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ABSTRACT

A "Multiple Interacting Continua" method (MINC) is presented which is applicable for numerical simulation of heat and multi-phase fluid flow in multidimensional, fractured porous media. This method is a generalization of the double-porosity concept. The partitioning of the flow domain into computational volume elements is based on the criterion of approximate thermodynamic equilibrium at all times within each element. The thermodynamic conditions in the rock matrix are assumed to be primarily controlled by the distance from the fractures, which leads to the use of nested grid blocks. The MINC concept is implemented through the Integral Finite Difference (IFD) method. No analytical approximations are made for the coupling between the fracture and matrix continua. Instead, the transient flow of fluid and heat between matrix and fractures is treated by a numerical method. The geometric parameters needed in a simulation are preprocessed from a specification of fracture spacings and apertures, and the geometry of the matrix blocks.

The MINC method is verified by comparison with the analytical solution of Warren and Root. Illustrative applications are given for several geothermal reservoir engineering problems.

INTRODUCTION

In this paper, we present a numerical method for simulating transient, non-isothermal, two-phase flow of water in a fractured porous medium. The method is based on a generalization of a concept originally proposed by Barenblatt et al. (1960) and introduced into the petroleum literature by

Warren and Root (1963), Odeh (1965), and others in the form of what has been termed the "double-porosity" model. The essence of this approach is that in a fractured porous medium, fractures are characterized by much larger diffusivities (and hence, much smaller response times) than the rock matrix. Therefore, the early system response is controlled by the fractures whereas the late time response is influenced by the matrix. In seeking to analytically solve such a system, the aforesaid workers grouped all the fractures into one continuum and all the matrix blocks into another, resulting in two interacting continua, coupled through a mass transfer function determined by the size and shape of the blocks as well as the local difference in potentials between the two continua. Later, Kazemi (1969) and Duguid and Lee (1977) incorporated the double-porosity concept into a numerical model. For a more detailed description of the concept and its application, see Streltsova-Adams (1978), Evans (1981) and Pruess and Narasimhan (1981).

Very little work has been done in investigating non-isothermal, two-phase fluid flow in fractured porous media. Moench (1978) used the discrete fracture approach to study the behavior of fissured, vapor-dominated geothermal reservoirs. The purpose of the present work is first to generalize the double-porosity concept into one of many interacting continua. We shall then incorporate the "multiple-interacting-continua" model (MINC) into a simulator for non-isothermal transport of a homogeneous two-phase fluid (water and steam) in multi-dimensional systems. Our approach is considerably broader in scope and more general than any previous models discussed in the literature. The MINC method permits treatment of multiphase fluids with large and variable compressiblity, and allows for phase transitions with latent heat effects, as well as for coupling between fluid and heat flow. The transient interaction

between matrix and fractures is treated in a realistic way. Although the model can permit alternate formulations for the equation of motion, we shall assume that macroscopically, each continuum obeys Darcy's law; in particular we shall use the "cubic law" for the flow of fluids in a fracture (Witherspoon et al., 1980). While the methodology presented in this paper is generally applicable to multi-phase compositional thermal systems, our illustrative calculations were restricted to geothermal reservoir problems.

The numerical method chosen to implement the MINC-concept is the Integral Finite Difference Method (IFD; Narasimhan and Witherspoon, 1976). In this method, all thermophysical and thermodynamic properties are represented by averages over explicitly-defined finite subdomains, while fluxes of mass or energy across surface segments are evaluated through finite difference approximations. An important aspect of this method is that the geometric quantities required to evaluate the conductance between two communicating volume elements are provided directly as input data, instead of generating them from data on nodal arrangements and nodal coordinates. Thus, a remarkable flexibility is attained by which one can allow a volume element in any one continuum to communicate with another element in its own or any other continuum. Inasmuch as the interaction between volume elements of different continua is handled as a geometric feature, the IFD methodology does not distinguish between the MINC method and the conventional porous medium type approaches to modeling. Therefore, one can combine, in the same flow region, discrete fractures in one part, MINC formalism in another part, and a homogeneous porous medium in yet another.

The MINC scheme for non-isothermal two-phase flow has been incorporated into a geothermal reservoir simulator called SHAFT79 (Pruess and Schroeder,

1980). The SHAFT79 code has been thoroughly validated against analytical and numerical solutions for flow in porous media. For purposes of this paper, we shall further validate the model against the well-known Warren and Root (1963) solution for flow to a well in a double-porosity representation of a naturally fractured aquifer. Some of the calculations reported below were carried out with a compositional simulator for thermal processes, called MULKOM, which is presently under development at LBL. The geothermal version of MULKOM has been validated against SHAFT79.

Our presentation is divided into five parts. We first state mass- and energy-conservation equations in integral form. Subsequently, the geometric description (discretization) of reservoir systems for purposes of numerical modeling is discussed. Employing the criterion of "approximate thermodynamic equilibium" within each computational volume element, we then introduce the basic concepts as well as the numerical implementation of the MINC method. Remaining parts deal respectively with the validation of the MINC-algorithm, and with illustrative applications to problems of interest to geothermal reservoir engineering.

GOVERNING EQUATIONS

For modeling purposes, it is customary to idealize geothermal reservoirs as systems of porous rock saturated with a single-component fluid in liquid and vapor form. The basic equations governing mass and energy transport in these systems can be written as:

$$\frac{d}{dt}\int_{V_n} \Phi\rho \, dv = \int_{\Gamma_n} \underset{\sim}{F}\cdot\underset{\sim}{n} \, d\Gamma + \int_{V_n} q dv \qquad (1)$$

$$\frac{d}{dt}\int_{V_n} U\,dv = \int_{\Gamma_n} \underset{\sim}{G}\cdot\underset{\sim}{n}\,d\Gamma + \int_{V_n} Q dv \qquad (2)$$

Equations (1) and (2) are general conservation statements, and hold true for arbitrary subregions V_n, irrespective of whether V_n is "large" or "small". Also, V_n can have arbitrary (irregular) shape, and arbitrary topological structure. Our rationale for presenting the governing equations in integral rather than differential form is that this avoids reference to a global system of coordinates, and therefore provides great flexibility in the geometrical description of the flow region. This flexibility is particularly useful in the modeling of fractured porous media, as will be shown below. It is to be noted that the validity of Equations (1) and (2) is not restricted to porous media. Indeed, these equations hold for arbitrary mixtures of rock and fluid. In particular, these equations are valid for fractured porous media and for regions containing only fluid and no rock (and vice versa).

To complete the formulation given in Equations (1) and (2), we specify mass and energy flow terms, and define the internal energy for a rock/fluid mixture.

We assume that mass flux is given by Darcy's law:

$$\underset{\sim}{F} = \sum_{\substack{\beta=\text{liquid,}\\ \text{vapor}}} \underset{\sim}{F}_\beta = -k\sum_\beta \frac{k_\beta}{\mu_\beta}\rho_\beta\left(\nabla p - \rho_\beta \underset{\sim}{g}\right) \qquad (3)$$

Here we neglect capillary pressure, as is customary for geothermal applications, although its inclusion is quite straightforward. We note that equation (3) is applicable to porous media as well as to fractures. Experimental and

theoretical work (e.g., Witherspoon et al., 1980) has established that laminar flow in fractures is very closely governed by Darcy's law, with fracture permeability given by

$$k_f = \frac{\delta^2}{12} \tag{4}$$

Energy flux contains conductive and convective terms

$$\underset{\sim}{G} = -K\nabla T + \sum_{\beta} h_\beta \underset{\sim}{F}_\beta \tag{5}$$

and the volumetric internal energy of the rock/fluid mixture is

$$U = \Phi\rho u + (1 - \Phi)\rho_R C_R T \tag{6}$$

The governing equations need to be complemented with a description of the thermophysical properties of the fluid filling the void space. This is based on the concept of "local thermodynamic equilibrium". It is assumed that the transient evolution of reservoir systems proceeds through a (continuous) sequence of quasi-equilibrium states "at each point". A thermodynamic variable "at a point" is but an appropriate average value of that quantity over a suitably defined finite subdomain to which the given point is interior. Although the point is usually assumed to be at the center of gravity of the subdomain, its appropriate location may be a function of time (Narasimhan, 1978; Narasimhan, 1980). Using the concept of local thermodynamic equilibrium, an "equation of state" can be employed to express all thermophysical parameters as functions of a set of primary dependent variables. For a single-component (one- or two-phase) fluid there are two primary variables, which in our simulator SHAFT79 are chosen to be specific internal energy u and density ρ.

Space-discretization of Equations (1) and (2) can be obtained by introducing appropriate volume averages:

$$\int_{V_n} x \, dv = x_n V_n \qquad (7a)$$

where x is a volume-normalized extensive quantity such as porosity, density, enthalpy, saturation, or moisture content and x_n is the average value of x over V_n. Surface integrals are approximated as a discrete sum of averages over surface segments A_{nm}:

$$\int_{\Gamma_n} \underset{\sim}{F}\cdot\underset{\sim}{n} \, d\Gamma = \sum_m F_{nm} A_{nm} \qquad (7b)$$

The sum in (7b) extends over all V_m sharing a surface segment with V_n. Boundary conditions are treated by introducing appropriate boundary elements (for Dirichlet-type conditions), or sinks and sources (for Neumann-type conditions). Approximating time derivatives as first-order finite differences, we have the following algebraic equations:

$$\left(\phi_n^{k+1}\rho_n^{k+1} - \phi_n^k\rho_n^k\right) - \frac{\Delta t}{V_n}\left\{\sum_m F_{nm}^{k+1} A_{nm} + V_n q_n^{k+1}\right\} = 0 \qquad (8)$$

$$\left(U_n^{k+1} - U_n^k\right) - \frac{\Delta t}{V_n}\left\{\sum_m G_{nm}^{k+1} A_{nm} + V_n Q_n^{k+1}\right\} = 0 \qquad (9)$$

To assure unconditional stability, we have used a fully implicit formulation, with all fluxes and variable sources evaluated at the new time level, t^{k+1}. Practical solution of these equations requires specification of the geometric partitioning, or mesh $\{V_n;\ n = 1, \ldots, N\}$.

GEOMETRIC DESCRIPTION (MESH)

Equations (8) and (9) are valid for completely arbitrary sub-regions V_n, including the entire reservoir system. These equations hold whether the reservoir is fractured or porous or both. For a complete description, the reservoir volume V may be partitioned into arbitrary volume elements V_n (n = 1, ..., N), such that (e.g., Narasimhan, 1978)

$$\bigcup_{n=1}^{N} V_n = V \qquad (10)$$

However, Equations (8) and (9) are useful only if the allowable partitions

$$\mathbb{P}V = \{V_n;\ n = 1, \ldots, N\} \qquad (11)$$

are suitably restricted on the basis of geometric and thermodynamic considerations. Generally speaking, the partitioning of the reservoir volume must be made in such a way that the flux terms F_{nm}, G_{nm} can be expressed as functions of the average values of the primary thermodynamic variables in the volume elements V_1, ..., V_N. This can be achieved if there is approximate thermodynamic equilibrium within each volume element at (almost) all times. The classical equations of motion assume (on empirical grounds) that F_{nm} is directly related to the gradient of average fluid potentials while conductive heat flux is directly related to the gradient of average temperature. We will here follow these classical equations of motion, but our formulation can permit other, more general expressions.

The expression for mass flux is given by,

$$F_{nm} = \sum_{\beta} \left(\frac{kk_\beta}{\mu_\beta}\right)_{nm} (\rho_\beta)_{nm} \left[\frac{P_m - P_n}{d_{nm}} - (\rho_\beta)_{nm} g_{nm}\right] \qquad (12)$$

An analogous definition holds for the energy flux G_{nm} which has a conductive and an advective component:

$$G_{nm} = K_{nm} \frac{T_m - T_n}{d_{nm}} + \sum_{\beta} (h_\beta)_{nm} (F_\beta)_{nm} \qquad (13)$$

In (12) and (13), different weighting procedures (e.g., harmonic weighting, spatial interpolation, upstream weighting) can be employed to evaluate the various "interface quantities," labeled with subscripts "nm". Equations (12) and (13) are appropriate for isotropic media, and for orthotropic media in which a principal axis of permeability coincides with the outer normal to the surface segment $d\Gamma$. For a more general orientation of the segment, Equations (12) and (13) can be extended by considering a set of non-collinear points for approximating gradients of pressures and temperatures.

Equations (8), (9), (12), and (13), together with an equation of state for the pore fluid, provide a flexible "integral finite difference" (IFD) formulation for mass and heat flow in geothermal reservoirs, which is applicable to fractured and/or porous media. The chief advantage of the IFD method is geometric flexibility. The entire geometric description of the system to be modeled is contained in a list of element volumes V_n, interface areas A_{nm}, and nodal distances d_{nm}, which is provided as input data. This allows modeling of one-, two-, or three-dimensional, regular, or irregular systems with the same ease. The partitioning of the system is subject to only two constraints: (i) within each volume element there exists approximate thermodynamic equilibrium at (almost) all times, and (ii) for adjacent volume elements the line connecting nodal points coincides with a principal axis of permeability, and the interface area is perpendicular to this line.

In porous media, the variables of thermodynamic state are slowly varying functions of position. Therefore, approximate thermodynamic equilibrium exists within each "small" portion of the flow region. As a general rule, then, volume elements would be chosen as "small" simply-connected regions. Larger volume elements are acceptable in regions where spatial variations in thermodynamic conditions are weak, or where little detail is desired, e.g., at greater distance from wells.

Conditions can be substantially different in fractured porous media. Extremely sharp gradients in fluid pressures, temperatures, and other thermodynamic variables may occur close to the fractures so that, to satisfy the requirement of thermodynamic equilibrium, one has to employ volume elements with extremely small volumes. Typically these elements will have widths of the order of a few fracture apertures. Additionally, the fractures themselves need to be modeled as very small volume elements. While this approach is conceptually straightforward, it requires detailed geometrical information which is seldom available from the field, and the computing effort increases dramatically with the increase in the number of volume elements to be handled. From a practical view point, therefore, the need for a simpler, more viable tool of analysis is clearly indicated.

We shall now generalize the above considerations for a fractured porous medium with several sets of fractures in three dimensions. Our discussion is based on a schematic idealized model of fractured porous media, similar to the conceptual model of double-porosity systems originally proposed and developed by Barenblatt et al. (1960), Warren and Root (1963), and others. While these authors used the double-porosity model to obtain analytical solutions, we have taken a more general numerical approach. This makes

possible the treatment of coupled fluid and heat transport without invoking quasi-steady approximations for interporosity flow.

MULTIPLE INTERACTING CONTINUA (MINC)

Consider a fractured porous reservoir, which we idealize as having three perpendicular sets of infinite, plane, parallel fractures of equal aperture δ and spacing D (Figure 1). The reservoir is made up of identical elementary units, in our idealization assumed to be cubes, which are bounded by fractures on all faces. The assumption of a regular fracture network is actually not essential for the method presented here. Generalizations to arbitrary assemblages of fractures which may be described in a statistical fashion are possible. This will not be pursued here, however, for the simple illustrative purpose on hand.

We envision a process with large differences in thermodynamic conditions (temperatures, pressures, vapor saturations) between fractures and rocks, such as depletion of a boiling reservoir zone. The process to be modeled involves two aspects: (i) the global movement of fluid and heat through the reservoir, which proceeds almost entirely through the fracture system, and (ii) fluid and heat flow between rock matrix and fractures. While thermodynamic conditions may vary strongly over small distances in the vicinity of the fractures, it appears reasonable to expect that spatial variations within the fracture system, or within certain portions of the matrix, may be slow and amenable to volume averaging. Thus we can lump appropriate portions of the flow region (e.g., certain groups of well-connected fractures or portions of porous blocks) into several distinct continua which interact with each other. Variations in thermodynamic conditions will be much less pronounced in the direction of a fracture than perpendicular to it. As a first approximation,

we shall assume that thermodynamic conditions in the matrix will depend primarily upon the distance from the nearest fracture. Therefore, we shall partition the flow domain into computational volume elements in such a way that all interfaces between volume elements in the matrix are parallel to the nearest fracture. We emphasize that this choice of partitioning implies a restriction in the possible functional form of the time-dependence of thermodynamic state in the matrix. Namely, we assume that the equipotential surfaces (of pressure, temperature, and other variables of thermodynamic state) are characterized by having constant distance from the nearest fracture. This approximation to the interaction between reservoir geometry and thermodynamic state is quite analogous to the approximation made by Gibbs in his theory of capillarity for the boundary layer between two liquid phases (Gibbs, Collected Works, 1948). Partitioning based on this assumption gives rise to a pattern of nested volume elements, which, for the two-dimensional case, is shown in Figure 2. Each volume element has a definite thermodynamic state assigned to it. Modeling of heat and fluid flow in such a system of nested volume elements, or interacting continua, is straightforward within the framework of the IFD method. The geometric description of the problem in terms of a set of elemental volumes V_n, interface areas A_{nm}, and nodal distances d_{nm} can be readily obtained from a specification of the volume fractions ϕ_j ($j = 1, \ldots, J$) occupied by the interacting continua. Assuming that $j = 1$ refers to the outer (fracture) continuum, we have

$$\phi_1 = \left[D^3 - (D-\delta)^3\right] / D^3 \approx 3\delta/D \qquad (14a)$$

$$V_j = \phi_j D^3 \qquad (j = 1, \ldots, J) \qquad (14b)$$

$$A_{j,j+1} = 6D^2 \left(\sum_{i=j+1}^{J} \phi_i \right)^{2/3} \qquad (j = 1, \ldots, J-1) \qquad (14c)$$

$$d_{1,2} = \frac{D}{4} \left[\left(\sum_{i=2}^{J} \phi_i \right)^{1/3} - \left(\sum_{i=3}^{J} \phi_i \right)^{1/3} \right] \qquad (14d)$$

$$d_{j,j+1} = \frac{D}{4} \left[\left(\sum_{i=j}^{J} \phi_i \right)^{1/3} - \left(\sum_{i=j+2}^{J} \phi_i \right)^{1/3} \right] \qquad (14e)$$

$$(j = 2, \ldots, J-2)$$

$$d_{J-1,J} = \frac{D}{4} \left(\phi_{J-1} + \phi_J \right)^{1/3} - \frac{3D}{20} \phi_J^{1/3} \qquad (14f)$$

In the last equation, we have adopted an interface distance for the innermost element V_J in accordance with the quasi-steady value of the interporosity flow parameter α as given by Warren and Root. Other choices are possible, but the impact on simulated results is negligible. The volume fraction ϕ_1 occupied by the fractures is determined from fracture spacing and width (Equation 14a). The other volume fractions are arbitrary in principle (except for the constraint $\sum_{j=1}^{J} \phi_j = 1$), and are chosen such as to provide good resolution where needed. Thus, the outer elements near the fractures (j = 2, 3, ...) will be closely spaced, with larger distances appropriate for the elements away from the fractures (j = J, J-1, ...).

In a mesh with nested volume elements such as shown in Figure 2, the matrix acts as a "one-way street" for fluid and heat flow, with all flow occurring outward into the fractures as production causes pressures and temperatures to decline in the fracture system. For some reservoir processes, e.g. in composi-

tional problems arising in enhanced oil-recovery operations, this appproach will have to be modified to permit throughflow through the matrix. A more flexible coupling between fractures and matrix is currently being investigated.

It is desirable to further generalize the partitioning scheme outlined above. In reservoir regions where spatial variations in thermodynamic conditions are weak, it is not necessary to have separate volume elements within each of the elementary units depicted in Figure 2. Corresponding nested volumes in neighboring units, which are identified by an index number in Figure 2, will have approximately the same thermodynamic conditions, and therefore can be lumped together into one computational volume element. The geometric parameters pertaining to such a system of multiple interacting continua can be readily obtained from Equations (14b-f) by means of a simple scaling operation. Note that the number of elementary units contained in V_n is given by

$$\sigma = V_n/D^3 \tag{15}$$

so that the volume of continuum j within V_n is

$$V_{nj} = \sigma V_j = \phi_j V_n \tag{16a}$$

Each interface area occurs σ times, so that

$$A_{nj,nj+1} = \sigma A_{j,j+1} \tag{16b}$$

The nodal distances obviously are independent of the number of elementary units so that

$$d_{nj,nj+1} = d_{j,j+1} \tag{16c}$$

Strictly speaking, the scaling laws given in equations (16a) through (16c) are applicable only if V_n actually contains an integral number of elementary units (σ = 1, 2, 3, ...). It is very convenient, however, to apply the same scaling to grid blocks of arbitrary shape or size, including the situation where $V_n << D^3$ (i.e., $\sigma << 1$). Of course, in regions where small grid blocks ($V_n << D^3$) are desirable for spatial resolution, e.g. near wells, one should attempt to model individual fractures. A description based on average fracture spacings is appropriate in the more distant portions of the reservoir, where less detail is available and desirable. However, we believe it useful to be able to extend a fracture description based on average parameters (spacings, apertures, orientations) to small volume elements, because this is applicable in cases where no detailed information about individual fractures near a well is available; furthermore, this is applicable for generic studies of fractured reservoir behavior. Thus we arrive at the following three-step procedure for defining a computational mesh for a fractured reservoir:

(i) Define a mesh $\{V_n; n=1, ..., N\}$ based on considerations of global geometry, global variations of reservoir conditions and parameters, and desired spatial resolution, just as would be done for a porous medium ("primary mesh").

(ii) Use Equations (14a-f) to obtain the geometric parameters for a set of nested volume elements within one elementary unit of the fractured reservoir.

(iii) Apply the scaling laws given in Equations (15) and (16) to partition each volume element V_n of the primary mesh into a sequence of interacting continua elements $\{V_{nj}; j = 1, ..., J\}$, and obtain volumes V_{nj}, interface areas $A_{nj,nj+1}$, and nodal distances $d_{nj,nj+1}$ appropriate for the "secondary mesh".

We have written a pre-processor program to perform the calculation of the geometric quantities for a fractured porous reservoir mesh. The program reads a set of volumes and "connections" (i.e., interface areas and nodal distances) for a "primary" mesh $\{V_n;\ n = 1, \ldots, N\}$. This mesh can have regular or irregular (polyhedral) grid blocks in one, two, or three dimensions, with an arbitrary number of interfaces between grid blocks. All primary ("interblock") connections are left unchanged, and these are assigned to the fracture continuum j = 1. Fracture permeability and porosity are taken into account by means of the equivalent continuum properties:

$$k(j=1) \approx 2k_f\,\delta/D = \delta^3/6D \qquad (17)$$

$$\phi(j=1) = \phi_1 \qquad (18)$$

Each volume element is subdivided into J continua, and the appropriate J-1 "intrablock" connections are appended to the list of interblock connections according to Equations (15) and (16). The mesh defined by the partition $\{V_{nj};\ n = 1, \ldots, N;\ j = 1, \ldots, J\}$ is then used for modeling reservoir behavior by means of Equations (8), (9), (12), and (13).

VALIDATION

From the foregoing discussion, it is clear that any simulator based on the IFD - method is immediately capable of modeling a system of multiple interacting continua. All that is required to apply such a simulator to fractured porous media is appropriate preprocessing of the geometrical information. No programming changes are necessary in the simulator itself, as the basic equations solved for each volume element are the same for porous as for fractured media.

Equations (8), (9), (12), and (13) are readily implemented on a computer. Our single-component, two-phase geothermal reservoir simulator SHAFT79 employs Newton-Raphson iteration to account for the severe non-linearities arising in phase transitions between liquid water, two-phase mixtures, and steam. The reservoir fluid is assumed to be pure water substance, and its thermophysical properties are represented by the steam-table equations as given by the International Formulation Committee (1967). The linear equations arising at each iteration step are solved directly, using Gaussian elimination and sparse storage techniques. The linear algebra is performed with the Harwell subroutine package "MA28", which efficiently handles non-symmetric matrices with random sparsity structure (Duff, 1977). This feature is ideally suited for our purposes, as it permits exploitation of the geometric flexibility of the IFD method to the fullest extent. Volume elements can be connected in any way desired, without any limitations as to band width or dimensionality of the problem.

In order to validate the MINC algorithm, we have performed several simulations of isothermal fractured reservoir problems, for which approximate analytical solutions are available from the work of Warren and Root (1963). Test calculations showed that fine spatial resolution and small time steps were required for a good agreement between the numerical simulations and the analytical results of Warren and Root. Parameters for one of the problems run are given in Table I. It is an isothermal single-phase injection problem, with results plotted in Figure 3. The agreement for pressure transients is good, with slight discrepancies at very early times due to the discrete approximation which we use in the simulation to represent the flux boundary condition at the wellbore radius (r_w = 0.10 m). This approximation tends

to overestimate flux and sandface pressure at early times. From the slope of the semilog straight lines (Figure 3) we deduce $kh = 2.07 \times 10^{-14}$ m^3, in excellent agreement with the value $kh = 2.08 \times 10^{-14}$ m^3 used in the simulation. The vertical displacement $\Delta p = 2.05 \times 10^4$ Pa corresponds to a storativity parameter $\omega = e^{-2\Delta P_d} = .211$. This agrees reasonably well with the proper value $\omega = \phi_2 C_2/(\phi_1 C_1 + \phi_2 C_2) = .193$, considering that ω depends very sensitively (exponentially) upon Δp. We also used simulated results to compute the time dependence of the interporosity coupling parameter α as defined by Warren and Root. We found that near the sandface (at $r = 1.125\ r_w$) α rapidly approaches the quasi-steady value of 60 m^{-2} which holds for a problem with three equidistant fracture sets and spacing $D = 1$ m. Deviations from quasi-steady behavior disappear at about the time when the pressure response changes from the early time to the late time straight line.

APPLICATIONS

We have applied the MINC method to a variety of problems which are of interest in geothermal reservoir engineering. We present results for: (i) two-phase flow to a well penetrating a highly fractured reservoir with low matrix permeablity; (ii) depletion of a fractured, boiling geothermal reservoir; and (iii) production from and injection into a fractured geothermal reservoir, using a five-spot well pattern. In the present context, these calculations are intended to be illustrative rather than exhaustive. We wish to indicate typical problems which can be investigated with the MINC method, and our discussion of the results will be brief. A more detailed analysis of the reservoir problems presented here can be found in Pruess and Narasimhan (1981) and Pruess (1981).

(i) Flow to a Well in a Boiling Fractured Geothermal Reservoir

There has been considerable controversy in the earlier literature about distribution and amount of pore water in vapor-dominated geothermal reservoirs (Truesdell and White, 1973; Weres, Tsao, and Wood, 1977). At present, a general consensus appears to exist that the saturation of distributed water must be rather "small" (perhaps in the range from 20%-50%) in order that liquid water may be nearly immobile. Conventional wisdom has held that higher water saturations and a mobile liquid phase are not possible in vapor-dominated reservoirs, because these systems produce slightly superheated steam, and vertical pressure gradients are close to vapor-static.

We performed a study of radial flow in a vapor-dominated geothermal reservoir to determine whether the fractured nature of these systems would permit large water saturations in the matrix. In systems containing steam/water mixtures, there is a one-to-one correspondence between pressure gradients (which drive mass flow) and temperature gradients (which drive conductive heat flow). Heat is transferred from the matrix to the fractures by means of fluid convection, and by means of heat conduction. If matrix permeability is low, conductive heat transfer can become very significant in comparison with convective heat transfer, giving rise to an increase in flowing enthalpy of the two-phase mixture. It is therefore possible that superheated steam is discharged into the fractures, even though liquid water may be mobile and flowing in the matrix.

Our simulation of this process employs parameters applicable to The Geysers vapor-dominated reservoir in Sonoma and Lake Counties, California (see Table II). Water saturation in the matrix is assumed to be 70% for

this study, but similar results are obtained for higher water saturations (in excess of 90%). We have taken great care to include a realistic description of wellbore storage and effective wellbore radius (skin), and we apply a realistic production rate (Koenig and Sanyal, 1981). The basic computational mesh is one-dimensional, and the connection between the wellblock, the fracture elements, and the rock matrix is schematically depicted in Figure 4. We emphasize that fractures are modeled as very small volume elements (see Equation 14a) with only fluid and no rock present, rather than as "smeared out" regions of large permeability. This treatment slows computations down by several orders of magnitude, due to throughput limitations in very small grid elements, but it is important for a realistic modeling of the rather subtle interplay between fluid and heat flow near the matrix/fracture interface. Full details of our parameter choices and the computational procedure are given in Pruess and Narasimhan (1981).

Figure 5 shows results for the transient evolution of production enthalpy for three different values of matrix permeability, compared with a uniform porous medium calculation. After an initial period dominated by wellbore storage (approximately 800 seconds), the fractured reservoirs show a much more rapid rise in enthalpy than the porous medium, due to more localized boiling. As the production-induced pressure drop diffuses outward into the fracture system, an increasing share of produced fluid is replenished by leakage from the matrix. This stabilizes enthalpies, and it prevents the fractures from drying up in the higher permeability cases ($k_m = 10^{-15}$ m^2, 10^{-16} m^2), so that a steam/water mixture continues to be produced. For the low matrix permeability of $k_m = 10^{-17}$ m^2, however, the fractures are depleted rapidly of fluid reserves, giving rise to superheated conditions.

This case is the most difficult computationally, as it involves propagation of a sharp steam/two-phase interface through small grid elements in and adjacent to the fractures. This severely limits attainable time steps, and we have only been able to simulate a few hours of physical time. Nonetheless, we feel that it is apparent from the calculations that the low matrix permeability of $k_m = 10^{-17}$ m^2 is quite sufficient to sustain the applied realistic production rate. We conclude from this study that production of superheated steam is compatible with the presence of highly mobile liquid water in a low-permeability matrix.

(ii) Depletion of a Boiling, Fractured Geothermal Reservoir

We consider a closed rectangular reservoir, 7 km x 3 km in extent, with a well field of 1 km^2 area. For the numerical simulation, the field is discretized into 21 volume elements of 1 km x 1 km. This mesh is identical to that used by Bodvarsson et al. (1980) for a study of the Baca geothermal field in New Mexico. Field evidence suggests a kh of about 1.83×10^{-12} m^3, presumably dominated by fracture permeability. We conducted a parameter study of this field, treating it as a fractured porous medium and investigating the effect of matrix permeability upon system response. In order to avoid the need for handling fracture elements of extremely small volume, we abandoned the cubic-law relation (4) in favor of treating fractures as somewhat extended regions of approximately 0.2 m width. Field-estimated values of permeability and porosity were assigned to the fracture continuum. The parameters used in the calculations are given in Table II. The volume fractions used for the MINC mesh were $\phi_1 = 0.01$, $\phi_2 = 0.04$, $\phi_3 = 0.15$, $\phi_4 = 0.40$, and $\phi_5 = 0.40$. We believe that the "smearing out" of the fractures will have some influence on the early response of the system but not on the late-time behavior.

Some results of the simulations are given in Figs. 6 and 7. Two basic depletion patterns are observed, depending on whether matrix permeability is low or high. For low matrix permeability ($k_m = 10^{-17}$ m^2) boiling is confined to the vicinity of the fractures, while high k_m ($k_m = 9\times10^{-17}$ m^2; 10^{-15} m^2) is accompanied by boiling occurring deep within the matrix, analogous to what has been observed for porous media (Pruess et al., 1979).

The combined influence of matrix permeability and flowing enthalpy leads to interesting two-phase mobility effects. Consider, for example, the two cases: (D, k_m) = (150 m, 9×10^{-17} m^2), and (50 m, 1×10^{-17} m^2). In both cases the matrix can provide the same mass flow support at the initial vapor saturation of 0.01. In the case of small matrix permeability, heat conduction causes a substantial increase in enthalpy (Fig. 7). This, in turn, leads to a reduction in mobility followed by localized boiling and a rapid decline in fluid pressure. Decreasing the fracture spacing to D = 5 m provides a ten-times larger matrix-fracture contact area, but it has little influence on conductive enhancement of flowing enthalpy or pressure decline. Therefore, it appears that matrix permeability is a far more important parameter than fracture spacing in controlling reservoir longevity. It might appear from Fig. 6 that porous-medium-type reservoirs will always have greater longevity than equivalent fractured reservoirs. This conclusion is not valid, however, since the results in Fig. 6 and 7 are influenced by discretization effects. Thus, in the porous medium case, mass and energy are produced from the entire wellblock, while in the fractured porous medium case production is made from a small portion (1% by volume) of the well block. The next example shows that fractured reservoirs may have greater longevity in some cases than equivalent porous media reservoirs.

(iii) Five-Spot Production and Injection

For a more realistic assessment of two-phase reservoir depletion, we investigated a five-spot production-injection strategy for the reservoir discussed in the previous example. The mesh as given in Fig. 8 takes advantage of flow symmetry. The MINC volume fractions, ϕ_j, were the same as in the above depletion example. The production-injection rate was 30 kg/s, corresponding to the more productive wells in the Baca reservoir.

The results show that without injection, pressures will decline rapidly in all cases. The times after which production-well pressure declines below 0.5 MPa are: 1.49 yrs for a porous medium; 2.70 yrs for D = 150 m, k_m = 9×10^{-17} m^2; and 0.44 yrs for D = 50 m, k_m = 1×10^{-17} m^2. Note that the fractured reservoir with large k_m (9×10^{-17} m^2) has a greater longevity than a porous reservoir.

The results obtained for 100% injection demonstrate the great importance of injection for reservoir pressure maintenance in fractured reservoirs with low permeability. Simulations of 90 years for the porous medium case and 42 years for the case D = 50 m, k_m = 10^{-17} m^2, showed no thermal depletion or catastropic pressure declines in either case. These times are significantly in excess of the 30.5 years needed to inject one pore volume of fluid. Our results show that the temperature profiles for the porous-medium case and the fractured-porous-medium case agree remarkably well, showing an excellent thermal sweep for the latter (see also Bodvarsson and Tsang, 1981). We are currently investigating thermal sweep efficiency in dependence upon fracture spacing, as well as the nature of thermal breakthrough when a prominent, short-circuiting fault or fracture exists between the production and injection wells.

CONCLUSIONS

1. We have presented a "multiple interacting continua" method (MINC) for numerically modeling fluid and heat flow in fractured porous media. This method is conceptually similar to, and is an extension of, the well-known double-porosity approach.
2. The MINC method can be implemented with any simulator based on an "integral finite difference" formulation. The geometric description of fractured reservoirs is handled by a pre-processor program. Systems of different dimensionality with regular or irregular geometry and fractured or porous regions can be treated on equal footing.
3. The MINC method was validated by comparison with the analytical solution of Warren and Root. Practical applications are demonstrated for a number of geothermal reservoir problems.
4. We find that fractured geothermal reservoirs may produce superheated steam even if the rock matrix has large water saturation and a mobile liquid phase. The depletion behavior of fractured geothermal reservoirs is more strongly dependent upon matrix permeability than upon fracture spacing. Full reinjection is essential for pressure maintenance in fractured liquid-dominated reservoirs with low permeability, and excellent thermal sweeps appear possible.
5. We are currently investigating applications of the MINC-method to compositional thermal problems in reservoirs with irregular and statistical fracture distributions.

NOMENCLATURE

A: Area $[L^2]$

A_{nm}: Area of interface between volume elements n and m $[L^2]$

C: Fluid compressibility $[LT^2/M]$

C_f: Fracture compressibility $[LT^2/M]$

C_R: Specific heat of rock $[L^2/T^2{}^oTEMP]$

d_{nm}: Distance between nodal points n and m [L]

$d\Gamma$: Surface segment $[L^2]$

D: Fracture spacing [L]

$\underset{\sim}{F}$: Mass flux vector $[M/TL^2]$

F_{nm}: Mass flux from volume element m into n $[M/TL^2]$

$\underset{\sim}{F}_\beta$: Mass flux of β-phase $[M/TL^2]$
(β = liquid, vapor)

$\underset{\sim}{g}$: Gravitational vector $[L/T^2]$

$\underset{\sim}{G}$: Energy flux vector $[M/T^3]$

g_{nm}: Normal component of gravitational acceleration between volume elements n and m $[L/T^2]$

G_{nm}: Energy flux from volume element m into n $[M/T^3]$

h: Specific enthalpy $[L^2/T^2]$

h: Height of reservoir [L]

h_β: Specific enthalpy of β-phase $[L^2/T^2]$

k: Superscript, indicates k^{th} time step

k: Absolute or intrinsic permeability $[L^2]$

k_f: Absolute permeability of a fracture $[L^2]$

k_m: Absolute permeability of rock matrix $[L^2]$

k_β: Relative permeability with reference to the β-phase

k_2: Equivalent continuum permeability for fractures $[L^2]$

K: Heat conductivity $[L/T^3{}^oTEMP]$

$\underset{\sim}{n}$: Unit normal vector, pointing inward

p: Fluid pressure $[M/LT^2]$

$\mathbb{P}$: Symbol designating partition

q: Rate of mass production per unit volume $[M/L^3T]$

Q: Rate of energy production per unit volume $[1/LT^3]$

q_n: Rate of mass production per unit volume from element n $[M/L^3T]$

Q_n: Rate of energy production per unit volume from element n $[1/LT^3]$

r_e: Outer radius of reservoir [L]

r_w: Wellbore radius [L]

r_w': Effective wellbore radius [L]

s: Skin

S_ℓ: Liquid saturation

$S_{\ell r}$: Irreducible liquid saturation

S_{sr}: Irreducible vapor saturation

t: Time [T]

T: Temperature $[^oTEMP]$

u: Specific internal energy of the fluid $[L^2/T^2]$

U: Internal energy contained in the rock-fluid mixture per unit volume of the medium $[M/LT^2]$

v,V: Volume [L^3]

V_n: Volume of element n [L^3]

x: Any volume-normalized extensive quantity

x_n: Average value of x over volume element V_n

α: Interporosity coupling parameter (Warren and Root)[$1/L^2$]

β: Phase ($\beta = \ell$: liquid; $\beta = v$: vapor)

Γ_n: Closed surface bounding volume element n [L^2]

δ: Aperture of a fracture [L]

λ: Interporosity flow parameter (Warren and Root)

μ: Viscosity [M/LT]

μ_β: Viscosity of phase β [M/LT]

ρ: Mass density [M/L^3]

ρ_R: Mass density of rock [M/L^3]

ρ_β: Mass density of β-phase [M/L^3]

Φ: Porosity

Φ_f: Porosity of fracture

Φ_m: Porosity of rock matrix

Φ_1: Porosity of matrix continuum ($\simeq \Phi_m$)

Φ_2: Porosity of fracture continuum

ϕ_j: Fractional volume of the j^{th} continuum

ω: Double-porosity storativity parameter

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TABLE I: Isothermal Water Injection into a Double-Porosity System

T	= 240 °C	μ	= 1.123×10^{-4} Pa·s
P	= 6.078 MPa	C	= 1.273×10^{-9} Pa^{-1}
q	= .025 kg/s		
D	= 1.0 m		
δ	= 5×10^{-5} m		
h	= 1.0 m		
Φ_f	= 1.0		
Φ_m	= .05		
k_f	= 208×10^{-12} m^2		
k_m	= 10^{-17} m^2		
Φ_1	= .05		
Φ_2	= 15×10^{-5}	λ	= 2.827×10^{-4}
k_2	= 208×10^{-16} m^2	ω	= .1926
C_f	= 10^{-7} Pa^{-1}		

Mesh: r_w = 0.10 m
16 elements with $\Delta r = 3.5 \times 10^{-2}$ m
100 elements with $\Delta r_{m+1} = f \cdot \Delta r_m$, such that
r_e = 5000 m

TABLE II: Parameters Used in Simulations

	Radial Flow Problem	Depletion Problem
Formation		
rock grain density	ρ_R = 2400 kg/m^3	2600 kg/m^3
rock specific heat	C_R = 960 J/kg°C	950 J/kg °C
rock heat conductivity	K = 4 W/m°C	2.22 W/m °C
porosity	ϕ = .08	.10
permeability x thickness	kh = 13.4x10^{-12} m^3	1.83x10^{-12} m^3
reservoir thickness	h = 500 m	305 m
matrix permeability	k_1 = 10^{-15} m^2, 10^{-16} m^2, 10^{-17} m^2	10^{-15} m^2, 9x10^{-17} m^2, 10^{-17} m^2
Fractures		
three orthogonal sets		
aperture	δ = 2x10^{-4} m	(a)
spacing	D = 50 m	5 m, 50 m, 150 m
permeability per fracture	$k_f = \delta^2/12$ = 3.3x10^{-9} m^2	(a)
equivalent continuum permeability	$k_2 \approx 2k_f\delta/D$ = 26.8x10^{-15} m^2	6x10^{-15} m^2
equivalent continuum porosity	$\phi_2 \approx 3\delta/D$ = 1.2x10^{-5}	.10
Relative Permeability		
Corey-curves	$S_{\ell r}$ = .30, S_{sr} = .05	$S_{\ell r}$ = .30, S_{sr} = .05
Initial Conditions		
temperature	T = 243 °C	300 °C
liquid saturation	S_ℓ = 70%	99%
Production		
wellbore radius	r_w = .112 m	
skin	s = -5.18	
effective wellbore radius	$r_w' = r_w e^{-s}$ = 20.0 m	
wellbore storage volume	V_w = 27.24 m^3	
production rate	q = 20 kg/s	82.5 kg/s[b]; 30 kg/s[c]
Injection		
rate	---	30 kg/s[c]
enthalpy	---	5x10^5 J/kg

(a) fractures modeled as extended regions of high permeability, with a widty of ≈ .2 m
(b) rectangular reservoir
(c) five-spot

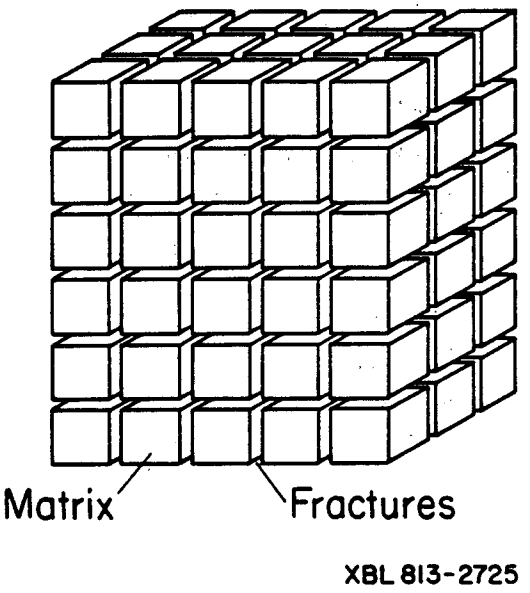

Fig. 1 Idealized model of a fractured porous medium.

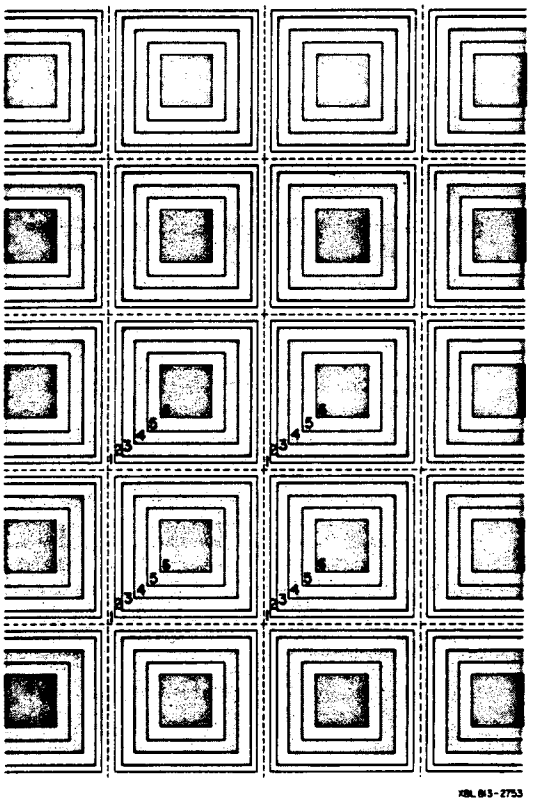

Fig. 2 Basic computational mesh for fractured porous media (two-dimensional case).

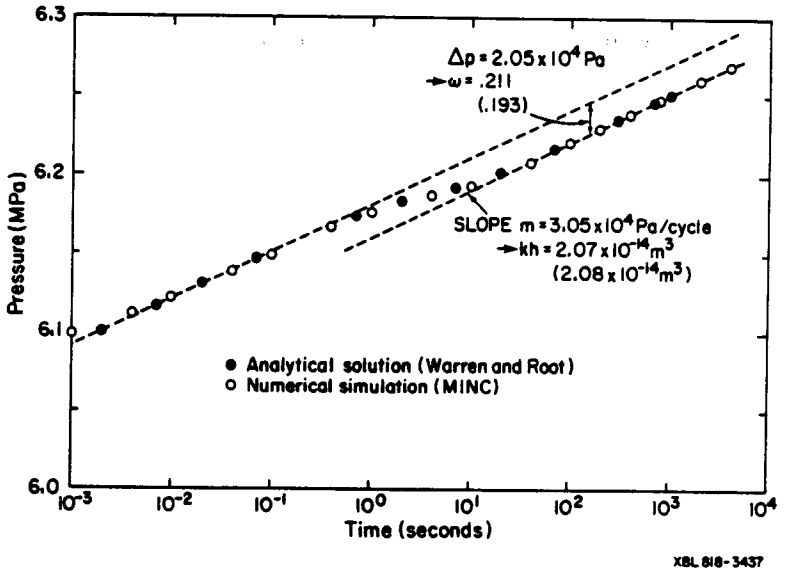

Fig. 3 Pressure transients for Warren/Root problem.

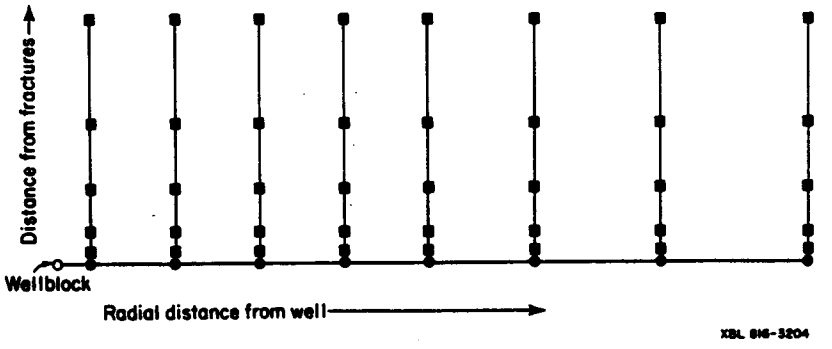

Fig. 4 Schematic diagram of mesh used for radial flow problem.

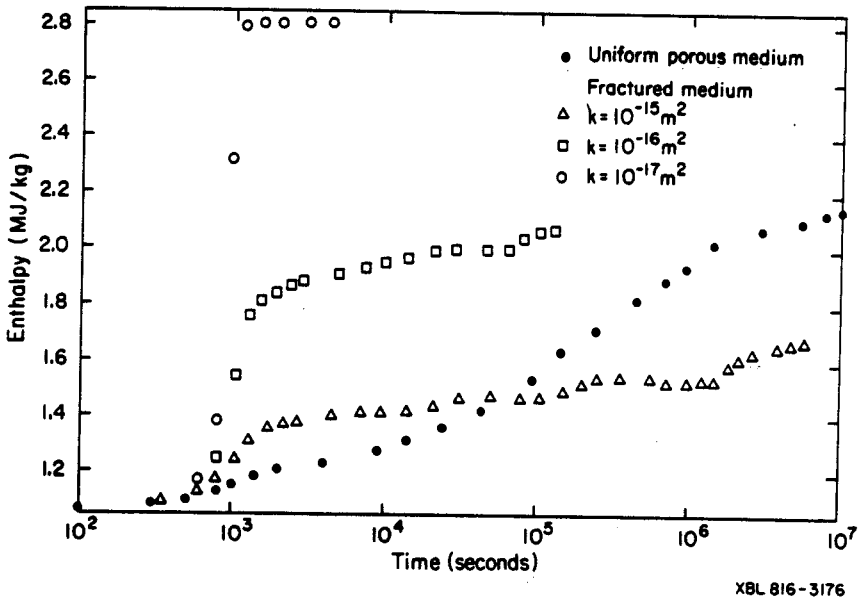

Fig. 5 Enthalpy transients for radial flow problem.

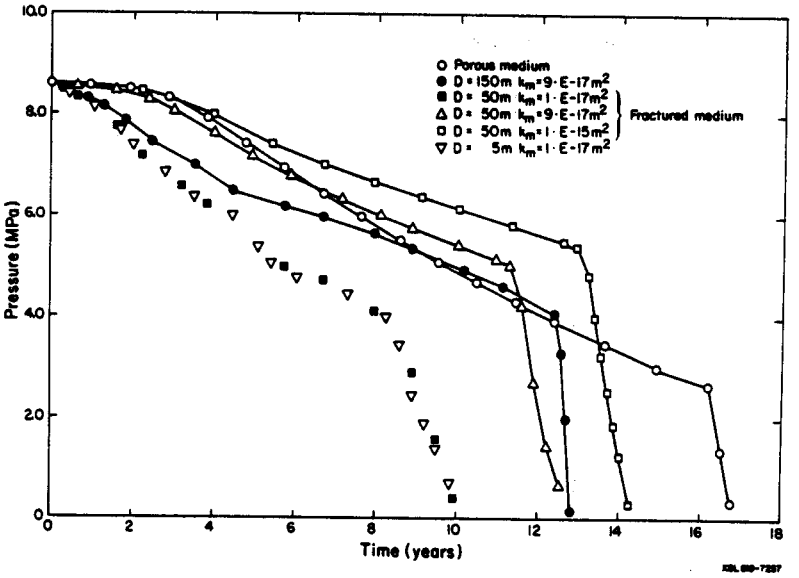

Fig. 6 Pressure decline for areal depletion problem.

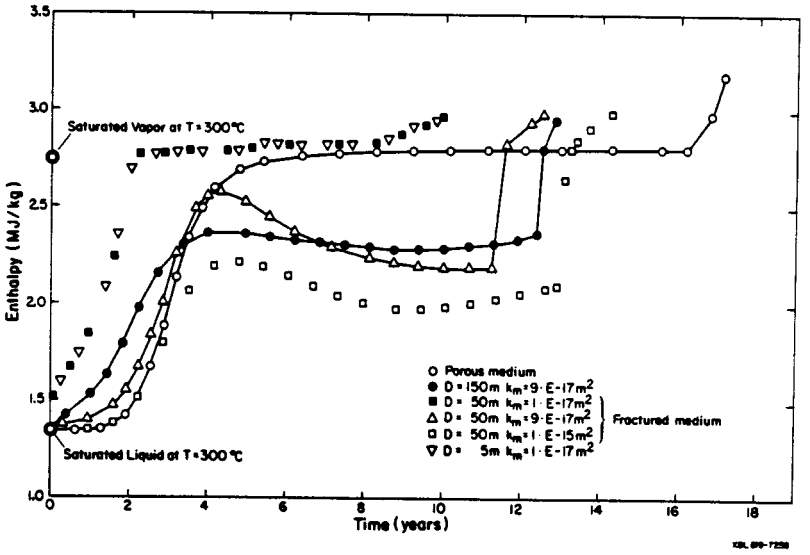

Fig. 7 Produced enthalpy for areal depletion problem.

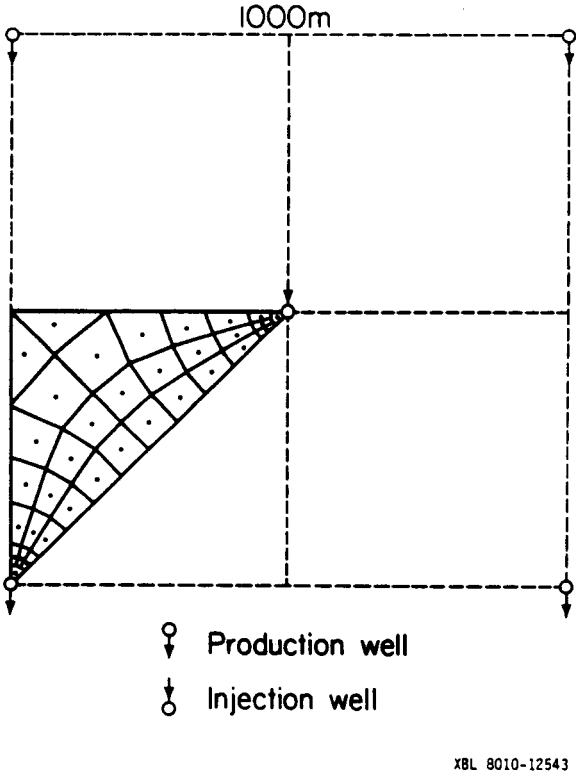

Fig. 8 Mesh for five-spot well pattern.