350
10.22-80

10. 1901

DOE/ER/04657-2

MASTER

**FUNDAMENTALS AND TECHNIQUES OF NONIMAGING OPTICS FOR
SOLAR ENERGY CONCENTRATION**

Final Report

By
Roland Winston
Joseph J. O'Gallagher

May 20, 1980

Work Performed Under Contract No. AS02-78ER04657

The Enrico Fermi Institute
The University of Chicago
Chicago, Illinois

59C

U.S. Department of Energy

Solar Energy

DISTRIBUTION OF THIS DOCUMENT IS UNLIMITED

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

DISCLAIMER

"This book was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof."

This report has been reproduced directly from the best available copy.

Available from the National Technical Information Service, U. S. Department of Commerce, Springfield, Virginia 22161.

Price: Paper Copy \$7.00
Microfiche \$3.50

FUNDAMENTALS AND TECHNIQUES OF NONIMAGING OPTICS
FOR SOLAR ENERGY CONCENTRATION

Roland Winston, Principal Investigator
Joseph J. O'Gallagher, Co-Investigator

Final Report
Department of Energy

Contract No. DE-AS02-78ER04657
(Originally ER-78-S-02-4657)

May 20, 1980

A handwritten signature, possibly 'mJ', is located in the bottom right corner of the page.

ABSTRACT

Nonimaging optics is a new discipline with techniques, formalism and objectives quite distinct from the traditional methods of focusing optics. The freedom from the restrictions associated with image forming systems permits the design of new families of concentrators. These new systems achieve or closely approach the maximum concentration permitted by the Second Law of Thermodynamics for a given angular acceptance and are often called "ideal". Application of these new principles to solar energy over the past seven years has led to the invention of a new class of solar concentrators, the most well known version of which is the Compound Parabolic Concentrator or CPC. Properly designed nonimaging concentrators greatly relax the angular tolerances associated with a given concentration and in the case of traditional CPC's eliminate the need for diurnal tracking of the sun. In order to develop the principles of the new field outside of the constraints of a narrowly defined hardware development program and to explore the large number of fundamental questions which arise in applications to solar energy, a program in basic research in nonimaging optics was established at the University of Chicago in 1978. This program has been supported for two years by the Department of Energy, Division of Advanced Energy Projects under the Office of Energy Research. This report describes the accomplishments of this effort during the period January 1, 1978 - December 31, 1979. A new formalism for analyzing nonimaging systems in terms of a quantity called the geometrical vector flux has been developed. This has led not only to a better understanding of the properties of ideal concentrators but to the discovery of several new concentrator designs. One of these new designs referred to as

the "trumpet" concentrator has several advantageous features when used as a secondary concentrator for a point focussing dish concentrator. A new concentrator solution for absorbers which must be separated from the reflector by a gap has been invented. The properties of a variety of new and previously known nonimaging optical configurations have been investigated: for example, Compound Elliptical Concentrators (CEC's) as secondary concentrators and asymmetric ideal concentrators. A thermodynamic model which explains quantitatively the enhancement of effective absorptance of "gray body" receivers through cavity effects has been developed. The classic method of Liu and Jordan, which allows one to predict the diffuse sunlight levels through correlation with the total and direct fraction was revised and updated and applied to predict the performance of nonimaging solar collectors. The conceptual design for an optimized solar collector which integrates the techniques of nonimaging concentration with evacuated tube collector technology was carried out and is presently the basis for a separately funded hardware development project. This basic research program to expand conceptual application of nonimaging optics to solar concentration is being continued in with support from the Department of Energy Division of Engineering, Mathematical and Geophysical Sciences Programs.

Table of Contents

Abstract	i
Table of Contents	iii
List of Figures	v
I. Introduction	1
II. Basic Principles of Nonimaging Optics	4
A. The Thermodynamic Limit	4
B. Phase Space Representation	5
C. The CPC for Flat Absorber	6
D. The Design Principle for Constructing "Ideal" Light Concentrators	6
E. Current State of Solar Applications	10
III. Review of Basic Objectives	12
IV. Review of Recent Advances	14
A. Category I Activities: Theoretical Studies, Design and Analysis	14
1. Development of a new formalism for nonimaging optics	14
2. The "trumpet" concentrator	18
3. Solution for ideal solar concentrators with reflector gaps	19
4. Properties of asymmetric ideal concentrators	23
5. Thermodynamic analysis of enhancement of effective absorptance of "gray body" receivers through cavity effects	24
6. Design considerations for nonimaging second stage optics combined with point focusing primaries	29

B. Category II Activities: Experiments in New Concept Applications	31
1. Fabrication and testing of a laboratory prototype concentrator for a tubular absorber with reflector gaps	31
2. The use of low concentration nonimaging optics with evacuated absorber systems	33
3. A survey of diffuse radiation and its long term correlation with standard hemispherical insolation data	37
V. Summary	40
VI. References	42

List of Figures	Page
1. Profiles of four different CPC configurations.	7
2. CPC from geometrical vector flux formalism.	17
3. The new "trumpet" concentrator derived from the geometrical vector flux formalism.	20
4. Modified CPC solution with reflector gap.	22
5. Typical reflector configurations for cylindrical tubular absorbers.	26
6. Enhancement of effective absorptance by "modified cavity effect".	28
7. Parabolic dish - CEC - two stage concentrator system.	30
8. Measured angular response for modified CPC with large reflector gaps.	34
9. The effect of low concentration on thermal performance up to 300°C (evacuated selective absorber).	35
10. Measured thermal performance for 5X ($\pm 8^\circ$) CPC with evacuated absorber.	36
11. Profile of low concentration CPC integrated with evacuated cylindrical tubular collector (conceptual design).	38

I. Introduction

During the past 12 to 13 years a new discipline referred to as "nonimaging" optics¹ has undergone very rapid development. The early work was done in the late 1960's by Hinterberger and Winston² and Winston³ at the University of Chicago in an effort to maximize the light gathering power of a Cerenkov radiation detector in a high energy physics experiment. In widely different contexts, the origins of the subject may also be traced to related work in Germany by Ploke⁴, and in the Soviet Union by Baranov⁵. The first application of the basic principles to the problem of solar energy concentration^{6,7} led to the conception of what has come to be known as the Compound Parabolic Concentrator or CPC, whose non-tracking capability and relaxed optical tolerances are now well known. A program to develop CPC's for solar applications was established at the Argonne National Laboratory in collaboration with the University of Chicago. This has led to the emergence into the market place of one version of the CPC by several manufacturers.

In the course of these hardware development efforts several further configurations based on nonimaging principles were conceived, resulting in a growing family of so called "ideal" concentrators with projected use across a broad range of solar applications. Examples included 1) second stage ideal concentrators (Compound Elliptical Concentrators or CEC's⁸), 2) generalized ideal concentrator profile solutions for a wide variety of energy absorber shapes⁷, 3) asymmetric ideal concentrators⁹, and 4) combined lens-mirror non-imaging designs¹⁰. In the meantime active research into the new optical discipline was begun at the University of Sydney¹¹, and the University of New South Wales¹² in Australia and at Imperial College^{13,14} in London, while the interest in the Soviet Union was revived¹⁵⁻¹⁸.

In order to broaden the base of fundamental optical principles underlying this new field and at the same time to search for new solutions and investigate general properties of these new concentrator designs beyond the scope allowed by the collector hardware development programs, a basic research program at the University of Chicago was begun in 1978. Support for this program during 1978 and 1979 has been provided by the Department of Energy, Division of Advanced Energy Projects. As a result of this effort several additional critical discoveries have emerged which further improve our ability to utilize thermodynamically optimized optics in all phases of solar concentration design. Among the highlights of these new developments are:

- 1) the establishment of a basic formalism for nonimaging optics based on the definition of a new quantity referred to as the "geometrical vector flux"^{19, 20},
- 2) use of the new formalism to discover a previously unknown concentrator design¹⁹ referred to as the "trumpet" concentrator,
- 3) better models for the incorporation of the contribution of diffuse insolation to long term concentrating collector performance predictions²¹,
- 4) a thermodynamic basis for interpreting and quantitatively predicting the performance enhancement observed for a stationary collector comprised of an absorber which is partially surrounded by a back-reflector with less than unit concentration (modified "cavity enhancement")²²,
- 5) a systematic study of the design principles for designing second

stage nonimaging concentrators for use with image forming primaries,³² and

- 6) the conceptual design for a nonimaging concentrating evacuated tube solar collector.

This report is a summary of some of the details of these and other developments which have taken place with the support of DOE Contract DE-AS02-78ER04657 (formally ER-78-S-02-4657). Subsequent sections will

- a) review the principles of thermodynamically ideal nonimaging concentrators,
- b) summarize the general scientific objectives of the basic research program,
- and c) review the major results of this effort.

II. Basic Principles of Nonimaging Optics

A. The Thermodynamic Limit

If one wishes to concentrate radiation, that is to gather it over some large collecting aperture A_1 and direct it to a smaller absorbing area A_2 , it is evident qualitatively that there must be some sacrifice in view angle (the smaller absorbing surface cannot "see" the full field of view comprising the hemisphere visible to the larger aperture). The quantitative relationship between this reduction in view angle and increasing concentration ratio $C = A_1/A_2$ is not intuitively obvious but can be derived in a straightforward manner from thermodynamic arguments⁹. These arguments are based on the fact that in thermodynamic equilibrium the absorber must re-radiate back to the environment the same amount of energy as it receives. In particular for a given acceptance half angle ($\pm\theta_a$) at the collecting aperture this condition defines a minimum absorber area and corresponding maximum possible geometrical concentration given by

$$C \equiv A_1/A_2 \leq \frac{1}{\sin^2 \theta_a} \quad (1)$$

if the concentration is done in only two dimensions (trough-like geometries) and

$$C \equiv A_1/A_2 \leq \frac{1}{\sin^2 \theta_a} \quad (2)$$

if the concentration takes place in three dimensions (cone-like geometries). This is referred to as the thermodynamic limit since if one could make a concentrator which transferred all the incident radiation to an absorber smaller than that given by Equations (1) or (2) it would not have sufficient

area to re-radiate this energy in thermodynamic equilibrium and its temperature would begin to rise above its surroundings in violation of the Second Law.

Any concentrator system which can attain this limit is referred to as "ideal". Concentrating systems based on imaging or focusing optics fall short of this limit by a factor of 3 to 4. The CPC and other concentrator shapes determined by the design principle described below (Section D) actually achieve this limit in two dimensions and closely approach it in three dimensions.

B. Phase Space Representations

Equations (1) and (2) can also be derived by application of the principles of étendue or phase space conservation^{3,6}. A bundle of rays propagating in the z direction can be characterized by a distribution of points in a four-dimensional (two position and two directional coordinates) phase space. Phase space conservation then requires that the volume, U , representing an ensemble of rays propagating through an optical system along the z axis must remain constant. If one then considers a distribution uniform in position across an aperture $A_1(z_1)$ and consisting of rays isotropically filling an acceptance angle θ_a with rays of $\theta \leq \theta_a$ and asks what is the smallest cross sectional area $A_2(z_2)$ to which these rays can be reduced while allowing the directional distribution to expand to fill the semi-circle in direction coordinate space, one is lead directly to Equations (1) and (2) in those geometries.

Neither the thermodynamic nor the phase conservation arguments tell us actually how to achieve the maximal or "ideal" concentration but simply that these are the limits which no optical system can exceed.

C. The CPC for Flat Absorber

If one considers the two-dimensional geometry defined by two parallel surfaces (A_1 and A_2 in Fig. 1a) perpendicular to a common optical axis, one can show that reflectors connecting these two surfaces with parabolic segments as shown in Fig. 1a actually achieve the limit defined by Equation (1). The left hand reflecting surface is a segment of a parabola with its axis tilted by an angle θ_a with respect to the optical axis and positioned so that its focus lies at the lower edge of the right reflector. All rays entering the entrance aperture at an angle θ_a will then strike one reflector and be brought to a focus at the opposite edge of the exit aperture. All rays entering between $\pm\theta_a$ must be reflected to points between the lower mirror edges. This is the simplest manifestation of an "ideal" concentrator in two-dimensions. A figure of revolution formed by rotating the shape of Fig. 1a about the concentrator axis does not perform as a true ideal concentrator essentially because in three dimensions, rays which are not in the meridian plane have an additional quantity (angular momentum) which must be conserved as well as the phase space so that the system is overdetermined. However, such 3-dimensional cones come very close to ideal performance.

D. The Design Principle for Constructing "Ideal" Light Concentrators

A generalized principle which will allow one to determine the two-dimensional cross-section for a reflecting trough which yields an ideal concentrator in any geometry for a non-concave curved absorber was developed in the early work at Chicago. This is alternatively described as the "edge ray principle" or more specifically as the "maximum slope principle".

Figure 1

Profiles of four different CPC configurations (a) flat absorber, (b) fin absorber, (c) wedge absorber, (d) tube absorber.

The edge-ray principle is a fundamental concept which is essential to use in generating any new form of nonimaging flux concentrator. We can explain its meaning best by comparison with image-forming optical systems, e.g. camera lenses, slide projector lenses, etc. In these systems there is an axis of symmetry on which the centers of all the lens components lie; likewise the object, e.g. a scene to be photographed or a slide to be projected, has usually its center or most important point on or near the axis. The designer of the image-forming system then attends first to the region near the axis: the basic properties such as focal length and magnification are determined by the properties of the lens components near the axis and it follows that the image quality or sharpness must be best near the axis. By adding or changing lens components the designer can improve the image quality away from the axis but it inevitably deteriorates gradually until the image is so fuzzy at some distance from the axis that the optical system is unusable beyond that point. This, of course, determines the "field of view".

Now in nonimaging concentrators there is a complete opposite emphasis. These systems can have either an axis or a plane of symmetry (3D or 2D in the technical language), or they may even have no symmetry at all, but in any case they are designed to collect entering flux over a certain angular range and larger aperture and concentrate it so that it emerges from a smaller exit aperture or strikes an absorber with a surface smaller than the entry aperture. The design principle to be followed in order to get the maximum concentration is that all rays entering at the extremes of the angular range must emerge just grazing the edge of the exit aperture or absorber surface. Sometimes

additional subsidiary conditions may be imposed on the extreme rays^{8,23}.

This is a concise statement of the edge ray principle. It can be seen that this is the complete opposite of the design of image-forming systems: the edge-ray principle ignores rays from the regions near the axis (if there is an axis) or near the center of the entering angular range; there is no question of sharpness of images anywhere; all that is required is that extreme entering rays meet certain operationally defined conditions, for example, graze the edge of the exit aperture as they emerge, whereas in image-forming systems the extent of the field of view gradually evolves post hoc when the center of the field has been taken care of. The edge-ray principle is the basis of the design of all efficient nonimaging concentrators, together with the laws of reflection and refraction. The "maximum slope principle" is a more specific statement of the ideal concentrator design principle particularly applied to systems where the active optical surfaces are reflectors. It says that each element of the reflector profile curve should be made to assume the maximum possible slope (i.e. rate of increase of the entrance aperture with increasing concentrator depth) consistent with the requirement that rays incident on the aperture at the desired extreme angle (i.e. "edge ray") illuminate the absorber. Given a particular absorber shape and a desired angular acceptance θ_a , the application of this principle yields a differential equation which allows generation of ideal two-dimensional reflector shapes⁷. Solutions for the other common absorber shapes are shown in Figures 1b, c and d.

E. Current State of Solar Applications

The basic advantages of nonimaging thermodynamically ideal optical concentration are well known. However, we reiterate them here for the sake of completeness. For a given acceptance angle Equations (1) or (2) permit concentration ratios 3 to 4 times those achievable using focusing optics (e.g. parabolic troughs or dishes, traditional lenses, etc). At a given concentration ratio, the corresponding wider acceptance angles permit:

- (1) non-tracking operation up to a concentration ratio of $\approx 10\times$,
- (2) greatly relaxed optical tolerances,
- (3) collection of the circumsolar radiation and a larger fraction of the diffuse component of insolation, and
- (4) reduced sensitivity to small angle scattering from dust or scratches on the reflector or cover.

Because of these advantages, nonimaging concentrators have attracted wide interest. As noted already, one version of a solar thermal CPC is already being produced commercially. Originally developed at Argonne National Laboratory, this is a low concentration $1.5\times$ fully fixed reflector coupled to an evacuated tube absorber. Higher concentration prototypes for both evacuated and non-evacuated absorbers are under development by a variety of institutions. CPC's have been identified nationally as a prime candidate for industrial processes heat applications and as having good potential for electric power generation. A second stage nonimaging concentrator design is being employed in two fixed mirror-tracking receiver concentrators and new uses of nonimaging terminal concentrators in point focusing geometry are being investigated by Jet Propulsion Laboratory²⁴ and the Electric Power Research Institute²⁵. In addition, CPC's and two stage nonimaging devices are being

used with photovoltaics both in hollow reflector and solid transparent dielectric (total internal reflection)²³ designs. All of this activity, generated in just under 5 years, can be put in perspective by considering that focussing concentrator designs have existed for centuries.

III. Review of Basic Objectives

As distinct from specific hardware development projects the basic research program has been directed towards exploring the broad range of possible solar applications for nonimaging systems and studying several general questions which are of particular interest to such designs because of their unique features. In particular there are seven areas in which specific objectives have been identified. Briefly, these are:

- A. The Fundamental Theory of Nonimaging Optics,
- B. Theoretical and experimental investigation of the general properties of CPC's and new solutions for other families of ideal concentrators,
- C. Analysis of the effects of nonspecularity in the optical elements of nonimaging systems,
- D. Analysis of phase-space distributions and their transformation by nonimaging systems,
- E. Investigation by theory and experiment of the spatial and directional distribution of concentrated sunlight,
- F. A survey of the properties of terrestrial sunlight as relevant to nonimaging concentrators, and
- G. Development of special techniques complementary to the application of nonimaging concentrators for solar energy.

Activities in these basic research areas fall naturally into two distinct but complimentary activity categories: (I) theoretical studies, design and analysis and (II) experiments in new concept applications. The former includes theoretical efforts to find new solar concentrator solutions, optical analysis

of promising designs including computer ray tracing, optimization studies, etc. The second category is devoted either to fabrication and performance testing of laboratory models of selected designs relevant to the development of advanced CPC and other near "ideal" nonimaging solar concentrators or to the collection and analysis of solar insolation data, particularly relevant to nonimaging concentrators.

The seven areas listed above were chosen to define relatively broad topics which would allow flexibility for fruitful research in unexplored areas. Significant progress has been made in almost every area. In particular, the development of the geometric vector flux formalism^{19,20}, described in Section IV could not have been anticipated. This is not only a major advance in the fundamental theory but provides a framework which a) develops in a simple and elegant way a new derivation of the CPC profile and b) has identified a totally new concentrator design not evident through application of the previous formalism. In the applications area, new designs suitable for coupling to practical evacuated solar absorber tubes have been developed and prototype concentrators tested. The integration of nonimaging optical systems with low loss materials (high reflectance and absorptance) which make the best use of the intermediate levels of concentration attainable continues to be one of the most urgent and promising areas for research. Furthermore with the first low concentration CPC's now being deployed, the development of performance models and their verification with a reliable data base for the response of nonimaging concentrators to a variety of insolation characteristics has become increasingly relevant.

IV. Review of Recent Advances

As described in Section III, activities under project can be grouped as belonging to one of two categories; either Category I: including theoretical or analytical activities or Category II: experimental or developmental activities. A review of the research efforts separated by category is given below.

A. Category I Activities: Theoretical Studies, Design and Analysis

1. Development of a new formalism for nonimaging optics. A major step has been achieved in the theoretical basis for describing nonimaging concentrators. Since the first developments in this field, it has been customary to characterize ideal light collectors in terms of the conservation of phase space volume occupied by accepted light rays. That is, one characterizes the direction of a light ray by a vector $\vec{p}$ whose components are the optical directional cosines (direction cosines multiplied by the refractive index n). Since there are actually only two degrees of freedom ($p_x^2 + p_y^2 + p_z^2 = n^2$) the direction of any ray can be characterized by a point in the p_x, p_y plane. Thus a bundle of rays incident on an aperture A_1 and propagating in the Z direction can be characterized by a distribution of points in the four-dimensional $p_x(Z), p_y(Z), x(Z), y(Z)$ phase space. The principle of phase space or étendue conservation requires that the volume in phase space for an ensemble of rays propagating through an optical system along the Z -axis must remain constant. That is the quantity

$$U(Z) = \int_{Z = \text{constant}} dx \, dy \, dp_x \, dp_y \quad (3)$$

is conserved along Z .

It is this relationship which leads directly to Eqs. (1) and (2) for the maximum geometrical concentration for a given angular acceptance. However, this formalism does not suggest any solution as to how this maximum is to be achieved.

A further study by Winston and Welford¹⁹ of the properties of the differential element of "étendue" has led to the introduction of a new quantity, the "geometrical vector flux" which is related to the étendue but has some remarkable properties which provide great insight into the behavior of ideal concentrators. In particular for a given distribution of light rays through a point (x,y,z) we define three quantities

$$\begin{aligned} J_x &= \iint dp_y dp_z \\ J_y &= \iint dp_z dp_x \\ J_z &= \iint dp_x dp_y \end{aligned} \tag{4}$$

These three quantities behave like the components of a vector, $\vec{J}$, whose vector field behavior exhibits interesting properties. For example, $\vec{J}$ is conserved (zero divergence).

It is fairly easy to calculate the vector $\vec{J}$ for simple configurations of Lambertian (isotropically emitting) sources, particularly in two dimensions, and it is found that the lines of flow of $\vec{J}$ have an important property. This is that a small element of mirror placed so that the lines of flow are in its surface do not disturb the flow locally. It may disturb them elsewhere in space; however, it is found that with particularly simple symmetrical arrangements of sources there is no disturbance of the flow lines anywhere. For these cases it

can be seen that if mirrors are placed along the flow lines, and if the direction of the light is reversed, an ideal concentrator is obtained!

That is, in two dimension two mirrors so placed will transform whatever phase space distribution exists at their defined aperture into an isotropic distribution at the absorber (source). This, by definition, is an ideal concentrator.

A Two Dimensional Example: Generation of the Classical 2D CPC. To initiate the CPC design we start with an array of Lambertian source elements (in two dimensions) as in Fig. 2. The source is, in effect, an infinite wedge of angle 2θ truncated symmetrically at the top, QQ' . The flow lines and also the loci of constant $|\vec{J}|$ are easily calculated from first principles and they are shown in the diagram. The system is one of those mentioned above for which a mirror placed along a flow line does not disturb the flow anywhere. Essentially, this is because, not merely is there zero component of $\vec{J}$ perpendicular to the flow lines, but the rays to any point P on a flow line are in detailed balance; by this is meant that for each ray incident on one side there is a corresponding ray on the other side at the position of reflection in the mirror.

If now we imagine mirrors placed along the flow lines originating at Q and Q' we have a concentrator: also, these flow lines turn out to be exactly the parabolic loci used in the 2D CPC and the concentration ratio agrees exactly. In the diagram the wedge semi-angle is $\theta = 30^\circ$ and this gives a (2D) concentration ratio of 2; this is verified on the diagram which is drawn to scale. Thus we have recovered the design of the 2D CPC but we initiated in in an entirely different way from the classical derivation based on the edge ray principle.

Figure 2 Derivation of CPC shape from geometric vector flux formalism. The lines of flow of J are found from the Lambertian source $R'Q'QR$ (a semi-infinite, truncated wedge). The concentrator reflector surfaces are placed along the lines of flow QT and $Q'T'$.

The CPC examples above illustrates the power of the new formalism. As a more dramatic illustration of the potential provided by the new development, consider the fact that in examining the behavior of $\vec{J}$, a new ideal concentrator design was discovered (see next sub-section).

The method based on the geometrical vector flux for initiating concentrator designs is entirely new and it is more powerful than the earlier methods based on raytracing. This is because raytracing deals with a selected small set of rays, usually in a meridian plane, whereas the vector flux approach takes account of the whole ensemble of rays. Thus we may expect that whenever a compromise has to be reached because of the known limitations in designing a perfect system the vector flux method will give information regarding the optimum design. We also remark that the lines of $\vec{J}$ obey a law of refraction differing from Snell's law. This can be expressed as

$$n \sin(\theta \pm u) = n' \sin(\theta' \pm u') \quad (5)$$

where $|\vec{J}| = 2n \sin(u)$, $|\vec{J}'| = 2n' \sin(u')$ and the other symbols have their customary meaning.

2. The "trumpet" concentrator. An entirely new ideal concentrator configuration¹⁹ was discovered immediately upon calculating the vector flux distribution for a source comprised of a simple strip or disk seen on edge as shown by the vertical line segment (FF') in Fig. 3. From elementary geometry it can be shown that in two dimensions

$$|\vec{J}| = 2s \sin \theta \quad (6)$$

where θ is the half-angle subtended by source as seen from the point at which $|\vec{J}|$ is being evaluated and the direction $\vec{J}$ points away from the source along

the bisector of the subtended angle. From this it follows that the configuration in Fig. 3 yields lines of constant $|\vec{J}|$ which are circles with FF' as a chord and lines of flow for $\vec{J}$ which are confocal hyperbolas with F and F' as foci.

Consider then the case in which the source is such a disk and suppose we place a mirror in the form of a hyperboloid of revolution coincident with a set of flow lines shown (AB and $A'B'$). We truncate this mirror so that its open end is a circle indicated by the two dimensional projection AA' . If we consider first the inside surface as a mirror, the properties of the vector flux field guarantee that this forms a nonimaging concentrator. All rays are redirected by the mirror so as to strike a new smaller disk, defined by the inner points BB' . The concentrator in effect, takes all rays from the virtual source FF' which pass the entry aperture and concentrates them onto the smaller disk aperture BB' . By introducing a suitable lens at the entry aperture the source can be placed at infinity so that, in effect, the trumpet concentrator serves as a second stage for a combined system somewhat similar to other two stage designs^{8,10}, but with substantially different properties and configurations. The concentrator works for skew as well as meridian rays so that two or three dimensional applications are possible.

3. Solution for ideal solar concentrators with reflector gaps. In designing a thermally efficient solar absorber one is often led to the requirement that it be enclosed by a glass shield for vacuum containment or as a convection barrier. On the other hand Winston and Welford²⁶ have shown that reflectors which extend to contact the absorber profile at at least one point are a necessary condition for coupling any absorber to an ideal concentrator.

Figure 3 The new "trumpet" concentrator derived from the vector flux formalism. The reflector surfaces AB and $A'B'$ are confocal hyperbolae which corresponds to the lines of flow of $\underline{J}$ arising from the source FF' and redirect the flux to BB' .

Therefore one cannot design a truly "ideal" concentrator for such an absorber. One approach to a practical collector design has been simply to calculate the reflector profile for the given absorber but to terminate the reflectors at the point at which they intersect the obstruction²⁷. However, in this case the optical losses become quite large if the gap size is comparable to the absorber dimensions. Winston²⁸ has analyzed this problem and developed a new solution which attains the thermodynamical maximum in flux concentration on the absorber for a given design acceptance angle.

The comparison between the old and new solutions is illustrated by the two mirror profile curves shown in Fig. 4. Figure 4a shows the conventional approach as described above for the case in which a vacuum enclosure around the absorber requires the mirror to be terminated leaving a gap $g = r_2 - r_1$ equal to the absorber radius itself. This design has an entrance aperture width $A_1 = \pi r_1 / \sin \theta_a$ and also has a "gap loss" of $\sim 22\%$ calculated from the formulas given in Reference 27. Thus the actual energy reaching the absorber even for otherwise perfect optics will be only 78% of what it could be for the design acceptance angle θ_a . For the solution in Fig. 4b, the profile curve is generated by constructing a "virtual absorber" by laying out tangent lines to the absorber from the point D (the point along the extension of the desired optic axis where it meets the obstruction). The reflector curves then begin at D and are the ideal solution obtained by applying the fundamental design principle⁷ to the virtual absorber (D B A A' B' D). This new solution will accept no radiation outside $\pm \theta_a$, but it is not an ideal concentrator in the usual sense since it will still reject some rays entering

Figure 4 a) conventional CPC solution with reflector gaps, b) new solution for maximal flux concentration onto absorber.

the aperture within $\pm\theta_a$ due to the gap. Although these gap losses as a fraction of aperture area are not reduced dramatically below that of the old solution, the new design has an aperture $A_1 > \pi r_1 / \sin\theta_a$ such that the net flux concentration seen by the absorber is $1/\sin\theta_a$ and is therefore the maximum permitted for this acceptance angle by the Second Law of Thermodynamics.

The above modification in design is not simply a subtle improvement. It should be emphasized that both of the fundamental motivations behind concentration, thermal and economic, have to do with the receiver (heat loss/unit area and cost/unit area). This new design increases the total amount of energy within a given acceptance angle which is directed onto a given receiver from roughly 4/5 of the thermodynamic maximum to the maximum allowed; a relative increase of about 25%. It achieves this by using somewhat more reflector and a somewhat oversized aperture for which both the cost and the heat losses are substantially smaller than for the absorber.

4. Properties of asymmetric ideal concentrators. One sub-class of ideal concentrators whose existence has been known since the initial symmetric CPC design principles were developed, but which had not been studied in great detail, consists of shapes optimized for collection between two extreme rays which are not symmetrically oriented with respect to an optic axis (normal to an aperture plane). These have been discussed briefly by Rabl⁹ and in somewhat more detail by Mills and Guitronich^{12,29}. The latter authors have pointed out that such devices may be advantageous in certain applications. We have carried out a comparative study relative to CPC's. The limiting case of asymmetric concentrators (referred to as Extreme Asymmetric Concentrators)

consists of only one curved mirror designed for a tubular absorber. The major advantage of these designs is that they have an effective concentration which varies with incidence angle and at the same time conserves phase space over well defined limits so that for the same peak concentration they have a broader angular range of partial acceptance than a CPC (although their average concentration is only 1/2 that of the CPC whose angular response is constant). This "triangular" response can be oriented such that the solar angle of incidence is near the peak concentration value for a long time near noon and yet the absence of a sharp cut-off allows practical non-tracking operation (tilt adjusting perhaps every few days) at higher peak concentration ratios ($\approx 10\times$) than are considered practical with CPC's. Our analysis shows that it is in this high concentration region that the main potential for this family of concentrators lies. The savings in mirror area coupled with the non-tracking option combine to suggest attractive possibilities. At peak concentration $\sim 5\times$ where seasonal load matching has been suggested¹² the asymmetric concentrator has comparable or larger mirror area (and half the average concentration) of the corresponding CPC; and thus appears from this study to be limited to applications where the load matching capability is of significant economic benefit.

5. Thermodynamic analysis of enhancement of effective absorptance of "gray body" receivers through cavity effect. Many of the selectively coated evacuated absorbers in use today have a low absorptance of about 0.7 (including the effect of reflections off the glass outer jacket). It has been noted recently that the optical performance of collectors using

such receivers can be improved by recapturing through multiple reflections back to the absorbing surface some of the energy which would otherwise be lost. Although this effect was first observed experimentally and then interpreted through ray traces³⁰ an analysis of the effect in terms of thermodynamic limits on radiative transfer yields a great deal of insight. The analysis carried out by our group²² gives quantitatively accurate predictions on the magnitude of the enhancement which could otherwise be attained only through complicated ray trace calculations. The effect is essentially a modified cavity effect. It is simply illustrated for the case of circularly cylindrical absorbers (such as commercial evacuated tubes) placed in front of a reflector of some shape such as the typical configurations illustrated in Fig. 5. For simplicity the mirror reflectance is assumed to be total in this illustration. If the collector is to have hemispherical acceptance, the tubes must be spaced such that the frontal projected area A_A is less than the total circumferential area A_R of the tubes (tube spacing $s < \pi d$ where d is the absorber diameter); that is the geometric concentration ratio $C = A_A/A_R$ must be less than unity. One then considers the problem of radiation transfer between the aperture surface A and the energy receiver surface R with the radiation exchange factors between A and R subject to the following constraints

$$\begin{aligned} E_{AA} + E_{AR} &= 1 \\ E_{RA} + E_{RR} &= 1 \end{aligned} \quad (7)$$

where E_{IJ} = the fraction of radiation leaving surface I which eventually reaches surface J . The aperture A is a black body while the receiver is a "gray" emitter with absorptance $\alpha = \epsilon = 1 - \rho$. In thermodynamic equilibrium the radiation transfer from A to R must be exactly equal to that from R to A

Figure 5 Typical reflector configurations for evacuated cylindrical absorbers.

so that we have the reciprocity relation

$$A_A E_{AR} = \epsilon A_R E_{RA} \quad (8)$$

Therefore the net effective absorptance of the aperture is

$$\begin{aligned} E_{AR} &= \frac{\epsilon}{C} E_{RA} \\ &= \frac{\alpha}{C} (1 - E_{RR}) \\ &\equiv \alpha_{\text{eff}} \end{aligned} \quad (9)$$

Equation (9) emphasizes the possibly counter-intuitive fact that to maximize the effective absorptance of the aperture surface one must minimize the self exchange factor E_{RR} which in turn can be shown to satisfy the inequality

$$E_{RR} \geq \frac{\alpha (1 - C)}{1 + \rho (1 - C)} \quad (10)$$

After substituting in Equation (9) we obtain an upper bound on the effective absorptance

$$\alpha_{\text{eff}} \leq \frac{\alpha}{\alpha + \rho C} \quad (11)$$

Note that in the limit that $\rho \rightarrow 0$, α_{eff} approaches unity. Also for $C \rightarrow 1$ α_{eff} becomes α as one expects. The dependency of this upper bound on concentration is shown in Fig. 6 compared with the results of ray trace for the involute configuration shown in Fig. 5d. The ray trace results begin to fall below the upper bound as the tubes approach the limit where they would be physically touching. Starting with our result, Bassett and Derrick at Sidney University have applied an elegant scattering formalism to show that the upper bound (Eq. 11) is a general result³¹.

Figure 6 Enhancement of effective absorptance by the modified cavity effect for $C < 1$.

6. Design considerations for nonimaging second stage optics combined with point focusing primaries. It is well known that single stage concentrators based on focusing optics fall far short of the thermodynamic limit given by Equations (1) and (2). In particular, point focusing paraboloids will achieve at best about $1/3$ and more typically between $1/5$ and $1/10$ of the maximum possible concentration if one wants to intercept all of the concentrated radiation. (The geometric concentration of such systems can be increased slightly if one is willing to accept some energy loss from spillover of the beam.) According to the formalism developed several years ago⁸, the combination of a second stage nonimaging concentrator of the CEC type can bring the overall system performance up to the thermodynamic limit to the extent that the primary can be regarded as a Lambertian source at a finite distance from the secondary. Of course, for focusing primaries in general the distribution of light reflected is not Lambertian in the aperture plane of the primary due to aberrations which introduce a gain in étendue. This results in a corresponding loss of maximum concentration which even an optimally designed second stage can never recover.

An analysis of the design problems involved in optimizing the primary-secondary match and of the relative importance of the actual shortfall for several configurations for the primary and a range of f /numbers has been carried out³². For the specific case of a point focusing paraboloid primary, illustrated in Fig. 7, the concentration of the primary alone decreases with increasing f number above $f/0.6$ while the combined system concentration increases with increasing f /number. As might be expected the loss in attainable concentration is most severe at short focal lengths (the second stage

Figure 7 Schematic representation (not to scale) of nonimaging CEC (BC' and $B'C$) coupled to a focusing primary. The CEC aperture is matched to the "waist" of the primary focussed radiation. Since the parabola does not represent a true Lambertian source the overall system concentration still falls somewhat below the thermodynamic limit.

can only improve the concentration ratio by a factor of ~ 1.5 from less than a quarter of the thermodynamic limit to a little more than a third of the limit). It is only when the f/number exceeds $f/2$ that the second stage is very effective increasing the concentration (for example, a gain of a factor of 10 from less than $1/10$ of the limit to about 90% of the limit can be achieved). However, at these longer focal lengths the CEC and overall system length becomes somewhat unwieldy for practical applications and the best compromise system seems to lie near $f/1.0$. In this case an overall concentration ratio close to 80% of the thermodynamic limit can be achieved as compared with only about 17% of the limit without a nonimaging final stage.

B. Category II Activities: Experiments in New Concept Applications

1. Fabrication and testing of a laboratory prototype concentrator for a tubular absorber with reflector gaps. The derivation of the optimized solution for absorbers with reflector gaps described in (1) above is based on thermodynamic and radiation transfer methods and does not yield a detailed description of the optical behavior (i.e., what rays are actually accepted or rejected). This information can only be obtained through computer ray tracing and testing of optical models. For this reason one particular design was selected for detailed study. Preliminary ray tracings have been carried out and a functioning prototype has been fabricated and undergone optical and thermal testing.

The common reason for large gaps relative to absorber size is the absolute value of mechanical tolerances that must be maintained regarding the alignment of long absorber tubes and the surrounding glass envelopes. This becomes very important as one tries to design smaller absorbers and

hence reduce the overall scale of the concentrator to manageable proportions. For example, the gap (g) in commercially available glass (Dewar type) evacuated absorber tubes (manufactured by General Electric or Owens-Illinois) is ~ 3.8 mm (0.15 inches) corresponding to a value $g/r = 0.176$ which yields a very small "gap loss" of $L = 0.02$ using the formula in Reference 27, so that the conventional CPC solution is quite adequate. However, if one couples such a receiver to a CPC of large numerical concentration, say $5X$, the resulting system is nearly 79 cm. wide and 1 meter deep. The desire to maintain such a concentration ratio on a less cumbersome scale is one of several reasons motivating the design of smaller diameter absorber tubes which, however, have typical values of $g/r \sim 1$ and correspondingly larger values of L .

We selected for study the case corresponding to one particular experimental absorber tube design having a steel tube 1.27 cm. (0.5 inch) diameter surrounded by a 2.40 in (0.945 in.) O.D. tube yielding $g/r = 0.89$. If the conventional CPC reflectors were terminated at the glass, this would result in a calculated loss of 18.8% due to the gap. A profile curve using the new solution allowing for gaps, and optimized for a design acceptance angle of $\pm 8^\circ$ was determined. For the new solution, one calculates a somewhat reduced gap loss of 15.9% with a correspondingly larger aperture. A prototype trough consisting of a 1.5 meter long trough was fabricated. The reflector was truncated to yield a new flux concentration of $4.9X$, i.e., a geometrical concentration of $5.75X$ allowing for the calculated gap loss mentioned above. This results in a $\sim 5X$ flux concentration for a small receiver which can be compared with previous conventional $5X$ systems. The trough prototype was

tested with our existing test loop facilities and yielded an optical efficiency and angular acceptance, shown in Fig. 8 which was in exact agreement with predictions.

2. The use of low concentration nonimaging optics with evacuated absorber systems. The new technology developed for spectrally selective surfaces in vacuum which have remarkably low heat losses is ideally matched to intermediate level nonimaging optical concentration. Since geometric concentration ratio is often regarded as a "linear" figure of merit, the fact that such low concentration systems can meet or exceed the thermal performance of parabolic trough collectors with concentration ratios greater than 20 up to temperatures of $\sim 300^\circ\text{C}$ is often not recognized. What one does in the design of such systems is to trade off the lower heat loss of the evacuated absorber for decreased concentration. The calculated behavior of evacuated collector thermal performance as a function of increasing concentration is shown in Fig. 9. The heat losses up to 300°C are already so low (the efficiency versus temperature curve is nearly flat even at concentrations from 3X to 5X) that further increases in the concentration ratio are only marginally effective and in fact in no way "linearly" related to the collector performance.

The validity of the calculations shown in Fig. 8 is confirmed by the test results of our evacuated 5X CPC shown in Fig. 10. For comparison the best parabolic trough concentrators tested at Sandia to date³³ (the HEXCEL) is also shown.

Figure 8 The measured angular response for a modified CPC with reflector gaps. The response function agrees with theoretical predictions.

Figure 9 The most significant gains in thermal performance are achieved at low concentration. With an evacuated selective absorber performance improvement above 5X is marginal below 200°C (572°F).

Figure 10 Measured thermal performance for a 5X evacuated tube CFC (12 annual tilt adjustments). The performance agrees with predictions (Figure 9) and is comparable to that of a tracking trough.

Based on analysis similar to that discussed above we have shown that even the low concentration of 1.5X - 1.8X compatible with fully stationary (fixed all year) operation results in a very low heat loss. When combined with a system where optical losses are reduced to a minimum a fully stationary collector for temperatures up to 300°C with efficiency greater than 50% results. We have carried out a detailed conceptual design of such a collector system based on the integration of the CPC reflector profile into the glass vacuum outer jacket by shaping standard 2" diameter fluorescent glass tubing. The concept, illustrated in Fig. 11, produces a system with an optical efficiency of greater than 70% and is the basis of a contract for a proof test panel hardware development effort at the University which is now in effect.

3. A survey of diffuse radiation and its long-term correlation with standard hemispherical insolation data. The type of data which forms the bulk of the insolation data base available for simulation of collector performance at individual locations consists of accumulated hourly, daily and long-term averages of the hemispherical insolation on a horizontal plane. Calculations of performance requiring separate characterization of the direct and diffuse components have relied on the classic correlation established by Liu and Jordan³⁴. However, since some inaccuracies in correlations have recently been indicated, and a knowledge of the diffuse components is essential to projections of CPC performance, a detailed analysis of these relationships has been carried out by our group²¹. The major conclusions are summarized below.

Advanced Integrated Stationary CPC

a)

b)

Figure 11 Profile curves showing how CPC reflector surface can be integrated directly into the glass vacuum jacket which can greatly reduce optical losses.

- a) The validity of the Liu and Jordan approach was confirmed but numerical inaccuracies in their correlations were found due in part to reliance on uncorrected measurements using pyronometer plus shade ring for the diffuse component.
- b) New parameters characterizing the correlations were established.
- c) The new correlations imply that the diffuse component is significantly larger than predicted by Liu and Jordan.
- d) The new model gives a complete description of the long-term average solar radiation on any surface. It has been found to predict radiation availability for solar collectors to better than 3%.
- e) The model provides a framework for optimal matching between solar radiation and ideal concentrators.

V. Summary

A program at the University of Chicago was begun in January, 1978, to conduct basic research in the field of nonimaging optics for solar energy. In this first two years under support from the Office of Advanced Energy Projects the program has been extremely fruitful, having led not only to the invention of new concentrator designs for absorbers enclosed by a transparent barrier, but to the development of an entirely new formalism for describing nonimaging optical systems. This formalism led immediately to the discovery of other new forms of ideal or near ideal concentrators and shows promise of providing the basis of a greatly improved understanding of these systems. The conceptual design for an optimized evacuated stationary nonimaging concentrator conceived during this work is the basis for a hardware development project now underway. In addition, as part of another project being carried out by our group, the nonimaging second stage concentrator designs, which were discovered in this effort, have been shown to be capable of increasing the concentrations of existing parabolic dish concentrators by about a factor of two without significant optical losses. A fundamental study of the characteristics of diffuse radiation and its correlation with the insolation data base, particularly important for nonimaging concentrators, has been completed and some preliminary experiments regarding the distribution of the diffuse component have been initiated. New designs have been selected for further study using computer ray tracing, long term projections of energy collection performance and in certain laboratory models for performance measurements.

These developments serve to emphasize the rapid progress that has been made in a field which is essentially totally new. This existing program, now firmly established after two years, is the basis for a continuing program now supported by the Division of Engineering, Mathematical and Geophysical Sciences Programs with the objectives of broadening the theoretical foundation for the field and maximizing the potential for additional solar applications.

VI. References

1. Welford, W. T. and R. Winston, The Optics of Nonimaging Concentrators, (Academic Press, New York, 1978)
2. Hinterberger, H. and R. Winston, "Efficient Light Coupler for Threshold Cerenkov Counters", Rev. Sci. Inst. 37, 1094 (1966).
3. Winston, R., "Light Collection within the Framework of Geometrical Optics", J. Opt. Soc. Am. 60, 245 (1970).
4. Ploke, J., "Light Collectors with High Concentration Efficiency", Optik 25, 31 (1967).
5. Baranov, V. K. and G. K. Melnikov, Soviet Journal of Optical Technology 33, 408 (1966).
6. Winston, R., "Principles of Solar Concentrators of a Novel Design", Solar Energy 16, 89 (1974).
7. Winston, R. and H. Hinterberger, "Principles of Cylindrical Concentrators for Solar Energy", Solar Energy 17, 255 (1975).
8. Rabl, A. and R. Winston, "Ideal Concentrators for Finite Sources and Restricted Exit Angles", Appl. Opt. 15, 2880 (1976).
9. Rabl, A., "1976 Comparison of Solar Concentrators", Solar Energy 18, 93 (1976).
10. Collares-Pereira, M., A. Rabl, and R. Winston, "Lens Mirror Combinations with Maximal Concentration", Appl. Opt. 15, 2677 (1977).
11. Bassett, I. M. and G. H. Derrick, "The Collection and Dissemination of Light with the Aid of Diffuse Reflectors", Optica Acta (in press) (1979).
12. Mills, D. R. and J. E. Guitronich, "Asymmetrical Non-imaging Cylindrical Solar Concentrators", Solar Energy 20, 57 (1978).
13. Welford, W. T., Aberrations of the Symmetrical Optical System, (Academic Press, New York, 1974).

14. Barnett, M. E., "Phase Space Conservation in a Compound Parabolic Optical Concentrator", preprint submitted for publication (1979).
15. Baranov, V. K., "Parabolicylindric Reflector Unit and Its Properties", Soviet Journal of Optical Technology (Geliotekhnika) 11, Nos. 3/4, 45, (1975).
16. Baranov, V. K., "Method of Calculating the Profiles of Focons and Foclines", Soviet Journal of Optical Technology (Geliotekhnika) No. 6, 12, 24 (1976).
17. Baranov, V. K., "Combination of Focons and Foclines with Radiation Receivers", Soviet Journal of Optical Technology (Geliotekhnika) No. 1, 13, 32, (1977a).
18. Baranov, V. K., "Concentration of Diffuse Radiation", Soviet Journal of Optical Technology (Geliotekhnika) No. 2, 13, 30-36 (1977b).
19. Winston, R. and W. T. Welford, "Geometrical Vector Flux and Some New Nonimaging Concentrators", J. Opt. Soc. Am. 69, 532 (1979).
20. Winston, R. and W. T. Welford, "Ideal Flux Concentrators as Shapes that Do Not Disturb the Geometrical Vector Flux Field" A New Derivation of the Compound Parabolic Concentrator", J. Opt. Soc. Am. 69, 536 (1979).
21. Collares-Pereira, M. and A. Rabl, "The Average Distribution of Solar Radiation - Correlations between Diffuse and Hemispherical and between Daily and Hourly Insolation Values", University of Chicago Report EFI-78-33, to be published in Solar Energy (1979).
22. O'Gallagher, J., A. Rabl, R. Winston and W. McInlre, "Absorption Enhancement in Solar Collectors by Multiple Reflections", to be published in Solar Energy (1980).
23. Winston, R., "Dielectric Compound Parabolic Concentrators", Appl. Opt. 15, 291 (1976).

24. "Thermal Power Systems Research and Development Project", Advanced Technology Development, Semi-annual Progress Report, Jet Propulsion Laboratory, 5102-67 (1978).
25. Electric Power Research Institute Technical Report No. ERH 78, contact Ed DeMeo, (Feb. 1977).
26. Winston, R. and W. T. Welford, "Two-dimensional Concentrators for Inhomogeneous Media", J. Opt. Soc. Am. 68, 289 (1978).
27. Rabl, A., N. B. Goodman and R. Winston, "Practical Design Considerations for CPC Solar Collectors", Solar Energy 22, 373 (1979).
28. Winston, R., "Ideal Flux Concentrators with Reflector Gaps", Appl. Opt. 17, 1668 (1978).
29. Mills, D. R. and J. E. Guitronich, "New Ideal Concentrators for Distant Radiation Sources", Solar Energy 23, 85 (1979).
30. McIntire, W., private communication (1979).
31. Bassett, I. M. and G. H. Derrick, "An Upper Bound on the Efficiency of A Collector of Diffuse Radiation onto a Graze Absorber", to be published in Opta Actica (1980).
32. Winston, R. and W. T. Welford, "Design of Nonimaging Concentrators as Second States in Tandem with Image-forming First Stage Concentrators", submitted for publication to Applied Optics.
33. Dudley, V. E. and R. M. Workhaven, "Performance Testing of the Hexcel Trough Solar Collector", Sandia Laboratories Report SAND78-0381 (1978).
34. Liu, B. Y. H. and R. C. Jordan, "The Interrelationship and Characteristic Distribution of Direct, Diffuse and Total Solar Radiation", Solar Energy 4, 1 (1960).
35. Chandrasekhar, S., private communication (1978).