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TITLE THE TRINUCLEONS: PHYSICAL OBSERVABLES AND MODEL PROPERTIES

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The Trinucleons: Physical Observables and Model Properties

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Abstract

Our progress in understanding the properties of ^3H and ^3He in terms of a nonrelativistic Hamiltonian picture employing realistic nuclear forces is reviewed. Trinucleon model properties are summarized for a number of contemporary force models, and predictions for physical observables are presented. Disagreement between theoretical model results and experimental results are highlighted.

1. INTRODUCTION

Michael Moravcsik noted that for a sizeable fraction of the attendees at a given conference in this series it was a first time experience. For those in that category and those outside the field of nuclear physics, let me provide a brief introduction to the subject.

From the earliest days of the field - Rutherford e^- scattering from the α particle, photodisintegration of the deuteron, or Thomas' observation that zero-range nuclear forces would lead to the collapse of the triton to a point (infinite binding) - few-nucleon systems have played a special role. Together with the deuteron, these systems have been the focus of research by a distinct group of scientists, a number of whom are present. The goal has been to solve numerically the Schrödinger equation for nuclear Hamiltonians based upon "realistic" force models (potentials). This has been accomplished for the $A=3$ ground states and is the subject of this review. In addition, we shall hear this week about progress in solutions for a variety of three- and four-nucleon problems including scattering and reactions.

The traditional approach to nuclear physics can be defined in terms of the constraints placed upon the complexity of the model assumptions:

- Nuclei consist only of nucleons, and other degrees of freedom are suppressed.
- Nucleons within the nucleus move slowly, and nonrelativistic dynamics prevails.
- Nucleons interact primarily via pairwise (two-body) forces,

$$H = \sum_i \frac{p_i^2}{2m} + \sum_{i < j} V_{ij}.$$

This is an enormous simplification of the physical problem but, in fact, accounts amazingly well for much experimental data. Much research over the past decade has focussed upon extensions of this model: meson exchange currents, three-body forces, $NN-N\Delta$ coupling, quark-gluon substructure of hadrons, relativistic dynamics, I will address

only the inclusion of two of these aspects in the picture, meson exchange currents and three-nucleon forces.

By solving the nonrelativistic Schrödinger equation for Hamiltonians containing only pairwise forces, one can test the validity of this traditional model of nuclear physics in a quantitative manner. It works surprisingly well. The nucleon dynamics is included exactly within the context of the model. No uncontrolled approximations are made. Disagreement between theory and experiment is an indication of missing physics; thus, numerical accuracy in the solution is essential. It is this aspect of the calculations that gives few-nucleon-systems investigations their special role and significance. Furthermore, once accurate models of few-nucleon systems are constructed, we can test our understanding of external weak and electromagnetic interactions in nuclei as well as benchmark calculational methods which may prove effective in many-nucleon systems.

Before turning to numerical results, let us examine the basis of the nonrelativistic dynamics and pairwise forces assumptions. The triton and alpha particle have charge radii of the size of 1.5-1.8 fm. (This is the same order as the pion Compton wavelength: $\lambda_\pi = 1/m_\pi \approx 1.4$ fm.) Using the uncertainty principle one can estimate Δp from the size of the system to be $\hbar/\Delta r$ or some 110-130 MeV/c. A similar estimate for the size of Δp comes from model values for the triton kinetic energy (≈ 40 MeV): $\langle p \rangle \approx 160$ MeV/c. Thus, the average momentum in the system is of the order of the pion mass, and

$$\left(\frac{v}{c}\right)^2 \approx \left(\frac{\langle p \rangle / M}{c}\right)^2 \approx \left(\frac{m_\pi}{M}\right)^2 \approx 2\%.$$

We emphasize that this is an average which should reflect accurately the relativistic corrections to the low energy observables that are averages over the nuclear volume. However, processes involving high momentum components of the wave function (e.g., elastic electron scattering at high momentum transfer) clearly can suffer much larger relativistic effects. Tjon will explore this in more detail.

The NN force is complex (reflecting many possible spin and orbital couplings) and isospin dependent (differing for nn , pp , and np systems). The rich complexity of isospin-spin-orbital configurations makes solving the three-nucleon equations rather difficult. The following aspects of the NN interaction are important for realistic force models:

- V_{nn} (V_{pp}) is weaker than V_{np} ; i.e., ^3H and ^4He cannot be described by a single radial function of a single symmetry as one finds in a molecular calculation.
- The spin-triplet force contains a strong tensor component that couples neighboring orbital waves like 3S_1 - 3D_1 ; i.e., OPEP is essential, providing 3/4 of the triton potential energy, so that central force model calculations may be misleading.
- The short range part of the NN force is highly repulsive; i.e., the probability of NN overlap at such distances is very small, and these holes in the wave function require accurate modeling.

Thus, a realistic NN force must fit the scattering data well and contain 1) OPEP type tensor components, 2) spin-isospin dependence, and 3) short range repulsion.

Even when a realistic NN force is used, we shall see that three-nucleon forces may play a role. Three-body forces arise in many branches of physics - atomic, molecular, condensed matter, gravitational, nuclear, In most cases they generate small but nonnegligible corrections to the properties of systems in which interactions are dominated by two-body forces. However, one should be aware that three-body forces are an artifact of restricting the model. A three-body force (3BF) appears in the nucleons-only picture, because certain degrees of freedom have been frozen out of the Hamiltonian. Thus, a 3BF in one language is a two-body force in a more complicated space. Sauer will discuss

the result of formally eliminating the ΔN channel from an $NN - \Delta N$ model of nuclear forces, an action which leads to an explicit NNN force in the triton. Looking ahead, we find the following circumstantial evidence for including a 3BF in model calculations:

- The triton binding energy $E_B(^3\text{H})$ is too small by 0.5 – 1.1 MeV.
- The $^3\text{H}/^3\text{He}$ rms charge radii are 0.1 – 0.2 fm too large.
- The asymptotic normalization ratio C_2/C_0 is 10% too small.
- The nd spin-doublet scattering length is a factor of 2 too large.

The model values for all of these observables can be improved by including a 3BF suitably parameterized to reproduce the triton binding energy.

Some brief remarks about the definition of three-nucleon forces is in order. In the nucleons-only Hamiltonian picture, a true 3BF depends in an irreducible way upon the simultaneous coordinates of three interacting nucleons. That is, one has V_{123} and not an interaction ($V_{12} + V_{23}$). Because OPEP describes accurately the long range part of the NN force, it is assumed in the 3BF models that we consider here that the long range part is determined principally by two-pion exchange ($2\pi\text{E}$); i.e., a static 3BF. The strong short range repulsion of the NN interaction is not sufficient to remove entirely the sensitivity to the short range properties of the $2\pi\text{E}$ 3BF. This uncertainty in the short range part of the 3BF (due to heavier meson exchange, etc.) is parameterized phenomenologically in terms of the size of the πN form factor – a cutoff similar to that determining the short range properties of the NN potential models. Holinde will discuss this in more detail. Because one cannot generate *a priori* a model of the NN force, one should not be surprised that we cannot generate *a priori* the complete NNN force.

Finally, we point out that although the 3BF contribution to the triton potential energy is small (≤ 1 MeV on a scale of ≈ 50 MeV), the 3BF produces sizeable nonperturbative effects in the wave function. First order perturbation theory fails to account for 3BF effects in $^3\text{H}/^3\text{He}$.

2. MODEL PROPERTIES

In the presentation to follow, I will discuss representative results from the large basis calculations of Friar, Payne, Gibson, ..., of Glöckle, Witala, ..., of Sasakawa, Ishikawa, ..., of Sauer, ..., and of Brandenburg, Kim, Picklesimer, We will see that the low energy observables scale with the $^3\text{H}/^3\text{He}$ binding energy, and that the “best estimate” is interpolated from such scaling plots. To paraphrase the expert in numerical analysis Hammond, “The purpose of computing is insight and not numbers.” Therefore, I will attempt to provide comments on our understanding of the physical observables in terms of the model parameters as we go.

Before we turn to these results, permit me a brief glimpse of history. Tjon [1] provided the initial Faddeev calculation of $^3\text{H}/^3\text{He}$ using a realistic potential model. The results of that charge form factor study are valid today: i) a triton model that is underbound has a radius which is too large; ii, the diffraction minimum occurs at too high a momentum transfer in the nonrelativistic Hamiltonian picture. Afnan and students [2] produced the initial calculations employing the 1S_0 , 3S_1 - 3D_1 NN partial waves for all of the then existing realistic force models. He employed an Adhikari-Sloan expansion (now advocated by Koike [3]) for which convergence can be systematically improved. The EST expansion was later used to generate such solutions for contemporary NN potentials [4]. Most recently Sandhas and collaborators [5] have demonstrated that a low-rank W-matrix

expansion yields quality small basis solutions for the Paris potential, and it holds promise for speedy $nd \rightarrow nnp$ breakup calculations.

2.1 Binding Energies

In order to understand the nomenclature used to describe realistic force model calculations, let me briefly review details of the 5-channel solution, which results from retaining the $j^\pi \leq 1^+$ NN potential partial waves ($^1S_0, ^3S_1$ - 3D_1). The Schrödinger wave function is expressed as the sum of three Faddeev amplitudes

$$\Psi(\mathbf{x}, \mathbf{y}) = \psi(\mathbf{x}_1, \mathbf{y}_1) + \psi(\mathbf{x}_2, \mathbf{y}_2) + \psi(\mathbf{x}_3, \mathbf{y}_3),$$

where $\mathbf{x}_i$ is the coordinate of the interacting (j, k) pair and $\mathbf{y}_i$ is the coordinate of the spectator nucleon i with respect to the c.m. of the pair (j, k). The Faddeev amplitude is expressed as

$$\psi(\mathbf{x}_i, \mathbf{y}_i) = \frac{\chi(\mathbf{x}_i, \mathbf{y}_i)}{x_i y_i},$$

where

$$\chi(\mathbf{x}_i, \mathbf{y}_i) = \sum_{\alpha} \chi_{\alpha}(x_i, y_i) |[(\ell_{\alpha}, s_{\alpha})j_{\alpha}(L_{\alpha}, S_{\alpha})J_{\alpha}]JM(t_{\alpha}, T_{\alpha})T\rangle.$$

Here the lower case letters (j_{α}) denote the interacting pair, while the upper case letters (J_{α}) denote the spectator. Truncating the NN partial waves to $j^\pi \leq 1^+$ leads to a total of 5 three-body channels listed in Table 1. Note that $\Psi(\mathbf{x}, \mathbf{y})$ contains $L = 0, 1, 2$ partial waves, because of the "permuted" amplitudes $\psi(\mathbf{x}_2, \mathbf{y}_1)$ and $\psi(\mathbf{x}_3, \mathbf{y}_3)$.

Table 1

Three-body channels resulting from the $^1S_0, ^3S_1$ - 3D_1 NN partial waves.

α	$(\ell_{\alpha}, s_{\alpha})j_{\alpha}$	$(L_{\alpha}, S_{\alpha})J_{\alpha}$	$(t_{\alpha}, T_{\alpha})T$
1	(0,0)0	(0,1/2)1/2	(1,1/2)1/2
2	(0,1)1	(0,1/2)1/2	(0,1/2)1/2
3	(2,1)1	(0,1/2)1/2	(0,1/2)1/2
4	(0,1)1	(2,1/2)3/2	(0,1/2)1/2
5	(2,1)1	(2,1/2)3/2	(0,1/2)1/2

Benchmark results exist from several groups [6-10] utilizing various NN force models [11-15] to calculate the ^3H binding energy E_B : RSC (7.02 MeV), AV14 (7.44 MeV), Nijmegen '78 (7.49 MeV), Paris (7.31 MeV), Bonn-r (8.23 MeV), and Bonn-c (8.27 MeV). The spread in these binding energies is understood as follows: i) RSC vs. AV14 reflects the 300 keV difference that arises from fitting the spin-singlet potential to pp rather than np scattering data; ii) Nijmegen vs. Paris reflects the difference in χ -square fits to the NN data; iii) Nijmegen vs. RSC reflects the difference due to the smaller deuteron D-state probability (P_D) arising in the OBE (one boson exchange) models, where the quadrupole moment constraint on P_D is relaxed; iv) Bonn-r (an OBE model) vs. Nijmegen reflects the very different ϵ_1 mixing parameter in that spin-triplet partial wave that leads to a much smaller P_D value for the OBE approximation to the full Bonn interaction.

The complete realistic NN force model calculations can be seen to converge when large basis results are obtained. The relation between the NN partial wave truncation and the number of three-body channels included in the calculation is included in Table 2 along with binding energy results for four different potential models. Note that the

$j^\pi \leq 1^-$ partial waves are repulsive (compare 10 with 5) and the higher odd parity partial waves are not significant (compare 9 with 18). However, the odd parity $j = 2$ partial waves are repulsive for the RSC and AV14 models but are attractive in the OBE models. Finally, variational bounds for $5 \leq j \leq 8$ indicate that these higher partial waves contribute only an additional 10 keV to the triton binding energy [16].

Table 2

^3H binding energy (in MeV) as a function of the number of three-body channels.

j^π	# channels	RSC	AV14	Paris	Nijmegen
1^+	5	7.023	7.441	7.311	7.489
2^+	9	7.210	7.569	7.415	7.597
1	10	6.856	7.252	7.119	7.289
2	18	7.231	7.572	7.388	7.541
3	26	7.342	7.667	7.469	7.615
4	34	7.346	7.670	7.467	7.625

It is obvious that these NN realistic forces underbind the triton. Adding a static 3BF [17-20] to the RSC and AV14 static NN potentials does enhance the ^3H binding. Anecdotal results [21-22] for a πN form factor cutoff of $\Lambda = 5.8m_\pi$ are given in Table 3. We see that 34 channels are required to obtain a converged solution. That the results are sensitive to the cutoff Λ is illustrated by results for the RSC/TM model as a function of Λ (in units of m_π): $E_B(\text{MeV}) = 7.46$ ($\Lambda = 4.1$), 8.86 ($\Lambda = 5.8$), 11.16 ($\Lambda = 7.1$). Thus, one can dial the 3BF to yield the observed triton binding energy of 8.48 MeV. A partial test of the efficacy of such a procedure is to calculate the ^4He binding energy. Wiringa will discuss this in more detail.

Table 3

^3H binding energy (in MeV) for various 3BF models as a function of the number of three-body channels and for the πN form factor cutoff $\Lambda = 5.8m_\pi$.

model	5-chan.	9-chan.	18-chan.	34-chan.
AV14/TM	8.26	8.94	9.49	9.36
AV14/BR	8.52	9.47	9.26	9.42
AV14/UA	7.70	8.34	8.77	8.99
RSC/TM	7.55	8.33	8.93	8.86
RSC/BR	7.86	8.97	8.91	9.09
RSC/UA	7.22	8.00	8.43	8.70

2.2 Wave Function Probabilities

The wave function probabilities P_S , $P_{S'}$, P_P , and P_D [$\sim L = 0, 1, 2$ for $\Psi(\mathbf{x}, \mathbf{y})$] are not physical observables. However, they provide a useful characterization of the wave function generated by a particular nuclear force model [6,23]. Qualitatively, one finds $P_D \equiv 0$ unless there exists a tensor force. In a central force approximation, $P_{S'} \equiv 0$ implies that $V(^1S_0) \equiv V(^3S_1)$. For realistic models, the following characteristics obtain.

The existence of a strong tensor force implies a D-state component in the wave function. In the simplest picture of the triton, one can think of the three nucleons as forming

an isosceles triangle. (The base between the two like neutrons exceeds the distance separating the np pairs, because the nn force is weaker.) The average np interaction is $3/4V_t + 1/4V_s$. Thus, 3/2 of the NN interactions in the triton are deuteron like, and one would therefore anticipate

$$P_D(^3\text{H}) \approx \frac{3}{2} P_D(^2\text{H})$$

for a given model. This relationship is found to hold remarkably well. For the static potential models (RSC and AV14), one finds P_D to be approximately 1.46 - 1.47 times $P_D(^2\text{H})$. Including a 3BF enhances P_D by 10-20%.

The S' (mixed symmetry) state provides a measure of the difference that exists between the ^3H and ^3He wave functions due to the fact that the NN force is charge dependent. (If $V_t = V_s$, then the isosceles triangle becomes equilateral and $P_{S'} \equiv 0$.) The S' state corresponds to an excitation in oscillator language from the $(1s)^3$ configuration to the $(1s)^2(2s)$ or $(1s)(1p)^2$. This implies that $P_{S'}$ should vary as the inverse square of the binding energy. This is born out well in model calculations; that is, $P_{S'}$ decreases as the triton binding energy increases according to $E_B^{-2.1}$. For the physical value of the triton binding, one finds $P_{S'}(^3\text{H}) \approx 1 - 1.1\%$. Extending this analysis to ^4He , one would expect the S' state probability in the α particle to be much less than 1%.

The P-state probability is relatively small for all NN force models considered. (For the RSC and AV14 models, $P_P \approx 0.08\%$ for 5-channel through 34-channel solutions.) However, the addition of a $2\pi E$ 3BF to the model induces a large enhancement (factor of 2-3) in $P_P(^3\text{H})$. Thus, it has been suggested that the vector polarization in Nd scattering would be one observable which might yield direct evidence of 3BF effects. Utilizing the vector polarization in Nd scattering as a nuclear filter should provide a window through which to observe such small effects that would otherwise be masked by the more dominant components of the nuclear interaction.

3. PHYSICAL OBSERVABLES

We summarize briefly the model results for various physical observables in comparison with their experimental values. In all examples of low energy observables, the model results scale with the binding energy of the system [23]. Thus, we are able to interpolate among model NN and $NN + NNN$ results to obtain "best estimates" of the physical observable in question.

3.1 Coulomb Energy

The $^3\text{H}/^3\text{He}$ binding energy difference of 764 keV can be thought of as coming from Coulomb repulsion between the two protons in ^3He plus a charge symmetry breaking ($V_{pp} \neq V_{nn}$) contribution:

$$E_B(^3\text{H}) - E_B(^3\text{He}) = E_C + \Delta E_{CSB}.$$

The Coulomb energy from model calculations [24] is

$$E_C = \left\langle \frac{\alpha}{r_{ij}} \right\rangle + E_C^{2nd} \\ \approx 648 \pm 4 \text{ keV}.$$

It can be shown that the rather implausible approximation [25-27]

$$\left\langle \frac{1}{r} \right\rangle \approx \frac{1}{\sqrt{3}} \left\langle \frac{1}{R} \right\rangle,$$

where R is the hyperradius of the trinucleon, is an approximation which works to better than 1%. The interest in this approximation is that it permits one to change variables and then estimate E_C using the experimental charge form factor data:

$$E_C^{FF} \approx 638 \pm 10 \text{ keV}.$$

This is smaller than the E_C estimate from model calculations above, because the experimental charge form factor exhibits a large negative secondary maximum which is not reproduced in the model triton calculations. We will explore this further.

The charge symmetry breaking exhibited in the $^3\text{H}/^3\text{He}$ binding energy difference is, therefore, of the order of 100 keV. [Recall that $V_{nn} \neq V_{np}(^1S_0)$ in the model calculations accounts for 300 keV in the binding energy difference of the various model tritons.] Of the 100 keV difference between E_C and the ground state energy of 8.48 MeV, 70 keV can easily be accounted for by such charge-symmetry-breaking mechanisms as ρ - ω mixing, π - η mixing, The np mass difference in the kinetic energy and nonstatic Coulomb effects actually account for 30-40 keV. Thus, charge symmetry breaking is a small but reasonably well understood aspect of the trinucleon system [28-31].

3.2 Charge Densities and Electron Scattering Form Factors

The difference between model values of E_C and the hyperspherical result estimated directly from measured elastic electron scattering charge form factors [32-34] is symptomatic of the problem illustrated in Fig. 1. The model impulse approximation calculations [35-40] fail to reproduce the magnitude of the negative secondary maximum of the form factor. This model discrepancy at large momentum transfer translates into the following problem in terms of the model charge density [41]. The Fourier transform of the charge form factor data generates a central depression in this "quasi experimental charge density" which is not predicted in nonrelativistic Hamiltonian model calculations of $\rho(r)$. This is illustrated in Fig. 2, where model densities with and without the inclusion of a 3BF are plotted along with the Fourier transform of the charge form factor. The relation

Figure 1. ^3He charge form factor compared with RSC model calculations.

Figure 2. Model ${}^3\text{He}$ charge densities compared with "experiment."

$$\rho(0) = \frac{1}{2\pi^2} \int_0^\infty F(q^2) q^2 dq$$

indicates why the central depression in the density is related to the size of the negative secondary maximum in the form factor. If one examines the S-state component of the wave function, it is easily shown that

$$\rho(0) \approx \int_0^\infty dx_1 x_1^2 \Psi(x_1, 0).$$

That is, only the collinear configuration contributes to $\rho(0)$. Therefore, one might postulate that a 3BF which is repulsive in such a configuration would alleviate the problem. However, the phenomenological 3BF models included here primarily increase the binding energy, which concomitantly compresses the density and stretches the form factor. That is, they fail to improve the fit to the secondary maximum.

Several caveats are in order[41]. The connection of the model charge density and the elastic electron scattering charge form factor at large momentum transfer through nonrelativistic quantum mechanics contains several approximations. No complete, unambiguous relativistic calculations have been performed. The *ad hoc* addition to the form factor of a component that vanishes at $q^2 = 0$ and is negative in the range of the diffraction minimum and secondary maximum would clearly improve the agreement. However, any meson exchange current corrections to the nonrelativistic model are of relativistic order and, therefore, model dependent and ambiguous. Only the impulse approximation is clearly defined. Although the hole in ${}^3\text{He}$ near the origin is controversial, it represents less than 1% of the charge. However, it does signify that our model wave functions will significantly underestimate the magnitude of the secondary maximum even though such nonrelativistic calculations are unlikely to be valid at this level of accuracy.

Finally, we note that the ${}^3\text{H}$ model calculations do exhibit a depression of the central density. However, this is an effect arising from antisymmetry of the nucleons. The two

Figure 3. Schematic model description of trinucleon configurations: a) symmetric S state, b) ${}^3\text{He}$, and c) ${}^3\text{H}$.

neutrons in the $S = 3/2$, $L = 2$ component of Ψ must reside in an odd parity state as then must the proton. The charge density contribution from such a state must necessarily vanish near the origin. Let us now turn to the low momentum transfer characteristics of the charge data, which do exhibit reasonable agreement between theory and experiment.

3.3 Charge Radii

One observes in the charge form factor plots that the model results for small values of q^2 provide a reasonable description of the data. This is the region which determines the charge radii. Is the relative size of ${}^3\text{H}/{}^3\text{He}$ understood? Recall that V_{nn} is weaker than V_{np} . Thus, like nucleons reside farther apart than do unlike nucleons, so that

$$\langle r^2({}^3\text{He}) \rangle > \langle r^2({}^3\text{H}) \rangle,$$

even in the absence of any charge symmetry breaking [23,24]. This is illustrated in Fig. 3, where the like nucleons in the base of the isosceles triangle in panels (b) and (c) is larger than that of the equilateral triangle in (a). This difference between ${}^3\text{He}$ and ${}^3\text{H}$ as indicated in (b) and (c) is a reflection of the isospin-spin-space correlation in the wave function introduced by the S' state (even in a simple model calculation employing only central forces).

Coulomb repulsion does push the protons farther apart in ${}^3\text{He}$, as one's intuition would suggest. However, the Coulomb interaction also modifies the tail of the wave function. The net effect of these competing mechanisms [24] is to increase the proton radius in ${}^3\text{He}$ by about 0.04 fm compared to the neutron radius in ${}^3\text{H}$. In Fig. 4 we see a comparison of model results with the experimental measurements. Because the NN force models alone underbind the triton, the corresponding radii are too large by 0.1 – 0.2 fm. The addition of a 3BF accounts for the points in the neighborhood of the physical energy and above. Interpolation of the model results implies

$$\langle r^2({}^3\text{H}) \rangle^{1/2} = 1.58 \text{ fm} \quad \text{and} \quad \langle r^2({}^3\text{He}) \rangle^{1/2} = 1.77 \text{ fm}.$$

The absolute electron scattering results do not have the precision necessary to test these predictions. A precision ratio measurement of the ${}^3\text{He}/{}^3\text{H}$ radii would provide a more significant test of the model prediction than present absolute measurements permit.

Figure 4. Plot of model trinucleon charge radii as a function of binding along with the experimental charge radii.

3.4 Magnetic Properties

Deviation of the ${}^3\text{H}/{}^3\text{He}$ magnetic moments from the Schmidt limits provided an early indication of possible nucleon modifications due to subnucleon degrees of freedom, when nucleons are embedded within the nuclear medium. In one language these effects are due to meson exchange currents (MEC).

Isoscalar MEC are expected to be small, of the same order as relativistic corrections to the traditional model. Indeed, the isoscalar deuteron does not appear to suffer significant magnetic moment modifications. Likewise, the isoscalar magnetic moment combination for ${}^3\text{He}$ and ${}^3\text{H}$ appears to be reasonably described in impulse approximation by [42]

$$\mu_s = \frac{\mu_p + \mu_n}{2} [1 - 2P_D] + \frac{1}{2} P_D,$$

where $\mu_p = 2.793$ and $\mu_n = -1.913$ in nuclear magnetons. For $P_D \approx 7\text{-}10\%$, one finds $0.40 \leq \mu_s \leq 0.41$. Three-body forces reduce μ_s , as P_D is enhanced by 1%. Thus, one needs a correction of $\approx 5\%$ to reproduce the experimental value of $\mu_s = 0.426$.

Meson exchange current corrections [43] to the isovector magnetic moment are expected to be much larger [44,45]. In this case, the lowest order terms (see Fig. 5) are not of relativistic order. One can decompose the isovector exchange current into $J = J_{imp} + J_{ex} + \Delta J$, where J_{imp} is the standard impulse approximation current, J_{ex} represents the pion exchange terms shown in Fig. 5, and ΔJ contains heavier meson and two-pion-exchange contributions. The impulse result for the magnetic moment is given by

$$\mu_v^{imp} = \frac{\mu_p + \mu_n}{2} [1 - \frac{4}{3} P_{S'} - \frac{2}{3} P_D] + \frac{1}{6} P_D$$

with a typical value in the range of $-2.14 \leq \mu_v^{imp} \leq -2.18$.

Figure 5. Pion-exchange-current contributions: a) seagull or pair diagram, b) true absorption diagram, and c) Δ -isobar diagram.

Calculation of the three terms comprising J_{ex} is sensitive to the value of the cut-off in the πN vertex. For the preferred value of $5.8m_\pi$ and 34-channel wave functions, one obtains a prediction [42] for the isovector magnetic moment in the range $-2.44 \leq \mu_v \leq -2.50$ compared to the experimental value of $\mu_v = -2.553$. Heavy meson exchange, multi-pion exchange, and relativistic corrections are missing. Including a 3BF in the model improves the agreement some, as the binding is increased. Although the percentage discrepancy between theory and experiment is smaller here than for μ_s , the 5% discrepancy in the latter case likely provides the better estimate of the magnitude of the omitted corrections.

Similar meson exchange current contributions to the magnetic elastic electron scattering form factors appear to provide the semiquantitative corrections to the impulse approximation result necessary to agree reasonably with the data [33,36,45].

3.5 Asymptotic Normalization Constants

The wave function probabilities are not physical observables. However, the asymptotic normalization constants (vertex constants in the language of dispersion theory) can be addressed experimentally. They echo the internal dynamics of the wave function through the overall normalization. A recent review by Weller and Lehman [46] summarizes the situation in detail.

The asymptotic normalization constants are defined relative to an S-state wave function generated by a zero range force, with (or without) a Coulomb interaction between the charged (or uncharged) clusters. Because finite range effects reduce the model wave function near the origin below the zero range function $[\exp(-\beta r)/r]$, the S-state asymptotic normalization constant $C_S \geq 1$. Coulomb effects are of two types and tend to cancel. Coulomb repulsion pushes probability density away from the origin, which increases C_S^C . But the binding energy of the system is also reduced, which decreases C_S^C . The D-state asymptotic normalization C_D reflects the tensor nature of the NN force. However, C_D is inherently much less than unity, because the D-state probability appears implicitly.

The general expression for the asymptotic form of the ^3He wave function takes the form [47]

$$\begin{aligned} \Psi(^3\text{He}) \rightarrow C_S^C N_W \frac{W_{-\alpha, 1/2}(2\beta y)}{y} \times [L = 0] \\ + C_D^C N_W \frac{W_{-\alpha, 5/2}(2\beta y)}{y} \times [L = 2], \end{aligned}$$

where the $W(2\beta y)$ are exponentially decaying Whittaker (Coulomb) functions describing ${}^3\text{He}$ in the zero range approximation but reduce to simple exponentials for the triton [$W_{-\kappa,1/2} \rightarrow \exp(-\beta y)$], N_W is the normalization (an analytic function) given by

$$N_W^{-2} = \int_0^\infty dy [W_{-\kappa,1/2}(2\beta y)]^2,$$

κ is defined in terms of the fine structure constant α ($= 1/137$) as $\kappa = 2M\alpha/3\beta$, and

$$\beta^2 = \frac{4M}{3} [E_B(A=3) - E_B({}^2\text{H})]$$

having a value of 0.1766 fm^{-2} for ${}^3\text{He}$ and 0.2102 fm^{-2} for ${}^3\text{H}$.

There exist weak experimental limits on C_S and C_S^C . The ratio of D-state to S-state constants η ($= C_D/C_S$) has been estimated by analytic continuation to the appropriate pole of T_{20} for the reaction ${}^4\text{He}({}^2\text{H}, {}^3\text{H}){}^3\text{He}$. However, the related distorted wave Born approximation parameter D_2 (the spectroscopic factor) has received more attention experimentally. This quantity is defined by

$$D_2 = -\frac{\int dy y^4 u_2(y)}{15 \int dy y^2 u_0(y)},$$

where

$$u_2 \sim C_D N_{ZR} e^{-\beta y} [1 + \frac{3}{\beta y} + \frac{3}{(\beta y)^2}] / y$$

and

$$u_0 \sim C_S N_{ZR} e^{-\beta y} / y,$$

so that we have

$$D_2 \approx -(\frac{C_D}{C_S} / \beta^2) \quad \text{and} \quad D_2^C \approx -(\frac{C_D^C}{C_S^C} / \beta^2) f(\kappa),$$

where $f(\kappa)$ is an analytic function. Experimentally, we find the following estimates [16,47] for these asymptotic normalization constants as of 1988:

$$\begin{aligned} 2.6 \leq (C_S)^2 \leq 3.3 & \quad 2.4 \leq (C_S^C)^2 \leq 3.5 \\ \eta = 0.050 \pm 0.006 & \quad \eta^C = 0.036 \pm 0.006 \\ D_2 = -0.20 \pm 0.04 & \quad D_2^C = -0.25 \pm 0.04 \end{aligned}$$

A more recent transfer reaction estimate [48] of η^C yielded a result of 0.046 ± 0.005 . These values can be compared with model results for ${}^3\text{H}$ [8,22,47] and for ${}^3\text{He}$ [47]. Best estimates have been extracted by interpolation among numerous calculations using the RSC and AV14 nuclear forces, plus the addition of a 3BF. For example, Fig. 6 exhibits such model results for η and η^C along with the published data for η . Theoretical predictions for the asymptotic normalization parameters are:

$$\begin{aligned} C_S \approx C_S^C &= 1.85 \pm 0.04 \\ \eta &= 0.046 \pm 0.002 & \eta^C &= 0.043 \pm 0.002 \\ D_2 &= -0.229 \pm 0.005 & D_2^C &= -0.238 \pm 0.005 \end{aligned}$$

The model calculations verify that Coulomb effects tend to cancel. Unfortunately, the experimental uncertainties are large enough that there is no clear discrepancy between

Figure 6. Theoretical η and η^C results for selected two-body and three-body force models and the published experimental estimate of η .

experiment and theory, although the nominal values are not in agreement. Clearly, the theorists' challenge has been laid down. A new result from TUNL [49], $D_2 = -0.217 \pm 0.010 \text{ fm}^2$ and the corresponding value of $\eta \approx 0.043 \pm 0.002$, would appear to provide such a test for ^3H . In fact, closer examination of the calculations for η indicates that the Paris and Nijmegen OBE models produce a value for η that is smaller than that quoted above based upon the static RSC and AV14 model results, in better agreement with the new TUNL value. However, any clear test of the important S-state constants is still missing.

3.6 Nd Scattering Lengths

The zero energy nd spin-doublet scattering length ($^2a_{nd}$) is closely correlated with the triton binding energy, as was first suggested by Phillips. The nd spin-quartet scattering length ($^4a_{nd}$) is determined primarily by the properties of the deuteron. The generally accepted experimental values [50-52] for these scattering lengths are

$$^2a_{nd} = 0.65 \pm 0.04 \text{ fm} \quad \text{and} \quad ^4a_{nd} = 6.35 \pm 0.02 \text{ fm}.$$

The correlation between $^2a_{nd}$ and $E_B(^3\text{H})$ is illustrated in Fig. 7. Model values [53] for the nd doublet scattering length, as interpolated from such a plot, are 0.57 fm (AV14) and 0.66 (RSC). That is, the agreement between theory and experiment is quite good. Similarly, one finds reasonable agreement between theory and experiment for the quartet scattering lengths, 6.38 fm (AV14) and 6.30 fm (RSC), in the case of nd scattering.

The agreement between theory and experiment for the pd scattering lengths is not as good. The pd scattering data are known less precisely, because the Coulomb barrier greatly suppresses the cross section (most of which is pure Coulomb and therefore uninteresting). In addition, data exist only for c.m. energies above 400 keV; the estimates of the (zero energy) scattering lengths were obtained by extrapolation from the available data. The values for $^4a_{pd}$ cluster around 11.5 fm [54-56]. The theoretical results [53] are quite

Figure 7. *Nd* spin-doublet scattering lengths plotted as a function of the corresponding trinucleon binding energy. The $^2a_{nd}$ datum coincides with RSC/TM model result for which the 3BF was adjusted to yield the ^3H binding. The $^2a_{pd}$ datum falls well off the theoretical curve.

different. As noted above for the *nd* case, the model numbers depend primarily upon the deuteron properties. Because the Pauli repulsion for three nucleons with spins aligned, 3BF effects and the influence of higher partial waves are minimal. We find:

$$^4a_{pd} = 13.76 \pm 0.05 \text{ fm (AV14)} \quad \text{and} \quad ^4a_{pd} = 13.52 \pm 0.05 \text{ fm (RSC)}.$$

This substantial disagreement with experiment is likely correlated with the corresponding disagreement for the spin-doublet case. The reported doublet scattering lengths have nominal values of 1.3 fm, 2.73 fm, and 4.00 fm. (The quoted errors in the three measurements are smaller than the spread in the values.) The theoretical best estimate for the *pd* doublet scattering length is

$$^2a_{pd} \approx 0,$$

as interpolated from the curve drawn through the model results plotted in Fig. 7. The introduction of the Coulomb force in the *pd* calculation converts the Phillips line for *nd* scattering lengths into a curve, so that the introduction of a repulsive force between the two protons, which consequently lowers the ^3He binding compared to that of ^3H , does not increase the value of the $^2a_{pd}$ compared to that for $^2a_{nd}$ as would happen if one had introduced an additional repulsion in the *pp* strong interaction. This result runs counter to intuition and disagrees with the reported experimental numbers, which have been obtained by extrapolating from measurements made above 400 keV. Because of curvature in the effective range expansion for energies below 300 keV, the extrapolation to zero energy is suspect [57]. Although low energy *pd* scattering experiments are difficult, one would hope that these theoretical predictions might inspire further efforts in this area.

4. Summary Remarks

The three-nucleon problem has proven to be an excellent laboratory in which to test our traditional approach to nuclear physics. Progress in understanding the physics of the trinucleons in terms of a nonrelativistic Hamiltonian picture employing realistic nuclear forces has been enormous. Solving the exact equations which describe the three-nucleon system enables one to make meaningful comparison between theory and experiment and to probe for novel features of the observables. The need to include meson degrees of freedom (meson exchange currents) in understanding magnetic observables is apparent. Circumstantial evidence (primarily the failure of the pairwise force assumption to reproduce the triton binding energy) exists for the need to include three-body forces in the picture. Addition of a phenomenological 3BF adjusted to yield the correct trinucleon binding energy improves the agreement between model and experiment for a number of physical observables. However, no consistent model of NN plus NNN forces exists, and the binding energy discrepancy viewed as a fraction of the potential energy in the triton is of the same order as simple estimates of relativistic effects. Further progress in the bound-state problem demands hard work, both precision calculations and precision measurements.

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