853
12-7-77

1645

LA-7014-MS

Informal Report

UC-20d and UC-21

Issued: November 1977

MASTER

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UNITED STATES
DEPARTMENT OF ENERGY
CONTRACT W-7405-ENG. 36

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Springfield, VA 22161

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SCALING OF REACTOR CAVITY WALL LOADS AND STRESSES

by

Ihor O. Bohachevsky

ABSTRACT

Scalings of reactor cavity wall loads and stresses are determined by deriving an analytic expression in terms of relevant parameters for each loading induced in the reactor cavity walls by fuel pellet microexplosion and by deriving associated expressions relating resulting stresses to shell thicknesses. Also identified are problems that require additional investigations to obtain satisfactory explicit stress estimates for the reactor cavity walls.

I. INTRODUCTION

Fusion reactors intended to operate on the principle of inertial fuel confinement will require a strong reaction chamber to contain pellet microexplosions inside and to withstand the pressure developed in lithium blanket due to neutron energy deposition from the outside. The need for lithium blanket requires at least two concentric structural shells; the lithium pressure will be an external load on the interior shell and an internal one on the exterior shell.

The sphere is an ideal containment vessel, and it offers most opportunities to obtain explicit results. Therefore, spherical shape will be used in this study to obtain least thicknesses of structural shells required to withstand stresses developed during cyclic reactor operation.

To be useful in the conceptual design stage, stress analysis of reactor vessel shells should provide realistic estimates of static and dynamic stresses resulting from all loadings encountered during reactor operation and should provide information about the dependence of these stresses on relevant parameters such as fuel pellet mass, energy yield, pulse duration, blanket and shell thickness, etc.

The purpose of the present report is:

1. To derive an analytic expression in terms of relevant parameters for each loading induced in

the reactor cavity walls by fuel pellet microexplosion;

2. To derive associated expressions relating resulting stresses to shell thicknesses; and
3. To identify problems that require further investigation to obtain more satisfactory but still explicit stress estimates for the reactor cavity walls.

The reactor vessel model adopted for the purpose of this report consists of two concentric spherical shells separated by a layer of liquid, usually lithium, as shown in Fig. 1. In general, shell thicknesses will be small in comparison to shell radii and blanket thicknesses; the specific assumptions or approximations, however, will be stated as needed. The notation indicated in Fig. 1 will be used consistently.

Most of the results in this report will be stated for a single shell; two shells with a blanket in between will be used only to derive loading due to blanket thermal expansion. To the best of our knowledge, the phenomenon of wave reflections between the two shells has not been treated satisfactorily and will require a separate analysis to obtain useful explicit results. Recently, we have formulated an approach and method to obtain the desired solution; results will be presented in a subsequent report.

Fig. 1. Reactor vessel model.

In the present report the material is organized as follows: the next Sec. II contains a summary of shell loadings in terms of parameters relevant to systems analysis; in the following Sec. III, functional relations between shell stresses and thicknesses are derived and discussed. In Sec. IV the results are summarized and the outstanding problems identified.

II. STRUCTURAL SHELL LOADING

Fuel pellet microexplosion and blanket thermal expansion generate internal and external mechanical loads on the structural shell; spatial and temporal temperature gradients inside the shell generate thermal loads. Expressions for the magnitudes of these loads will now be derived and discussed.

A. Mechanical Loads

Considered are internal loads on the interior shell induced by fuel pellet microexplosion and the external load induced by blanket expansion.

A third type of load is the weight of the lithium blanket and of structural shells themselves. A simple calculation, however, shows that this load is negligible in comparison with the first two; shell thickness required to support in tension a 1-m-thick lithium blanket around a hemisphere of 5-m radius is only 0.76×10^{-2} cm.

1. Internal Load

Internal load is the pressure generated by pellet microexplosion. In reactor cavities operated at high vacuum, this pressure is generated by direct impact of pellet debris into the wall; when significant background gas density exists in the cavity prior to fuel microexplosion, a blast wave develops and its reflection generates pressure on the wall. A third mechanism that generates pressure on the wall is the recoil caused by ablation or evaporation of the wall or of its protective layer induced by the absorption of radiation and pellet debris generated during the pellet burn.

a. Conversion of debris kinetic energy into pressure

For background densities below 10^{14} to 10^{13} atoms/cm³ (i.e., mean-free path 1 - 10 cm), a blast wave does not develop, and it may be assumed that pellet debris proceeds undisturbed to the vessel wall where its kinetic energy is converted into pressure on the wall. The debris acquires kinetic energy by retaining a fraction of reaction energy yield:

$$\frac{1}{2} Mv^2 = fY, \quad (1)$$

where M is the pellet mass, v is the average debris velocity, f is the fraction of energy yield retained in the debris, and Y the total fuel pellet energy yield. From Eq. (1), the debris momentum is

$$Mv = \sqrt{2fYM}. \quad (2)$$

Assuming that this momentum is given up to the wall uniformly in time during the pulse duration T , the force developed is:

$$F = \frac{\sqrt{2fYM}}{T},$$

and thus the pressure on the wall, p_w , is given by:

$$p_w = \frac{\sqrt{fYM}}{2\sqrt{2\pi R_1^2 T}}, \quad (3)$$

where R_1 is the interior shell radius. Equation (3) shows that the pressure on the cavity wall exerted by the pellet debris is proportional to the square

root of the product of pellet mass and yield, inversely proportional to the time spread of debris arrival, and decreases as the inverse square of the cavity radius.

A typical value of p_w for $M = 1g$, $Y = 500$ MJ (5×10^{15} erg), $f = 0.20$, $R_1 = 5$ m, and $T = 100$ μ s (obtained from numerical debris expansion simulation) is $p_w = 10^6 / \sqrt{5\pi}$ dynes/cm² or ≈ 0.14 atm.

b. Blast wave reflection

When the ambient density in the cavity exceeds approximately 10^{14} atoms/cm³ the fuel microexplosion will generate a spherical blast wave. A satisfactory theory of this phenomenon existed for distances large in comparison with fuel pellet dimension¹ and it has been extended recently by Freiwald and Axford² to describe blast wave at distances at which the mass swept by the blast wave is comparable to the pellet mass. Wall pressure generated by this phenomenon will now be derived.

From standard references on supersonic flow and shock waves³ the pressure behind a strong shock wave (blast wave), p_{bw} , is

$$p_{bw} = \frac{2}{\gamma + 1} \rho_0 \dot{R}^2, \quad (4)$$

where γ is the ratio of specific heats, ρ_0 is the ambient density, and $\dot{R}$ is the speed of the wave.

At the cavity wall, when the blast wave swept a much larger mass of the ambient gas than the fuel pellet mass, its propagation is described accurately by the Taylor-Sedov similarity solution:^{1,2}

$$\dot{R} = \sqrt{\frac{3}{4\pi}} \left(\frac{\gamma + 1}{2} \right) \frac{\sqrt{fY}}{\sqrt{\rho_0}} R^{-3/2}. \quad (5)$$

Substituting Eq. (5) into Eq. (4) we obtain:

$$p_{bw} = \frac{3}{4\pi} \frac{\gamma + 1}{2} fY R^{-3}. \quad (6)$$

Finally, the pressure on the cavity wall is the pressure behind a normally reflected strong shock wave:³

$$p_w = \left[\frac{\gamma + 1}{\gamma - 1} + 2 \right] p_{bw}.$$

The factor in the above expression is the pressure increase arising from conversion of the kinetic energy of the ordered gas motion into internal energy. Using Eq. (6) the expression for wall pressure is:

$$p_w = K(\gamma) fY R_1^{-3}, \quad (7)$$

where $K(\gamma) = \frac{3}{4\pi} \frac{\gamma + 1}{2} \left[\frac{\gamma + 1}{\gamma - 1} + 2 \right] = 2.29$ for $\gamma = 1.40$. Thus the blast wave pressure on the wall is directly proportional to the yield, inversely proportional to the cube of the cavity radius, and independent of the background density.

For $\gamma = 1.4$, $f = 0.20$, $Y = 500$ MJ, and $R_1 = 5$ m, Eq. (7) yields $p_w = 1.93 \times 10^7$ dynes/cm² or 18.3 atm; this value is more than ten times greater than the pressure exerted by the plasma debris in the case of vacuum in the reactor cavity, and it increases linearly with yield as opposed to only a square root dependence in the vacuum case. This result demonstrates the effectiveness of vacuum in containing fusion explosions when the debris mass, M , is small.

When the blast wave cannot be approximated by the Taylor-Sedov similarity solution, the modified theory of Freiwald and Axford² should be used; it does not admit a simple representation for the wall pressure but requires numerical evaluation of relevant expressions.

c. Ablation-evaporation recoil

Ablation and evaporation of wall material following absorption of x-ray pulse or plasma debris will exert pressure on the wall analogously to the action of a rocket engine. The magnitude of this pressure will now be estimated.

Assuming that a negligible amount of energy is required to raise a thin layer of wall material to boiling or sublimation temperature, the vapor temperature increase, ΔT , is given by:

$$\Delta T = \frac{xY - Hm}{c_{ps} m}, \quad (8)$$

where x is the fraction of yield in x-rays, H is the heat of vaporization, m is the mass of material evaporated, and c_{ps} is the heat capacity of vapor at constant pressure. When the heated vapor expands to initial ambient temperature, the maximum attainable

velocity, v_m , is given by Bernoulli's Law:

$$v_m^2 = 2 c_{ps} \Delta T. \quad (9)$$

Therefore, the momentum in the direction away from the wall is:

$$I = \eta m \sqrt{2 c_{ps} \Delta T}, \quad (10)$$

where η is the effectiveness coefficient to account for the fact that not all vapor moves with the maximum velocity v_m . In his unpublished work Freiwald⁴ used the value $\eta = 0.15$ ($0.15 v_{max}$ is the average gas velocity in a Riemann wave expanding into vacuum). Substituting Eq. (8) into Eq. (10), we find:

$$I = \sqrt{2} \eta \sqrt{m(xY - Hm)}. \quad (11)$$

For the purpose of engineering estimates, it is convenient to work with the maximum value of I which occurs when the amount of material evaporated, m , is given by:

$$m = \frac{xY}{2H}. \quad (12)$$

With this value of m , the maximum value of momentum I_m is:

$$I_m = \frac{\eta xY}{\sqrt{2H}}. \quad (13)$$

Now, postulating as in Sec. IIA that the above momentum builds up uniformly during a time interval τ , the force, i.e., rate of change of momentum, is I_m/τ , and therefore the pressure p_w is:

$$p_w = \frac{\eta xY}{4\pi R_1^2 \tau \sqrt{2H}}. \quad (14)$$

Thus, the wall pressure due to evaporation recoil is directly proportional to the energy yield and inversely proportional to the square of the cavity radius, pulse duration, and square root of the heat of evaporation.

In the case of a wetted wall reactor cavity, Eq. (14) is valid also for the pulse that obtains when the debris energy is deposited in liquid lithium; in this case, x should be replaced with f and the appropriate pulse duration τ used. The fraction of energy in debris, f , is much larger than in x-rays, however the pulse length for debris is also much longer than for the x-rays and nonoverlapping with it, hence, the x-ray pulse will probably exert stronger pressure.

An idea of the magnitude of the pressure generated by blow-off may be obtained by evaluating Eq. (14) for: $\eta = 0.50$, $x = 0.05$, $Y = 500$ MJ, $H = 21.5$ kJ/g (lithium), $R_1 = 5$ m, $\tau = 1 \mu s$.

With these values substituted, Eq. (14) yields $p_w = 6.05 \times 10^7$ dynes/cm² or 60.5 atm.

2. External Load

External loads on a shell are imposed by pressures in the liquid lithium blanket; these pressures are either static induced by thermal expansion or dynamic arising from wave propagation and reflection.

a. Thermal expansion of the blanket material (static)

We assume that all of the neutron energy generated in the fuel pellet microexplosion is deposited in the blanket material; its temperature rise, ΔT , therefore is given by:

$$\Delta T = \frac{(1 - f - x)Y}{c_{pf} V \rho_f}, \quad (15)$$

where V is the blanket volume and ρ_f the blanket material density. Since the blanket volume can be well approximated by

$$V = 4\pi R_1^2 \Delta$$

(see Fig. 1), its temperature rise is given by:

$$\Delta T = \frac{(1 - f - x)Y}{4\pi R_1^2 \Delta c_{pf} \rho_f}. \quad (16)$$

The volumetric thermal expansion is given by:

$$\frac{\Delta V}{V} = b \Delta T, \quad (17)$$

where b is the coefficient of thermal expansion.

If we now assume that the structural shells do not expand but exert a pressure necessary to counteract the thermal expansion, then this pressure is given by:

$$p = \beta \frac{\Delta V}{V}, \quad (18)$$

where β is the adiabatic bulk modulus.

Combining Eqs. (16), (17), and (18), the following expression for the blanket pressure obtains:

$$p = \frac{\beta b}{c_{pf} \rho_f} \frac{(1 - f - x) Y}{4\pi R_1^2 \Delta}. \quad (19)$$

The first factor is a purely material constant and the second indicates that blanket pressure is directly proportional to the yield and inversely proportional to the blanket volume.

For a lithium blanket with: $\beta = 109 \text{ kb}$ ($1.09 \times 10^{11} \text{ dynes/cm}^2$), $b = 1.39 \times 10^{-4} / \text{K}$, $c_{pf} = 3.55 \text{ J/g K}$, $\rho_f = 0.52 \text{ g/cm}^3$, $Y = 500 \text{ MJ}$, $1 - f - x = 0.75$, $R_1 = 5 \text{ m}$, $\Delta = 1 \text{ m}$, Eq. (19) yields the pressure $p = 9.75 \text{ atm}$.

b. Pressure wave loading (dynamic)

The presently available results describing propagation and reflection of pressure waves in the liquid blanket are not in a form which would allow us to write down a comprehensive summary of the loads induced by these phenomena. An investigation is in progress which will result in explicit expressions for the loads on the shells caused by pressure wave reflections; the results will be presented in a separate report.

B. Thermal Load

1. Static

Static thermal loading is the steady state temperature gradient between the two surfaces of the shell. The associated stress arises because of the incompatibility of thermal expansions and contractions of the different parts of the shell; its magnitude will be calculated in the next section.

2. Dynamic

The dynamic thermal load arises when the energy is deposited near the surface at a rate exceeding that at which heat can be conducted away into the shell material. The resulting spatial and temporal

temperature distribution was investigated previously⁵; here we will summarize the results.

When the energy flows into a solid at a rate ϕ per unit area per unit time during an interval $0 \leq t \leq \tau$; then at the end of the pulse the surface temperature increase, ΔT_s , is:

$$\Delta T_s = \frac{\phi \tau \varphi(u)}{\delta \rho c_{ps}}, \quad (20)$$

where δ is the depth of the solid layer in which energy is deposited and:

$$\varphi(u) = \text{erf}(u) - 2u^2 \text{erfc}(u) + \frac{2}{\sqrt{\pi}} u e^{-u^2}, \quad (21)$$

with $u = \delta / 2\sqrt{\kappa \tau}$, κ being the thermal diffusivity $\kappa = k / \rho c_{ps}$, k = thermal conductivity of the solid. When τ is replaced by an arbitrary time $t < \tau$, Eq. (20) gives surface temperature at that time and can be differentiated to yield the rate of surface temperature increase. However, to estimate loads and stresses within approximations employed throughout this report it will be sufficient to assume that the increase ΔT_s is either instantaneous or accumulates linearly in time.

Equation (20) can be simplified significantly in cases of small and large u . For small u , i.e., $u \leq 0.75$, linear terms in power series expansions of exponential and error functions approximate them to within a few percent accuracy and therefore, in this case:

$$\Delta T_s = \frac{xY}{2\pi R_1^2 \sqrt{\pi \tau \rho c_{ps} k}}, \quad (22)$$

where the fact:

$$\phi = \frac{xY}{4\pi R_1^2 \tau} \quad (23)$$

has been used.

For large u , i.e., $u \geq 1.75$ $\text{erf}(u) \approx 1$ and $\text{erfc}(u) \approx 0$, hence $\varphi(u) \approx 1$ and thus:

$$\Delta T_s = \frac{xY}{4\pi R_1^2 \delta \rho c_{ps}}. \quad (24)$$

It is clear from Eqs. (22) and (24) that the case of small u is dominated by heat conduction away from the surface and the case of large u by heat storage in the shell of thickness δ .

For u in the intermediate range, i.e., $0.75 \leq u \leq 1.75$, Eq. (21) should be used to evaluate surface temperature increase.

This concludes the summary of shell loadings. In the next section, stresses resulting from these loads are estimated.

III. SHELL STRESSES

A. Static stresses

Considered will be membrane stress, critical buckling pressure, and thermal stress induced by a linear temperature distribution.

1. Membrane stress

Explicit expressions exist for tangential and radial stresses in arbitrarily thick walls of a spherical pressure vessel subjected to uniform internal or external pressure (Ref. 6, p. 308). However, fusion reactor vessel walls will have to be relatively thin in order not to perturb significantly the neutronics of the blanket or the economics of the reactor and, therefore, thin-shell approximation will suffice in the conceptual design stage. This approximation has the advantage of great simplicity and will serve well the purpose of this presentation: to display explicitly the dependence of stresses on vessel and pellet parameters.

The membrane stress is tensile for internal and compressive for external pressure loads. In general, these loads will not be in phase at all times and, therefore, the assumption that they cancel each other to some degree cannot be made.

The formula for membrane stress is an expression of the fact that the total pressure on the hemisphere is balanced by the shell stress along the equator, i.e.,

$$2\pi R_1 \delta_1 \sigma = \pi R_1^2 p_w,$$

where σ is the stress.

From the above relation, the expression for shell thickness, δ_1 , is

$$\delta_1 = \frac{R_1 p_w}{2\sigma}, \quad (25)$$

where σ is the allowable stress.

In the case of internal debris loading, substitution of Eq. (3) gives the following expression for the required shell thickness:

$$\delta_1 = \frac{\sqrt{f y M}}{4\sqrt{2\pi} R_1 \tau}. \quad (26)$$

To obtain numerical estimates throughout the remainder of this section we will use the value $\sigma = 50,000 \text{ psi} = 3.4 \times 10^3 \text{ atm} = 3.4 \times 10^9 \text{ dynes/cm}^2$.

Thus, for the pressure $p_w = 0.14 \text{ atm}$ exerted by debris impact, Eq. (26) gives $\delta_1 = 1.05 \times 10^{-2} \text{ cm}$; such a small value indicates that other considerations dominate the design requirements.

In the case of internal blast wave loading, substitution of Eq. (7) into Eq. (25) yields the following dependence:

$$\delta_1 = \frac{K(Y) f y}{2R_1^2 \sigma}. \quad (27)$$

For $p_w = 18.3 \text{ atm}$ calculated in this case, Eq. (25) gives the value of $\delta_1 = 1.35 \text{ cm}$, which is a reasonable magnitude for a structural shell of 5-m radius.

For ablation recoil loading, substitution of Eq. (14) into Eq. (25) yields the following dependence:

$$\delta_1 = \frac{\eta x y}{8\pi R_1 \sqrt{2H\tau\sigma}}. \quad (28)$$

For the pressure load of 60.5 atm calculated in this case, Eq. (25) yields the value $\delta_1 = 4.45 \text{ cm}$, which is a significant thickness.

For the external load applied by blanket thermal expansion, substitution of Eq. (19) into Eq. (25) yields:

$$\delta_1 = \frac{\beta b}{c_p f p_f} \frac{(1 - f - x) y}{8\pi R_1 \Delta \sigma}. \quad (29)$$

For the blanket pressure of 9.75 atm, Eq. (25) gives $\delta_1 = 0.72 \text{ cm}$.

2. Elastic buckling

External pressure applied by the blanket thermal expansion may cause the shell to buckle. The critical elastic buckling pressure for a sphere is (Ref. 6, p. 354):

$$P_c = \frac{2E\delta_1^2}{R_1^2 \sqrt{3(1-\nu^2)}}, \quad (30)$$

where E is the Young's modulus of elasticity and ν the Poisson's ratio.

Solving for the thickness δ_1 and substituting the expression for the external pressure from Eq. (19) we obtain:

$$\delta_1 = [3(1-\nu)^2]^{1/4} \left(\frac{\beta b}{c_{pf} \rho_f} \right)^{1/2} \left[\frac{(1-f-x)y}{8\pi E \Delta} \right]^{1/2}. \quad (31)$$

This result indicates that the critical thickness required to withstand buckling is independent of shell radius. Using $E = 2 \times 10^{12}$ dynes/cm² = 2×10^6 atm (29.3 $\times 10^6$ psi) and the blanket pressure of 9.75 atm calculated previously, Eq. (31) yields the value $\delta_1 = 0.862$ cm.

3. Thermal Stress Induced by a Steady Linear Temperature Gradient

Exact expressions for radial and tangential stresses in arbitrarily thick walls of a spherical vessel resulting from a temperature difference ΔT across the wall have been derived by Kent (Ref. 7, Appendix 2). In the thin-shell approximation used in this study and with the hypothesis that the temperature gradient is constant inside the wall, Kent's results indicate that the maximum stress occurs at the inner surface, is tangential, and is given by:

$$\sigma = \frac{E\alpha\Delta T}{2(1-\nu)}, \quad (32)$$

where α is the linear coefficient of thermal expansion. It is interesting to observe that this stress is independent of the shell thickness and radius.

For stainless steel $\alpha = 14.4 \times 10^{-6}$ /K and therefore for $\Delta T = 200$ K the stress is $\sigma = 41.2 \times 10^8$ dynes/cm² = 4.12×10^3 atm (60 564 psi).

B. Dynamic Stresses

Dynamic stresses arise from instantaneous or rapid application of thermal or mechanical loads. Most of the results in this area have been obtained for specific problems and are not generally applicable. We will formulate approaches and derive

results that are directly applicable to fusion reactor vessel analysis. Additional investigations will be indicated in the remarks concluding this report.

1. Thermal Loads

Kent⁷ derived general expressions for the state of stress in the walls of spherical and cylindrical vessels produced by temperatures varying with time. His expressions will be applicable to cavity wall stress analysis when combined with solution of the heat equation describing temperature history in the cavity wall. With the thin-shell approximation, the results probably can be simplified sufficiently to serve the purpose of this report.

A general result that is applicable to the first wall analysis is the expression for the stress developed in response to an instantaneous surface temperature increase of magnitude ΔT_s K. In this situation the surface layer is prevented from expanding by the bulk of material behind it. The result is a compressive stress of magnitude equal to the stress developed in a thin plate heated by ΔT_s degrees and prevented from expanding by rigid boundary constraints.⁷ This stress is given by (Ref. 6, p. 374):

$$\sigma = \frac{E\alpha\Delta T_s}{1-\nu}, \quad (33)$$

and therefore is twice the value induced by linear temperature distribution across a thin shell. This is physically plausible: in an unconstrained shell with linear temperature gradient, half of the strain is tension and half compression while in the thin-layer case all strain is of one kind.

The stress given by Eq. (33) is present only instantaneously and decreases rapidly as more material is heated.⁷ In terms of pellet parameters σ for small u is obtained by substituting Eq. (22) into Eq. (33):

$$\sigma_u \leq 0.75 = \frac{E\alpha x Y}{(1-\nu) 2\pi R_1^2 \sqrt{\pi \tau_{pc} k}}, \quad (34)$$

and for large u by substituting Eq. (24) into Eq. (33):

$$\sigma_u \geq 1.75 = \frac{E\alpha x Y}{(1-\nu) 4\pi R_1^2 \delta_{pc} k}. \quad (35)$$

2. Response of a Thin Shell to a Pressure Pulse

a. Ringing

In this section, we determine the response of a single shell to a pressure impulse without coupling to the fluid in the blanket. For completeness we will sketch the analysis leading to results directly applicable to systems work. The coupling of the shell to the fluid in the blanket will be investigated analogously in the next section.

The differential equation describing the response of a thin elastic shell to a pressure pulse is the statement that the product of mass and acceleration equals the sum of the forces. Thus for a unit area of the shell:

$$\rho \delta_1 \frac{d^2 w}{dt^2} = \pm p_w - \frac{2\sigma \delta_1}{R_1}, \quad (36)$$

where w is the radial displacement (+ outward), $+p_w$ refers to internal, $-p_w$ to external pressure pulse, and the last term is the component of the membrane stress σ directed radially inward.

In the elastic regime,

$$\sigma = \frac{E\epsilon}{1-\nu}, \quad (37)$$

where ϵ is the strain; the geometric compatibility between the strain and radial displacement w requires that:

$$\epsilon = \frac{w}{R_1}. \quad (38)$$

Substituting Eqs. (37) and (38) into Eq. (36), we obtain for $w(t)$ a harmonic equation with a forcing term:

$$\rho \frac{d^2 w}{dt^2} + \frac{2E}{(1-\nu)R_1^2} w = \pm \frac{p_w}{\delta_1}. \quad (39)$$

The natural frequency or ringing of the shell is described by the solution of the homogeneous part of Eq. (39); we will determine that solution now and simultaneously attempt to determine the solution of the full Eq. (39).

Toward this end, we postulate (to be verified later) that the period of the shell vibration will

be long in comparison to the pressure pulse duration τ . In such case, the right-hand side of Eq. (39) may be treated as a Dirac δ -function and thus incorporated into initial conditions.

The initial conditions for Eq. (39) are therefore statements that the initial displacement is zero, and initial velocity satisfies the equality between the pressure impulse and the change in momentum, i.e.,

$$\left. \begin{aligned} w(0) &= 0 \\ \frac{dw}{dt} \Big|_0 &= \frac{p_w \tau}{\rho \delta_1} \end{aligned} \right\}. \quad (40)$$

The solution of Eqs. (39) and (40) is:

$$w(t) = \sqrt{\frac{1-\nu}{2}} \frac{R_1 p_w \tau}{\delta_1 \sqrt{E\rho}} \sin \sqrt{\frac{2E}{(1-\nu)R_1^2 \rho}} t, \quad (41)$$

from which the vibration period t_p is:

$$t_p = \pi R_1 \sqrt{2(1-\nu) \frac{\rho}{E}}. \quad (42)$$

For $R_1 = 5$ m, $\rho = 7.8$ g/cm³, and $E = 2 \times 10^{12}$ dynes/cm², $t_p = 3.67 \times 10^{-3}$ s. It remains to show that this value of t_p is indeed large in comparison to the pressure pulse duration τ .

Numerical simulation of debris expansion⁸ show that the pressure pulse duration due to debris impact is between 10^{-5} and 10^{-4} s, hence indeed small in comparison to t_p .

Pressure pulse duration for blast wave reflection has been estimated by D. A. Freiwald⁹ to be approximately 10^{-5} s, hence small in comparison to t_p .

Pulse lengths associated with x-ray and neutron energy depositions are measured in nanoseconds and therefore the corresponding pressure pulses are not likely to exceed 10^{-5} s either; however, exact estimates will be made in connection with the determination of wave motion in the shell and blanket.

Thus in all cases the pressure pulse is short relative to the natural shell period and Eq. (41) is a valid description of the shell response. Further-

more, in the elastic regime the superposition principle holds and solutions similar to Eq. (41) determined for different pulses can be added with appropriate phasing to obtain the general shell behavior. This observation is validated by the results of Neudecker and Merson;¹⁰ they calculated numerically stresses in spherical shells of fusion reactors and found a behavior that resembles closely superposition of trigonometric functions.

The above analysis does not include damping which must be taken into account through the interaction between the shell and liquid blanket.

To obtain an expression for the stress explicitly, we substitute Eqs. (37) and (38) into Eq. (41); the result is:

$$\sigma = \sqrt{\frac{1-\nu}{2}} \frac{p_w r \sqrt{E\rho}}{\delta_1} \sin \left[\frac{1}{R_1} \sqrt{\frac{2}{1-\nu}} \sqrt{\frac{E}{\rho}} t \right] \quad (43)$$

This formula is valid for any impulse generating mechanism (e.g., absorption of α -particles). It shows that the stress amplitude is directly proportional to the product of the impulse ($p_w \tau$) and the impedance $\rho \sqrt{E/\rho}$, inversely proportional to the shell thickness δ_1 , and is independent of the shell radius.

For an example, we consider the case of blast wave loading with $p_w = 1.83 \times 10^7$ dynes/cm², $\tau = 10^{-5}$ s (τ is estimated by Freiwald $\tau \sim \text{const } R/R$ where the const is small), $\delta_1 = 1.35$ cm, $E = 2 \times 10^{12}$ dynes/cm², $\rho = 7.8$ g/cm³. The maximum stress from Eq. (43) is $\sigma = 4.07 \times 10^7$ dynes/cm² = 40.7 atm = 598.3 psi; such a small value indicates that static design requirements may be too conservative. It should be noted that this stress estimate is only as good as the estimate of the total impulse $p_w \tau$.

b. Damping

When the structural shell is surrounded by a fluid, natural oscillations may be significantly damped because the shell kinetic energy is transferred to the fluid. This energy loss mechanism is much more effective than the internal damping in the shell material and dominates the behavior; it will be investigated now.

In an unbounded fluid medium, the pressure pulses generated by an oscillating sphere carry the

energy away in the form of sound waves; in a fluid blanket of finite thickness the pulses are reflected at the outer shell, and the wave interaction pattern must be determined to obtain a complete description of the phenomena.

To derive a simple formula and thus explicitly investigate the transfer of energy from the shell to the fluid, in this section we determine the solution valid before the time when the first reflected wave reaches the inner shell. The analysis required to obtain a complete solution will be discussed later.

The governing equation is obtained as Eq. (36), with the addition on the right-hand side of the term representing pressure exerted by the fluid on the shell. With the assumption that the fluid is inviscid and the propagation of spherical waves can be approximated with locally plane wave fronts, this term is $\rho_f c_f (dw/dt)$, where c_f is the sound speed in the fluid and the product $\rho_f c_f$ is known as the acoustic impedance of the medium. Thus, the equation for the radial displacement in this case is:

$$\rho \delta_1 \frac{d^2 w}{dt^2} = p_w - \frac{2\sigma \delta_1}{R_1} - \rho_f c_f \frac{dw}{dt}; \quad (44)$$

substitution of relations (37) and (38) reduces it to:

$$\rho \frac{d^2 w}{dt^2} + \frac{\rho_f c_f}{\delta_1} \frac{dw}{dt} + \frac{2E}{R_1^2 (1-\nu)} w = \frac{p_w}{\delta_1}. \quad (45)$$

The arguments that were used to analyze Eq. (39) apply in this case also (subject to a posteriori verification), consequently we assume that the right-hand side is a Dirac δ -function and solve the homogeneous part of Eq. (45) with initial conditions given by Eq. (40).

Substitution of the complex exponential form

$$w = C e^{i\omega t} \quad (46)$$

reduces Eq. (45) to an algebraic equation for:

$$\omega^2 - \frac{\rho_f c_f}{\rho \delta_1} i\omega - \frac{2E}{\rho R_1^2 (1-\nu)} = 0. \quad (47)$$

For a liquid lithium blanket with $\rho_f = 0.53 \text{ g/cm}^3$ and $c_f = 4.54 \times 10^5 \text{ cm/s}$, and for the values of parameters used in the previous section, the value of the shell blanket coupling term $\rho_f^2 c_f^2 / 4\rho^2 \delta_1^2 = 1.31 \times 10^8$ while the isolated shell response is characterized by $2E/\rho R_1^2 (1 - \nu) = 2.93 \times 10^6$. Therefore, the inequality:

$$\frac{\rho_f^2 c_f^2}{4\rho^2 \delta_1^2} \gg \frac{2E}{\rho R_1^2 (1 - \nu)} \quad (48)$$

will be true in most cases of interest. Condition (48) implies that both roots of Eq. (47) are imaginary,

$$\omega_{1,2} = \left\{ \frac{\rho_f c_f}{2\rho \delta_1} + \left[\frac{\rho_f^2 c_f^2}{4\rho^2 \delta_1^2} - \frac{2E}{\rho R_1^2 (1 - \nu)} \right]^{1/2} \right\} i, \quad (49)$$

which means that the solution $w(t)$ is exponential and not oscillatory.

The solution of Eq. (45) with initial conditions (40) is now given by:

$$w(t) = C(e^{-\omega_1 t} - e^{-\omega_2 t}), \quad (50)$$

where the ω s no longer contain the factor i and:

$$C = -\frac{P_w \tau}{2\rho \delta_1} \left[\frac{\rho_f^2 c_f^2}{4\rho^2 \delta_1^2} - \frac{2E}{\rho R_1^2 (1 - \nu)} \right]^{-1/2}. \quad (51)$$

The corresponding stress is given as in Eq. (43) by

$$\sigma = \frac{E}{1 - \nu} \frac{w}{R_1}. \quad (52)$$

The maximum stress σ_m occurs at time t_m which is determined from Eq. (50) to be

$$t_m = \frac{\ln(\omega_1/\omega_2)}{\omega_1 - \omega_2}. \quad (53)$$

Above result is explicit but not sufficiently simple to be used conveniently. Considerable

simplification can be effected with an approximation suggested by inequality (48).

When condition (48) is satisfied, the isolated shell response $2E/\rho R_1^2 (1 - \nu)$ may be neglected in comparison to the shell-blanket coupling term $\rho_f^2 c_f^2 / 4\rho^2 \delta_1^2$ except in ω_2 where the ratio of these terms is the dominant contribution. With such approximation and an appropriate expansion of the square root, the result simplifies to:

$$C = -\frac{P_w \tau}{\rho_f c_f}, \quad (54)$$

$$\omega_1 = \frac{\rho_f c_f}{\rho \delta_1}, \quad (55)$$

and

$$\omega_2 = \frac{\delta_1}{R_1} \frac{2E}{R_1 (1 - \nu) \rho_f c_f}. \quad (56)$$

The terms now have clear physical interpretation: C is the ratio (negative) of the impulse to the acoustic impedance of the blanket medium, ω_1 is the ratio of that impedance to the shell mass per unit area, and ω_2 is the product of two ratios, shell thickness to radius and shell elastic stiffness to the blanket acoustic impedance.

For the values of parameters used in this and previous sections, we have: $C = -0.76 \times 10^{-3} \text{ cm}$, $\omega_1 = 2.28 \times 10^4 \text{ s}^{-1}$, $\omega_2 = 1.39 \times 10^2 \text{ s}^{-1}$; $w(t)$ for these parameters is plotted in Fig. 2. The result shows that the shell displacement rises to its peak value of $7.3 \times 10^{-4} \text{ cm}$ at $2.2 \times 10^{-4} \text{ s}$ and decays to less than $2 \times 10^{-4} \text{ cm}$ in 10^{-2} s . Thus, the characteristic response times are longer than the loading pulse duration, and the approximation of the impulse $P_w \tau$ with a δ -function is justified.

The corresponding shell stress given by Eq. (52) is $\sigma_m = 4.17 \times 10^6 \text{ dynes/cm}^2 = 4.17 \text{ atm} = 61.32 \text{ psi}$; this value is nearly ten times smaller than the value obtained for the same loading of an isolated shell without the liquid blanket. Such result is physically obvious: the massive liquid blanket inhibits shell expansion and thus reduces strain and stress.

The above-determined solution is valid up to the first transit time, t_r , of the compression wave through the blanket which is given by:

Fig. 2. Shell displacement in the presence of liquid blanket..

$$t_r = \frac{2\Delta}{c_f} \quad (57)$$

For $\delta = 100$ cm, $t_r = 4.41 \times 10^{-4}$ s; Fig. 2 shows that at this time the stress reached its maximum value but relaxed only insignificantly because of the large natural decay time. Therefore, wave reflections in the blanket medium between the two structural shells may have a significant effect on the subsequent stress history.

IV. SUMMARY

Scalings of reactor wall stresses are summarized in Table I where along each parameter is entered its exponent in the expression for static and maximum dynamic membrane stress induced by four loading mechanisms: debris impact, blast wave reflection, evaporation recoil, and blanket thermal expansion. Dynamic stress in Table I refers to an isolated ringing shell; the expressions for dynamic stress in a shell coupled to a liquid blanket, given by Eqs. (49) through (53), are not amenable to such convenient summary because of their complexity.

TABLE I
SCALING OF REACTOR VESSEL WALL STRESSES

Parameter	Loading Mechanism							
	Debris Impact		Blast Wave Reflection		Evaporation Recoil		Blanket Expansion	
	Stat	Dyn	Stat	Dyn	Stat	Dyn	Stat	Dyn
M	0.5	0.5	/	/	/	/	/	/
r	0.5	0.5	1	1	1	1	1	1
z	0.5	0.5	1	1	/	/	(1-f-w)	(1-f-w)
x	/	/	/	/	1	1	(1-f-w)	(1-f-w)
R ₁	-1	-2	-2	-3	-1	-2	-1	-2
δ ₁	-1	-1	-1	-1	-1	-1	-1	-1
δ ₂	/	0.5	/	0.5	/	0.5	/	0.5
H	/	/	/	/	-0.5	-0.5	/	/
R	/	0.5	/	0.5	/	0.5	/	0.5
1-ν	/	0.5	/	0.5	/	0.5	/	0.5
δ	/	/	/	/	/	/	-1	-1
ρ _f	/	/	/	/	/	/	-1	-1
c _{pf}	/	/	/	/	/	/	-1	-1
b	/	/	/	/	/	/	1	1
β	/	/	/	/	/	/	1	1
r	-1	/	/	/	-1	/	/	1
η	/	/	/	/	/	1	/	/

V. CONCLUDING REMARKS

The results described in this report have been useful in:

1. Obtaining rapid estimates of loads and stresses expected in the walls of (micro)explosion containment vessels,
2. Providing immediate information on the variation of these loads and stresses with different parameters that characterize the pellet-vessel combination,
3. Revealing areas and topics that require further investigation to facilitate effective analyses of laser fusion applications.

These areas and topics are:

1. Firm and accurate estimates of the loading pulse durations,
2. Nonlinear wave motion in the blanket between the two shells with the coupling to the elastic response of the shells and investigation of the possibility of resonance conditions,
3. Elastic stress wave motion in the structural shells,
4. Extension of the results to designs containing more than two structural shells and one blanket region.

Investigations of these problems are being pursued and the results will be reported as they become available.

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