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THE SCALES AND UPPER BOUNDS OF CLASSICAL MONOPOLE SOLUTIONS*

Cosmas K. Zachos
High Energy Physics Division
Argonne National Laboratory
Argonne, Illinois 60439

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ABSTRACT

The masses of GUT monopoles are bounded from above by a limit roughly twice the Prasad-Sommerfield limit, and thus depend only weakly on Higgs masses and the electroweak scale.

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I would like to report on some observations made together with T. Kirkman and A. N. Schellekens, on the mathematical structure of the classical monopole solutions. These observations may guide our intuition on the structure of the relevant physical objects. For lack of time, I will concentrate on results: the technical underpinnings of our formal argument are to be found in Refs.[1,2].

The equations of the Grand Unified Theories describing monopoles involve several mass scales which typify the various gauge bosons and Higgs scalars of these systems.^{3,4,5} However, the monopole solutions to these equations depend extremely weakly on the masses of the Higgs particles and the electroweak breaking scale, regardless of their size. We may highlight this by finding the finite limits which bound monopole masses from above. What makes these upper bounds especially interesting is that they are quite close to the lower bounds (the corresponding Prasad-Sommerfield/Bogomolnyi⁶ limits)--they are roughly twice these bounds.

To obtain the upper limits, we increase the concavity of the Higgs potential indefinitely in all group directions, maintaining the Higgs field vacuum values fixed. This essentially* freezes the scalar field variables to their vacuum values. In consequence, all effects of the scalar potential drop out, and the resulting drastic reduction of variables permits a straightforward numerical solution of the variational problem.^{1,2}

* Actually, there is a slight technical complication involved here: certain Higgs radial variables must vanish at the origin for the monopole Ansätze to be meaningful, as dictated by the topology of the solutions. Nevertheless, simple asymptotics imply that these variables in fact drop to zero from their finite vacuum value in a region of radius $O(1/M_{\text{Higgs}})$; this lowers the energy by $O(1/M_{\text{Higgs}})$. Hence both the size of the transition region and the decrement of the energy vanish in the special limit of infinite Higgs masses.

The above limit of frozen Higgs variables is an upper bound, for the following reason: when the masses of the gauge bosons (equivalently, the v.e.v.'s of the scalars) are kept fixed, the mass of a monopole is a nondecreasing function of the Higgs masses (equivalently, the Higgs self-couplings).¹ In the limit of infinite Higgs masses, all scalar variables freeze and their potential vanishes. There is also an alternative way to rationalize this upper bound: For any Higgs self-couplings, constraining scalar variables in the variational problem raises the energy and decouples the potential--naturally, if left unconstrained, these functions will vary so as to lower this energy.

The resulting upper bound for the $SO(3)$ monopole ('t Hooft-Polyakov) is $1.7867 m$, where m is the Prasad-Sommerfield/Bogomolnyi lower bound $m = M_W/\alpha$. For the spherically symmetric^{3,4} $SU(5)$ monopoles we find² the following upper bounds:

$1.7867 m$ for the single strength pole

$2.0741 \times 2m$ for the double strength pole

$2.3155 \times 3m$ for the triple strength pole

where m is the PSB lower bound $m = 3M_X/8\alpha$; the limit for the single $SU(5)$ pole reduces to the 't Hooft-Polyakov problem, which leads to the identical numerical bound. These numbers are purely empirical, and we have no fundamental understanding of them. Note that in this limit, the higher strength monopoles in this family are above threshold for decay into single strength monopoles.

Just as in the case of the lower limit (PSB), in this (upper) limit the

specific form of the Higgs potential is irrelevant, as long as it constrains the vacuum to a unique absolute minimum. Several variables which couple to the rest of the system only through the potential drop out as a consequence. In particular, in the SU(5) monopoles examined, the variable for the Higgs 5 completely decouples and there is no trace of the associated electroweak breaking scale M_W left in the problem. We should not therefore expect a significant dependence on the electroweak scale for finite Higgs masses, regardless of the magnitude of this scale: it would turn out to be unimportant even if it happened to be enormous ($\approx$ GUT scale)! The above argument then provides formal support for the conventional statement that the only essential scale for the GUT monopoles is set by just M_X , and it further justifies the use of the PSB limit in astrophysical estimates.

Eckert et al.⁷ have recently provided some illustrations to the points just made. They solve for the single SU(5) monopole in a wide range of different finite Higgs masses and couplings. They find that the upper bound can in fact be approached close to saturation. In addition, they verify the weak dependence of the monopole mass and structure on $\sigma \equiv$ electroweak scale/GUT scale ($\approx M_W/\sqrt{2}M_X$): as σ increases from 0 to 1, the monopole mass in general decreases less than 5%. Finally, they also observe that all radial variables turn out to be monotonic in r throughout their entire investigation.

This technique is easy to generalize. For instance, for this family of SU(N) spherically symmetric monopoles, the strength l pole relies on a spin $l/2$ representation of the crucial SU(2) subgroup of SU(N) which controls the topology. The corresponding limit can be taken by freezing out all scalar functions and solving for only l vector variables numerically.

Of course, the infinite coupling limit we have been taking is not meant to be physical. However, it brackets any classical case of interest conveniently, and likewise those situations which involve only irrelevant quantum corrections to the effective action (e.g. modifications of the Higgs potential which do not affect the symmetry breaking pattern).

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FIGURE CAPTION

The scalar and vector field radial functions for the 't Hooft-Polyakov monopole (Ref.1) in the upper and lower (PSB) limits. r is measured in units of $1/M_W$. The vector function K in the lower limit is the dotted line, and it moves monotonically with the Higgs mass to the solid line in the upper limit. The scalar function H/r in the lower limit is the dashed line, and it moves to the dash-dot step function in the upper limit. The corresponding curves for all values of the Higgs mass lie inside these envelopes.