout for Brigham and Morrow's problem from CYCN.

KCYC	TIME	DELTE	ENERGY	DE	TEMPERATURE	PRESSURE	PHASE	RAT	NUTS
24	.161159E+07	.756217E+04	.254681E+07	-.409738E+04	.251538E+03	.411055E+07	-.999133E+00	.364408E-07	1

KCYCF	BEGIN TIME	DELTF	DENSITY	DD	DEF	PRESSURE	PHASE	RATF	NUTSF
0	.160403E+07	.100000E-11	.213820E+02	.335321E-16	.838303E-10	.411055E+07	.999133E+00	.289307E-01	1

KCYCF	BEGIN TIME	DELTF	DENSITY	DD	DEF	PRESSURE	PHASE	RATF	NUTSF
1	.160403E+07	.756217E+04	.214370E+02	.550239E-01	.136993E+06	.411055E+07	.999133E+00	.568978E-01	8

***** T A G ***** N = 1 J = 1 SAT= .100000E+01 QUAL= .100000E+01 EX= .260904E+07 DX= .195387E+02

KCYC	TIME	DELTE	ENERGY	DE	TEMPERATURE	PRESSURE	PHASE	RAT	NUTS
25	.161999E+07	.839511E+04	.250276E+07	-.429593E+04	.251520E+03	.411043E+07	-.999063E+00	.370108E-07	1

* BRIGHAM/MORROW FINE MESH ≥20≥

ENERGY CYCLE OUTPUT DATA 25 ECYCLES

==

KCYC	KWIT	NLTS	NUTX	NUTSUM	MAX. DE	SLIMIN	SLIMAX	DESLMX	RATG	RATGS
25	3	1	4	62	.429593E+04	.105678E+07	.621192E+11	.113414E+12	.370108E-07	.364408E-07

TOTAL TIME	CUM. HEAT GEN.	CUM. HT. GAIN	AV. GAIN RATE	TIME STEP	HEAT GEN.	HEAT GAIN	GAIN RATE	WGHT. FACTOR
.161999E+07	-.227394E+15	-.227394E+15	-.140268E+09	.839511E+04	-.117962E+13	-.117962E+13	-.140513E+09	.570000E+00

==

ELEM.	INDEX	E	DE	DDE	T	P	PHZX	CD/DE	DTDE	DD	TOTAL DE
F 1	1	.260904E+07	.596576E+01	.710623E-03	.251858E+03	.393149E+07	.100000E+01	-.137757E-03	.442645E-03	-.821824E-03	-.238926E+11
F 2	2	.260791E+07	.490040E+01	.583721E-03	.251859E+03	.397538E+07	.100000E+01	-.137104E-03	.441358E-03	-.671867E-03	-.201389E+11
F 3	3	.260502E+07	.203474E+01	.242373E-03	.251863E+03	.401794E+07	.100000E+01	-.136614E-03	.440099E-03	-.277975E-03	-.847236E+10
F 4	4	.260384E+07	-.102798E+01	-.122450E-03	.251866E+03	.404400E+07	.100000E+01	-.157504E-03	.439387E-03	.161912E-03	.157385E+09
F 5	5	.260343E+07	.437463E+00	.521093E-04	.251866E+03	.405440E+07	.100000E+01	-.158469E-03	.439178E-03	-.693242E-04	.317097E+10
F 6	6	.260301E+07	-.325321E+01	-.387513E-03	.251867E+03	.406479E+07	.100000E+01	-.156037E-03	.432733E-03	.507621E-03	.351489E+09
F 7	7	.260260E+07	.125977E+01	.150060E-03	.251867E+03	.407516E+07	.100000E+01	-.155896E-03	.432502E-03	-.196394E-03	.379399E+10
F 8	8	.260218E+07	-.254818E+01	-.303531E-03	.251869E+03	.408552E+07	.100000E+01	-.155339E-03	.432272E-03	.395832E-03	-.115803E+10
F 9	9	.260178E+07	.586071E+00	.698110E-04	.251872E+03	.409583E+07	.100000E+01	-.155463E-03	.432043E-03	-.911124E-04	.344195E+10
F 10	10	.260135E+07	-.103497E+02	-.123282E-02	.251874E+03	.410613E+07	.100000E+01	-.125964E-03	.431813E-03	.130369E-02	-.660047E+12
F 11	11	.254276E+07	-.429593E+04	-.511719E+00	.251520E+03	.411043E+07	.999063E+00	-.142853E-04	.389205E-04	.613687E-01	.511265E+12
F 12	12	.109121E+07	-.140531E+01	-.167396E-03	.251974E+03	.411541E+07	.234626E-01	-.187701E-01	.211644E-03	.263779E-01	-.779084E+12
F 13	13	.109056E+07	-.322397E+00	-.384029E-04	.251988E+03	.411623E+07	.562955E-02	-.196281E-01	.212005E-03	.632803E-02	-.142112E+12
F 14	14	.109044E+07	-.122732E+00	-.146194E-04	.251993E+03	.411648E+07	.218788E-02	-.204657E-01	.212049E-03	.251179E-02	-.525773E+11
F 15	15	.109041E+07	-.267693E-01	-.318868E-05	.251994E+03	.411653E+07	.121717E-02	-.204657E-01	.212049E-03	.547855E-03	-.113792E+11
F 16	16	.109041E+07	-.563665E-02	-.671421E-06	.251994E+03	.411655E+07	.101234E-02	-.204657E-01	.212049E-03	.115358E-03	-.239194E+10
F 17	17	.109041E+07	-.107867E-02	-.128487E-06	.251994E+03	.411655E+07	.973350E-03	-.204657E-01	.212049E-03	.220757E-04	-.457584E+09
F 18	18	.109041E+07	-.180169E-03	-.214612E-07	.251994E+03	.411655E+07	.966924E-03	-.204657E-01	.212049E-03	.368729E-05	-.764252E+08
F 19	19	.109041E+07	-.239302E-04	-.285051E-08	.251994E+03	.411655E+07	.966101E-03	-.204657E-01	.212049E-03	.489752E-06	-.101508E+08
F 20	20	.109041E+07	-.149268E-05	-.177804E-09	.251994E+03	.411655E+07	.966060E-03	-.204657E-01	.212049E-03	.305489E-07	-.633132E+06
F 21	21	.109041E+07	.503050E-06	.599218E-10	.251994E+03	.411655E+07	.966282E-03	-.204657E-01	.212049E-03	-.102953E-07	.213397E+06
F 22	22	.109041E+07	.284987E-06	.339468E-10	.251994E+03	.411655E+07	.966094E-03	-.204658E-01	.212049E-03	-.583248E-08	.120890E+06
F 23	23	.109041E+07	.106375E-06	.126710E-10	.251994E+03	.411655E+07	.966097E-03	-.204657E-01	.212049E-03	-.217704E-08	.451231E+05
F 24	24	.109041E+07	.525857E-08	.626385E-12	.251994E+03	.411655E+07	.966099E-03	-.204657E-01	.212049E-03	-.107621E-09	.892261E+04
F 25	25	.109041E+07	.225766E-09	.268925E-12	.251994E+03	.411655E+07	.966099E-03	-.204658E-01	.212049E-03	-.462046E-11	.383088E+03

Table 17 (continued)

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*   BRIGHAM/MORROW FINE MESH 2202

FLOW DOMAIN DATA                    ELAPSED TIME SINCE START OF SIMULATION =  .16200E+07    =  .51369E-01 YEARS
                                                    SINCE START OF RUN     =  .16200E+07
                                                    SINCE LAST PRINT-OUT   =  .16200E+07

VOLUME, MASS, ENERGY AND HEAT BALANCES

      FLUID    VOLUME  IN PLACE       =  .10000E+09
      LIQUID   VOLUME  IN PLACE       =  .49882E+08
      VAPOR    VOLUME  IN PLACE       =  .50118E+08

      FLUID     MASS   IN PLACE       =  .40721E+11
      LIQUID    MASS   IN PLACE       =  .39715E+11
      VAPOR     MASS   IN PLACE       =  .10064E+10

      MARGINAL  MASS PRODUCTION       =  .80242E+08
      MARGINAL  MASS PRODUCTION RATE  =  .49533E+02
      CUMULATED MASS PRODUCTION       =  .80242E+08      FLUID MASS FRACTION PRODUCED  =  .19667E-02
      AVERAGE   MASS PRODUCTION RATE  =  .49533E+02

      SOLID  INTERNAL ENERGY          =  .55866E+18      SOLID  HEAT CONTENT           =  .55866E+18
      FLUID  INTERNAL ENERGY          =  .45925E+17      FLUID  HEAT CONTENT           =  .46331E+17
      LIQUID INTERNAL ENERGY          =  .43303E+17      LIQUID HEAT CONTENT           =  .43509E+17
      VAPOR  INTERNAL ENERGY          =  .26218E+16      VAPOR  HEAT CONTENT           =  .28232E+16

      MARG.   SOLID  INT. ENERGY LOSS =  .29911E+14      MARG.   SOLID  HEAT LOSS      =  .29911E+14
      MARG.   FLUID  INT. ENERGY LOSS =  .12154E+15      MARG.   FLUID  HEAT LOSS      =  .12568E+15
      MARG.   RESER. INT. ENERGY LOSS =  .15145E+15      MARG.   RESER. HEAT LOSS      =  .15559E+15
      MARG.   RESER. ENERGY LOSS RATE =  .93490E+08      MARG.   RESER. HEAT LOSS RATE =  .96047E+08

      CUMUL.  SOLID  INT. ENERGY LOSS =  .29911E+14      CUMUL.  SOLID  HEAT LOSS      =  .29911E+14
      CUMUL.  FLUID  INT. ENERGY LOSS =  .12154E+15      CUMUL.  FLUID  HEAT LOSS      =  .12568E+15
      CUMUL.  RESER. INT. ENERGY LOSS =  .15145E+15      CUMUL.  RESER. HEAT LOSS      =  .15559E+15
      AVERAGE RESER. ENERGY LOSS RATE =  .93490E+08      AVERAGE RESER. HEAT LOSS RATE =  .96047E+08

MASS AVERAGES

      FLUID  SPECIFIC INTERNAL ENERGY =  .11278E+07      FLUID  SPECIFIC ENTHALPY      =  .11378E+07
      LIQUID SPECIFIC INTERNAL ENERGY =  .10904E+07      LIQUID SPECIFIC ENTHALPY      =  .10955E+07
      VAPOR  SPECIFIC INTERNAL ENERGY =  .26051E+07      VAPOR  SPECIFIC ENTHALPY      =  .28051E+07

VOLUME AVERAGES

      FLUID  DENSITY                  =  .40721E+03
      LIQUID DENSITY                  =  .79617E+03
      VAPOR  DENSITY                  =  .20081E+02

      LIQUID TEMPERATURE              =  .25199E+03
      VAPOR  TEMPERATURE              =  .25185E+03

      LIQUID PRESSURE ( N/M2 )        =  .41165E+07
      LIQUID PRESSURE ( ATM. )        =  .41976E+02
      VAPOR  PRESSURE ( N/M2 )        =  .40168E+07
      VAPOR  PRESSURE ( ATM. )        =  .40960E+02

      P/Z             ( N/M2 )        =  .48728E+07
      P/Z             ( ATM. )        =  .49688E+02
      P/Z             ( PSIA )        =  .70674E+03
      Z                               =  .82435E+00
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Table 17 (continued)

```
*   BRIGHAM/MORROW FINE MESH >20<

DENSITY CYCLE OUTPUT DATA                                    25 DCYCLES

 KCYC  KCYCF  KCYCFC KWIT NUTSF NUTXF NUTSUMF     MAX. DD        SLIMIN         SLIMAX         DDSLMX         RATGF          RATGSF
   25     2     208    3     6    48     1699   .544057E-01   .705125E+03   .197940E+04   .167539E+02   .125271E+00   .440451E-01

 TOTAL TIME  CUM. MASS GEN.   CUM. GAIN  AV. GAIN RATE    TIME STEP      MASS GEN.      MASS GAIN      GAIN RATE  WGHT. FACTOR
.161999E+07  -.809993E+08  -.809993E+08  -.500000E+02   .419755E+04  -.419755E+06  -.419755E+06  -.500000E+02   .716709E+00

ELEM.  INDEX      D            CD           DDD            T            P            PHZ           CE           DPDD          B            VB

F   1     1  .195379E+02-.811418E-03-.966537E-07 .251858E+03 .393136E+07 .100000E+01 .538992E+01 .199199E+060.            .567513E+05
F   2     2  .198154E+02-.655550E-03-.780871E-07 .251859E+03 .397527E+07 .100000E+01 .478079E+01 .198644E+060.            .567633E+05
F   3     3  .200868E+02-.265422E-03-.316163E-07 .251863E+03 .401789E+07 .100000E+01 .194220E+01 .198101E+060.            .567730E+05
F   4     4  .202531E+02 .226930E-04 .270313E-08 .251866E+03 .404401E+07 .100000E+01-.152113E+00 .195925E+060.            .567787E+05
F   5     5  .203181E+02 .653702E-04 .778671E-08 .251866E+03 .405441E+07 .100000E+01-.417043E+00 .195812E+060.            .567817E+05
F   6     6  .203833E+02-.395880E-04-.471561E-08 .251867E+03 .406479E+07 .100000E+01 .245717E+00 .195695E+060.            .567856E+05
F   7     7  .204481E+02-.432990E-03-.515765E-07 .251867E+03 .407509E+07 .100000E+01 .277419E+01 .195576E+060.            .567897E+05
F   8     8  .205123E+02-.142705E-02-.169990E-06 .251869E+03 .408529E+07 .100000E+01 .912228E+01 .195459E+060.            .568383E+05
F   9     9  .205748E+02-.374076E-02-.445588E-06 .251872E+03 .409524E+07 .100000E+01 .239954E+02 .195344E+060.            .568944E+05
F  10    10  .206351E+02-.847456E-02-.100946E-05 .251873E+03 .410477E+07 .100000E+01 .524327E+02 .195232E+060.            .569486E+05
F  11    11  .215048E+02 .677955E-01 .807560E-05 .251500E+03 .411038E+07 .998977E+00-.468191E+04 .183129E+060.            .570227E+05
F  12    12  .777844E+03-.132282E+00-.158762E-04 .251974E+03 .411539E+07 .236349E-01 .589151E+01 .766593E+03 .675759E+04 .535247E+02
F  13    13  .791744E+03-.530727E-01-.632186E-05 .251988E+03 .411622E+07 .569805E-02 .226773E+01 .732461E+03 .869076E+04 .773331E+00
F  14    14  .794447E+03-.197367E-01-.235121E-05 .251993E+03 .411648E+07 .221222E-02 .807346E+00 .674063E+03 .910991E+04 .455825E-01
F  15    15  .795218E+03-.442942E-02-.527619E-06 .251994E+03 .411653E+07 .122286E-02 .181242E+00 .674063E+03 .923175E+04 .773134E-02
F  16    16  .795381E+03-.937190E-03-.111635E-06 .251994E+03 .411654E+07 .101354E-02 .383526E-01 .674063E+03 .925766E+04 .438646E-02
F  17    17  .795412E+03-.181216E-03-.215859E-07 .251994E+03 .411655E+07 .972583E-03 .741746E-02 .674063E+03 .926261E+04 .388911E-02
F  18    18  .795417E+03-.307973E-04-.366848E-08 .251994E+03 .411655E+07 .966964E-03 .126104E-02 .674063E+03 .926343E+04 .380940E-02
F  19    19  .795418E+03-.423506E-05-.504468E-09 .251994E+03 .411655E+07 .966106E-03 .173533E-03 .674063E+03 .926352E+04 .379927E-02
F  20    20  .795418E+03-.313838E-06-.373825E-10 .251994E+03 .411655E+07 .966061E-03 .129002E-04 .674063E+03 .926354E+04 .379874E-02
F  21    21  .795418E+03 .684274E-07 .815087E-11 .251994E+03 .411655E+07 .966082E-03-.278699E-05 .674063E+03 .926354E+04 .379899E-02
F  22    22  .795418E+03 .440850E-07 .525127E-11 .251994E+03 .411655E+07 .966094E-03-.189120E-05 .674063E+03 .926354E+04 .379912E-02
F  23    23  .795418E+03 .169607E-07 .202030E-11 .251994E+03 .411655E+07 .966097E-03-.693443E-06 .674063E+03 .926354E+04 .379917E-02
F  24    24  .795418E+03 .849385E-09 .101176E-12 .251994E+03 .411655E+07 .966099E-03-.347374E-07 .674063E+03 .926354E+04 .379919E-02
F  25    25  .795418E+03 .364294E-10 .433936E-14 .251994E+03 .411655E+07 .966099E-03-.148992E-08 .674063E+03 .926354E+04 .379919E-02
```

Table 18. Input deck for running a depletion problem for a reservoir with uniform initial vapor saturation.

```
BRIG2,3,400,170000.466712,PRUESS
*INPUT 66008 20.14.07  06 NOV 78   VIA KP00002
LIBCOPY,DIABLO,LGO,CYCNG.
REWIND,LGO.
FETCHGS,TABLE=PSSBACKUP/DIABLO/FLUTA83,11186.
LINK,X,PP=(PL=999999).
LIBRITE,DIABLO,SAVE/R3.BE1000,143.
EXIT.
DUMP,0.
FIN.
REWIND,INPUT.
COPYSBF,INPUT,OUTPUT.
```

```
*   BRIGHAM AND MORROW  (HALF WATER / HALF STEAM)
*CYCLES
 1  99   1  .7
RONP1     2300.      .10           1.E-13                      0.0          1232.

START
 8 21000 600  5000000000202220     STEP      F  1     RONP1     00000     000
0.                  0.                          1.50E+7
01000.    01.0                                                             .61
                                          25.       .2
ELEMENT INFORMATION
F   1    4    1RONP1     1.0E8
F   6    4    1RONP1     1.0E8

CONNE
F   1F   2    8    1    1    1   50.00    50.00    1000000. 0.

GENER
F   1COI      0    0    3    1     MASS    -  50.0

INCON
F   1    9    1         252.    .5

ENDCYCLES
*SPLIT
```

Table 19. Printout for the uniform saturation depletion problem after 83.9% of the reservoir fluid has been produced.

```
*   BRIGHAM AND MORROW  (HALF WATER / HALF STEAM)

ENERGY  CYCLE OUTPUT DATA                                   1000 ECYCLES

================================================================================================================================

  KCYC          KMIT  NUTS  NUTX  NUTSUM     MAX. DE       SLIMIN        SLIMAX        DESLMX         RATG          RATGS
  1000             3     1     2    1228   .985447E+03   .819650E+07   .130967E+12   .128174E+15   .259668E-10   .261011E-10

  TOTAL TIME  CUM. HEAT GEN.  CUM. HT. GAIN AV. GAIN RATE   TIME STEP     HEAT GEN.     HEAT GAIN      GAIN RATE  MGMT. FACTOR
 .635294E+09  -.959954E+17   -.959954E+17   -.140679E+09   .332826E+05  -.465946E+13  -.465946E+13  -.139997E+09   .570000E+00

================================================================================================================================

ELEM.  INDEX      E            DE           DDE           T            P           PHZX        DD/DE        DTDE          CO         TOTAL DE

F   1     1 .167480E+07 .365447E+03 .296085E-01 .221800E+03 .242250E+07 .981631E+00-.390140E-04 .606964E-04-.345044E-01-.180518E+13
F   2     2 .151506E+07 .315744E+02 .275142E-02 .223045E+03 .247396E+07 .971798E+00-.631674E-04 .740418E-04-.578451E-02-.633578E+11
F   3     3 .138760E+07 .129963E+03 .390483E-02 .224223E+03 .252361E+07 .958491E+00-.113686E-03 .974584E-04-.147749E-01-.427116E+12
F   4     4 .130769E+07 .991783E+02 .297988E-02 .225212E+03 .256654E+07 .944845E+00-.177934E-03 .390228E-04-.176472E-01-.498896E+12
F   5     5 .126126E+07 .619586E+02 .186159E-02 .225992E+03 .260204E+07 .932990E+00-.253504E-03 .108394E-03-.157067E-01-.438459E+12
F   6     6 .123426E+07 .396262E+02 .119060E-02 .226581E+03 .263119E+07 .923867E+00-.357795E-03 .113364E-03-.141780E-01-.393017E+12
F   7     7 .121976E+07 .393615E+02 .118325E-02 .227023E+03 .265479E+07 .917110E+00-.382383E-03 .117049E-03-.159588E-01-.421486E+12
F   8     8 .120681E+07 .421674E+02 .126755E-02 .227412E+03 .267259E+07 .911063E+00-.397188E-03 .119863E-03-.167963E-01-.471549E+12
F   9     9 .119904E+07 .377384E+02 .113389E-02 .227667E+03 .268403E+07 .907248E+00-.415185E-03 .123701E-03-.156686E-01-.428779E+12
F  10    10 .119501E+07 .357528E+02 .107422E-02 .227800E+03 .268962E+07 .905234E+00-.421288E-03 .124563E-03-.150822E-01-.421618E+12

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Table 19 (continued)

```
*   BRIGHAM AND MORROW  (HALF WATER / HALF STEAM)

FLOW DOMAIN DATA                    ELAPSED TIME SINCE START OF SIMULATION =  .68529E+09
                                                  SINCE START OF RUN       =  .68529E+09
                                                  SINCE LAST PRINT-OUT     =  .23628E+07

VOLUME, MASS, ENERGY AND HEAT BALANCES

     FLUID    VOLUME  IN PLACE       =  .10006E+09
     LIQUID   VOLUME  IN PLACE       =  .64552E+07
     VAPOR    VOLUME  IN PLACE       =  .93545E+08

     FLUID     MASS   IN PLACE       =  .65792E+10
     LIQUID    MASS   IN PLACE       =  .53704E+10
     VAPOR     MASS   IN PLACE       =  .12126E+10

     MARGINAL  MASS PRODUCTION       =  .11814E+09
     MARGINAL  MASS PRODUCTION RATE  =  .49999E+02
     CUMULATED MASS PRODUCTION       =  .34265E+11      FLUID MASS FRACTION PRODUCED  =  .67892E+00
     AVERAGE   MASS PRODUCTION RATE  =  .50000E+02

     SOLID  INTERNAL ENERGY          =  .50045E+16      SOLID  HEAT CONTENT           =  .50045E+16
     FLUID  INTERNAL ENERGY          =  .83711E+16      FLUID  HEAT CONTENT           =  .86383E+16
     LIQUID INTERNAL ENERGY          =  .52125E+16      LIQUID HEAT CONTENT           =  .52295E+16
     VAPOR  INTERNAL ENERGY          =  .31543E+16      VAPOR  HEAT CONTENT           =  .33966E+16

     MARG.  SOLID  INT. ENERGY LOSS  =  .21667E+15      MARG.  SOLID  HEAT LOSS       =  .21667E+15
     MARG.  FLUID  INT. ENERGY LOSS  =  .11441E+15      MARG.  FLUID  HEAT LOSS       =  .11472E+15
     MARG.  RESER. INT. ENERGY LOSS  =  .33107E+15      MARG.  RESER. HEAT LOSS       =  .33138E+15
     MARG.  RESER. ENERGY LOSS RATE  =  .14012E+09      MARG.  RESER. HEAT LOSS RATE  =  .14025E+09

     CUMUL. SOLID  INT. ENERGY LOSS  =  .58311E+17      CUMUL. SOLID  HEAT LOSS       =  .58311E+17
     CUMUL. FLUID  INT. ENERGY LOSS  =  .37727E+17      CUMUL. FLUID  HEAT LOSS       =  .37879E+17
     CUMUL. RESER. INT. ENERGY LOSS  =  .96038E+17      CUMUL. RESER. HEAT LOSS       =  .96190E+17
     AVERAGE RESER. ENERGY LOSS RATE =  .14014E+09      AVERAGE RESER. HEAT LOSS RATE =  .14036E+09

MASS AVERAGES

     FLUID  SPECIFIC INTERNAL ENERGY =  .12724E+07      FLUID  SPECIFIC ENTHALPY      =  .13118E+07
     LIQUID SPECIFIC INTERNAL ENERGY =  .97061E+06      LIQUID SPECIFIC ENTHALPY      =  .97377E+06
     VAPOR  SPECIFIC INTERNAL ENERGY =  .26013E+07      VAPOR  SPECIFIC ENTHALPY      =  .28011E+07

VOLUME AVERAGES

     FLUID  DENSITY                  =  .65792E+02
     LIQUID DENSITY                  =  .83194E+03
     VAPOR  DENSITY                  =  .12963E+02

     LIQUID TEMPERATURE              =  .22647E+03
     VAPOR  TEMPERATURE              =  .22562E+03

     LIQUID PRESSURE ( N/M2 )        =  .26278E+07
     LIQUID PRESSURE ( ATM. )        =  .26796E+02
     VAPOR  PRESSURE ( N/M2 )        =  .25896E+07
     VAPOR  PRESSURE ( ATM. )        =  .26406E+02

     P/Z             ( N/M2 )        =  .29883E+07
     P/Z             ( ATM. )        =  .30472E+02
     P/Z             ( PSIA )        =  .43343E+03
     Z                               =  .86656E+00
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Table 19 (continued)

* BRIGHAM AND MORROW (HALF WATER / HALF STEAM)

DENSITY CYCLE OUTPUT DATA 1000 DCYCLES

KCYC	KCYCF	KCYCFC	KNIT	NUTSF	NUTXF	NUTSUMF	MAX. DD	SLIMIN	SLIMAX	DDSLMX	RATGF	RATESF
1000	1	2000	3	2	10	5591	.353710E-01	.100000E+25	.100000E-23	.100000E-23	.332026E+25	.100000E+09

TOTAL TIME	CUM. MASS GEN.	CUM. GAIN	AV. GAIN RATE	TIME STEP	MASS GEN.	MASS GAIN	GAIN RATE	WGHT. FACTOR
.685294E+09	-.342647E+11	-.342647E+11	-.500000E+02	.332826E+05	-.166413E+07	-.166413E+07	-.500000E+02	.570000E+00

ELEM.		INDEX	D	DD	DDD	T	P	PH?	DE	CPDD	E	VR
F	1	1	.272321E+02	-.353710E-01	-.106275E-05	.221797E+03	.242257E+07	.981674E+00	.101100E+04	.785679E+050.		.689433E+05
F	2	2	.355716E+02	-.605899E-02	-.182044E-06	.223945E+03	.247394E+07	.971506E+00	.959998E+02	.562424E+050.		.607795E+05
F	3	3	.467087E+02	-.140128E-01	-.445062E-06	.224221E+03	.252356E+07	.958508E+00	.130324E+03	.401547E+050.		.606161E+05
F	4	4	.548699E+02	-.176766E-01	-.531107E-06	.225210E+03	.256647E+07	.944867E+00	.393634E+02	.303905E+050.		.604838E+05
F	5	5	.679046E+02	-.157230E-01	-.472408E-06	.225991E+03	.260197E+07	.933009E+00	.680328E+02	.246071E+050.		.603800E+05
F	6	6	.754481E+02	-.141885E-01	-.426304E-06	.226580E+03	.263112E+07	.923884E+00	.396610E+02	.201345E+050.		.603021E+05
F	7	7	.810044E+02	-.150683E-01	-.452739E-06	.227022E+03	.265472E+07	.917129E+00	.394120E+02	.193039E+050.		.602484E+05
F	8	8	.859936E+02	-.157618E-01	-.503619E-06	.227410E+03	.267253E+07	.911983E+00	.422042E+02	.185330E+050.		.601975E+05
F	9	9	.891579E+02	-.156790E-01	-.471110E-06	.227665E+03	.268398E+07	.907267E+00	.377720E+02	.181379E+050.		.601637E+05
F	10	10	.908303E+02	-.150700E-01	-.452789E-06	.227798E+03	.268957E+07	.905252E+00	.357756E+02	.175350E+050.		.601450E+05

NOMENCLATURE

A_{nm}	interface area between volume elements n and m, m^2
a	general element of area, m^2
C_S	specific heat of rock, J/°C kg
d_n, d_m	distance of interface from center of volume elements, m
$\underset{\sim}{F}$	mass flux, kg/m^2s
$\underset{\sim}{F}_\ell$	flux of liquid ,kg/m^2s
$\underset{\sim}{F}_v$	flux of vapor, kg/m^2s
F_{nm}	mass flux between volume elements n, m, kg/m^2s
$\underset{\sim}{G}$	energy flux, J/m^2s
G_{nm}	energy flux between volume elements n, m, J/m^2s
$\underset{\sim}{g}$	vector of gravitational acceleration, m/s^2
g_{nm}	component of gravitational acceleration perpendicular to the interface between volume elements n, m, m/s^2
h	specific enthalpy of fluid, J/kg
h_ℓ	specific enthalpy of liquid, J/kg
h_v	specific enthalpy of vapor, J/kg
h_S	specific enthalpy of rock, J/kg
K	thermal conductivity of rock/fluid mixture, J/ms°C
K_n, K_m	average thermal conductivity in volume elements n, m J/ms°C
K_{nm}	average thermal conductivity at interface between volume elements n, m, J/ms°C
K_S	thermal conductivity of rock, J/ms°C

k	absolute permeability, m^2
k_n, k_m	average absolute permeability in volume elements n, m, m^2
k_{nm}	absolute permeability at interface between volume elements n, m, m^2
k_ℓ	relative permeability of liquid, fraction
k_v	relative permeability of vapor, fraction
$(k/\nu)_T$	total kinematic mobility, s
M	rate of fluid production, kg/s
$\underset{\sim}{n}$	unit normal on surface element
p	pressure, N/m^2
p_n, p_m	average pressure in volume elements n, m, N/m^2
p_o	pressure at initial time, N/m^2
q	volumetric rate of mass generation, kg/m^3s
q_n	average volumetric rate of mass generation in volume element n, kg/m^3s
Q	volumetric rate of energy generation, J/m^3s
Q_n	average volumetric rate of energy generation in volume element n, J/m^3s
R	ratio of density to energy change, $(kg/m^3)/(J/kg)$
R_n	average ratio of density to energy change in volume element n, $(kg/m^3)/(J/kg)$
r_i	radial length of volume element i of radial grid, m
S	volumetric vapor saturation, fraction
S_{res}	residual immobile volumetric liquid saturation, fraction

T	temperature, ^{o}C
T_n, T_m	average temperature in volume elements n, m, ^{o}C
t	time, s
Δt_u	time step for energy equation, s
Δt_ρ	time step for density equation, s
t_o	initial time, s
u	specific internal energy of fluid, J/kg
u_n, u_m	average specific internal energy of fluid in volume elements n, m, J/kg
u_{nm}	average specific internal energy of fluid at interface between volume elements n, m, J/kg
u_ℓ	specific internal energy of liquid, J/kg
u_v	specific internal energy of vapor, J/kg
u_S	specific internal energy of rock, J/kg
u_{up}	average specific internal energy of upstream volume element, J/kg
u_{down}	average specific internal energy of downstream volume element, J/kg
V	volume of reservoir, m^3
V_n, V_m	volume elements of reservoir, m^3
(V_n)	surface of volume element V_n, m^2
Z	gas law compressibility factor, dimensionless

Δ	increment
ε	fluid energy per volume, J/m^3
κ	weighting factor for mobilities, dimensionless
λ	weighting factor for energies, dimensionless
ρ	fluid density, kg/m^3
ρ_n, ρ_m	average fluid density in volume elements n, m, kg/m^3
ρ_{nm}	average fluid density at interface between volume elements n, m, kg/m^3
ρ_o	initial fluid density, $kg/m,^3$
ρ_ℓ	density of liquid, kg/m^3
ρ_v	density of vapor, $kg/^3$
ρ_α	density of phase α, kg/m^3
ρ_S	density of rock, kg/m^3
τ	general volume element, m^3
θ	time weighting factor, dimensionless
μ_ℓ	viscosity of liquid, Ns/m^2
μ_v	viscosity of vapor, Ns/m^2
μ_α	viscosity of phase α, Ns/m^2
ϕ	porosity, dimensionless
ϕ_n	porosity of volume element n, dimensionless

SUBSCRIPTS

α	liquid or vapor phase
down	downstream
exp	explicit
ℓ	liquid
n, m	volume elements
S	rock
res	residual
ρ	referring to density
u	referring to energy
up	upstream
v	vapor

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