DO E/ET/27148-1

MASTER

AN ANALYTIC COMPARISON OF TIME- AND FREQUENCY- DOMAIN ELECTROMAGNETIC METHODS

by

C.H. Stoyer
Geophysics Department
Colorado School of Mines
Golden, CO 80401

DISCLAIMER

This book was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

REA

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

Abstract

The time- and frequency-domain methods of electromagnetic geophysical prospecting are compared to determine the similarities and differences and to recommend system choices for particular field situations. Only the wire-loop configuration is considered, as this array is quite standard in geothermal prospecting. Comparisons are carried out using hardware and physical considerations, a large catalog of 3-layer model curves, 2D-3D model calculations, and by comparing Fourier transforms and layered inversions of Field data from the Randsburg KGRA in California.

The results generally indicate that Frequency-domain methods offer better resolution and more practical hardware design for long-offset shallow applications. They also have a much better backup in terms of modelling tools for interpretation and history of experience. Transient methods are better suited for deeper probing, for both long- and short-offset applications. Frequency-domain methods are limited by the primary field, which is subject to distortion from near-surface inhomogeneities; this is also the case for early-time transients. Transient measurements in late time are limited by ambient electromagnetic noise and dynamic range of receiving equipment.

Mapping of field data across domain lines by Fourier transformation proved to be disastrous. This was predicted by the results of layered model calculation studies. It seems that, although the domains are analytically equivalent, they are effectively isolated by practical considerations.

In many cases, the quality of data and ensuing interpretations will depend more on the experience and skill of the field crew and interpreter, than on whether a time- or frequency-domain method was used.

INTRODUCTION

For the past 30 years, electromagnetic methods have played an important role in mining geophysics, especially in Sweden and Canada. By far, the majority of work has been done in frequency-domain, perhaps because the mathematical expressions which are used to interpret the data are derived in the frequency-domain. Periodically, however, time-domain methods have been developed and employed, often to avoid some sort of difficulty which is encountered in practical frequency-domain situations.

Of course, there are many methods which do not clearly fit into the time- or frequency-domain. For the purposes of this study, we shall provide two definitions for guidance. Strictly speaking a time-domain method is one in which the source is a delta function in time (or a step if a derivative is measured) and a frequency-domain technique is one in which the source is a delta function in frequency (continuous harmonic of a single frequency in time). Loosely speaking, a time-domain method is one in which the source is constant in time (or off) during the measurement period, and a frequency-domain method is one which measures both a primary field from the source and a secondary field. The latter definition is far more practical than the former, and it also includes methods which satisfy the former.

Thus, there are two basic differences between the methods. First, since we live and work in the time-domain (the frequency-domain does not really exist outside of the mathematics) all measurements must be made of voltage varying with time, whether

or not they are transient or sinusoidal. This implies that a frequency-domain method requires repeated measurements at many frequencies in order to obtain a sounding, while a transient method can produce a sounding in one fell swoop. Practically, of course, transients must be stacked, so this difference may not be as advantageous to the time-domain method as it first seems.

The second difference is that of a strong primary field which is present in the frequency-domain, but absent in the time-domain. This is the most important reason for moving to time-domain methods. Specifically, the Barringer Input system was designed to avoid the rather large spurious responses which arise from the receiver "bird's" movement in the primary field, and there is currently a general trend towards transient methods for deep probing for geothermal applications and for mining applications in Australian (thick conductive overburden) environments. The latter point concerns the effect of near-surface variations' distortion of the primary field, and the large power requirements for geothermal probing applications.

From a physical point of view, currents flowing the ground in frequency-domain methods are concentrated near the source, and are therefore closely tied to the surface. This requires that the offset distance be much larger than the desired probing depth, otherwise the primary field will be much larger than the secondary field, which contains the desired information. In the time-domain, the currents which influence the measurements come from near-surface and resistive bodies at early times, and from deep and conductive bodies at late times. There is therefore a

strong tendency for the influence of the overburden and the conductive target to be separated in time, whereas this is not true in frequency.

The interpretation tools available for frequency- and time-domain methods are also widely different. Since the great majority of the algorithms for computing model responses have been carried out in the frequency-domain, the time-domain has limited interpretation backup. This trend, however, is changing, albeit somewhat slowly. In addition, transient data, because of the depth separation in time, can sometimes be qualitatively interpreted using elementary physical principles. For the time being, however, frequency-domain does have an advantage from the standpoint of experience and modelling capability.

In the following sections, we will compare the time- and frequency-domain from the standpoints of hardware, computer models of layered and 2D-3D situations, and field results. We will restrict our study to the wire-loop configuration, that is an x-directed grounded current dipole source, and a flat-lying loop receiver. This is one of the most common configurations, and we were able to obtain field data for wire-loop soundings. For the most part, these comparisons agree with our original premise that shallow applications demand frequency methods, deep studies demand time-domain methods, and medium depth applications can be satisfied by either, depending on the availability of hardware and experienced personnel.

Some Basic Concepts

Analytically speaking, the time- and frequency- domains are equivalent, and the Fourier Transform forms the connection between the two. The equivalence of the two are practically dependent on our being able to carry out this Fourier transform. However, difficulties arise in implementing this transform both in analytical, numerical and measurement situations.

The transform is somewhat simplified by the consideration of causality. Since the transient response must be zero for times less than zero (the onset is defined at $t=0$), it follows that the transient must consist of equal even and odd components, such that they reinforce each other for positive times and cancel for negative times. This means that we can calculate the transient by either the cosine transform of the real part or the sine transform of the imaginary part.

This symmetry indicates that the in-phase and quadrature components of the amplitude and phase contain redundant information. If, for instance, only the in-phase component were known, one would simply Fourier transform to the time domain via the cosine transform, and then transform back to frequency using the sine transform. Then we would also know the quadrature component as well as the in-phase. A similar relationship exists between amplitude and phase. One can begin an argument with the amplitude and contour integration, and end up with the phase via the Hilbert transform. Thus, in principle, there is no reason to

measure both the amplitude and phase or both in-phase and quadrature components of the frequency-domain fields.

In practice, however, the situation is somewhat different. Later, we will discuss the extreme difficulty of carrying out these Fourier transforms. The analytically equivalent domains of time and frequency are effectively isolated by practical considerations, unless stable and precise data are available. Thus, if Fourier transforms can be carried out analytically, the equivalence is preserved, totally. If they must be carried out numerically, the data set must be complete, which means it must be both abundant and precise. This requirement effectively rules out any such treatment of measured data, as we shall see later in a test.

Thus, amplitude and phase information are not redundant in a practical sense, since the phase can effectively look-ahead to detect changes in the amplitude spectrum which would occur, if the data set were not band-limited. Also, from these considerations, we will not expect to be able to create transients from anything but complete, precise, and abundant data, nor shall we expect to obtain phase from amplitude or quadrature from in-phase field data.

Equipment

From the standpoint of source equipment, the time-domain has a distinct advantage for deep probing, where the measurement times are long. For shallow probing, extremely short current transitions are necessary to prevent distortion of the

transient. Switching times of several milliseconds are easily achieved with simple mechanical switches, so that the source equipment is relatively simple, available, and virtually all of the power is delivered to the ground. For instance, the Colorado School of Mines has a system capable of several hundred amperes, which consists of little else but a generator, a set of switches, and some diodes. Switching transistors can be used for shorter times, and still consume little power. Nevertheless, switching a direct current instantaneously is not an easy achievement, so there are practical limits.

A frequency-domain source must use transistors to control the current, and about half the power is consumed by the amplifier. There are also power limits on the amplifier. However, high-frequency currents can be achieved relatively easily. While at Penn State, I developed a frequency-domain source for shallow work with little difficulty. The operating frequency was as high as 100 kHz. For shallow work, power is never a major consideration.

The quality of the measurements depends considerably on the receiver equipment. Time-domain measurements require a wide-band amplitude sensitive recording device. Since the definition of wide-band changes with probing depth, better results can again be obtained for deep probing than for shallow probing. An amplitude-sensitive system can also be used for very shallow probing with frequency-domain methods, with the added advantage that narrow-band filters can be used to remove noise. Otherwise, the signal can be digitized and matched-filtered on a computer.

Provided that a clock or source signal is provided, the phase can now be calculated. Phase can also be determined using analog circuits. As we shall see later, the additional information contained in the phase allows for deeper probing.

In either case, a digital receiving system is probably the best way of obtaining the best frequency- or time-domain measurements, provided the software for processing the data is well-designed. Except in the case of amplitude-sensitive frequency-domain systems, analog filters are to be avoided in either domain. They tend to introduce phase distortion which must affect both domains identically. Low-pass filters are necessary, however, to prevent aliasing of the digital signal.

Thus from the standpoint of equipment, time-domain seems to have an advantage in deep probing, while frequency domain is better for shallow probing.

Model Calculations

The mathematics of electromagnetic sounding is heavily biased towards the frequency-domain. Calculations for plane-layered, two- and three- dimensional problems are carried out almost without exception in frequency, and transient responses must be obtained by Fourier-transforms of frequency-domain responses. These Fourier transforms are extremely difficult to calculate, and care must be taken to avoid numerically noisy results, especially in late time when the transient is decaying rapidly.

It is probably safe to say that the calculation of

electromagnetic transients with reasonable accuracy was quite difficult until Anderson (1975) provided digital filtering algorithms for computing the Fourier cosine transform. Kauahikaua and Anderson (1977) published a complete FORTRAN program for the grounded wire source, which includes both frequency- and time-domain algorithms. This program package has been extensively tested by us for the case of a loop receiver (vertical component of the magnetic field) and has been found to be quite satisfactory, at least for measurable fields. The program fails for very late times of the transient, however, since no care was taken to stabilize the numerically noisy calculations which are inevitable when the transient has decayed many orders of magnitude from its initial value. So we undertook to develop a new program, based on Anderson's (1975) transform filters, which could provide extremely accurate results over an extensive time range, to be used in this study. We have no intention of degrading Kauahikaua and Anderson's program, however, since it forms the basic forward section of our own transient inversion program.

During the process of program development, we learned several things which are pertinent to this study. Although means were taken to insure numerical stability in the frequency-domain calculations (these, remember, form the kernel function for the time-domain calculations). The real problem was to stabilize the transient calculations. The problem stems from the fact that the Hankel transform integral for the total field does not converge. For large values of the Hankel wavenumber, g (which

relates to small r), The integrand rapidly approaches that for a homogeneous half-space with a resistivity equal to that of the first layer. The most obvious thing to do is to subtract the primary field for a half-space of resistivity ρ_1 . Now, the integral converges, and the primary response, which is expressed analytically, can be added later, either in the time or the frequency domain.

The total field frequency-domain response for a loop receiver of unit area and a wire source of unit moment is given by $\mu_0 \partial H / \partial t$, where

$$H_z(\omega) = \frac{Y}{4\pi r} \int_0^{\infty} g(1 + R_l(g)) J_1(gr) dy. \quad (1)$$

Here, J_1 is the Bessel Function of the first kind and order 1, and R_l is the Transverse electric reflection coefficient, found by

$$R_l = \frac{N_o - Y_l}{N_o + Y_l}$$

where $N_i = \sqrt{g^2 + k_i^2} / j\omega\mu$ is the intrinsic admittance of a layer, $k_i = \sqrt{-\frac{j\omega\mu_0}{\rho_i}}$ and N_i is the admittance at the top of a layer based on the input impedance concept (Wait, 1966):

$$Y_i = N_i \frac{Y_i + 1 + N_i \tanh(\sqrt{g^2 + k_i^2} h_i)}{N_i + 1 + \tanh(\sqrt{g^2 + k_i^2} h_i)} \quad (3)$$

The time-domain fields can be calculated by Fourier transforming the frequency-domain expression:

$$h_z(t) = \frac{Y}{4\pi r} \int_0^{\infty} \cos \omega t \, d\omega \int_0^{\infty} g[1+\text{Re}(R+(g))] T_1(gr) dg \quad (4)$$

Then, the primary field, that for any homogeneous earth of resistivity ρ can be expressed as

$$H_p(\omega) = \frac{\rho Y}{2\pi j\omega\mu r^5} (3-3+3jkr-k^2r^2) e^{-jkr} \quad (5)$$

In the frequency-domain, or as

$$h_p(t) = \frac{3Y\rho}{2\pi\mu_0 r^5} [\text{erf}(u) - 2(u+2u^3/3)e^{-u^2}/\sqrt{\pi}] \quad (6)$$

in the time-domain, where $u = \sqrt{\mu_0 r^2/4t\rho}$

Now, the integrand in equation (1) does not converge. Neither does the integrand in the integral expression for the primary field:

$$H_p(\omega) = \int_0^{\infty} g \left(1 + \frac{N_0 - N_1}{N_0 + N_1} \right) J_1(gr) dg \quad (7)$$

Both of these kernels tend to a value of g as the Hankel wavenumber, g , tends to infinity. Of course, we need never numerically evaluate the above expression, since we can do this analytically (equations 5 and 6). If we subtract a primary field for resistivity ρ_1 from the layered-earth integrand, it will converge, but it will also converge if we subtract the primary field for any resistivity. Thus, numerically stable calculations of the transient fields will require that we choose a homogeneous earth resistivity which varies with time such that it provides a signal which closely follows the transient from the layered medium. This will ensure that the secondary field, calculated by a double transform (Hankel inside a Fourier) will never be large compared to the primary field, calculated by an exact analytical expression. Of utmost importance, the case of primary and secondary fields which are added to form a total field which is orders of magnitude smaller than either of the two components, is completely avoided.

This algorithm requires about four to six times the computational effort of the standard method which utilizes the first layer resistivity, so it is not recommended for general practice. This is why we are using Kanahikaua and Anderson's algorithm in our current inversion and modelling program. However, it should be used when exact curves spanning many orders

of magnitude in time are desired.

Layered Calculations

The digital filters devised by Anderson (1975) have a log-domain sampling of approximately 12 points per decade, for both Hankel and Fourier transforms. The calculation of a single frequency point requires the calculation of about 30 to 50 Hankel-domain points. The calculation of a single time-domain point requires about 90 to 130 frequency domain points (each of which constitutes a Hankel transform). Of course, the frequency domain points are saved in an array so that they need not be calculated again when additional time points are requested, and additional frequency points are calculated (and saved) as required. In our experience, no more than four to six additional frequency points are needed, once the first time point is calculated; this means that all additional points require minimal computational effort. Thus, the Fourier integral transform for electromagnetic fields spans eight to ten orders of magnitude in frequency. Also, the notion that early time and late time are equivalent to high and low frequency, respectively, is cast in a shadow of doubt. We are reminded that a single point in time is a weighted average of all frequency components, as specified by Fourier transform theory. It naturally follows that a Fourier transform to frequency from a transient should be just as difficult to carry out numerically as that from frequency to time.

Computations for more Complex Models

As soon as we begin to consider models which are more complex than a stratified earth, we realize that, in the context of a time- and frequency- domain comparison, we are in deep trouble. First of all, we must limit ourselves to realistic models, those which concern an inhomogeneity buried in a conducting half-space, which makes electrical contact with the host medium, and which includes the response to a three-dimensional source field. To consider anything less, has been shown by a thirty-year history to be misleading when applied to practical problems. Thus, many analytical approaches, which are attractive because they might form the basis for a stable Fourier transform, must be discarded.

Numerical modelling methods can handle realistic models, but there have been many questions raised about their precision. From my own experience, I can easily predict that the use of finite-element, finite-difference, and integral-equation model results in a Fourier transform would be most laborious, and would produce disastrous results. All three of these methods encounter great difficulty at high frequencies so that, while they are adequate for frequency-domain modelling at modest frequencies (and with great care), they are useless for modelling electromagnetic transients.

At least one modelling method satisfies our requirements of being a conductor, in electrical contact with its conducting

semi-infinite hosts, which can be expressed in analytical form and calculated with sufficient accuracy to withstand a Fourier transform from frequency to time without being seriously degraded by numerical noise. This is the model of a buried thin axial conductor described by the primary fields which are expressed as before, and the secondary fields are expressed as a triple Fourier integral (Tsubota and Wait, 1980):

$$e_x(t) = -2\mu_0/\pi \int_0^\infty \cos \omega t \, dt \int_{-\infty}^\infty \tilde{I}(k_x)$$

$$\left[\frac{1}{u+u_0} - \frac{k_x^2}{k^2 u_0} \right] \exp(u_0 z - u d) \cos(k_y y)$$

$$\cos(k_x x) \, dk_x \, dk_y$$

where

$$u = \sqrt{k^2 - k_x^2 - k_y^2}$$

and $\tilde{I}$ is given in terms of the primary field and the axial conductor's impedances:

$$\tilde{I}(k_x) = E_x^p / [Z_c(k_x) + Z_e(k_x)].$$

The conductor's characteristic impedance is

$$Z_c = \frac{j\omega \mu_0}{2\pi c} \frac{I_0(jk_c c)}{I_1(jk_c c)}$$

where ρ_c is the resistivity of the conductor and c is its radius. The external impedance is given by

$$Z_e = \frac{(k_x^2 - k^2)\rho}{2\pi} \{ K_0(jc\sqrt{k^2 - k_x^2})$$

$$- K_0(2jd\sqrt{k^2 - k_x^2}) + \frac{2k^2}{k^2 - k_x^2} \cdot$$

$$\int_{-\infty}^{\infty} \left[\frac{1}{u+u_0} - \frac{k^2}{k_0^2 u + k^2 u_0} \right] \exp(-2ud) dk_y$$

I_0 , I_1 , and K_0 are modified Bessel functions, d is the depth of burial of the axial conductor. Unsubscripted values are for the homogeneous host of resistivity ρ , while those subscripted with zero are for air.

Actually, the theory was developed for the electric field from a vertical magnetic dipole, but reciprocity allows us to reverse the role of the source and receiver (but not the positions), so that the voltage is induced in the horizontal loop while the current is injected into the wire. Although, it would be more interesting to have a purely three-dimensional model as well, such a model capable of withstanding Fourier transformation to the time domain was not available, so we have used this 2D-3D model, incorporating a two-dimensional structure and three-dimensional fields instead. Indeed, the distortion introduced by this 2-D "test conductor" is small enough and that of a 3-D "test conductor" would likely produce little distortion of the fields,

although this distortion would likely be confined to early times.

Computation Times

All calculations were carried out on our CDC CYBER 720 Computer. Each frequency-domain curve required about 15 seconds and each time-domain curve required about 90 seconds CPU time for layered models. For the 2D - 3D calculations, the frequency domain calculations used in the time-domain Fourier transform were retained, so that only one set of calculations was required. Each of these required about 12 minutes of CPU time per curve. Thus the total computational effort represented in this study is about 7 hours.

Visual Representations of Calculated Curves

Unless a cryogenic magnetometer is used in the field, the measurements will be proportional to the time-derivative of the magnetic induction, $\partial B / \partial t = \mu \partial H / \partial t = V / A$, where V is the voltage induced in a loop receiver of effective area A . Assuming the electronics in the receiver are distortion-free, this will result in an amplitude factor of ω and a phase shift of 90° . This factor and phase shift can easily be removed, so it seems reasonable to display the frequency-domain results in terms of the amplitude and phase of the total field. This is also the most common representation.

Under the same assumptions, the transient induced in a loop

from a current step in the source wire will rise instantaneously to an initial value, and will decay rapidly in the late time stage. We could simply display the voltage induced in a loop or unit area for a unit current step in a unit-length source wire, but transients are customarily not displayed in this fashion. A convenient way of normalizing the transient without introducing any distortion is to use the early-time apparent resistivity.

Time-domain Apparent Resistivity

There is no straightforward means of expressing electromagnetic measurements in terms of an apparent resistivity, as is the case for direct current or magnetotelluric sounding methods. However, if one considers the asymptotic expressions for the fields, apparent resistivity formulas valid for specific time (or frequency) ranges can be devised. For instance, consider equation (6) for very small values of t . As $t \rightarrow 0$, this expression becomes

$$h_p(t) = \frac{3\gamma\rho}{2\pi\mu_0 r^5}.$$

Thus, for a source moment $I dx$ and a receiver loop area A , the voltage induced in the loop can be expressed as an apparent resistivity by inverting this expression:

$$\rho_a = \frac{2\pi V r^5}{3AI \partial x}$$

This is a very convenient way of normalizing field curves.

However, electromagnetic transients eventually decay with time, and shortly before this decay begins the early-time apparent resistivity is no longer valid and is reduced to a simple normalizing factor.

Equation (6) also has a late-time asymptote which is not so obvious by inspection. However, by considering the low-frequency asymptote of the frequency-domain fields and using the Fourier transform, Kaufman and Goldman (1967) give the expression

$$\rho_a = \left[\frac{r A I dx \mu^{5/2} \sin 2\alpha}{40 \pi \sqrt{\pi} t^{5/2} V(t)} \right]^{2/3}$$

where $\alpha = \cos^{-1} (y/r)$.

Since it is customary to represent time-domain curves in terms of these apparent resistivities, we shall use these formulas here. This is not meant to imply that apparent resistivity cannot be defined for frequency-domain measurements. For instance, the high-frequency asymptote of the induced voltage in the frequency-domain yields the same formula as the time-domain early-time formula. However, it is not customary to display frequency-domain curves in this way, and so we will not use it. Figure 1 shows transient apparent resistivities for a family of curves as described in the Figure. Note that the early time formula is quite descriptive of the model for times which precede the rapid decay, while the late time formula becomes descriptive at this point. One can easily visualize (but not calculate) a "true" apparent resistivity curve

which looks exactly like a 2-layer Schlumberger curve over the same model, if the horizontal scale were $2 \log AB/2$, instead of time.

Curves for a Stratified Earth

A rather extensive graphical catalog of three-layer sounding curves were calculated for both frequency- and time-domain fields. These were calculated for a three-layer model having first and last layers of 100 ohm-m resistivity, and a second layer resistivity which varies ($\rho_2 = 1, 3, 10, 30, 100, 300, 1000, 3000, \text{ and } 10000$) to form a family of curves. Ratios of second layer to first layer thickness were 0.5, 1, 2, 3, and infinity (2-layer model). Offset distances ranged from very long to very short, with R/h_1 ratios of 100, 33, 10, 3, and 1. Frequency-domain curves are represented in the standard form of amplitude and phase of magnetic field (unnormalized) while transients are represented in the form of apparent resistivities (using both early- and late-time formulas), which is also standard.

These curves are displayed on real-time and real-frequency scales, rather than normalized scales. This is done to give the reader a feel for the practical problems which might arise in the field when these are measured. There are, however, two methods of normalizing time and/or frequency, and it is important to realize that they are considerably different in practical value.

Probably the most familiar normalization involves the offset distance:

$$\tau = t_p / \mu R^2 \text{ and } b = \omega \mu R^2 / \rho.$$

These have the convenience of making the response of a homogeneous half-space invariant under changes in either R or ρ . They have, however, little to do with the depth to which the fields are sensitive. Instead of using the dimensionless τ or b , the expressions

$$\Delta = \sqrt{t_p / 2\mu} \text{ and } \delta = \sqrt{2\rho / \mu\omega}$$

have significantly more meaning. Here, Δ is the diffusion depth and δ is the familiar skin depth. In the following catalog of curves, these can be used with surprising accuracy to predict the behavior of the sounding curves in a qualitative sense.

Each of the 25 sounding curves is shown on a separate page in the Appendix, but for convenience of visual comparison, they are reduced and shown on three separate pages in Figures 2, 3, and 4 for transient, amplitude, and phase, respectively. These curves are not meant to be read. They are presented in this very reduced form only to facilitate comparisons among the various curve families. This would be quite cumbersome if the curves were on 75 separate pages. The curves in the Appendix are related to these curves by standard row-column matrix notation. For instance, the family shown in row 2, column 3 or figure 2 shows up as figure 2-3,3 in the appendix.

It is only fair to caution the reader at this point that the

late-time apparent resistivities can be misleading. They are not misleading in the sense that they portray a distorted perception of the earth model, but they are misleading in the sense that, while they are aesthetically very pleasing and informative, they may not be measurable. Similar apparent resistivities can be formulated for the frequency-domain fields, and these would be non-measurable over certain regions as well.

A quick visual comparison of the three figures 2, 3, and 4 indicates that these are the time-domain response is indicative of structure over the entire range of parameters while the frequency-domain response seems to differ insignificantly from that of a homogenous earth for short-offset situations

($r < h_1$). Some caution must be exercised here, and we turn our attention to the early-time apparent resistivity curves (always lower at any particular time than the late-time curves). A comparison of responses for $r = h_1$ shows that the transients for a conductive layer are measurable with little difficulty, while the frequency-domain amplitudes and phases have almost insignificant differences in response. Phase can be measured to several milliradians, or several parts per thousand, however. But the curves across the top of the page indicate that the frequency-domain response even for low frequencies is severely influenced by a shallow conductive layer, while the transient response for late time is influenced to a proportionately lesser degree. Except for the two-layer case (extreme right column of curves), the transients refuse to respond strongly to a resistive layer, and the frequency data are not much better. This is to be

expected, since electromagnetic fields are notorious for ignoring resistive zones in the subsurface.

For shallow or long-offset situations, the frequency-domain response seems to be significantly greater than the transient response (top of page). Thus, frequency-domain methods seem to be better suited for shallow or long-offset situations while time-domain methods seem to be more applicable in deep or short-offset applications. In either case, a resistive bed will be only indicated by the data and its resistivity will never be known unless the contrast is relatively low. This is where the late-time apparent resistivity curves mislead us into believing that a resistive half-space can be defined. The dynamic range and signal level requirements to obtain the resistive late-time curves are so severe, however, that in many cases they are unobtainable. For shallow resistive beds, frequency-domain data should be able to provide better definition than time-domain data.

Calculated Curves from 2D and 3D Model

The previously described model which incorporates 2D conductors in 3D source fields (Wait and Tsubota, 1980) was used to calculate several curves to show the effect of near-surface lateral inhomogeneity on time- and frequency-domain fields. The geometry is shown in figure 5. The model used is a buried axial conductor of radius $c = 0.01$ buried at a depth of $d = 0.1$ or 0.2 meters. The wire source is offset 1 meter from the buried conductor ($y_w = 1$). Receiver positions are $x = 0.5, 0.3, 0.2,$

0.1, 0.05, -0.05, -0.1, -0.2, -0.3, and -0.5, where positive positions represent a displacement from the buried conductor towards the source, while negative positions represent a displacement away from the source. Transients are displayed in terms of apparent resistivities, as before, but times are now normalized by the factor $\rho/\mu R^2$. Amplitudes of frequency-domain fields are also normalized so that their amplitude is independent of offset distance, and are shown versus the normalized frequency or induction number $b = \omega \mu R^2 / \rho$. Phases in degrees are also shown versus normalized frequency.

The curves are displayed in Figures 5 through 11 for the shallow (.1m depth) conductor, and in 12 through 17 for the deeper (.2m depth) conductor. Inspection of these curves shows several things.

First of all, there is a distinct difference in field behavior depending on whether or not the source and receiver straddle the conductor. When they are on one side of the buried conductor, the transients exhibit a late-time reversal, which is not consistent with a layered structure. The frequency-domain data show a maximum in the amplitude spectrum and positive phases at low frequency. This is also not consistent with a layered structure.

When the wire source and loop receiver straddle the conductor, the secondary field produced by the buried inhomogeneity are in the opposite sense of the aforementioned case. Now, there are no reversals in the transients and the frequency-domain curves do not exhibit obviously non-layered

phenomena. There is some possibility that a single sounding in this case could be interpreted in terms of a layered earth with an intermediate resistive layer and a conductor at depth.

The curves for the deeper conductor show a quite similar behavior, but it is, of course, much less pronounced. Time-domain reversals occur later in time, as do the onset of the perturbations in both the reversing and non-reversing cases. Similarly, the frequency-domain response is cut off at lower frequencies. For both the amplitude and phase curves. The difference is about a factor of ten in both cases, even though the burial depth has only increased by a factor of two.

These curves can be viewed from two perspectives. If we are searching for the buried two-dimensional features, the time-domain curves for late times seem to yield a much larger anomalous response than the frequency-domain response for low frequencies. This is obvious in the case of a reversing transient, but what is not so obvious is that the late time slope for nonreversing transients is different from that for a layered structure, and indicates a late-time apparent resistivity (not shown) which would decrease with time. In contrast, the frequency-domain curves show far less distortion for the deeper conductor than do the transients. This is because the transient response at late times is purely secondary (due to the conductor) while the low-frequency response is strongly influenced by the host medium.

Another perspective from which to view these curves is that of rejecting near-surface inhomogeneities so that the underlying

layered medium might be realistically preceived. Unfortunately, this model does not allow us to study the problem from this perspective, since, although the inhomogeneity is shallow, it is infinite in length and therefore will distort both low-frequency and late-time information, the latter more than the former. It is unfortunate that a realistic, fully three-dimensional model capable of providing transient results with good precision is not available. Without such a calculations, there is no firm support for the contention that deep probing is less sensitive to geologic noise in the time-domain than in the frequency-domain. Indeed, these results support the opposite point of view. We can only be certain that a three-dimensional inhomogeneity would respond quite differently than a two-dimensional one.

Comparisons between Measured Data

One of the points of the proposal for this project was a comparison of frequency- and time-domain data recorded at identical source-receiver positions. Included in this comparison was a Fourier transform from time data to frequency representation and from original frequency data to transient representation. After careful consideration of the problem of calculating transient model curves, it was realized that such a mapping between domains would be futile, and this proved to be the case.

Acquiring both time- and frequency-domain data taken at identical source-receiver stations turned out to be non-

trivial. It seems that no group has access to both frequency- and time-domain equipment, and they are usually heavily biased in favor of one or the other. However, the U.S. Geological Survey proved to be a large and diverse enough group to contain a strong advocate of each method. I was very pleased to obtain a set of frequency-domain data from Walter Anderson (1978) and a set of transient data from James Kauhikaua (1979). Both sets of data were taken in the Randsburg, California, KGRA during late November and early December, 1977. Five stations were occupied by both parties for the same source position.

It has always been my suspicion that a field test of time-versus frequency-domain electromagnetic probing could easily reduce to a competition between field crews and equipment design. It is not clear which crew is likely to have the advantage in this situation, because although transient equipment should be easier to design, very good frequency-domain equipment is available off-the-shelf. In addition, the historical experience of frequency-domain sounding far outweighs that of the transient methods, at least in the Western world. I have judged that the frequency-domain crew had the advantage in the Randsburg survey, although the results are mixed; some support the conclusions of this report and some do not.

One disturbing feature of the transient data was that the magnetic field (not its derivative) was measured during source turn-on. Thus, rather than decaying with time, these transients rise with time. The feature of the late-time's rejection of near-surface geologic noise is likely negated by this approach.

Also, in order to display these transients in the standard apparent resistivity (early-time) form, they had to be time-differentiated. This was first tried using a spline interpolation, but the data are admittedly noisy (standard deviations are 10 to 50% of the measured value) and this did not work very well. So each curve was fit to an order eight polynomial, which was then used to calculate the time-derivatives. Another drawback of the time-domain curves is that the source current was only ten amperes (peak-to-peak square wave), which probably accounts for the noise. In situations like this one, currents of several hundred amperes are quite easily achieved. In spite of all this, the time-domain curves did not fare much worse than the frequency-domain curves in this comparison.

Coincident frequency- and time-domain data were taken at stations 1, 5, 6, 8, and 9. Each data set was mapped onto the corresponding set in the other domain, and both the original and mapped data are shown in Figures 18 through 23. Inversion results (Kauahikaua, 1979) are Figure 24. For the most part, there is no correspondence between the original transients and those mapped from the frequency data or between the frequency-domain data and that mapped from the transients. There are, however, some exceptions. Sounding 6 is very distorted in all cases. Anderson (personal communication) complained about the structure's being very complex under this particular station, and mentioned that he felt very uncomfortable about the layered interpretation. The transient for this station is very short;

this is an artifact of my processing and my fit to an order-8 polynomial in the log domain. Most of this transient was discarded. Kauahikaua, however, managed to obtain a good fit to a three-layer model which shows a conductor of depth. There is also a similarity between the results from both domains for Station 8. The inversion results show that, considering Kauahikaua digitized data rate of 200 samples per second, there is a consistency in the upper section, whereas the transient shows a deep conductor which is apparently too deep for the frequency-domain data to be affected. Inversions for sounding 5 agree roughly, while those for soundings 1 and 9 are in disagreement. It is also interesting to note that Anderson (1978) shows Schlumberger interpretations for station 8, which agree only roughly with the electromagnetic results. Since no field amplitude information was supplied with the data, it is no wonder that the amplitudes of the curves mapped from one domain to the other do not agree.

It could be argued that better results might have been obtained if the curves were extended using half-space formulas so they might span the desired frequency range. Since none of the curves approaches this asymptote, however, extension of the curves would be difficult at best, and would require addition of bias on the part of myself. It could also be argued that the layered models interpreted from the data could be used to calculate a curve in the other domain for matching. This might be interesting, but it does not serve the purpose which was intended here.

The results are fairly clear. The frequency-domain, which sampled the higher frequencies discarded by a slow sampling rate in the time-domain are quite sensitive to the shallow structure, and are able to provide a significant amount of detail in the interpretations. The time domain data, however, in spite of their shortcomings, seem to be sensitive to deeper structure which is missed by the frequency data. Both sets of data, however, suffer from geologic noise; the transient at sounding 6 seems to be able to sense a deep conductor in spite of the noise, but this may be an artifact of lateral variations. The transient data seems to lump the shallow structure into one "average" layer, and in cases where the probing depth interpreted from the inversions is similar, the frequency-domain data are capable of resolving more detail in the structure. This could perhaps be resolved by utilizing a more powerful source for the transient measurements.

Other Methods

Aside from the time-versus frequency-domain consideration, there are many methods which do not really fit neatly into either category, and many more could be devised. The use of a square wave which is too short in period to give transient results, the use of truncated sine waves, chirp signals, triangular waves and assorted other waveforms which are neither sinusoidal nor "step/delta" functions are really mixed-domain methods. The consideration of these is beyond the scope of this report, but we

mention them lest they become conspicuous by their absence.

A proposed method which is intended to bring time-domain virtues to shallow frequency-domain prospecting is the dual-frequency method, described by Botha (1980). This technique, which proposes to measure the difference in field from two frequencies; for example:

$$\Delta H = H(\omega_1) - \frac{\omega_1}{\omega_2} H(\omega_2)$$

is intended to remove the primary field's influence from the quadrature component, thereby allowing the low-frequency information to be perceived without the distortions of local inhomogeneities on the primary field. Apparent resistivity formulas for low frequencies yield curves which are virtually identical to those of late-time transients. The method is intended to be used for shallow work, where transient-methods fall short because of the difficulty of designing an adequate source-switching mechanism. Although the method has not been tested, it promises to be a frequency-domain method which removes the objections raised by proponents of transient methods. It is therefore appropriate to mention it in the context of time- and frequency-domain comparison, the hope that it may shed some light on the inherent differences between conventional methods in these domains. Basically, it offers an analog means of rejecting the primary field which may be comparable to the ability of transient methods to do the same.

Summary, Conclusions and Recommendations

We have considered both time- and frequency-domain methods of electromagnetic sounding from a standpoint of hardware, mechanics, layered and two-dimensional earth model responses, comparisons of real interpreted field measurements, and physical concepts. The hardware requirements make it difficult to carry out deep (> 1 km) soundings with frequency methods, or to carry out shallow (< 100 m) soundings with transient methods. Although the mechanics of the survey are different, it is difficult to detect a distinct bias toward either method for good quality soundings. If good quality is not required, the bias is in favor of transient methods. Layered earth responses show that frequency-domain methods are subject to geologic noise for short offset, but they appear to give better resolution for long offsets than time-domain. The 2D and 3D model calculations show that the response of near-surface but infinitely extensive inhomogeneities is far greater in the time-domain. The field data were taken in a medium-offset situation (offset/probing depth of about 4 to 10), which of course also included long offset situations as well. These results showed that the frequency-domain fared far better in the long-offset (shallow) situation, but the remainder of the results reduced to a competition between hardware and data acquisition technique. This was to be expected. Consideration of physical concepts predicts that transient sounding is better for deep probing, but that

shallow probing is better done in the frequency domain.

Although the frequency and time-domain are analytically equivalent, there are practical considerations which render them different. There is a strong primary field in the frequency-domain which makes low frequency responses more sensitive to shallow structure than to deep. This primary field can be avoided in the time-domain by using proper field techniques, so that late times, the majority of the currents are flowing in deep structure, and the response from near-surface is small.

The measurement of both low-frequency and late-time information is difficult, but in different respects. With low-frequencies, the quadrature component becomes quite small compared to the primary in-phase field. Thus geologic as well as instrument noise is likely to limit this measurement to several milliradians (or several parts per thousand) at best. In the time domain, the late-time curve decays rapidly (as $1/t^{5/2}$ for a homogeneous earth), and again we are faced with the problem of digging a signal out of the noise. This noise, however, is random and is probably not strongly dependent on instrumentation or geologic structure. In this case, it is the ambient electromagnetic noise which limits the range of the measurement.

We found the Fourier transform for electromagnetic fields incredibly difficult to carry out in practice. This restricts the calculation of transients to models for which an analytical expression can be derived, and purely numerical models are ruled out unless they are directly solved in the time-domain. One important point is that there seems to be little hope for

carrying our Fourier transforms on real data. Hence, although the domains are analytically equivalent, they are effectively isolated from each other by practical considerations

We can make several recommendations from this study. For shallow applications, time-domain methods are useless because of difficulties in practical design of field equipment. For deep studies, frequency-domain methods run amuck for the same reasons, and because of a relatively strong primary field which is strongly responsive to near-surface structure. For medium depths (100 m to 1000 m and perhaps beyond), the competition between the two methods reduces to a competition between field systems and crews. Anyone who is presently obtaining satisfactory results in either domain should retain their status quo, as the differences between the time- and frequency-domain are far less than the differences between an experienced worker and a novice one. I would like to specifically not recommend that a field contest between time- and frequency-domain advocates be arranged unless it can be done at minimal cost. I feel such results would be likely to reduce to a contest between two field crews and would probably not shed much real light on the real differences unless it were very carefully organized.

References

- Anderson, W.L. 1975, Improved digital filters for evaluating Fourier and Hankel transform integrals: U.S. Geological Survey Dept. USGS-GD-75-012, 119 p. (NTIS PB-238-199).
- Anderson, W.L., 1978, Interpretation of electromagnetic extra-low-frequency soundings in the Randsburg, California, Known geothermal Resource Area: USGS open-file report 78-562.
- Bothá, W., 1980, Basis of the theory and interpretation of the dual frequency method: Ph.D. thesis T-2329, Colorado School of Mines, Golden, Colorado, 185 p.
- Kauahikaua, J., and Anderson, W.L., 1977, Calculation of standard transient and frequency sounding curves for a horizontal wire source of arbitrary length: U.S. Geological Survey Report USGS-GD-77-007 (NTIS P-234-119)
- Kauahikaua, James, 1979, Interpretation of time-domain electromagnetic soundings in the Randsburg, California, Known Geothermal Resource Area: USGS open-file report 79-244
- Kaufman, A.A. and Goldman, M.M., 1969, Grounded dipole over horizontally stratified anisotropic media: Academy of Science, USSR, NOVA SIBIRSKY, CCCP.
- Tsubota, K., 1979, The frequency and the time domain responses of a buried two-dimensional inhomogeneity: Ph.D. thesis T-2130, Colorado School of Mines, Golden, Colorado, 166 p.
- Tsubota, K. and Wait, J.R., 1980, The frequency and time-domain responses of a buried axial conductor: Geophysics, v. 45, pp. 941-951.
- Wait, J.R., 1966, Fields of a horizontal dipole over a stratified anisotropic half-space: IEEE Trans. Ant. and Prop. vol. AP-14 pp. 790-792.

Figure Captions

- Figure 1 Family of two-layer curves for time-domain sounding illustrating the concepts of early- and late-time apparent resistivity.
- Figure 2 Transient apparent resistivity for 3-layer earth for $\rho_1 = 100$, ρ_2 variable $\rho_3 = 100$ ohm-m h_1, h_2, r variable
- Figure 3 Amplitude of frequency-domain response for 3-layer earth for $\rho_1 = 100$, ρ_2 variable $\rho_3 = 100$ ohm-m h_1, h_2, r variable
- Figure 4 Phase of frequency-domain response of 3-layer earth for $\rho_1 = 100$, ρ_2 variable $\rho_3 = 100$ ohm-m h_1, h_2, r variable
- Figure 5 Model geometry for the wire-loop transient response of a thin buried axial conductor.
- Figure 6 Transient response of shallow axial conductor when receiver and source are both to the right of the conductor. Time is normalized: $\tau = t\rho/\mu r^2$
- Figure 7 Amplitude response of shallow axial conductor when receiver and source are both to the right of the conductor. Frequency is normalized: $b = \omega\mu r^2/\rho$
- Figure 8 Phase response of shallow axial conductor when receiver and source are both to the right of the conductor. Frequency is normalized: $b = \omega\mu r^2/\rho$
- Figure 9 Transient response of shallow axial conductor when receiver and source straddle the conductor. Time is normalized: $\tau = t\rho/\mu r^2$
- Figure 10 Amplitude response of shallow axial conductor when receiver and source straddle the conductor. Frequency is normalized: $b = \omega\mu r^2/\rho$
- Figure 11 Phase response of shallow axial conductor when receiver and source straddle the conductor. Frequency is normalized: $b = \omega\mu r^2/\rho$
- Figure 12 Transient response of deep conductor when receiver and source are to the right of the conductor. Time is normalized: $\tau = t\rho/\mu r^2$
- Figure 13 Amplitude response of deep axial conductor when receiver and source are to the right of the conductor. Frequency is normalized: $b = \omega\mu r^2/\rho$

- Figure 14 Phase response of deep axial conductor when receiver and source are to the right of the conductor. Frequency is normalized: $b = \omega \mu r^2 / \rho$
- Figure 15 Transient response of deep axial conductor when receiver and source straddle the conductor. Time is normalized: $\tau = t \rho / \mu r^2$
- Figure 16 Amplitude response of deep axial conductor when source and receiver straddle the conductor. Frequency is normalized: $b = \omega \mu r^2 / \rho$
- Figure 17 Phase response of deep axial conductor when source and receiver straddle the conductor. Frequency is normalized: $b = \omega \mu r^2 / \rho$
- Figure 18 Original time-domain data, fit to polynomial of order 8 and time-differentiated (Kauahikaua, 1979).
- Figure 19 Original amplitude data. (Anderson, 1978).
- Figure 20 Original phase data (Anderson, 1978).
- Figure 21 Time-domain representation of frequency-domain data.
- Figure 22 Amplitude representation of time-domain data.
- Figure 23 Phase representation of Frequency-domain data.
- Figure 24 Inversion results of time- and frequency-domain data, after (Kauahikaua, 1979)

Figure 1.

Figure 2. Apparent resistivity versus time for 3-layer models of resistivities 100, variable, and 100 ohm-m. Thicknesses range from 100 to 1000 m. layer two has resistivities of 1, 3, 10, 30, 100, 300, 1000, 3000, and 10000 ohm-m for each family.

Figure 3. Amplitude versus frequency for 3-layer resistivities of 100, variable, and 100 ohm-m. Each family is for second layer resistivities of 1, 3, 10, 30, 100, 300, 1000, 3000, and 10000 ohm-m. Thicknesses range from 100 to 1000 m.

Figure 4.

Phase versus log frequency for 3-layer models of resistivities 100, variable, 100 ohm-m, and thicknesses ranging from 100 to 1000 m. Second layer resistivity for each family is 1, 3, 10, 30, 100, 300, 1000, 3000, and 10000 ohm-m.

Figure 5. Model geometry for the wire-loop transient response of a thin buried axial conductor.

Figure 6.

log Amplitude of Vertical H

Figure 7.

Phase of Vertical H (deg.)

log Frequency

Figure 8.

log Apparent Resistivity (Ohm-m)

Figure 9.

log Amplitude of Vertical H

Figure 10.

Phase of Vertical H (deg.)

log Frequency

Figure 11.

Figure 12.

log Amplitude of Vertical H

Figure 13.

Phase of Vertical H (deg.)

log Frequency

Figure 14.

Figure 15.

log Amplitude of Vertical H

Figure 16.

Phase of Vertical H (deg.)

log Frequency

Figure 17.

log Apparent Resistivity (Ohm-m)

Figure 18.

Figure 19.

Phase of Vertical H (deg.)

log Frequency (Hz)

Original FDEM data

Figure 20.

Figure 21.

Figure 22.

Phase of Vertical H (deg.)

log Frequency (Hz)

Figure 23.

Figure 24. Inversion results from TDEM (Kauahikaua, 1979) and FDEM (Anderson, 1978). After kauahikaua (1979).

Appendix

This Appendix contains the 75 curves and figures 2, 3, and 4, numbered in standard matrix notation as figure 2-row, column 3-row, column and 4-row, column. For instance, the upper left hand corner of figure 2 is figure 2-1,1, while the upper right is figure 2-1,5. For the first four columns, a three-layer model was used. The first and third resistivities were 100 ohm-m for all cases. For column 5 of each figure 2, 3, or 4, two layer models were used, with the first layer resistivity being 100 ohm-m. Each family is labeled for h_1 , h_2 (where applicable) and R (source-receiver offset). The two layer models can be thought of as 3-layer models, with $h_2 = \infty$.

The top row of each group (For example, figures 2-1,1 through 2-1,5 are labeled for ρ_2 . Values of ρ_2 are 1, 3, 10, 30, 100, 300, 1000, 3000, and 10000 ohm-m. Using these as a guide, the labeling of all other curves should be obvious to the reader.

Figure 2-1,1

Figure 2-3, 1

Figure 2-4,1

Figure 2-5,1

Figure 2-1,2

Figure 2-2,2

Figure 2-4,2

Figure 2-1,3

Figure 2-2,3

Figure 2-3,3

Figure 2-4,3

Figure 2-5,3

Figure 2-1,4

Figure 2-2,4

Figure 2-3,4

log Apparent Resistivity (Ohm-m)

$h_1 = 100$
 $R = 10000$

Figure 2-1,5

log Time (seconds)

log Amplitude of Vertical H (Amp/m)

Figure 3-1,1

Figure 3-2,1

log Amplitude of Vertical H (Amp/m)

Figure 3-3,1

log Amplitude of Vertical H (Amp/m)

Figure 3-4,1

log Frequency (Hz)

log Amplitude of Vertical H (Amp/m)

Figure 3-5,1

log Frequency (Hz)

log Amplitude of Vertical H (Amp/m)

Figure 3-1,2

log Amplitude of Vertical H (Amp/m)

Figure 3-2,2

log Frequency (Hz)

log Amplitude of Vertical H (Amp/m)

Figure 3-3,2

Figure 3-4,2

Figure 3-5,2

Figure 3-1,3

log Amplitude of Vertical H (Amp/m)

Figure 3-2,3

Figure 3-4,3

Figure 3-5,3

log Amplitude of Vertical H (Amp/m)

Figure 3-1,4

log Amplitude of Vertical H (Amp/m)

Figure 3-2, 4

Figure 3-3,4

log Amplitude of Vertical H (Amp/m)

Figure 3-4, 4

Figure 3-5,4

log Amplitude of Vertical H (Amp/m)

Figure 3-2,5

log Amplitude of Vertical H (Amp/m)

Figure 3-3,5

log Amplitude of Vertical H (Amp/m)

Figure 3-5,5

log Frequency (Hz)

Phase of Vertical H (deg.)

log Frequency (Hz)

Figure 4-1.1

Phase of Vertical H (deg.)

log Frequency (Hz)

Figure 4-2,1

log Frequency (Hz)

Phase of Vertical H (deg.)

Figure 4-3,1

log Frequency (Hz)

Phase of Vertical H (deg.)

Figure 4-4,1

Phase of Vertical H (deg.)

log Frequency (Hz)

Figure 4-5, 1

Phase of Vertical H (deg.)

log Frequency (Hz)

$$h_1 = 100$$

$$\rho_2 = 1$$

$$h_2 = 100$$

$$R = 10000$$

Figure 4-1,2

log Frequency (Hz)

Phase of Vertical H (deg.)

Figure 4-2,2

Phase of Vertical H (deg.)

log Frequency (Hz)

Figure 4-3,2

Phase of Vertical H (deg.)

log Frequency (Hz)

Figure 4-4,2

Phase of Vertical H (deg.)

$$h_1 = 1000$$

$$h_2 = 1000$$

$$R = 1000$$

Figure 4-5,2

Phase of Vertical H (deg.)

log Frequency (Hz)

Figure 4-1,3

Phase of Vertical H (deg.)

log Frequency (Hz)

log Frequency (Hz)

Phase of Vertical H (deg.)

Figure 4-3,3

log Frequency (Hz)

Phase of Vertical H (deg.)

Figure 4-4,3

Phase of Vertical H (deg.)

log Frequency (Hz)

$h_1 = 1000$

$h_2 = 2000$

$R = 1000$

Figure 4-5,3

log Frequency (Hz)

Phase of Vertical H (deg.)

Figure 4-1,4

Phase of Vertical H (deg.)

log Frequency (Hz)

Figure 4-2, 4

log Frequency (Hz)

Phase of Vertical H (deg.)

Figure 4-3,4

Phase of Vertical H (deg.)

log Frequency (Hz)

Figure 4-4, 4

Phase of Vertical H (deg.)

log Frequency (Hz)

Figure 3-5,4

Phase of Vertical H (deg.)

log Frequency (Hz)

log Frequency (Hz)

Phase of Vertical H (deg.)

log Frequency (Hz)

Phase of Vertical H (deg.)

Figure 4-3,5

log Frequency (Hz)

Phase of Vertical H (deg.)

Figure 3-4,5

log Frequency (Hz)

Phase of Vertical H (deg.)

Figure 3-5,5