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ANOMALOUS ALPHA PARTICLE TRANSPORT IN THERMONUCLEAR TOKAMAK PLASMA

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ANOMALOUS ALPHA PARTICLE TRANSPORT IN THERMONUCLEAR TOKAMAK PLASMA*

ABSTRACT

Due to the strong localization of the fusion born α -particles in velocity and configuration space and their coupling to Alfvén waves in the background plasma the relaxation of α -s is anomalous. In a finite system, the enhanced electromagnetic fluctuations can produce rapid spatial losses of α -population and energy. These losses prevent the α -velocity distribution from attaining a stable collisional equilibrium, thus maintaining a steady state turbulence level. A self-consistent numerical quasilinear calculation is performed for the most dominant mode, showing the evolution of the α -distribution and yielding the anomalous loss rates.

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I. α -particles are born with a velocity distribution function

$f_\alpha \sim \delta(E - E_\alpha)$ where $E_\alpha = 3.5$ MeV and a spatial distribution

$n_\alpha = \frac{1}{4} \langle \sigma_f v \rangle n_d^2 t$, where $\langle \sigma_f v \rangle \propto T^{-2/3} \exp[-20/T^{1/3}]$ is strongly peaked in the hot reactor core, as is the square of the deuteron density n_d .

In fact, in the absence of anomalous α -diffusion typically 1/2 of the α -energy is contained inside 1/4 of the reactor radius. Thus one suspects that this very collisionless hot α -component has a large amount of free energy which, because of the velocity ordering

$$v_{thi} < c_A < v_\alpha < v_{the} \quad (1.1)$$

can be released via Alfvén-wave-instabilities. (Here, the subscripts stand for ion, Alfvén, alpha and electron velocity). Since the first papers by Kolesnichenko and Korablev [1] on "thermonuclear instabilities" in 1967, a large number of such linear instabilities have been found, reviewed recently by Lominadze [2]. We note that neutral beam driven instabilities have a much weaker, unstable wave particle interaction, since for that case $v_{beam} < c_{Am}$ (1.1).

In an infinite medium one expects these instabilities to exist transiently until the combination of turbulent and collisional spreading of the α -distribution produces a stable shape $\frac{\partial f_\alpha}{\partial E} < 0$. Practical toroidal reactor designs however are quite removed from the condition $\delta r_b/a \ll 1$, where δr_b is the α -banana width and a is the effective plasma radius. The particle confinement time τ_p will therefore be finite and if anomalous diffusion is strong enough to produce the inequality $\tau_p < \tau_s$ (where τ_s is the collisional slowing down time of a fast

α -particle), the stable shape $\frac{\delta f}{\delta E} < 0$ will not be attainable, thus leading to steady state turbulence and ensuing α -particle loss.

In this paper we will quantitatively pursue such a case for the linear instability recently discovered by Kaladze and Mikhailovskii [3]. For the sake of self consistency, we will calculate the particle confinement solely due to this instability, whereas in practice the sum of all loss mechanisms including scrape off and drift Alfvén mode losses [4] will be effective. A fully coupled numerical solution of the collisional time evolution of the α -distribution has been performed by Duchs and Pfirsch [5] who find regions of positive slope $\frac{\delta f}{\delta E}$ at lower energies of the numerous linear Alfvén type instabilities destabilized by α -particles the Kaladze-Mikhailovskii mode [3] appears to have particularly interesting transport characteristics: trapped α -s resonate with the shear Alfvén wave $\omega = C_A k_{\parallel}$ at the rather high α -bounce frequency $\omega_{b\alpha} = v_{\alpha} \epsilon / R q$, with maximum growth rate for radial wave lengths k_r^{-1} as large as an α -banana width r_b . Our quantitative analysis of the quasilinear evolution of α -distribution function shows indeed that besides the expected anomalous rapid slowing down in velocity space there occurs substantial radial α -diffusion on the same time scale.

In Section II, after summarizing the results of the linear theory of Ref. [3], the quasilinear model is developed analytically and numerically. Section III contains the numerical results showing the existence of a steady state turbulence level, the ensuing α -particle and energy losses and their scaling with fundamental reactor parameters. In Section IV, the numerical results of the previous section are related to qualitative

analytic scaling laws. Section V contains a brief discussion and perspective of the limitations of the present work.

II. Linear and QL Theory

Since the mode frequency is below the α -gyrofrequency we start with the usual drift kinetic equation [6] for the α -distribution function

$$\frac{\partial f}{\partial t} + v_{\parallel} \frac{B}{B} \cdot \nabla f + (\underline{v}_D + \underline{v}_E) \cdot \nabla f + \frac{\ell}{m} (\tilde{E}_{\parallel} v_{\parallel} + \tilde{E} \cdot \underline{v}_D) \frac{\partial f}{\partial \epsilon} = C(f) \quad (2.1)$$

where $\epsilon = m\sigma^2/2$. (Later, we will include a fusion source term on the RHS.)

For the shear Alfvén-wave, $\tilde{B}_{\parallel} \ll B_0$, and we may assume [7]

$$\tilde{E}_{\perp} \approx -\nabla_{\perp} \phi, \quad \tilde{E}_{\parallel} = -\nabla \phi - \frac{1}{c} \frac{\partial A_{\parallel}}{\partial t} \approx 0 \quad (2.2)$$

where we have

$$\phi \propto \exp[-im\theta + i\ell\xi - i\omega t + i k_r r] \dots \quad (2.3)$$

(Thus, following Kaladze-Mikhailovskii [3], in the local approximation.

The extension to a more rigorous treatment will be discussed in Section V.)

From (2.3), $k_{\parallel} = \ell q - m/Kq$, where $q(r)$ is the safety factor.

Linearizing (2.1) one observes that the instability is driven by ∇f_0 and $\frac{\partial f_0}{\partial \epsilon}$ dominates ∇f_0 , is certainly valid at early times in the α -particle evolution. We note strongly that inclusion of the drift instability term $\alpha \nabla f_0$ would only produce more anomalous transport. Thus, our results will be conservative.

We now estimate the driving terms in (2.1):

$$\frac{v_{\parallel}}{B} \tilde{B} \cdot \nabla f_0 \sim \frac{ck_{\theta} \phi}{B_0} \frac{f_0}{r}; \quad (2.4a)$$

$$\underline{v}_E \cdot \nabla f_0 \sim \frac{ck\theta\phi}{B_0} \frac{f_0}{r}; \quad (2.4b)$$

$$\underline{\tilde{E}} \cdot \underline{v}_D \frac{\partial f_0}{\partial \epsilon} \sim \frac{ckr\phi}{B_0} \frac{f_0}{R} \quad (2.4c)$$

where, in the last term, we evaluated the radial magnetic drift velocity in trapped particle space, anticipating a trapped particle mode. Thus, in order to emphasize the driving term (2.4c) over (2.4b) we choose the

$$k_\theta/k_r < \frac{r}{R} \quad (2.5a)$$

and because of the resonance condition $p\omega_{b\alpha} \approx k_\parallel C_A = 3$ and $k_r \sim \delta r_b^{-1}$ we also have, typically

$$k_\parallel/k_r < (r/R)^2 \quad (2.5b)$$

As mentioned in [3], this ordering justifies the neglect of the drift frequency $\omega_D = k_\theta v_D$ in the resonant denominator $(\omega - p\omega_b - \omega_D)^{-1}$, contained in the growth rate, since one finds $\omega_D/p\omega_b \sim \frac{r}{Rp} \frac{\rho a}{\delta r_b} \ll 1$ which is smaller than $(\omega - p\omega_b)/p\omega_b = \sqrt{\frac{R}{r}} \frac{C_A}{v_\alpha} \frac{\lambda q - m}{p}$, where we used $\omega = C_A k_\parallel$. Then, also using (2.2), (2.1) reduces to

$$\frac{\partial f}{\partial t} + v_\parallel \nabla_\parallel f + \underline{v}_D \cdot \nabla f + \frac{\lambda}{m} \underline{\tilde{E}}_r v_{Dr} \frac{\partial f}{\partial \epsilon} = c(f) \quad (2.6)$$

II.1. Linear Theory

The linearized Eq. (2.6) has been solved by K-M who obtained for the growth rate of this electromagnetic mode

$$\gamma_\alpha = \omega_2 \frac{C_A^2}{C^2} \text{Im } \epsilon_{11}^\alpha = \frac{2\pi\omega_{p\alpha}^2 C_A^2 (r/R)}{n_\alpha k_r^2 C^2 q R} \sum_p \int dv v^2 J_p^2(\tilde{n}_p) \frac{\partial f_0}{\partial v} \quad (2.7)$$

where $\eta_p = \sqrt{\frac{2R}{r}} \frac{k_r v}{\Omega_0} \kappa_p$; $\frac{\kappa_p}{2} = \left[1 - \sqrt{\frac{2R}{r}} \frac{\omega_q R}{pv} \right]^{\frac{1}{2}}$.

The quantity κ_p measures the shift of the wave frequency from $p\omega_b^\alpha$ where ω_b^α is the α -bounce frequency $\omega_b^\alpha = 2\pi/\phi q R d\theta/v_{||}(\theta)$. To display the dependence of γ_α on $k_r \delta r_b$, we use $f_0 = \dots$ in (2.7) whence [3] $\gamma_\alpha = \dots$. Fig. 2.2 shows γ_α vs. $k_r \delta r_b$ for $\dots$ and one concludes the $|p| = 1, 2$ are the most important bounce harmonics.

This increment due to the α -particles is partially offset by wave damping on the electrons. (The phase velocity $\propto \frac{\omega_{ba}}{\kappa_{||}}$ is much too fast for ion damping). Mazue and Mikhailovskii have recently shown the dominant damping mechanism to be collisional damping on trapped electrons [8]:

$$\gamma_e/\omega = -\frac{1}{\pi} \sqrt{R/r} (k_r \rho_i)^2 \left(\frac{v_e}{r\omega/R} \right)^{\frac{1}{2}} \left[\ln 8 \left(\frac{r\omega}{Rv_e} \right)^{\frac{1}{2}} \right]^{\frac{1}{2}} \quad (2.8)$$

Strictly valid only near mode rational surfaces $k_{||} < \frac{1}{Rq}$. (The extension to $k_{||} \sim 1/Rq$ has been performed by Mikhailovskii and Frenkel, to be published in Sov. Phys. Plasma Physics 1977).

From (2.7) and (2.8), these results for the threshold density at marginal stability

$$\frac{n_\alpha}{n_0} \approx (k_r \rho_\alpha)^2 (k_r \rho_i)^2 \sqrt{\frac{v_e R}{\omega r}} \left[\ln 8 \left(\frac{r\omega}{Rv_e} \right)^{\frac{1}{2}} \right]^{\frac{1}{2}} \quad (2.9)$$

and we point out that this value is of $\sigma(10^{-5})$ for practical situations, thus enabling instability before a significant amount of α -s has slowed down and spread the α -distribution in velocity space into a stable shape.

II.2. Quasilinear Theory

For our instability, the quasilinear relaxation of the α -distribution function occurs on a time scale τ_{QL} slower than $\omega^{-1} \sim \omega_{b\alpha}^{-1}$.

Applying a multiple time scale formation to the toroidal plasma yields in the standard fashion [9]

$$\frac{2\pi}{\omega_{b\alpha}} \frac{\partial f_0}{\partial t} = - \oint \frac{d\theta R q}{v_{||}} \frac{\ell}{m} v_{Dr} \sum_k \tilde{E}_r \cdot -k \frac{\partial \tilde{f}_k}{\partial \epsilon} \quad (2.10)$$

where, from the linear theory

$$\tilde{f}_k = - \frac{\ell}{m} \frac{\partial f_0}{\partial \epsilon} \int_{-\infty}^t dt^1 v_{Dr} \tilde{E}_r \exp \left[i\omega(t - t^1) - i \int_{t^1}^t dt^1 (k_{||} v_{||} + \underline{k} \cdot \underline{v}_D) \right].$$

Breaking the time history integral into a sum of bounce time intervals and following the manipulations of Ref. 3, one then obtains in the subspace of (deeply) trapped α -particles

$$\frac{\partial f_0}{\partial t} = \sum_k \frac{\ell^2}{m^2} |E_{rk}|^2 \frac{\partial}{\partial \epsilon} D_{\epsilon\epsilon} \frac{\partial f_0}{\partial \epsilon} \quad (2.11)$$

there

$$D_{\epsilon\epsilon} = \sum_p \sum_{-\infty}^{\infty} R^2 q^2 \frac{\omega_{b\alpha}^2}{4\pi} \delta(\omega - p\omega_{b\alpha}) \left| \int_{\theta}^{\theta^2} \frac{d\theta}{v_{||}} v_{Dr} \cos M \right|^2$$

with $M = \int_{\theta}^{\theta^2} \frac{d\theta}{v_{||}} qR(\omega - \underline{k} \cdot \underline{v}_D)$, and one finds

$$\int_{\theta_1}^{\theta_2} \frac{d\theta}{v_{||}} v_{Dr} \cos M = \frac{(-1)^{p_{\pi p}}}{qRk_r} J_p \frac{2k_r q}{\omega_{c\alpha}} \left(\sqrt{\frac{R}{r}} \sqrt{\epsilon \kappa} \right)$$

where the pitch angle visible $2\kappa^2 = 1 - \cos \theta_0 \ll 1$ for trapped particles and θ_0 is the turning point angle of the banana orbit. In the subspace of

circulating particles, this trapped particle instability produces no resonant diffusion, of course. It also produces no diffusion in perpendicular energy μB , since $\omega \sim \omega_{b\alpha} < \omega_{c\alpha}$. Thus we can integrate (2.11) over μ by using the properties of $\delta(\omega - p\omega_{b\alpha})$ and obtain for the flux surface averaged distribution function

$$F \equiv \int \frac{d\theta}{2\pi} \sum_{\alpha = \pm 1} \int \frac{2\pi b d\mu}{|v_{\parallel}|} f_0 ,$$

$$\frac{\partial F}{\partial t} = \sum_k |E_{rk}|^2 \frac{\partial}{\partial \epsilon} D_{\epsilon\epsilon} \frac{\partial F}{\partial \epsilon} \quad (2.12a)$$

with

$$D_{\epsilon\epsilon} = \sum_p \frac{|p|}{kr^2} J_p^2 \left(\frac{2k_r q}{\omega_{c\alpha}} \sqrt{\frac{R}{r}} \sqrt{\epsilon} \kappa_p \right) \frac{\pi^2}{\bar{K}} \frac{\sqrt{r/R}}{qR} \epsilon^{-1/2} \frac{\ell^2}{m^2} \quad (2.12b)$$

Here, $\bar{K} = \int_0^1 K(\kappa) du^2$ is a number, ($K(\kappa)$ being the 1. elliptic integral)

and $\kappa_p = \left(\frac{p\omega_{b\alpha}^0 - \omega}{p\omega_{b\alpha}^0/4} \right)^{1/2}$ where $\omega_{b\alpha}^0$ is the bounce frequency for $\kappa \rightarrow 0$, as

in Ref. 0.

Although we dropped the driving term $\alpha \nabla f_0$, thus neglecting unstable drift waves, the fluctuations of the present instability driven by $\partial f_0 / \partial \epsilon$ produces anomalous spatial transport in the radial direction through the $\underline{E}_\theta \times \underline{B}$ drift. Rather than attempting to solve a multidimensional quasi-linear equation containing all elements of the diffusion tensor (a task practically impossible even using a computer) we propose the following model kinetic equation for the evolution of the α -distribution function F :

$$\frac{\partial F}{\partial t} = \frac{\partial}{\partial \epsilon} D_{\epsilon\epsilon} \frac{\partial F}{\partial \epsilon} - \frac{D_{rr}}{d^2(t)} F + C(F) + S_f \delta(v - v_\alpha) / 4\pi v_\alpha^2 \quad (2.13)$$

where $D_{\epsilon\epsilon}$ follows from Eqs. (2.12), $S_f = n_d n_t \langle a_f v \rangle$ and $C(F)$ is the

Coulomb collision operator with details given in the appendix. Clearly, the rigorous form for the spatial diffusion term $\nabla \cdot D_{rr} \nabla F$ has been approximated using a time dependent (and later self consistently determined) radial scale length $d(t)$. D_{rr} is approximately calculated as follows.

First, following Hall/Sturrock Eq. (2.11) [10] define

$$D'_{rr} \equiv \int_{-\infty}^t dt' \left\langle \frac{dr_{GC}(t)}{dt} \frac{dr_{GC}(t')}{dt'} \right\rangle$$

where $\frac{dr_{GC}}{dt} = \frac{Cf\theta}{B}$ and $\langle \quad \rangle$ denotes the ensemble average over all phase angles between waves and particles. For trapped particles, we take for simplicity for the unperturbed orbits

where δr_b is the α -banana width $qv/\omega_{c\alpha} = 2r/R$. Then, writing $\tilde{E}_\theta = E_\theta \exp[-i\omega t' + ik_r t(\theta) + i\psi]$ and averaging over all phase angles ψ , one obtains $D'_{rr} = \pi \frac{C^2}{B^2} \sum_k \sum_p |E_{\theta k}|^2 \delta(\omega - p\omega_{b\alpha}) J_p^2(k_r \delta r_b)$.

For use in Eq. (2.13), this is integrated over μ , whence

$$D_{rr} = \frac{C^2}{B^2} \sum_k |E_{\theta k}|^2 \sum_p \frac{4\pi^2}{\sqrt{2K}} \frac{1}{\omega_{b\alpha} |p|} J_p^2(\mu_p) \quad (2.14)$$

At this point it becomes obvious that a numerical method must be employed to calculate the time evolution of $F_\alpha(r, \epsilon, t)$. We determine the radial scale length $d(t)$ introduced in Eq. (2.13) by solving the α -particle continuity equation

$$\frac{\partial n_\alpha}{\partial t} - \frac{1}{r} \frac{\partial}{\partial r} \overline{D_{rr}} \frac{\partial n_\alpha}{\partial r} = n_d n_t \langle \partial_f v \rangle \quad (2.15)$$

simultaneously with Eq. (2.13). In (2.15) the value for $(\bar{D}_{rr} n_\alpha)$ is obtained by integrating over the energy dependence of $D_{rr} F$ in Eq. (2.13). Since in the present work the radial variation of $|E_{r,\theta}|^2$ is not known, we suppress the radial dependence of D_{ee} and D_{rr} and evaluate all r -dependent quantities at $r = d_0$, the initial half width of $n_\alpha(r)$. From the numerical solution of (2.15) we extract $d(t)$, defined as the half width at half maximum of $n_\alpha(r, t)$. The initial condition for (2.15) is the burn profile

$$n_\alpha(r, t_0) = n_d n_t <\delta f v> t_0$$

where t_0 is the time at which the threshold density (2.9) is reached. The boundary condition for (2.15) is $n_\alpha(a, t) = 0$ for all times, since there exists a mirror plasma radius such that α -bananas at $r > a$ will be scraped off by the wall. The second boundary condition is $\frac{\partial n_\alpha}{\partial r} \Big|_{r=0} = 0$. Self consistent solutions for $d(t)$ are shown in Fig.

The quasilinear theory is completed by describing the evolution of the spectrum. One has

$$\frac{\partial E_k}{\partial t} = \gamma_k E_k, \quad k = (k_r, k_\theta) \quad (2.15)$$

where we take

$$\gamma_k = \gamma_\alpha + \gamma_e$$

as given in (2.7), (2.8), evaluated at $r = d$, and we point out that there is no explicit dependence of γ_k on k_θ or $k_\parallel$. This is a consequence of our neglect of the ∇f_0 driving term in Eq. (2.1), in accord with the ordering (2.5). This then consistent to restrict the numerical evolution

of $\underline{E}_k$ to the domain $\left\{ \begin{array}{l} a^{-1} < k_r < \rho_i^{-1} \\ a^{-1} < k_\theta < \frac{r}{R} k_r \end{array} \right\}$

where the cutoff for k_θ is effected via

$$\gamma_k = \begin{cases} \gamma_\alpha + \gamma_e & 0 \leq \theta \leq \frac{r}{R} k_r \\ 0 & \text{elsewhere} \end{cases} \quad (2.16)$$

At $t = 0$, we start $\underline{E}_k(k_r, k_\theta, r = \text{do}, t = 0)$ from an assumed thermal noise level. (This again is a conservative procedure. Some authors argue that due to resistive MHD-mode and drift mode activity the shear Alfvén spectrum is strongly enhanced over the thermal level even in the absence of α -particle instabilities [6].)

Finally, the $\sum_k$ in (2.12) and (2.14) is performed by integrating

over $\int_{a^{-1}}^{\rho_i^{-1}} dk_r \int_{a^{-1}}^{r k_r / R} dk_\theta$, according to (2.16). It then follows from

$E_r = i k_r \psi$, $E_\theta = i k_\theta \psi$ that

$$D_{rr}/D_{\epsilon\epsilon} \propto |E_{k\theta}|^2 / |E_{kr}|^2 \propto r^2 / R^2 \quad (2.17)$$

III. NUMERICAL RESULTS

In this section we present the quantitative solution of the full quasilinear model described in the previous section. The parameters are chosen for a relatively small demonstration reactor brought to ignition by use of auxiliary heating at technologically desirable magnetic field levels:

$$B = 50 \text{ Kgauss}, a/R = 120/400, q(a) = 2.5,$$

$$n_e = 5 \times 10^{13} \text{ cm}^{-3}, T_e = T_i = 15 \text{ keV}, I = 3.6 \text{ MAmpere}$$

At this value of plasma current, $\delta r_b/a \approx .3$ corresponding to acceptable classical α -banana confinement.

The four basic questions to be addressed are:

- i) Has the instability vanished after one collisional slowing down time τ_s or is there a steady state turbulence.
- ii) If so, what is the shape of $\frac{\partial F_\alpha}{\partial \epsilon}(\epsilon, t)$, and the width of the α -density profile
- iii) What is the α -particle and energy loss rate vs. time
- iv) How do these losses scale with B , q , n and T , at fixed aspect ratio $A = 10/3$.

Concerning question i) we find that neither coulomb nor anomalous slowing down can fill in the $\frac{\partial F}{\partial \epsilon} > 0$ region at lower energy because the anomalous radial diffusion rapidly eliminates the trapped α -particles

Defining an α -particle and energy confinement time through

$$\tau_p = d^2/D_{rr}, \quad \tau_\epsilon = \epsilon_0/D_{\epsilon\epsilon} \quad (3.1)$$

where d is the density profile halfwidth and $\epsilon_0 = 3.5$ MeV, we find numerically $\tau_p \geq \tau_E$. This will be discussed in the next section. Schematically, the α -particle diffusion paths in $\epsilon - r$ space are shown in Fig. 3.1.

The turbulence level $\frac{E^2}{8\pi nT}$ vs time is typically of $0 (10^{-6})$ at saturation as shown in Fig. 3.2.

Concerning question ii), the positive slope of $F_\alpha(\epsilon, t)$ vs energy ϵ is maintained in the resonance region which extends roughly to $\frac{1}{2} \epsilon_0$, see Fig. 3.3. In configuration space, $n_\alpha(r, t)$ has spread by a factor of ≥ 2 from its initial classical "burn profile" as shown in Fig. 3.4. This should affect the background plasma temperature profile, (not treated in this paper). The steady state slope $\partial F_\alpha / \partial \epsilon$ must be such that $\gamma_\alpha + \gamma_e = 0$, cf. Eqs.... and The steady state slope $\frac{\partial n_\alpha}{\partial r}$ must satisfy $D_{rr} \frac{\partial n_\alpha}{\partial r} = \int d_{rr} n_d n_t \langle \sigma + v \rangle$ at saturation amplitude contained in D_{rr} .

Concerning question iii), let us define the particle and energy loss coefficients R_N and R_E as follows.

The total α -energy lost from the system at time t is

$$\Delta \epsilon_\alpha(t) = \int_0^t dt d^{-2}(t) \int d^3v \frac{m_\alpha v^2}{2} D_{rr} f_\alpha$$

Classically, the total α -energy accumulated in the system at that time would be $\epsilon_0 \int_0^t dt$, where $\int_0 = n_d n_t \langle \sigma_p v \rangle$. Thus we define the energy loss coefficient

$$R_E = \Delta \epsilon_\alpha(t) / \epsilon_0 \int_0^t dt \quad (3.2a)$$

Since the total α -particle density present at time t is

$$N_\alpha(t) = \int_0^a 2\pi r dr n_\alpha(r, t)$$

we define the particle loss coefficient as

$$R_N = 1 - N_\alpha(t) / \int_0^t t \quad (3.2b)$$

R_E and R_N are defined such that without turbulent losses $R_E = R_N = 0$. Figs. 3.5 and 3.6 show $R_E(t)$ and $R_N(t)$ and we note their significant magnitude for the assumed reference design parameters. This will be discussed in the next section.

Concerning question iv) Fig. 3.7 shows the scaling of R_N with increasing B , at fixed q , n , T , a and R . This corresponds to an increase in plasma current I and therefore in the routine $\frac{a}{\delta r_b}$, as indicated in the figure. The scaling appears to be $\propto \frac{q}{\delta r_b}^2$.

Fig. 3.8 shows the scaling of R_N with increasing q , at fixed B , n , T , a and R . The scaling appears to be $\propto q^2$.

Fig. 3.9 shows the scaling of R_N with increasing density, all other parameters being fixed. The losses appear to be $\propto \frac{1}{n}$. Increasing density diminishes the collisional slowing down time $\tau_s \propto \frac{T_e^{3/2}}{n}$ and the electron damping bath stabilizing effects, but increases the α -particle production rate so, a destabilizing effect.

Fig. 3.10 shows the scaling of R_N with increasing plasma temperature, with all other parameters held constant. Since the temperature enters τ_s , γ_α , γ_e and S_0 in with different powers, it is hard to predict the observed scaling of R_N , which appears to be $\propto T\beta$.

Overall, Figs. 3.7-3.10 indicate that the anomalous α -losses can be reduced by working at large values of $a/\delta r_b$, small $q(a)$, high density and low temperature. All of these conditions are favored in a

super high magnetic field high density Ohmic Tokamak without blanket,
but maybe very difficult to achieve in a practical power reactor.

IV. Physical Discussion of Numerical Result

In this section, we will provide some physical comments on the results of the previous section, again following the sequence of the "basic questions" raised there:

i) Steady State Turbulence

The reason for the existence of a steady state as given before is contained in Fig. 3.1 and the numerical observation $\tau_p \geq \tau_e$. This is a surprising result, since classically it is known [11] that $\tau_p/\tau_v \sim a^2/\rho^2 \gg 1$. Here, $\tau_p = a^2/\rho^2 dt (c\tilde{E}/B_0)^2$; $\tau_v = v^2/dt (e\tilde{E}/m)^2$ a is the plasma radius and ρ the gyroradius. In the present case of a radially rather small reactor ($\delta r_b/a = .3$) the time for an α -particle to $E \times B$ drift to the wall is not larger than the time needed to thermalize from its birth energy of 3.5 MeV. Recalling the results for D_{rr} and D_{ee} Eq. () and from Eq. () the result $d^3k |E_{\theta k}|^2 \approx \epsilon^2 d^3k |E_{rk}|^2$, and using the definitions (3.1), we obtain indeed $\tau_p \geq \tau_e$.

The saturation amplitude can be estimated using the observation that the resonance region extends from ϵ_D to approximately $\frac{1}{2} \epsilon_D$ (where $\epsilon_D = 3.5 n_e V$) and assuming the basic fine scale for energy diffusion to be $10\gamma^{-1}$:

$$D_{ee} \equiv \frac{(\Delta\epsilon)^2}{\Delta t} \sim \frac{\gamma_{linear}}{10} \cdot (\frac{1}{2} \epsilon_D)^2$$

With D_{ee} given by (2.) we solve for $|E|^2$ obtaining

$$\frac{|E|^2}{8\pi n_0 T} = .1 \times \frac{\gamma_\alpha}{\omega} \left(\frac{v_\alpha}{c} \right)^2 \left(\frac{2\epsilon}{\beta q^2} \right) \quad (4.1)$$

The last factor is typically of $O(1)$, $\left(\frac{v_\alpha}{c} \right)^2 \approx 10^{-3}$ and, scaling $\frac{\gamma_\alpha}{\omega}$ from

Eq. (2.) by specializing to $F_\alpha \propto \delta(\epsilon - \epsilon_0)$ we obtain [3]

$$\frac{\gamma_\alpha}{\omega} \approx \pi \frac{n_\alpha \sqrt{T/R}}{n_0} \left(\frac{q^2 R}{r} \right) \sim 50 \frac{n_\alpha}{n_0} \quad (4.2)$$

so that (4.1) can be estimated to be

$$\frac{|E|_{\text{sat}}}{8\pi n_0 T} \approx (.5) 10^{-2} \frac{n_\alpha}{n_0} \quad (4.3)$$

in rough agreement with the numerical results, yielding typically 10^{-5} .

ii) Positive Slope of $\partial F_\alpha / \partial \epsilon$

Integrating the steady state Eq. 2. indefinitely over the energy yields

$$D_{\epsilon\epsilon} \frac{\partial F_\alpha}{\partial \epsilon} = \frac{\bar{D}_{rr} n_\alpha}{d^2} \quad (4.4)$$

or, using (3.1)

$$\frac{\partial F}{\partial \epsilon} \approx \frac{n_\alpha}{\epsilon_0} \frac{\tau_\epsilon}{\tau_p} > 0 \quad (4.5)$$

As mentioned before, this value for the slope must satisfy $\gamma_\alpha + \gamma_e = 0$ in steady state.

III) Scaling of α -Particle Loss Rate

In Section II we found

$$\bar{D}_{rr} = \sum_p \frac{C_p^2}{B^2} |E_\theta|^2 \left(8\pi^2 \frac{T_p^2}{|p|} \right) \frac{\sqrt{2R/R}}{\omega_{b\alpha}}$$

which, upon using (4.1) for the saturation amplitude, and (4.2) for $\frac{\gamma_\alpha}{\omega}$ becomes $\bar{D}_{rr} = \left(\sum_p \frac{16\pi^3 J_p^2}{10 |p|} \sqrt{2} \right) \frac{n_\alpha}{n_0} \frac{r}{R} \frac{v_\alpha^2}{\omega_{b\alpha}}$ where the numerical

factor is of 0(1).

Thus, roughly,

$$\bar{D}_{rr} \approx \frac{n_\alpha}{n_0} \sqrt{\frac{r}{R}} \bar{v}_\alpha R_q \quad (4.6)$$

where $\bar{v}_\alpha$ is the average α -velocity in steady state. However, this simple scaling is misleading since the value of $\frac{n_\alpha}{n_0}$ at saturation is an involved function of all parameters.

Using (4.6) in the α -particle confinement time τ_p these results

$$\frac{\tau_s}{\tau_p} = \frac{4\tau_s}{a^2} \frac{n_\alpha}{n_0} \sqrt{\frac{r}{R}} \bar{v}_\alpha R_q$$

With $\tau_s \equiv \hat{\tau}_s \frac{T^{3/2}}{n_0}$ one obtains a necessary condition for large anomalous losses, namely $\tau_s > \tau_p$, of the form

$$1 > (4\hat{\tau}_s) \sqrt{\frac{R}{a}} \left(\frac{n_\alpha}{n_0}\right) \left(\frac{\bar{v}_\alpha}{a}\right) T^{3/2} \quad (4.7)$$

Analogously, on numerical results, have indeed an increase of the losses with q and T , and a decrease with n_0 .

iv) Alpha Density at Saturation.

Using (4.6) and the numerical (and plausible) observation that the density profile width d at saturation equals about half the mirror radius, the steady state α -contributing equation becomes roughly

$$n_\alpha \frac{4\bar{D}_{rr}}{a^2} = n_d n_t \langle 6_f v \rangle$$

or, writing $n_d = n_t = n_0/2$

$$\left(\frac{n_\alpha}{n_0}\right)_{\text{sat.}}^2 = \frac{a n_0 T^2}{16 \bar{v}_\alpha q} \sqrt{\frac{a}{R}} \left(\frac{\langle 6_f v \rangle}{T^2}\right) \quad (4.8)$$

where we have split off the very slow function $\frac{\langle 6_f v \rangle}{T^2}$.

V. Unresolved Questions

Since an accurate assessment of anomalous alpha particle losses has a profound influence on reactor design, we provide a brief discussion of the limitations of the present theory and thereby hope to increase the understanding of the problem of alpha particle turbulence.

V.1. Radial Mode Structure

Just as the linear theory of Ref. 0, our work does not contain a solution to the radial eigenmode problem of the unstable shear Alfvén-wave in Tokamak-geometry. Mazur and Milhailovskii [12] have shown that rather radially localized Alfvén-wave solutions can be derived using Timofeev's method. Rosenbluth and Rutherford [5], on the other hand, have constructed nonlocalized solutions to the fourth order finite Larmor radius equation for this wave and find a certain region around the magnetic axis to be unstable. Current work by Catto [13] et al. also seems to prove the existence of Alfvén wave eigenmodes in the Tokamak. Rosenbluth et al. [5] consider mainly the destabilization of the plasma by an injected beam, and find the unstable region to be contained inside $0 \leq p \leq r_0$ where $\omega = (\kappa_{\parallel} C_A)_{R=r_0}$ defines the resonance point r_0 . In our case, the unstable region depends on the condition $\omega = p\omega_{b\alpha} = \kappa_{\parallel} C_A$ with $\omega_{b\alpha} = v_{\alpha} \sqrt{r/R}/R_q$ having a finite range of α velocities $v_{\alpha p}$. Fig. 5.1 shows schematically the extent of the resonance region.

Once the radial eigenmode problem is solved, it would be justified to replace our spatial diffusion term $-D_{rr}/d^2$ exactly by $\frac{1}{r} \frac{D}{Dr} r D_{rr} \frac{D}{Dr}$ i.e. increase the number of dimensions in the quasilinear code.

V.2. Large α -Banana Width

For the calculation of the standard trapped particle modes in the background plasma it usually suffices to approximate the banana-orbits by the field line trajectory when using $db = dl/v_4 \approx Rq d\theta$. This is obviously not a good orbit approximation for the very energetic α -banana. [14] A more precise calculation seems to necessitate a numerical approach even for the linear growth rate, where upon one loses any physical insight very early in the problem. If the α -trajectory samples a large radial extent of the plasma, the radial eigenvalue problem for the unstable Alfvén-wave may be significantly affected and would also be amendable only to numerical treatment.

V.3. Competing Turbulence Mechanisms

As explained in the introduction of Ref. (3) there exists a large number of linear α -particle instabilities. Clearly, inclusion of the spatial gradient of the α -distribution function provides enhanced radial cones in the later development stages of the originally (practically) singular (with respect to the energy variable) distribution function. If one admits high frequency unstable Alfvén waves (the "cyclotron thermonuclear instabilities" mentioned in Ref. (3)) the velocity space slowing down is enhanced thus perhaps eliminating our present instability turbulence condition $\tau_p \sim \tau_e$. In such a case, however, we have shown that strong velocity space anisotropies may be generated, [15] destabilizing a number of additional instabilities and thus decreasing the particle confinement time as τ_e is decreased. Particularly, we note the high frequency modes produce pitch angle scattering [9] with the possibility of anomalous

α -banana diffusion. (Recall that our radial α -transport was simply caused by $\underline{E}_\theta \times \underline{B}$ drifts!) Quantitative work similar to the present paper will be needed to investigate the possibility for steady state turbulence in this case.

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APPENDIX

Approximate Coulomb collision operator used since the quasilinear evolution occurs in the interval $\frac{1}{2} \epsilon_0 \leq \epsilon < \epsilon_0$ (where $\epsilon_0 = 3.5$ Mev) it implies to me the simplified collision operation

$$C(f_{\alpha\alpha}) = \frac{v s^{\alpha e}}{v^2} \frac{\partial}{\partial v} (v^3 f_{\alpha\alpha}) + \frac{1}{v^2} \frac{\partial}{\partial v} \frac{v_{\parallel}^{\alpha e}}{2} v \frac{\partial f_{\alpha\alpha}}{\partial v}$$

Here,

$$v s^{\alpha e} = v_{\alpha e} \frac{2\epsilon_{\alpha}}{T_e} G(v/v_e)/(v/v_2),$$

$$v_{\parallel}^{\alpha e} = 2 v_{\alpha e} G(v/v_e)/(v/v_{\alpha})^3$$

where $G(x) =$ is the Chandrasekhav function, $v_{\alpha} = \sqrt{\frac{2\epsilon_{\alpha}}{m_{\alpha}}}, \sqrt{\frac{2T_e}{m_{\alpha}}}$

and

$$v_{\alpha} = 4 N_e (e_{\alpha} e_e)^2 \ln N / (2\epsilon_{\alpha})^{3/2} m_{\alpha}^{1/2}.$$