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OCTOBER 1975

**Westinghouse Electric Corporation
Fusion Power Systems Department**

**STRUCTURAL ANALYSIS OF EQUILIBRIUM
AND
OHMIC HEATING COIL ASSEMBLIES
FOR THE TFTR**

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FOR THE TFTR

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ABSTRACT

The structural adequacy of the equilibrium and ohmic heating coils and their support systems for the TFTR device has been investigated. The capability of the coils to span ribs of the support structure has been established. The support structure has been found to be effective in resisting the magnetic forces in the coils. The bands encircling the outboard coils and the band tensioning devices have been found to perform adequately. The analysis is based on October 1975 conceptual design.

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1.0 INTRODUCTION

The ohmic heating (OH) and equilibrium field (EF) coil system is comprised of groups of copper windings surrounding the toroidal field coils. The inboard windings are supported from a central support structure which consists of three modules. The center module contains the center stack of coils. The top and bottom modules each consist of twenty ribs which the coil groups span. The three modules are tied in by main tie bolts (see Figure 1).

The outboard windings are supported from the toroidal field coils. These windings are split at the device parting plane and do not take any radial forces. The radial magnetic forces are transmitted to pretensioned stainless steel bands which enclose the outboard coil groups (see Figure 2).

The structural adequacy of the equilibrium and ohmic heating coils and their support systems has been investigated. The coils need to be periodically supported so that high stresses and deflections do not impair their functional adequacy. The loads in the coils and their supports come from the magnetic forces induced in the coils due to the interaction of the coil currents with the magnetic field.

The structural items that have been investigated in this report are the coil group span, center stack bolts, main tie bolts, support and structural connections between the coil groups and the support structure for the inboard coil assembly; and the coil support span, retaining bands and tensioning devices for the outboard coil assembly.

The analysis is based on October 1975 conceptual design.

Figure 1. Inboard Poloidal Coil Assembly

Figure 2. Outboard Poloidal Coil Assembly

2.0 FORCES IN THE EF AND OH COILS

The forces on the EF and OH windings resulting from the magnetic fields and the OH and EF currents have been calculated using a code known as "Dynamic Forces Code" supplied by the Princeton Plasma Physics Laboratory. These forces have been obtained as a function of time for three different equilibrium field indices of curvature; namely, $n = 0.05$, $n = 0.21$, and $n = 0.38$. For illustrative purposes, the computer force output for a typical winding element; namely, the pair of EF coils located 2.983 meters (117.5-in.) radially from the centerline of the machine, (See Figures 1 and 3), has been included in Table 2-1. This coil has been referred to as coil Number 7.

The structural analyses of the coils and support have been conducted based on the maximum values of the forces for a given winding element as obtained from the computer output of the forces and tabulated on Tables 2-2 and 2-3. These maximum values are based on 11 time intervals (ranging from $t = 0$ sec. to $t = 0.56$ sec.) for 3 different indices of curvature of the magnetic field. Note that the maximum values of forces occur either at $t = 0$ sec. or $t = 0.56$ sec. and correspond to an index of curvature of either 0.05 or 0.38.

FIGURE 3. LOCATIONS OF EF AND OH COILS

TABLE 2-1

COMPUTER PRINTOUT OF MAGNETIC FORCES

FORCE V.S. TIME FOR COIL NO. 7 , OH GROUP n = 0.05

(UNITS: SECONDS, POUNDS/INCH, AND POUNDS)

TIME	FR/L	FZ/L	FR	FZ
.000	-6.076+00	-3.773+01	-4.484+03	-2.784+04
5.000-03	-1.100+01	-1.752+01	-8.120+03	-1.293+04
1.500-02	-1.189+01	-2.737+00	-8.776+03	-2.020+03
2.500-02	-1.153+01	6.151+00	-8.506+03	4.540+03
3.500-02	-9.491+00	9.190+00	-7.005+03	6.782+03
4.500-02	-5.663+00	6.904+00	-4.179+03	5.095+03
5.500-02	-6.680-01	9.150-01	-4.930+02	6.753+02
7.000-02	8.470+00	-1.278+01	6.251+03	-9.429+03
9.000-02	2.258+01	-3.651+01	1.666+04	-2.694+04
1.450-01	4.310+01	-7.336+01	3.181+04	-5.414+04
5.600-01	1.021+02	-1.870+02	7.531+04	-1.380+05

FORCE V.S. TIME FOR COIL NO. 7 , OH GROUP n = 0.21

(UNITS: SECONDS, POUNDS/INCH, AND POUNDS)

TIME	FR/L	FZ/L	FR	FZ
.000	-6.076+00	-3.773+01	-4.484+03	-2.784+04
5.000-03	-8.846+00	-1.668+01	-6.528+03	-1.231+04
1.500-02	-8.872+00	-1.563+00	-6.547+03	-1.153+03
2.500-02	-8.225+00	7.434+00	-6.070+03	5.486+03
3.500-02	-6.599+00	1.031+01	-4.870+03	7.612+03
4.500-02	-3.877+00	7.599+00	-2.861+03	5.608+03
5.500-02	-4.530-01	9.986-01	-3.343+02	7.369+02
7.000-02	5.695+00	-1.385+01	4.203+03	-1.022+04
9.000-02	1.508+01	-3.943+01	1.113+04	-2.910+04
1.450-01	2.862+01	-7.899+01	2.112+04	-5.829+04
5.600-01	6.722+01	-2.005+02	4.961+04	-1.480+05

FORCE V.S. TIME FOR COIL NO. 7 , OH GROUP n = 0.38

(UNITS: SECONDS, POUNDS/INCH, AND POUNDS)

TIME	FR/L	FZ/L	FR	FZ
.000	-6.076+00	-3.773+01	-4.484+03	-2.784+04
5.000-03	-6.690+00	-1.535+01	-4.937+03	-1.162+04
1.500-02	-5.852+00	-3.831-01	-4.319+03	-2.872+02
2.500-02	-4.924+00	8.717+00	-3.634+03	6.433+03
3.500-02	-3.707+00	1.144+01	-2.736+03	8.442+03
4.500-02	-2.090+00	8.293+00	-1.543+03	6.120+03
5.500-02	-2.380-01	1.062+00	-1.757+02	7.996+02
7.000-02	2.921+00	-1.423+01	2.155+03	-1.102+04
9.000-02	7.577+00	-4.234+01	5.591+03	-3.125+04
1.450-01	1.415+01	-3.461+01	1.044+04	-6.244+04
5.600-01	3.240+01	-2.140+02	2.391+04	-1.560+05

TABLE 2-2
MAXIMUM FORCES FOR INBOARD COILS

Stack	Coil		Radius (in.)	Turns	Radial Force FR	Axial Force FZ (lb)	Time t (sec)	Index of Curvature n
	Type	No.						
Center	OH	33	29.6	2	103100	3326	0.	0.05
	OH	35	28.9	3	208200	13010	0	
	OH	37	28.9	6	402900	-18540	0	
	HY	87	28.9	6	395700	-79040	0	
	HY	89	30.4	1	26380	-25310	0	
	OH	39	28.1	2	168600	-52130	0	
	EF	79	28.9	1	73310	82000	0.56	
	OH	41	28.9	6	394600	97690	0	
	OH	43	28.9	6	402900	16160	0	
	OH	45	28.9	6	411500	-36940	0	
	OH	47	28.9	6	423300	-94550	0	
	OH	49	28.9	6	442000	-182800	0	
	OH	51	28.9	3	230800	-153000	0	
1	HY	83	35.8	3	103000	66650	0	0.38
	HY	85	35.8	2	-43380	-113700	0.56	
	OH	27	35.8	6	175500	243200	0.56	
	OH	29	35.8	6	303700	-247600	0	
	OH	31	35.8	3	159100	-147100	0	
2	HY	81	42.9	2	-62890	-37570	0.56	0.38
	OH	25	42.9	3	110600	37610	0.56	0.38
3	OH	21	49.2	4	160500	66630	0.56	0.38
	OH	23	49.2	2	92540	-67550	0.56	0.38
4	OH	19	56.0	4	140600	-21820	0.56	0.38
5	OH	17	62.6	4	114600	-47070	0.56	0.38
6	EF	77	74.1	1	82240	-11540	0.56	0.38
7	OH	15	81.4	2	98770	2251	0.56	0.05
8	OH	11	88.6	2	64360	25460	0.56	0.38
	OH	13	88.6	1	35910	-49260	0.56	0.38
9	EF	71	110.2	1	140500	111300	0.56	0.38
	EF	75	110.2	1	146300	-75460	0.56	0.38
	OH	9	110.2	1	69490	-80370	0.56	0.38
10	EF	69	117.5	2	154600	41990	0.56	0.05
	OH	7	117.5	2	75310	-138000	0.56	0.05

TABLE 2-3

MAXIMUM FORCES FOR OUTBOARD COILS

Coil Assembly	Coil		Radius (in.)	Turns	Radial Force FR (lb)	Vertical Force FZ (lb)	Time t (sec)	Index of Curvature n
	Type	No.						
1	OH	1	194.5	1	2056	-169000	0.56	0.05
	EF	55	194.5	2	74140	541400	0.56	0.05
	EF	57	194.5	2	61430	24230	0.56	0.05
	EF	59	194.5	1	22710	-146200	0.56	0.05
	EF	73	194.5	1	15470	-313700	0.56	0.05
2	EF	61	179.5	2	240100	68190	0.56	0.05
	OH	63	179.5	1	102400	-305200	0.56	0.05
3	OH	3	154.6	1	5508	-55870	0.56	0.05
	EF	65	154.6	1	32380	41310	0.56	0.05
	OH	5	146.1	1	43450	-100600	0.56	0.05
	EF	67	146.1	2	214900	-21640	0.56	0.05

3.0 ANALYSIS OF INBOARD ASSEMBLY

In this section, the adequacy of the coils to span the ribs of the support structure has been analyzed. The bolts used to hold both the center stack and the three modules of the support structure have been shown to adequately react the magnetic forces. The effectiveness of the structural connections between the coil groups and the ribs of the support structure has been evaluated.

3.1 ADEQUACY OF COILS TO SPAN RIBS OF SUPPORT STRUCTURE

Due to the radial component of the magnetic forces in the coils, hoop stresses will be sustained in the coils. Because the coils are connected to the ribs at twenty places along their circumferences, radial rib reaction forces induce bending stresses in the coils. The actual state of stress in the coils is therefore a combination of hoop and bending stress, due to the radial loads.

Due to the vertical (axial) components of the magnetic forces, bending and torsional stresses are induced in the coils.

For structural analysis of the coils, the adequacy of a single turn of coil has been considered as the most conservative approach. When the analysis shows this approach gives unacceptable results, the single coil is combined with the local coil group (stack). It has been assumed throughout that the ribs provide rigid support.

3.1.1 Effects of Radial Components of Magnetic Forces

The actual state of stress in the coil is a combination of two effects:

- a) Unrestrained coil subjected to radial magnetic force giving rise to hoop stress.

- b) Coil subjected to 20 radial point loads of magnitude equal to rib reaction forces (1/20 of the total radial load) giving rise to bending stresses.

The superposition of these two effects will be used to calculate the stress due to the radial load.

For the case (a), the hoop stress and deflections are computed in the following manner. If f_r denotes the radial force per unit length of the coil circumference, the hoop stress induced in a coil turn is simply

$$S_{\text{hoop}} = \frac{f_r R}{A} \quad (1)$$

where R is the radius of the coil turn in consideration, and A is the cross-sectional area.

The radial (hoop) deflection is given by,

$$\delta_r = \frac{f_r R^2}{AE} \quad (2)$$

where E is the modulus of elasticity.

For the case (b), the bending stresses have been calculated in the following way. The model used is that of a circular ring acted on by 20 concentrated radial forces spaced equally along the circumference. The solution is obtained from Reference (1) (Table VIII, Formula 9). The maximum bending moment occurs at the support point and is,

$$M_r \text{ max} = 1/2 F_r R (1/\theta - \cot \theta) \quad (3)$$

where:

F_r = rib reaction, assumed 1/20 of the total radial load (upper bound)

R = radius of the coil

2θ = angular span between ribs = 18°

The maximum bending moment is then calculated to be:

$$M_{r \max} = 0.0262 F_r R \quad (4)$$

The bending stress corresponding to this moment is then calculated to be,

$$S_{r \text{ bending}} = \frac{M_{r \max}}{Z} \quad (5)$$

where Z is the sectional modulus.

3.1.2 Effects of the Vertical Components of Magnetic Forces

The model used is that of a circular ring supported vertically on 20 point supports and acted on by a uniformly distributed vertical load corresponding to the vertical component of the magnetic force. From Reference (2), the maximum bending moment is found to occur at a support point and is,

$$\begin{aligned} M_{z \max} &= 1/2 F_z R (1/\theta - \cot \theta) \\ &= 0.0262 F_z R \end{aligned} \quad (6)$$

where F_z = rib reaction = 1/20 of the total vertical load.

The maximum bending stress is then given by

$$S_{z \text{ bending}} = \frac{M_{z \text{ max}}}{Z} \quad (7)$$

where Z is the sectional modulus.

From Reference (2), the maximum torsion induced in the coil is given by,

$$M_{t \text{ max}} = \frac{F_z R}{4} (1 - \sec \theta/2) \quad (8)$$

The shear stresses induced in the coil is given by (from Reference (8)),

$$S_{\text{shear}} = \frac{M_t}{\alpha_A b^2 h} \quad (9)$$

where α_A is the factor of torsion for a rectangular shaft of square cross section ($b = h$) and equals 0.208.

From Reference (2), the maximum deflection due to vertical span has been calculated. The formula for our particular case (twenty supports) reduces to,

$$\delta_z = 4.06 \times 10^6 \frac{VR^3}{EI} \quad (10)$$

where, V = total vertical load = $20 F_z$

The OH coils have a cross-section of 1.375 x 1.375 in. with a 0.375-in. coolant hole. The EF coils have a cross section of 2 x 2 in. with a 0.375-in. coolant hole. The sectional moduli of OH and EF coils have been calculated to be 0.43 in³ and 1.33 in³, respectively.

3.1.3 Stresses in Inboard Coils

The stresses in the inboard coils arising from radial and vertical loads have been calculated and tabulated in Table 3-1. In calculating the total stress, the effect of shear stresses has been neglected since they are found to be generally small and their maximum values do not occur at the same location where the others add. The total stress has been obtained by combining the maximum values of hoop and bending stresses.

Also from Table 3-1, we see that three outermost pairs of the symmetric stack of coils (stack 8, 9, and 10) contain coils that cannot be spanned freely on the ribs. The stresses in these coils exceed 117 MPa (17 ksi), the 100,000 cycle fatigue limit of OFHC copper. Therefore, these three pairs of stacks must obtain additional support. The stresses in all the other coils are found to be less than 117 MPa (17 ksi).

Since the forces in individual coils in a stack sometimes act in different directions due to opposite current flows, they would cancel each other when the coils are clamped together. Also, some of the coils in a stack can carry more than their own magnetic forces. Therefore, the approach was taken to clamp all the coils in a stack together. The individual stacks are now analyzed for stresses in the clamped condition. The vertical and radial deflections of the stacks are calculated based on net flexural rigidity of a stack and it is obtained by combining the areas and the flexural rigidities of individual coils comprising the stack. Each coil in a stack is then constrained to have the same radial and vertical deflections as the stack, and the stresses in the coils are determined from the deflections of the stack. The results are tabulated in Table 3-2 for the three outermost stacks.

TABLE 3-1

STRUCTURAL ANALYSIS OF COILS SPANNING RIBS

Stack	Coil		Radius (in.)	Turns	S_{hoop} (psi)	$S_{rbending}$ (psi)	$S_{zbending}$ (psi)	S_{shear}^* (psi)	S_{total} (psi)	δ_r (in)	δ_z (in)
	Type	No.									
1	HY	83	35.8	3	3069	3749	2383	57	9200	0.0065	0.0008
	HY	85	35.8	2	1939	2368	611	15	4918	0.0041	-0.0002
	OH	27	35.8	6	2615	3194	4353	104	10162	0.0055	0.0014
	OH	29	35.8	6	4525	5527	4432	106	14484	0.0095	-0.0015
	OH	31	35.8	3	4741	5791	5266	126	15800	0.0010	-0.0018
2	HY	81	42.9	2	2811	4114	2418	58	9343	0.0071	0.0012
	OH	25	42.9	3	3296	4824	1614	38	9734	0.0083	0.0008
3	OH	21	49.2	4	3587	6021	2458	59	12066	0.0104	0.0016
	OH	23	49.2	2	4137	6943	4985	119	16065	0.0120	0.0032
4	OH	19	56.0	4	3142	6004	916	22	10062	0.0104	0.001
5	OH	17	62.6	4	2561	5470	2210	53	10241	0.0094	0.002
6	EF	77	74.1	1	3364	6003	842	20	10209	0.0147	0
7	OH	15	81.4	2	4415	12261	275	7	16950	0.0211	0
8	OH	11	88.6	2	2877	8696	3384	81	14960	0.0150	0.0070
	OH	13	88.6	1	3210	9704	13093	312	26000	0.0167	-0.027
9	EF	71	110.2	1	5746	15250	12081	285	33077	0.0372	0.027
	EF	75	110.2	1	5984	15880	8191	193	30055	0.0388	0.018
	OH	9	110.2	1	6212	23356	26570	633	56138	0.0403	0.086
10	EF	69	117.5	2	3161	8947	2430	57	14538	0.0230	0.060
	OH	7	117.5	2	3366	13494	24323	561	41183	0.0220	0.089

*TORSIONAL

TABLE 3-2
ANALYSIS OF CLAMPED STACKS

Stack	Coil	Radius (In.)	Turns	S_{hoop} (psi)	$S_{rbending}$ (psi)	$S_{Zbending}$ (psi)	S_{total} (psi)	δ_r (in.)	δ_z (in.)
8	11	88.6	2	2988	8890	2110	13988	0.0156	0.0044
	13	88.6	1	2988	8890	2110	13988		
9	71	110.2	1	5938	15757	2168	23863	0.0385	0.0048
	75	110.2	1	5938	15757	2168	23863		
	9	110.2	1	5938	22326	1512	30286		
10	69	117.5	2	3225	9127	4529	16881	0.0223	0.0114
	7	117.5	2	3225	12930	3164	19320		

From Table 3-2, we observe that the coils are still highly stressed in stacks 9 and 10. The stress levels, however, have been significantly reduced by clamping the coils in the stacks. It is now clear that an added reinforcement is necessary for these coils. By making all of the coils in stack 9 of 2.125-in. x 2.125-in. with a 0.375-in. coolant hole and by making the coils in stack 10, 2-in. x 2-in. with a 0.375-in. hole, the stress levels have been calculated to be less than 117 MPa (17 ksi). The stresses in all the coils now being less than the 100,000 cycle fatigue limit of OFHC copper, and it is concluded that the coils are structurally adequate to span twenty ribs of the support structure.

3.2 ANALYSIS OF CENTER STACK RETAINING BOLTS

Two symmetric groups of centrally located coils form the center stack. This stack is retained by a set of bolts. The magnetic forces in various coil groups have been obtained in Section 2.0. From this data, the vertical (axial) components of the magnetic forces in the center stack have been calculated.

The stack is found to be in compression under vertical magnetic loads, at all times with a maximum value of 5.12×10^5 lb. The stack retaining bolts need to be preloaded such that the bolts never become loose during the course of operation. The preloading level depends on the operating loads as well as the structural stiffnesses of the stack and the bolts.

The stiffness of the center stack has been calculated in the following way. The turns are separated by insulation of thickness 0.062-in. The stiffness of an insulated turn of coil (which has been denoted by $\bar{k}_{\text{turn}}$) is obtained from the stiffness of a bare turn (denoted by k_{turn}) connected in series with two turns of insulation of stiffness k_{ins} .

Thus,

$$\frac{1}{\bar{k}_{\text{turn}}} = \frac{1}{k_{\text{turn}}} + \frac{2}{k_{\text{ins}}}$$

or

$$\bar{k}_{\text{turn}} = \frac{k_{\text{turn}} k_{\text{ins}}}{2 k_{\text{turn}} + k_{\text{ins}}} \quad (11)$$

The cross-sectional area of a turn of coil in the center stack is 1.375-in. x 1.375-in. with a 0.375-in. coolant hole. The mean radius of the center stack is 30-in. Assuming an elastic modulus of 17×10^6 psi for copper and 10^6 psi for the epoxy insulation, we have,

$$\begin{aligned} k_{\text{turn}} &= \frac{A_{\text{turn}} E_{\text{turn}}}{L_{\text{turn}}} = \frac{2\pi \times 30 \times 1.375 \times 17 \times 10^6}{1.375} \\ &= 3.2 \times 10^9 \text{ lb/in} \end{aligned}$$

and

$$k_{\text{ins}} = \frac{A_{\text{ins}} E_{\text{ins}}}{L_{\text{ins}}} = \frac{2\pi \times 30 \times 1.375 \times 10^6}{0.062} = 4.2 \times 10^9 \text{ lb/in}$$

Thus,

$$\bar{k}_{\text{turn}} = \frac{3.2 \times 4.2}{(2 \times 3.2) + 4.2} \times 10^9 = 1.27 \times 10^9 \text{ lb/in}$$

Since the stack is formed by 40 insulated layers of copper with 3 turns per layer, the stiffness of the stack is given by,

$$k_{\text{stack}} = \frac{\bar{k}_{\text{turn}}}{40} \times 3 = 0.95 \times 10^8 \text{ lb/in} \quad (12)$$

The following specifications have been used for the bolts:

Number of bolts	=	4
Diameter of bolts	=	1.5-in.
Teeth per inch	=	6 ⁽⁴⁾
Stress area per bolt	=	1.4041 sq-in. ⁽⁴⁾
Bolt material	=	SS-305
Elastic Modulus, E_{bolt}	=	29×10^6 psi
Yield Strength, S_{yield}	=	30 ksi
Endurance Limit, $S_{\text{endurance}}$	=	25 ksi
Length of bolt	=	63-in.

The stiffness of the bolts is then,

$$\begin{aligned}
 k_{\text{bolt}} &= \frac{4 A_{\text{bolt}} E_{\text{bolt}}}{L_{\text{bolt}}} \\
 &= \frac{4 \times 1.4041 \times 29 \times 10^6}{63} \\
 &= 2.59 \times 10^6 \text{ lb/in.}
 \end{aligned} \tag{13}$$

If P_{preload} is the preload on one bolt, and P is the total vertical magnetic load on the center stack, then the force on the bolt is,

$$P_{\text{bolt}} = \frac{k_{\text{bolt}} P}{4 (k_{\text{bolt}} + k_{\text{stack}})} + P_{\text{preload}} \tag{14}$$

The minimum preloading required on the bolts is when the bolt force is zero under maximum compressive load, $P_{\max}$. Thus,

$$0 = \frac{k_{\text{bolt}} P_{\max}}{4 (k_{\text{bolt}} + k_{\text{stack}})} + P_{\text{preload}} \quad (15)$$

k_{bolt} and k_{stack} are obtained from Equations (1) and (2), respectively. Substituting these values in Equation (4), we have with $P_{\max} = 5.12 \times 10^5 \text{ lb}$

$$P_{\text{preload}} = 3400 \text{ lb} \quad (16)$$

3.2.1 Preloading Effects For Center Stack Bolts

Following Reference (5), the applied torque T , is found to be related to the load P in the following way,

$$T = r_t P \left[\frac{\cos \theta \tan \alpha + \mu}{\cos \theta - \mu \tan \alpha} + \left(\frac{r_c}{r_t} \right) \mu \right] \quad (17)$$

where:

α = helix angle

θ = one-half the thread angle

r_c = moment arm to frictional force on bolt head

r_t = pitch radius of the thread

μ = coefficient of friction

First we assume that a very high coefficient of friction exists. We take $\mu = 0.25$, which corresponds to little or no lubrication. The torque is calculated for the required preload P with

$\mu = 0.25$. Next, with the obtained torque, the new load P^* is calculated using $\mu = 0.1$, which corresponds to excellent lubrication. The stresses on the bolt due to P^* are then checked.

For the standard 60° threads, the following values were obtained,

$$T = 0.73 r_t P \text{ for } \mu = 0.25 \quad (18)$$

$$T = 0.32 r_t P \text{ for } \mu = 0.10 \quad (19)$$

From Reference (4), the pitch diameter of 1.5-in. bolt is 1.3917-in. Thus,

$$r_t = \frac{1.3917}{2} = 0.696\text{-in.}$$

From Equation (18) for $P = 3400\text{-lb}$, T is calculated to be 144 ft-lb. With this torque, the new load P^* corresponding to $\mu = 0.1$ has been calculated from Equation (19) and is 7760-lb.

Hence, the peak stress in the bolts is P^* divided by the stress area of the bolt and is,

$$S_{\max} = \frac{7760}{1.4041} = 5527 \text{ psi (43.4 MPa)} \quad (20)$$

This value is less than the allowable value of $2 S_m \left[S_m = 103 \text{ MPa (15 ksi)} \right]$ for bolt material as per ASME Boiler and Pressure Vessel Code.

3.3 ANALYSIS OF MAIN TIE BOLTS

The purpose of the main tie bolts is to retain the three modules of the central support structure for EF and OH coils. The center stack is retained independently and the magnetic forces from

this stack are not transmitted to the main tie bolts. The total axial force acting on the main tie bolts therefore come from all the coil groups except the center stack. These forces have been calculated.

A maximum compressive force of 1.0×10^6 lb acts on the three modules of support structure. The structure bearing the bolts is a cylinder of inside radius 21-in. and outside radius 31-in. giving a cross sectional area of 1634 sq-in.

$$\begin{aligned}\text{Length of the part} &= \text{Length of the bolt} \\ &= 190\text{-in.}\end{aligned}\tag{21}$$

Thus the stiffness of the part is,

$$\begin{aligned}k_{\text{part}} &= \frac{A_{\text{part}} E_{\text{part}}}{L_{\text{part}}} = \frac{1634 \times 2 \times 10^6}{190} \\ &= 17.2 \times 10^6 \text{ lb/in.}\end{aligned}\tag{22}$$

where the modulus of elasticity for fiber-glass reinforced plastic has been taken to be 2×10^6 psi.

The following specifications have been used for the main tie bolts.

Number of bolts	=	4
Diameter of bolts	=	2-in.
Teeth per inch	=	$4.5^{(4)}$
Stress Area	=	$2.4917\text{-in}^2^{(4)}$
Bolt Material	=	SS-305

$$\text{Elastic Modulus, } E_{\text{bolt}} = 29 \times 10^6 \text{ psi}$$

$$\text{Yield Strength, } S_{\text{yield}} = 30 \text{ ksi}$$

$$\text{Endurance Limit, } S_{\text{endurance}} = 25 \text{ ksi}$$

Stiffness of the bolts,

$$\begin{aligned} k_{\text{bolt}} &= \frac{4 A_{\text{bolt}} \cdot E_{\text{bolt}}}{L_{\text{bolt}}} = \frac{3 \times 2.4971 \times 29 \times 10^6}{190} \\ &= 1.52 \times 10^6 \text{ lb/in.} \end{aligned}$$

If F_{preload} is the preload on one bolt and F is the total magnetic load, then the force on the bolt is given by,

$$F_{\text{bolt}} = \frac{k_{\text{bolt}} F}{4 (k_{\text{bolt}} + k_{\text{part}})} + F_{\text{preload}} \quad (23)$$

The minimum preload required on the bolts is when the bolt force is zero under maximum compressive operating load, F_{max} . Thus,

$$0 = \frac{k_{\text{bolt}} F_{\text{max}}}{4 (k_{\text{part}} + k_{\text{bolt}})} + F_{\text{preload}} \quad (24)$$

$$F_{\text{max}} = -1.0 \times 10^6 \text{ lb}$$

Substituting k_{part} and k_{bolt} from Equations (12) and (13), we have,

$$\begin{aligned} F_{\text{preload}} &= \frac{1.52 \times 1.0 \times 10^6}{4 \times (1.52 + 17.2)} \\ &= 20,300\text{-lb} \end{aligned} \quad (25)$$

3.3.1 Preloading Effects of Main Tie Bolts

From Reference (4), the pitch diameter of 2-in. bolts is 1.856-in. Thus,

$$r_t = \frac{1.856}{2} = 0.928\text{-in.}$$

For a coefficient of friction $\mu = 0.25$, the torque is calculated for $P = 20,300\text{-lb}$ (from Equation (18)) as, $T = 1146\text{ ft-lb}$. With this torque, the new load P^* corresponding to $\mu = 0.1$ has been calculated from Equation (19), and is $46,309\text{-lb}$.

Hence, the peak stress in the bolts is P^* divided by the stress area of the bolt and is,

$$S_{\max} = \frac{46,309}{2.4971} = 18,545\text{ psi}$$

This value is less than the allowable value of $2 S_m$ ($S_m = 103\text{ MPa}$ (15 ksi)) for the bolt material as per ASME Boiler and Pressure Vessel Code.

3.4 SUPPORT RIBS

The support structure for the inboard coils consists of twenty ribs equally spaced circumferentially around the centerline of the device. These ribs are cantilevered, from the top and bottom central support cylinders, and are made out of fiber-glass reinforced plastic. The mechanical stresses result in the ribs from the magnetic forces in the inboard coils. The assumed external loads on each of the twenty support ribs are equal to the magnetic loads distributed over $1/20$ of the circumference of various coil groups. The forces on the coil groups have been obtained in Section 2.0.

Each of the ribs has been analyzed as a cantilevered beam with non-uniform load distribution. A modulus of elasticity of 1.38×10^{10} MPa (2×10^6 psi) has been assumed for the material. The rib cross-section is an I-section with an 8-in. wide flange. The flange and web are each 1-in. thick.

The bending moments due to the magnetic loads and the dead weight of the coils have been calculated for different sections of the rib. The moments divided by the sectional modulus at a section gives the bending stress at that section. The radial loads give rise to membrane stresses in the rib which have been calculated. The maximum combined bending plus membrane stress has been computed to be 37 MPa (5400 psi), tension, at a radial location of 2.983 m (117.5-in.) from the device centerline. This is well below the 100,000 cycle fatigue limit of 103 MPa (15 ksi) for fiber-glass-reinforced plastic.

The maximum vertical deflection occurs at the outermost point on the rib and has been computed to be 1.04 cm (0.41-in.). From Table 3-2, at this section the coil vertical deflection has been calculated to be 0.0114-in. (stack no. 10). The support rib deflection when added to this deflection implies a maximum coil deflection of 1.07 cm (0.42-in.) in space. This occurs at a radial location of 2.983 m (117.5-in.).

3.5 STRUCTURAL CONNECTIONS BETWEEN COILS AND SUPPORT RIBS

The coil groups are tied down to the ribs of support structure by means of plastic cover plates and through bolts. For each coil group, four bolts have been used on a cover plate to anchor the coil stack to the ribs. The vertical components of the magnetic loads on the connecting structure due to various coil groups have been calculated for structural analysis.

3.5.1 Retaining Bolts

There are ten stacks of coils for each half of the support structure that span the ribs (Figure 1). The peak loads on each of the stacks have been calculated and the peak loads on individual bolts obtained. For the innermost stack, 1-in. diameter bolts have been used. All of the other stacks have 1/2-in. diameter bolts. The peak loads and stresses on the bolts have been shown in Table 3.3. The stress areas of the bolts, used for computing tensile stresses, have been obtained from Reference (4). The maximum tensile stress in the bolts is 11,646 psi (80.3 MPa) and is less than $2 S_m$ for bolting material SS-305 [$S_m = 103 \text{ MPa}$ (15 ksi)] as per ASME Boiler and Pressure Vessel Code. The bolts are thus adequate to retain the coil stacks.

3.5.2 Cover Plates

In the following, the cover plates have been analyzed for stresses and deflections. The model used is that of a rectangular plate with all edges supported. This case has been treated in Reference (6). The maximum stresses and deflections have been calculated based on peak loads from the coil of groups. The maximum stress is given by, ⁽⁶⁾

$$S_{\max} = \beta_w \left(\frac{b}{t} \right)^2$$

and the maximum deflection is given by, ⁽⁶⁾

$$\delta_{\max} = \alpha \frac{w b^4}{E t^3}$$

TABLE 3-3

STRESSES IN RETAINING BOLTS

Stack No.	Radial Location (in.)	Peak Transverse Stack Load (lb.)	Peak Load on a bolt (lb.)	Stress Area of bolt (in ²)	Peak Stress in bolt (psi)
1	35.8	445,400	5,568	0.6051	9,200
2	42.9	110,300	1,379	0.1416	9,740
3	49.2	119,800	1,498	0.1416	10,580
4	56.0	86,660	1,083	0.1416	7,648
5	62.6	108,600	1,358	0.1416	9,590
6	74.1	115,400	1,443	0.1416	10,190
7	81.4	20,830	260	0.1416	1,836
8	88.6	53,330	667	0.1416	4,710
9	110.2	131,900	1,649	0.1416	11,645
10	117.5	96,540	1,207	0.1416	8,524

where:

a = larger dimension of the plate

b = smaller dimension of the plate

t = thickness of the plate

w = loading intensity in lb/in^2

α = geometric parameter

β = geometric parameter

E = modulus of elasticity of material

The results are documented in Table 3-4. The modulus of elasticity for the plate material has been taken to be 2×10^6 psi corresponding to fiber-reinforced plastic.

The maximum computed stress in the plates is 25.7 MPa (3735 psi) and is less than the 100,000 cycle fatigue limit of fiber-glass-reinforced plastic of 113 MPa (15 ksi).

TABLE 3-4
ANALYSIS OF RETAINING PLATES

Stack No.	Plate Thickness (in.)	Dimensions Length x Width (in. x in.)	b/a	α from Ref. (4)	β from Ref. (4)	Peak Load (lb)	Maximum Stress (psi)	Maximum Deflection (in.)
1.	1.5	6.5 x 8	1.23	0.0639	0.4645	22270	3735	0.007
2.	1	6.5 x 8	1.23	0.0639	0.4645	5515	2081	0.006
3.	1	5 x 8	1.60	0.0906	0.5172	5990	1937	0.004
4.	1	5 x 8	1.60	0.0906	0.5172	4333	1400	0.003
5.	1	5 x 8	1.60	0.0906	0.5172	5433	1756	0.004
6.	1	4.75 x 8	1.68	0.0950	0.5378	5770	1842	0.004
7.	1	3.5 x 8	2.29	0.1175	0.6401	1042	292	0
8.	1	3.5 x 8	2.29	0.1175	0.6401	2667	747	0.001
9.	1	4.25 x 8	1.88	0.1054	0.5854	6595	2051	0.003
10.	1	4.25 x 8	1.88	0.1054	0.5854	4827	1501	0.002

4.0 ANALYSIS OF OUTBOARD ASSEMBLY

The assembly to be analyzed is located outboard of the toroidal field coils. This assembly consists of equilibrium field coil and ohmic heating field coil windings and is combined into six different groups. These groups are supported from the toroidal field coils. All the outboard turns are to have connectors at the device parting plane. Each group of windings is encased in a supporting ring and enclosed by a band which has tensioning devices located at the device parting plane. The tensioning devices are remotely actuated. These bands are used to support the radial forces

The items that need structural evaluation are the coil support rings, the steel bands encircling the coils, and the band joints (tensioning devices).

The maximum values for the magnetic forces are obtained from Table 2-3 and the structural analysis of the assembly is based on these loads.

4.1 ANALYSIS OF RETAINING BANDS

The radial components of the magnetic forces in different coil groups induce hoop stresses in the bands. If F_r denotes the net radial force per unit length of the band circumference due to all the coils in that group, the hoop stress induced in the band is simply,

$$S_{\text{band}} = \frac{F_r R}{A_{\text{band}}}$$

where R and A_{band} are the mean radius and the cross-sectional area of the band, respectively. The six retaining bands (3 for each symmetric half) have been shown in Figure 2 .

The circumferential deflection of the band is given by,

$$\delta_{\text{band}} = \frac{2\pi F_r R^2}{A_{\text{band}} E_{\text{band}}}$$

Where E_{band} is the modulus of elasticity of the band material. In the following, band thicknesses are proposed and stresses and deflections in the bands calculated. The results are given in Table 4-1. We have assumed here that the vertical component of the magnetic forces in the coils are not transmitted to the bands, but are reacted by the support rings.

From Table 4-1, we observe a maximum circumferential deflection of 0.324-in. in the coils. This implies a maximum connector displacement of 0.162 in. The maximum hoop stress in the band is 64 MPa (9270 psi) which is below the 100,000 cycle fatigue limit of AISI 305 Stainless Steel, the band material.

4.2 ANALYSIS OF SUPPORT RINGS FOR COILS

The coil stacks are encased in support rings which take the vertical component of the magnetic forces in the coils. The support rings are supported from the toroidal field coils. These rings have been investigated for spanning the toroidal field coils. The vertical loads induce bending stresses in the rings. The coil support rings are shown in Figure 2. For each ring the sectional moduls for the vertical loads has been calculated. The vertical reaction exerted by a toroidal field coil on a support ring is 1/20 of the net vertical load as obtained in Section 2.0. The maximum bending moment is related to the vertical TF reaction by Equation (6). The bending moment divided by the sectional moduls gives the bending stress. The stresses in the rings have been tabulated in Table 4-2. The maximum bending stress has been calculated for the ring at a radial location of 4.62 m (182-in.) and is 13.8 MPa (2010 psi) which is below the 100,000 cycle fatigue limit of 103 MPa (15,000 psi) for the ring material (fiber-glass reinforced plastic).

TABLE 4-1
RETAINING BAND ANALYSIS

Band	Radial Location (in.)	Cross Section (in. x in.)	Hoop Tension (lbs)	Hoop Stress (psi)	Circumferential Deflection (inches)
1	196.0	15.5 x 0.5	27,980	3,638	0.149
2	182.0	6.5 x 1.0	54,510	8,502	0.324
3	158.0	11.0 x 0.5	47,148	9,270	0.306

4.3 ANALYSIS OF BAND JOINTS

Two screw jacks of 15 ton capacity have been used for each band for tensioning purposes. From manufacturer's specifications, ⁽⁷⁾ the load bearing member is the screw of diameter 5.7 cm (2.25-in.) having 1.27 cm (0.5-in.) pitch acme thread. The root diameter of the screw is 5.1 cm (2-in.). The maximum load in the screw is the maximum hoop tension in the band; which from Table 4-2 is 54,510-lb. Therefore, the maximum tensile stress induced in the bolt is given by,

$$S_{\text{bolt}} = \frac{54510}{\pi/4 (2)^2} = 17,350 \text{ psi}$$

which is less than $2 S_m$ [$S_m = 103 \text{ MPa (15,000 psi)}$] for bolting material as per ASME Boiler and Pressure Vessel Code requirements. Hence, the tensioning devices are adequate in transmitting the radial magnetic loads from the coils.

TABLE 4-2
STRESS ANALYSIS OF COIL SUPPORT RINGS

Ring	Mean Radius (in.)	Sectional Modulus (in ³)	Net Vertical Load (lb)	TF Reaction (lb)	Bending Moment (lb-in.)	Bending Stress (psi)
1	196	68.7	64,000	3,200	16,433	239
2	182	28.1	237,000	11,850	56,506	2,010
3	158	122.2	136,800	6,840	28,315	232

5.0 REFERENCES

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