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PROPOSED APPARATUS FOR MEASURING INTERNAL FRICTION IN ROCKS
AT HIGH TEMPERATURES AND PRESSURES: A DESIGN ANALYSIS

B.P. Bonner

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PROPOSED APPARATUS FOR MEASURING INTERNAL
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ABSTRACT

From many designs, we have selected an apparatus that measures internal friction in rocks at high temperatures ($\sim 800^{\circ}\text{C}$) and pressures (~ 1.0 GPa). Steady oscillations (~ 1.0 Hz) are induced in a jacketed sample while coaxial capacitive transducers monitor the resulting radial strain. Sample strains are continuously compared to the deformation of a low-loss standard, which acts as a stress transducer. The stress state produced is uniaxial stress. We use the theory of viscoelasticity to partition the loss into components depending on pure shear and dilatation. The theoretical results emphasize the importance of ultimately measuring each loss independently.

INTRODUCTION

Matter dissipates mechanical energy. The energy dissipation rate is a fundamental material property. Nonetheless, we usually assume perfect elasticity in analyzing seismic data that form the basis for much of our knowledge of planetary interiors. The consequences of small but finite absorption are just now being recognized. Gross earth models produced by inversion of free oscillation and body wave data,^{1,2} as well as path-dependent corrections for the estimated yields of underground nuclear explosions,³ appear to be affected. Internal friction measurements (where the fractional energy lost/cycle = $2\pi/Q$)⁴ are a useful diagnostic tool for examining structures ranging from microscopic⁵ to planetary⁵ scale. Under certain circumstances, we use absorption data to predict other time-dependent effects, such as primary creep.⁶ Regardless of the need for these data, internal friction has not been measured for rocks under pressures and temperatures characteristic of the earth's deep crust and upper mantle. Our understanding of attenuation is still rudimentary⁷; the most recent evidence suggests that the loss mechanism may be linear for ambient

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conditions (in that the Fourier integral can be used to represent pulse propagation).⁸ This document presents and critically examines an experimental apparatus that measures internal friction at seismic frequencies and high temperatures and pressures.

EXPERIMENTAL TECHNIQUE REVIEW

A variety of established techniques is used to determine the anelastic properties of solids.^{9,10} Individual methods are best suited for measurements of high or low loss at either long or short characteristic times. The best approach is usually dictated by an estimate of the experimental values and their frequency and amplitude dependence. Although such data are rare for rocks, estimates based on the behavior of similar materials aided considerably in the proposed design. A brief review of experimental techniques with key references is presented below. These techniques were carefully considered in the elimination process that led to the experimental design presented in the next section.

Technique 1 — High Frequency Pulse Propagation^{11,12}

- Adaptable to high pressure and temperature environments.
- $Q_\alpha Q_\beta$ (loss ratio of compressional waves to shear waves) can be determined; same strain field as the seismic case.
- Model must be assumed to extract seismic data.
- Direct evidence for nonlinear behavior is in pulse shape.
- Rayleigh scattering may be important.
- Overriding consideration is that characteristic frequencies are 10^4 higher than seismic frequencies; therefore, Q may be affected by different mechanisms.

Technique 2 — Measuring Creep Function $\phi(t)$ ^{13,14}

- Measuring the creep function $\phi(t)$ directly, where $\epsilon(t) = \frac{\sigma_0}{M} [1 + \phi(t)]$; M = appropriate modulus, σ_0 = initial stress, and $\epsilon(t)$ = time-dependent strain.
- Linear behavior must be assumed for decomposing the data to describe periodic seismic disturbances; best for long characteristic times.

Technique 3 — Spring-Mass System — Free Oscillation Decay¹⁵

- Sample acts as a nonideal spring.
- Classical method of materials science.
- Samples tend to have one large dimension (tens of centimeters); not compatible with present high pressure-temperature apparatus.
- Method covers only a narrow-frequency range.
- Pure nonuniform shear can be conveniently applied with torsion.
- Method would be seriously considered for a "fresh start" experiment.

Technique 4 — Driven Oscillations — At Resonance^{16,17}

- Resonant frequencies in the kHz range for samples with reasonable dimensions.
- Recent work using a composite oscillator¹⁸ has reached 50 Hz in torsional resonance for their rods ~10 cm long.
- Best results are achieved for low-loss samples.

Technique 5 — Driven Oscillations — Off Resonance¹⁹⁻²¹

- Data can be acquired at seismic frequencies on samples small enough to fit into the available furnace-vessel assembly.
- Stress-strain loop shape gives direct information on the dominant absorption mechanisms (e.g., the question of linearity can be addressed).
- Creep function and losses can be measured simultaneously; best results are with lossy media.

EXPERIMENTAL DESIGN

Proposed Experimental Apparatus

Technique 5 (driven oscillations — off resonance) offers the best possibility for successful high pressure and temperature measurements in the seismic frequency band. In this experiment, we first subject the sample to the desired environmental (P, T) conditions. Then a stress, which is a known periodic time function (~0.1 to 10 Hz), is applied, and the resulting strain

is recorded. The energy lost per stress-strain cycle and the phase lag between stress and strain are measures of the internal friction of the material. Internal friction is defined by $Q^{-1} = 1/2\pi \Delta E/E$, where E = total elastic energy stored and ΔE = energy lost per cycle. The data reduction requires no further assumptions.

Theoretically an experiment producing pure shear or dilatation is most attractive. Experimental considerations dictate that this first preliminary experiment be a variation on uniaxial stress geometry. If the novel concepts (at high P and T) proposed for applying the load and measuring small strains prove successful, the experiment should be extended to a geometry involving pure shear.

The figures illustrate the design. Figure 1 depicts the internally heated, gas (argon) pressure vessel; this system has been used to 1600°C in 0.8 GPa argon (with small samples).²² The working dimensions are 38-mm diameter by 153 mm long.

Figure 2 shows the furnace assembly and dc solenoid in place on one high pressure plug. The solenoid supplies a programmable load (~ 250 NT force) to the sample. Furnace windings are on a thin (~ 1 mm) Al_2O_3 tube and are platinum-40% rhodium. Resistance heating is used. The temperature controllers and furnace power supply are conventional. The solenoid is driven by a high-powered operational amplifier that provides periodic loads superimposed on a dc bias.

Figure 3 is a cross section of the sample and load cell assembly inserted in the furnace bore. The specimen, 8.9-mm diameter by 17.8-mm long, is jacketed in annealed copper, sealed with electron beam welds, and placed in mechanical series with a molybdenum standard of the same dimensions. The standard cylinder has high mechanical Q at small fractions of the melting temperature²³ and serves as a stress transducer. Coaxial capacitance transducers determine the radial strains for the sample and standard. The coaxial geometry is most practical to fit inside the bore, to minimize convective heat losses, and to provide enough plate area for reasonable capacitances (5-20 pF). The reference capacitor (outer conducting rings) automatically balances changes in the dielectric (e.g., the pressure medium) at different temperatures and pressures. Note also that the outer electrodes extend beyond the center plate by 50% to reduce nonlinearities resulting from curving field lines near the plate edges.

Fig. 1. Cross section of the argon vessel showing the working volume.

Fig. 3. The sample, standard (load cell), and strain transducer assemblies in place on the high pressure plug. This assembly is inserted into the furnace bore.

The most recent data⁸ indicate that attenuation is a linear phenomenon, implying amplitude independence for strains of order 10^{-6} in rocks under ambient conditions. The existence of such a strain threshold has not been demonstrated for rocks under more realistic conditions of pressure and temperature. The proposed experiment should therefore be very sensitive.

The capacitance of a coaxial capacitor is $C = A/\ln(b/a)$, where A is a constant and b/a is the ratio of the outer to inner radius. The sensitivity of such a device has been evaluated numerically but cannot be presented in closed form. In the limit where b/a approaches 1.0 (in the proposed design $b/a = 0.97$), $\ln \frac{b+dr}{b}$ approaches dr , and the expression reduces to that for a parallel plate capacitor. Numerical results indicate the parallel plate expression is a lower limit for the sensitivity, and the derivation gives a result in closed form; the approximate analysis is given below.

Consider capacitors in series (Fig. 4a). In this configuration, the two outer plates are fixed; the inner plate moves with the specimen. In the undisplaced position for parallel plates,

$$d_1 = d_2 = d_0, \quad c_1 = c_2 = c_0 = \frac{a}{d_0}, \quad (1)$$

where a = permittivity $\times$ area, c_1 and c_2 are capacitances, and d_0 = plate spacing. When an increment D displaces the plate inward,

$$d_1 = d_0, \quad c_1 = \frac{a}{d_0}, \quad (2)$$

$$d_2 = d_0 - D, \quad c_2 = \frac{a}{d_0 - D}. \quad (3)$$

Thus,

$$c_0 d_0 = c_1 d_0 = c_2 (d_0 - D), \text{ or } c_0 = c_2 \left(1 - \frac{D}{d_0}\right). \quad (4)$$

Note that the constant a can be adjusted by adding trimming capacitances outside the vessel. Driving the bridge with an audio transformer (Fig. 4b) and summing currents I_i at the center plate yield

Fig. 4(a). Transducer assembly sketch (capacitors in series) indicating symbols used in the derivations.

Fig. 4(b). Driving and sensing circuit schematic.

$$I_1 + I_2 = E_0/Z_0, \quad (5)$$

where E_0 = voltage, Z_0 = load impedance, $i = \sqrt{-1}$, and ω = angular frequency. The currents in each arm are

$$I_1 = (E_1 - E_0)ic_1, \quad I_2 = (-E_1 - E_0)ic_2. \quad (6)$$

Solving for the voltages E_1 , omitting the algebra, yields

$$\frac{E_0}{E_1} = \frac{c_1 - c_2}{c_1 + c_2 + \frac{1}{i\omega Z_0}}. \quad (7)$$

Assuming $Z_0 \rightarrow \infty$ and substituting c_1, c_2 , from Eqs. (2) and (3),

$$\frac{E_0}{D} = \frac{E_1}{2d_0}. \quad (8)$$

For initial plate spacing of 0.25 mm and driving voltage $E_0 = 10$ V,

$$\frac{E_0}{D} = \frac{10 \text{ V}}{0.5 \text{ mm}} = \frac{20 \mu\text{V}}{\text{mm}} \quad (9)$$

for the transducer sensitivity. Since the specified electronics (lock-in amplifiers with phase-sensitive detection) can resolve 1 μV , the theoretical displacement sensitivity is $\sim 5 \times 10^{-11}$ m. For a specimen with a radius of 4.44 mm, the theoretical sensitivity for radial strain is $\sim 10^{-9}$. Strains of amplitude 10^{-6} could then be observed to better than 1%, and Q values of less than 500 (relative to the standard) should be measurable. The required strain sensitivity seems to be available.

Similar devices have application in other fields. A linear displacement transducer based on this design and using parallel plates²⁴ routinely resolves earth displacements of 10^{-7} m and is not used at full sensitivity. Direct observations of magnetostriction were made with capacitance transducers.²⁵

Displacement sensitivities of ~ 1 nm were claimed. In one instance, hysteretic stress-strain loops were observed with capacitors.²¹ Displacement sensitivity was not limited by any of the above factors, but by ground motion. Three component velocity data for a location a few hundred meters from the proposed site of the experiment have been integrated by A. Taylor, Lawrence Livermore

Table 1. Three component ground motion as a function of frequency (unpublished data, A. Taylor).

Vert. floor			N-S floor			E-W floor		
Freq., Hz	Vel., in./s	Disp., in.	Freq., Hz	Vel., in./s	Disp., in.	Freq., Hz	Vel., in./s	Disp., in.
12.8000	3.0585E-05	3.8029E-07	0.4000	5.1940E-05	2.0666E-05	0.4000	5.1940E-05	2.0666E-05
14.8000	3.2772E-05	3.5242E-07	10.8000	5.0177E-05	7.3943E-07	5.0000	5.1940E-05	1.6533E-06
17.5000	5.0758E-05	4.6162E-07	13.2000	5.0177E-05	6.0499E-07	9.0000	5.0758E-05	8.9759E-07
20.8000	3.1659E-05	2.4225E-07	19.8000	1.7002E-04	1.3666E-06	14.8000	5.1940E-05	5.5855E-07
27.2000	3.1659E-05	1.8525E-07	30.0000	8.9228E-05	4.7337E-07	17.8000	7.1697E-05	6.4106E-07
30.0000	5.0758E-05	2.6928E-07	60.0000	3.0939E-05	8.2067E-08	30.0000	8.9228E-05	4.7337E-07
			81.8000	5.0758E-05	9.8757E-08	89.4000	5.1940E-05	9.2467E-08
			84.4000	8.8207E-05	1.6633E-07	95.2000	1.1105E-04	1.8565E-07
			89.0000	7.0065E-05	1.2529E-07	98.3000	5.1940E-05	8.4095E-08
			93.6000	5.0758E-05	8.6307E-08			

Laboratory.²⁶ Data are listed in Table. 1. Although small location changes can radically affect the ground motion, the large magnitudes involved suggest that some vibration isolation may prove necessary. In the proposed experiment, capacitor parts are fixed to the vessel to minimize relative motion.

We now must estimate the stress needed to produce these strains and to determine if the solenoid can produce such a stress. For this purpose, we assume the validity of Hooke's law. Elastic moduli from low amplitude tests at high P and T are not well known. However, uniaxial deformation data²⁷ for several rock types indicate that Young's modulus ranges from ~50 GPa to ~100 GPa. The stress range to produce strain of 1×10^{-6} is 0.05 to 0.1 MPa. Several manufacturers produce suitable devices; stripped-down, electromechanical motors have functioned at high pressure.²⁸ Figure 5 is a reproduction of manufacturer's specifications (Ledex) for a solenoid tested in the pressure vessel bore. The ordinate shows force on the sample. Sufficient force is available to produce strains of order 10^{-4} in the sample, two orders of magnitude greater than the strain threshold for constant Q under room conditions. The vessel wall is used as part of a magnetic return path, yielding larger forces than the manufacturer's specifications.

The final topic of this experimental section is a discussion of energy loss to the pressure medium. Since the specimen, in general, has Poisson's ratios ≤ 0.5 , pressure-volume work is done on the pressure medium. The volume change ($\Delta V = 10^{-2} \text{ m}^3$) is miniscule, so the effect is negligible. Unfortunately, the loss calculation requires data on the nonideal behavior of argon at high pressure and temperature, which is unavailable. Viscous coupling occurs as

Fig. 5. Manufacturer's data for the solenoid tested in the pressure vessel bore. Duty cycle = on time/total time.

the jacket moves through the gas and contributes loss. Analysis²⁹ and experience^{29,18} suggest that this loss is significant at the low frequencies (~1 Hz) used in this experiment. The most persuasive argument against apparatus loss is experimental. Reference samples with known low internal friction will be tested to evaluate contributions to loss from the apparatus.

RELEVANCE OF THE MEASUREMENT TO SEISMOLOGY

Importance of Low Frequencies

The present experiment searches for absorption peaks in the seismic frequency band under realistic conditions. Experience with unflawed materials indicates absorption is strongly temperature-dependent; many mechanisms are thermally activated. For processes of this type, elementary reaction rate theory³⁰ shows that rate r of occurrence of the energy absorbing process = (frequency of attempts to jump the potential barrier needed to initiate absorption, f) $\times$ (probability of successful attempts):

$$r = f \times \exp\left(-\frac{\Delta G}{kT}\right),$$

where ΔG = free energy of activation for the energy absorbing process, and kT is the thermal energy. This implies temperature- and frequency-dependence. Absorption mechanisms dominant for the seismic case (10^{-3} - 10 Hz) may not be activated in experiments with different characteristic times.

Stress State in the Sample - Comparison with Seismic Waves

When a P wave passes through an elastic solid, each elemental volume is subjected to a state of uniaxial strain. An S wave produces simple shear. The proposed experiment produces a state of uniaxial stress. Boundary conditions are important, and the equivalent elastic disturbance propagates at $(E/\rho)^{1/2}$, where E = Young's modulus and ρ = density.

The mathematical formalisms of linear viscoelasticity can profitably be used to explore this difference. These theories seem to be good approximations in whole earth problems.^{1,2} In effect, the stress-strain hysteresis loop, which may have cusped ends if the mechanism is nonlinear, are approximated by ellipses.

The viscoelastic stress-strain relation can be represented by

$$\sigma = G_{\alpha} * \epsilon, \quad (10)$$

where G_α are material properties (the relaxation function for stress at constant strain) and $*$ is the generalized product (the Stieltjes convolution) defined by

$$a * b = \int_0^t a(t - \tau) \frac{db(\tau)}{d\tau} d\tau \quad (11)$$

Use of the convolution incorporates the strain history into the stress history as demonstrated by Boltzmann in 1876.³¹ The obvious structural analogy to elasticity can be pursued by applying integral transforms to relations like Eq. (10). The Laplace transform $\left[\bar{f}(s) = \int_0^\infty f(t)e^{-st} dt \right]$ of stress-strain relations, decomposed into isotropic and deviatoric parts, yields

$$\bar{\sigma}_{kk} = s \bar{G}_2 \bar{\epsilon}_{kk}$$

and

$$\bar{\sigma}_{ij} = s \bar{G}_1 \bar{\epsilon}_{ij}, \quad (12)$$

where conventional elastic tensor notation is followed and - indicates the transform, $\bar{\sigma}_{ij}$ = stress, $\bar{\epsilon}_{ij}$ = strain, G_1 is the relaxation in shear, and G_2 is the relaxation in pure compression. Relaxation functions appropriate to various stress states can be defined through the analogy stated in Eq. (12) by replacing the elastic moduli by the s multiplied (if the Laplace transform is used) transform of the viscoelastic relaxations.³² Uniaxial compression is characterized by Young's modulus ($E = \sigma_{11}/\epsilon_{11}$), which can be written as $f(G_1)$. Recasting in terms³² of the more familiar K and μ ,

$$\bar{E}(s) = \frac{9\bar{\mu}(s)\bar{K}(s)}{\bar{\mu}(s) + 3\bar{K}(s)} \quad (13)$$

For steady state oscillations, viscoelastic moduli are complex and are related to the relaxation functions by relations of the form Eq. (10) (Christensen, p. 204).³² The complex part of Young's modulus determines the loss³³ and is examined here to evaluate the loss in uniaxial compression: *

$$E_I = \frac{9k_I \mu_I}{3k_I + \mu_I} = 9K_I \times \frac{1}{\left(1 + \frac{3K_I}{\mu_I}\right)} = 3\mu_I \times \frac{1}{\left(1 + \frac{\mu_I}{3K_I}\right)} \quad (14)$$

* The technique of examining only the complex part of the modulus may lead to misleading results. This point will be further examined in subsequent publications.

Applying the binomial expansion $[(X + Y)^{-1} = X^{-1} - YX^{-2} + \dots \text{ for } X \gg Y]$ first to the middle expression for the case $\mu_I \gg 3K_I$, yields an expression for the loss in Young's modulus accurate to second order:

$$E_I \approx 9K_I \left[1 - \frac{3K_I}{\mu_I} + O(\mu_I^3) \right]. \quad (15)$$

Doing the same to the right-hand expression for the second case, $3K_I \gg \mu_I$, yields

$$E_I \approx 3\mu_I \left[1 - \frac{\mu_I}{3K_I} + O(\mu_I^3) \right], \quad (16)$$

where $O(\mu_I^3)$ designates third order terms.

The results of Eqs. (15) and (16) demonstrate the loss in Young's modulus changes greatly if μ_I or K_I dominate. If $\mu_I \gg K_I$, E_I depends to first order on K_I . The reverse is true if $K_I \gg \mu_I$. Second order terms depend on the ratio K_I/μ_I (or its inverse).

The expression Q_α/Q_β can be written as a function of K_I/μ_I by taking the ratio of the loss in the compressional modulus (e.g., $K + 4/3\mu$) to the loss in pure shear, giving

$$\frac{Q_\alpha}{Q_\beta} = \left(\frac{v_s}{v_\rho} \right)^2 \left(\frac{K_I}{\mu_I} + \frac{4}{3} \right), \quad (17)$$

where the velocity ratio appears since the loss in each modulus is calculated from the ratio of imaginary-to-real parts of that modulus.

The proposed experiment does not measure the loss in Young's modulus directly; radial strain is determined. Since Poisson's ratio $\gamma = -\epsilon_{22}/\epsilon_{11}$ and Young's modulus is defined by $\sigma_{11} = E\epsilon_{11}$, the stress can be written as a function of ϵ_{22} , $\sigma_{11} = (-E/\gamma \epsilon_{22})$. The proposed experiment actually determines the loss in E/γ . After defining γ and E in terms of K and μ and considerable algebra, one finds an expression analogous to Eq. (14):

$$\frac{E_I}{\gamma_I} = \frac{18\mu_I K_I}{3K_I - 2\mu_I}. \quad (18)$$

The result is of the same form as Eq. (14). The conclusion regarding dominance of K_I or μ_I is unchanged. The second order terms are twice as large as in Eqs. (15) and (16).

The foregoing analysis demonstrates a critical, long-term experimental goal is to separately observe loss in pure shear and dilatation. One possible solution would be to measure both ϵ_{11} and ϵ_{22} and solve equations analogous to Eqs. (14) and (15) simultaneously. Another approach, compatible with the proposed experiment, would be to independently change the confining pressure while monitoring the resulting time-dependent strain. The second method will be attempted and will at least put a lower limit on the loss in pure compression.

Sample Size Problem

Seismic wavelengths are on the order of kilometers and have larger characteristic dimensions than any laboratory sample. For low strain rates, the deformation proceeds nearly statically with each elemental volume subjected to a compression (or direct shear) test.³⁴ Q values are relevant if they meet homogeneity, isotropy, and kinetic energy requirements³⁵; the proposed experiment appears to meet these.

Many of the loss mechanisms proposed to explain internal friction are microscopic. Experience with unflawed materials demonstrates that small sample measurements detect such losses if grain size is smaller than the sample. Certain geophysical phenomena, such as fluid movement in the asthenosphere³⁶ that might dissipate seismic waves, are large scale and are not observed in this experiment.

Possible Significance of Nonisothermal Conditions

Experimental evidence exists that internal friction in rocks at seismic frequencies is nonisothermal,²¹ effecting potentially important consequences. Results from a linear nonisothermal theory³⁷ based on energy conservation and the entropy production inequality illustrate these consequences. Reproducing the details is not appropriate here, but an outline is presented. The first step is to derive an approximation to the free energy analogous to Eq. (10), including both strain and temperature history. When this expression is subjected to the constraints of the Clausius-Duhem inequality and two constitutive relations, and expression (containing two convolutions per term) for the rate of dissipation of energy Δ results:

$$\begin{aligned}
\Delta = & -\frac{1}{2} \int_{-\infty}^t \int_{-\infty}^t \frac{\partial}{\partial t} G_1(t - \tau, t - n) \frac{\partial e_{ij}}{\partial \tau} \frac{\partial e_{ij}(n)}{\partial n} d\tau dn \\
& - \frac{1}{6} \int_{-\infty}^t \int_{-\infty}^t \frac{\partial}{\partial t} G_2(t - \tau, t - n) \frac{\partial e_{kk}(\tau)}{\partial \tau} \frac{\partial e_{ij}(n)}{\partial n} d\tau dn \\
& + \int_{-\infty}^t \int_{-\infty}^t \frac{\partial}{\partial t} \phi(t - \tau, t - n) \frac{\partial e_{kk}(\tau)}{\partial \tau} \frac{\partial \theta(n)}{\partial n} d\tau dn \\
& + \frac{1}{2} \int_{-\infty}^t \int_{-\infty}^t \frac{\partial}{\partial t} m(t - \tau, t - n) \frac{\partial \theta(\tau)}{\partial \tau} \frac{\partial \theta(n)}{\partial n} d\tau dn, \quad (19)
\end{aligned}$$

where the first and second terms correspond to the isothermal loss and depend only on the relaxation functions. The functionals ϕ and m are time-dependent thermomechanical constitutive relations that may or may not have significant values for rocks at high pressure and temperature. One feature of the proposed experiment is the total work done by the solenoid can be estimated (corrected for Joule heating) and may provide an estimate of the last two terms.

CONCLUSION

The main purpose of this experiment is to estimate the influence of temperature, pressure, and general rock type on internal friction in rocks at realistic conditions (e.g., deep crustal and upper mantle). Data and experience obtained as a result of the preliminary experiment will be used to design future experiments with the goal of investigating the effect of partial melting on attenuation. The apparatus analyzed here seems best suited for an initial attempt at this goal.

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