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A STATISTICAL MODEL FOR ASSESSING THE RISK OF HAIL DAMAGE
TO ANY GROUND INSTALLATION

Technical Report, June 1978—March 1979

By
Michael Cox
Peter R. Armstrong

MASTER

September 1979

Work Performed Under Contract No. EM-78-C-04-4291

Altas Corporation
Santa Cruz, California

MASTER

U.S. Department of Energy

Solar Energy

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Michael Cox and Peter R. Armstrong

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500 Chestnut Street
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CONTENTS

List of Figures and TABLES	iv
Abstract	1
Introduction	2
1. Available Data	
a. Insurance Data	6
Crop Damage	
Property Damage	
b. Hailday Frequencies	7
c. Hailstone Sizes	10
d. Storm Severity	12
e. Hailstorm Duration	16
2. Previous Work	18
3. Proposed Model	22
APPENDIX A	
DATA EVALUATION	
Hailday Frequencies	27
APPENDIX B	
DATA EVALUATION	
Hailstone Size, Storm Severity	33
4. Conclusions	
a. User's Guide - Interpreting the Results	51
b. Sensitivity Analysis Results	60
c. User's Guide - Utilizing this Study	70
5. Hailstone Terminal Velocity and Kinetic Energy	72
6. Recommendations	81
APPENDIX C	
INDEPENDENCE OF STONE SIZE DISTRIBUTION AND	
STORM SEVERITY	87
APPENDIX D	
FORTRAN SOURCE CODE	91
7. References	105

LIST OF FIGURES AND TABLES

Figures

1 - Number of haildays in an average 20-year period	9
2 - Relative frequency distribution of hailstone size (taken from 1972 NHRE data)	13
3 - Hailstone size distribution (published in Ref. [6]) showing fitted exponentials to 1972 NHRE data	36
4 - Hailstone size distribution (1972 NHRE data) plotted as cumulative frequency. The straight line shows the exponential distribution fit by the authors of Ref. [6]	38
5 - Distribution of total number of stones per storm (vs. number of occurrences) used to characterize storm severity (NHRE data). The curve shows fitted gamma distribution	45
6 - Three cumulative hailstone size distributions gener- ated from three data sets (Colorado, Illinois, Al- berta)	48
7 - A larger view of the three cumulative hailstone size distributions shown in Figure 6, with the raw data extrapolated to larger diameters	49
Predictions of the risk model for . . .	
8 - Albuquerque, New Mexico	55
9 - Abilene, Texas	56
10 - Kansas City, Missouri	57
11 - Cheyenne, Wyoming	58
12 - Predictions of the risk model for values of $K < 210$. All four locations in TABLE 1 are plotted at an ultimate risk = .50	59

13 - Re vs. C_d for smooth spheres, taken from Ref [45]	74
14 - Hailstone terminal velocity vs. size, for three elevations	76
15 - Hailstone terminal kinetic energy vs. size, for three elevations	79

TABLES

1 - Representative Poisson and negative binomial fits to hailday data. These four locations were analyzed in the risk model	62
2 - Probable maximum hailstone size predicted by the risk model for 1 year on 1 m ² (i.e. ultimate risk = .50, A = 1, K = 1). The credibility of these results can be tested against raw NHRE data (discussed in text)	63
3 - Final predictions of the risk model for the four locations analyzed (those shown in TABLE 1), using one of the three size distributions shown in Figure 7, and for various values of A and K (respectively, area of exposure, m ² , and duration of exposure, years)	65

ABSTRACT

This report describes a statistical model which has been developed to determine the risk of damage by large hail to any ground installation (such as a solar flat plate collector). The model is based on data for the number of haildays per year, hailstone size distribution, and storm severity (expressed in number of hailstones per square meter per storm). Other than parameters derived from the raw meteorological data available, the parameters of the model are the number of years of surface exposure and the exposure area size. The end result is the probability of a hailstone of a given size striking a given surface area in a given number of years. The "maximum probable hailstone size" is used as a convenient index of hail risk.

The data upon which to base a prediction model are sparse at this time, covering few geographic locations; much of the information available is deficient in sampling consistency and/or sample size. For this reason, this report fully documents the derivation and use of the model for future applications, when more and better data are collected. The FORTRAN source code to calculate the risk model digitally is included in APPENDIX D. This model improves on previous work in: (1) the use of more thorough statistical procedures and a more rigorous accounting of "storm severity"; (2) a more thorough investigation into the two probability density distributions commonly used (Poisson and negative binomial) to describe hailday frequencies, and (3) an attempt to define more rigorously the distribution of hailstone sizes.

A sensitivity analysis was performed and conclusions are drawn from the results.

INTRODUCTION

Though hail is formed in storms across the United States, large hail is almost exclusively formed in thunderstorms lee of the Rockies, on the Great Plains, and in the midwest. The "hail belt", as these areas are referred to, is where property damage (e.g. solar flat plate collector glazing breakage) is most likely to occur. Although hail the size of golfballs sometimes occurs outside the hail belt (e.g. Los Angeles [43]), such reports are extremely rare, qualifying for front page newspaper headlines. Crop damage can result from storms with many small hailstones, which are observed relatively often even in California, Washington, Florida, and Maine. But, this type of hail is not likely to puncture roofing or break collector glazing. Therefore, the focus of this study has been on the occurrence of large hailstones, which have reportedly punctured cars and killed farm animals.

Thunderstorms form from atmospheric instability, when warm, moist air is forced to rise. Since temperature is colder with altitude, the rising air is even warmer with respect to the environment. It is, as a result, more buoyant, and accelerates as it rises. At the earth's surface, a low pressure zone is created and air is sucked into the developing thunderhead (explaining why winds generally blow in the direction of an approaching storm).

In the beginning stages of thunderstorm development, virtually all local air movement is upward. When the initial updrafts reach heights where they are no longer warmer than the surrounding air, their ascent slows and eventually stops. Water condenses at these high altitudes and a "cap" thus forms on the thunderhead.

The developing storm at this point can be observed from the ground, but precipitation has not yet begun. Water droplets formed in the cloud are supported by the updrafts and coalesce to larger sizes. Hail is formed when particulate matter is carried through a "supercooled" region of the thunderhead. In this region, "supercooled" water droplets (as cold as -4°F) are in liquid state, and readily adhere to "ice nuclei" and freeze [35]. Eventually, the weight of precipitation is too great to be

supported by the updrafts, and torrential rains and/or hail fall.

Since evaporation of this downpour cools the air, it becomes heavier with respect to the environment and strong downdrafts develop. At this stage of thunderstorm evolution, up and downdrafts are strongest. If a growing hailstone is blown from an updraft to a downdraft (say, by strong horizontal gusts) and is then recaptured by the updraft, it can be carried through the supercooled region of the cloud again, accreting more and more layers of ice. In this way, hailstones can be "bounced" up and down in a thunderhead many times, until they are heavier than can be supported by the updraft. As long as the fallspeed of a given stone is less than the updraft velocity, it will be supported and can continue to accrete layers of ice and grow. It is interesting to note that, twice in this century, hailstones 12 to 13 cm in diameter have been discovered, indicating updrafts of over 75 m/sec (which correspond to a Mach number of 0.2).

From this model of thunderstorm development, it is clear that a number of factors directly affect the size of hail that will be formed. For example, the steeper the "lapse" rate (the steeper the temperature differential with altitude), the faster will be the updrafts generated. More hailstones will therefore be supported longer and will reach larger sizes before falling. Another factor affecting hail size is the vertical extent of the thunderhead. A taller storm potentially allows a longer traverse for a growing hailstone. Thunderstorms in the "hail belt" often extend to altitudes above 12 km.

Three types of thunderstorms produce hail, and it might be argued that the hail in each case has different characteristics (such as the distribution of sizes). This will be discussed further in Section 1.c and APPENDIX B. Possibly the most severe of these is the "frontal" thunderstorm, which sometimes consists of a line of storm cells hundreds of miles long. At one time, these storms were quite hazardous to transcontinental flights because of the strong up and downdrafts, powerful horizontal gusts, and hail encountered. Frontal storms result when either a warm, moist air mass meets colder air and rises (warm front), or when cold wind undercuts a warm air mass, forcing it aloft (cold front). Two

fronts need not meet at ground level. Air masses can meet at higher altitudes, and are likely to produce taller thunderheads [35].

Orographic (mountain effect) storms result when warm, moist air blows into a mountain range and is forced to rise. This effect can be seen on the slopes and up to 200 miles east of the Rockies.

The "convictional" or "thermal" thunderstorm is a result of local surface heating. On warm, (generally summer) days, moist air close to the earth is often heated by thermal loss from the ground. The air naturally rises, encountering colder air above.

With the foregoing picture of hail formation, what is the risk of encountering a specified large (potentially damaging) hailstone? Immediately, it is clear that we must be more specific. The three types of storms occur with differing frequencies in different locations; and, if they do have varying characteristics (such as the size hail that is formed), that must be accounted for. Furthermore, variability is found in hail from location to location, regardless of the storm type [24,27]. With very detailed information--say, measurements of hailstone size from many storms in a given location--it would be possible to determine an average distribution of size per storm (in that location). Then, knowing the number of haildays that could be expected per year, it would be possible to determine the frequency of various sizes of stones in a typical year (or over a period of years). One crucial parameter remains--that storms occur with varying severity.* Since size distributions are most convenient when normalized with respect to the total number of stones counted, "storm severity" is not inherently measured in the data, and must be included as a separate parameter.

The approach in this model has been to "sift" through the avail-

*Severity can be measured in a number of ways: using storm duration as an index, or number of stones falling per m^2 per minute, or, as has been done in this study, total number of stones per m^2 counted per storm.

able and most reliable data (on distribution of size, number of haildays per year, and relative distribution of storm severity) to formulate a model which predicts the risk of encountering a specified large hailstone. Ideally, one would have records for each location under consideration and apply in each case the statistical procedures outlined in this report. However, such detailed records are not available, and it has been necessary to assume that the distributions of: (1) storm severity, and to a lesser extent, (2) hailstone size, are invariant with respect to geographic location. It is recognized that this not a completely accurate picture of hail, and if this model is applied to new locations (or if predictions in new locations are based on the results in this report), the validity of this assumption should be checked with local meteorologists (e.g. weather station personnel). The degree to which this assumption does not hold will be one degree of bias in the risk model's predictions.

Since hailfall is a geographically variable phenomenon, any accounting of local hail characteristics will increase the accuracy of a risk model's predictions. One input parameter of the model described in this report does vary with geographic location--the distribution of number of haildays per year. This parameter alone is probably insufficient to explain all observed geographic variation in hailfall; in using the model, judgment concerning the other parameters is required.

For example, it hails as often (on the average) in Salt Lake City as it does in Denver. Applying the statistics outlined here would yield roughly the same risk of hail damage in both cities. However, personnel of the Salt Lake City Weather Station report that the vertical extent of thunderstorms in the area is not great enough to produce large hail [41]. Stones greater than 2.5 cm in diameter are extremely rare. Extrapolation of the available stone size and storm severity records would clearly result in an inaccurate picture of the risk of hail damage in the Salt Lake City area.

1. AVAILABLE DATA

a. Insurance Data

To determine the incidence of damaging hail, one approach might be to examine crop and property damage data. This approach circumvents the analysis of hailfall characteristics altogether by studying the incidence of damage itself. The Crop-Hail Insurance Actuarial Association (CHIAA) and various insurance companies and organizations were contacted in a search for available damage statistics.

Crop Damage

At 1968 price levels, it is estimated that close to \$300 million worth of damage is done to crops each year, with the highest losses in the Central Plains of the U.S., followed by heavy losses in the Midwest. Careful records of claims paid for hail loss have been kept for about the past 40 years [25]. Also available are detailed records of premiums charged per \$100.00 of insured value. These reports have been compiled and expressed not only as dollar amounts, but as ratios of claims to premiums charged, and claims to total dollar amount insured; i.e. the figures have been normalized with respect to inflation and regional price differences and can be compared with figures from other years and/or geographical location.

However, there are major problems associated with correlating crop insurance data with possible risk to other susceptible property. Crop damage depends not only on weather related phenomena (such as storm severity, hail size, windspeed, etc.), but also on factors unrelated to hail, such as the plants' maturity when the hailstorm strikes, the type of crop, etc. [28].

Furthermore, crop damage is primarily a function of hailfall density; property damage is more a function of hail size. Small stones in a dense hailfall can be quite detrimental to crops, whereas structural property (buildings, flat plate collectors), with a higher damage threshold, is vulnerable only to larger

hail. For example, even the sturdiest plants can be obliterated by pea-sized hail [3]; golfball-sized hail does not necessarily cause more crop damage. In both cases, the same dollar amount in claims would be paid, and there is no way of determining from the insurance data which crop-damaging storms would also damage a solar collector.

Property Damage

By conservative estimates, \$32 million in property damage is caused each year by hail [26]. Not all of this is insured, but insurance companies have some raw figures on file in the form of individual claim and premium reports. However, presently available reports of property damage compile hail damage (unidentified as such) with other insurance categories, such as "Extended Coverage" "Fire," etc. [14,15,27]; according to all insurance companies contacted in this study, damage reportedly caused by hail is indistinguishable from other reported property damage.

Furthermore, property claims and premiums are tabulated in dollars, with no adjustment made to account for inflation [16,17], so compiled data from different locations and/or years cannot be compared. Also, claims are paid in lieu of the cost of replacement materials, which vary by geographic location. A claim paid in one county is not necessarily equal in damage with an equal claim paid elsewhere in the same year.

Although property damage reports could be a reliable index of possible damage to other ground installations, the data have not been tabulated in a useable fashion. Perhaps insurance reports of hail damage can still be salvaged from the filing cabinets. This is discussed further in Section 6.

b. Hailday Frequencies

The number of haildays in a given location varies from year to year. This is known as the hailday distribution. Clearly, the risk of encountering a damaging hailstone (or any size hailstone for that matter) is related to the number of days of hail, which

weather stations across the country have been recording since 1892*. Until 1934, weather records were published in the annual "Report of the Chief of the Weather Bureau" [20]. Since 1935, the data have been published in the "United States Meteorological Yearbook" [21].

Simplified charts and maps of the mean annual number of haildays are also widely available [e.g. 7,22,37]. An example is shown in Figure 1.

Given the mean annual number of haildays, λ , the probability of encountering x haildays in one year, $f(x)$, has been found by Thom [5] to be described by one of two discrete probability density functions. According to Thom, for a mean annual number of haildays of two or fewer, the Poisson distribution generally provides the better fit. The Poisson distribution is given by

$$f(x) = e^{-\lambda} \frac{\lambda^x}{x!}$$

In applying the Poisson distribution to discrete data, it must be assumed that the events being characterized are independent. For hailday data, this means that the occurrence of a hailday is independent of the occurrence of any other hailday. That is, that the occurrence of hail on any given day is no more or less probable because of the previous occurrence of hail at the same location. Thom hypothesized that for a mean annual number of haildays of two or fewer, the time interval between hailstorms is generally great; hence (Thom hypothesized), the storms are quite likely to be unrelated (i.e. independent) events. This was Thom's explanation for the better fit of the Poisson in these cases.

When the mean is greater than two, the time between hailstorms

*A hailday is defined by the U.S. Weather Bureau as any day on which hail falls on the ground.

FIGURE 1 -- Number of haildays in an average 20-year period.

is reduced accordingly, possibly leading to storm dependence on the same meteorological phenomena [5] (e.g. two hailstorms could result in one place on consecutive days as a result of high ambient temperature). Since virtually all hailfall occurs during the summer, the time between storms is considerably shorter than would appear from the yearly hailday data. If the haildays in a given location are not independent events, another discrete probability density distribution is required. In these cases, the negative binomial (which can be viewed as a compound Poisson, with a gamma distribution ascribed to the mean annual number of haildays [23]) is required. According to Thom, the negative binomial is applicable more often when the mean is greater than two.

The negative binomial probability density function is given by

$$f(x) = \frac{\Gamma(k + x)}{\Gamma(x + 1) \Gamma(k)} \cdot \frac{p^x}{(1 + p)^{k + x}}$$

where

$f(x)$ represents the probability of encountering x haildays in one year.

p and k are parameters of the distribution, derived from values of the sample mean, sample variance, and sample size.

$\Gamma(Q)$ represents a gamma function (defined as $\Gamma(Q) = (Q - 1)!$ for all Q , integer and real).

Both distributions and their application to hailday data are discussed further in APPENDIX A.

c. Hailstone Sizes

Two types of hailstone size data are available. The most widely published information is collected from cooperative observers, who record either the most common and/or maximum stone size during a storm.

More carefully gathered experimental data are also available for three locations in North America: Illinois, northeastern Colorado, and Alberta, Canada. In Illinois and Colorado, hailpads (made of styrofoam mounted horizontally on support stands) were used to record hailstone indentations. In both states, for each storm, the number of hailstones in given diameter intervals was tabulated. In Alberta, farmers kept hailstones frozen until researchers could pick them up.

There have been two major projects at the Illinois State Water Survey that have collected hailstone size data. The project, "Design of an Experiment to Suppress Hail in Illinois (DESH)", was conducted from 1969 - 1972. METROMEX, a study to determine an urban area's effect on precipitation and severe weather phenomena, was conducted from 1971 - 1975. Much of the raw data from these two projects has been punched onto computer cards (10,000 IBM cards from DESH; 4,000 from METROMEX); these are available for a small fee from the Water Survey.* Summary tables of the data are also available in many Survey publications [e.g. 8,9,18, 33].

The Colorado data were gathered in the National Hail Research Experiment (conducted by the National Center for Atmospheric Research in Boulder, Colorado). At present, raw data from 1972 - 1974 are available [6]. Total number and size of stones were tabulated for each hailpad, with a total sample size of 37 storms over the three summers, '72 - '74. In Vol. I of the "Final Report" are summary tables and graphs of the size distributions for each of the three years.

Size data for Alberta, Canada are available in the publications of R.H. Douglas [11,12,13]. Farmers with wire-mesh baskets collected a total of 67 useful stone samples from 1957 - 1963; these samples have been tabulated in histograms and also fit to an exponential of the form:

$$N_D = N_0 \exp(-m D)$$

*Illinois State Water Survey, Box 232, Urbana, Illinois, 61801.

where

N_0 = the ordinate intercept

m = the slope of the curve

D = the stone diameter

It is important to note the problems associated with the size data presently available. The frequency of occurrence of hailstones larger than 2.5 cm is quite low. In fact, very few stones this large have been collected in Colorado, Illinois or Alberta.*

In Figure 2 is shown the relative frequency distribution of 1972 NHRE data. As can be seen, the information on stones larger than 2 cm is scarce (i.e. it is difficult to determine from the available information the relative frequency of damaging hailstones). For this reason, the distribution of large (damaging hailstones) must be extrapolated from the distribution of smaller diameters. This can be done either graphically (extrapolation "by eye") or analytically (e.g. with the fitting of a probability distribution to the data) as was done by Douglas for Alberta hail.

The hailstone size distribution and its application in the risk model are discussed in more detail in APPENDIX B.

d. Storm Severity

So far, two parameters of hail have been discussed which must be considered in developing a risk model.

*A very few stones of up to 5 cm have been collected in Alberta, although the sample size of these stones is so small that random sampling error is significant.

FIGURE 2 -- Relative frequency distribution of hailstone size (taken from 1972 NHRE data)

The annual frequency of haildays in a given location is clearly a factor in predicting the occurrence of large, damaging hailstones; this frequency is not constant from year to year, but varies in a characteristic manner (generally being well described by one of two probability density distributions). Furthermore, the distribution of annual haildays varies with geographic location; this variability can be well characterized by the available data.

The second parameter so far discussed has been the size of hailstones that occurs in a storm. In developing a prediction model, we are interested in the occurrence of large hailstones. However, the frequency of occurrence of these stones must be extrapolated from the data on smaller diameters, since a sufficiently large sample has not been collected to characterize accurately the incidence of large hailstones. Furthermore, the distribution of hailstone size varies to some degree with geographic location, and is also related to the type of storm encountered (e.g. frontal or orographic).

With these two parameters, we might simply calculate the probability of encountering a large (potentially damaging) hailstone in a given period of time (using the hailday distribution for that location). However, if we used this approach, we would have to assume that the probability of encountering a hailstone D_1 cm large is the same in every hailstorm. Such an assumption would be invalid. In some storms, hailfall is more dense (and/or hailfall duration is longer) than in others. That is, the "severity" of hailfall varies from storm to storm in a given location, following a distribution which must be quantified to obtain an accurate picture of hail damage risk. Clearly, the more severe the storm (e.g. the longer the storm lasts, the more hail that falls, etc), the higher the probability of encountering a large, damaging hailstone.*

It should be noted that storm severity is not inherently characterized in the hailstone size distribution previously

*assuming that the hailstone size distribution is independent of the storm severity distribution (discussed further in APPENDIX C).

discussed. The size distribution is normalized with respect to the measures of storm severity mentioned above, and expresses the relative (or cumulative) frequency of hailstones of a given size, regardless of the size of the sample (that is, regardless of the severity, or number, of storms sampled). If one storm is more "severe" than another, the relative frequency of large stones is not greater (as just discussed). However, the absolute frequency of such stones is greater, since there have been more trials in which the event could have occurred (that is, the hailstorm has lasted longer or more hailstones have fallen, etc).

How should storm severity be characterized? This parameter could be measured in a number of ways (a few of which have been mentioned above). The duration of hailfall might be used as an index of storm severity, and it might be assumed that the longer a hailstorm lasts, the more severe it is. Such a distribution of hailfall duration might be derived from the available data; however, these data are quite sparse, and many of the figures available are only crude estimates made by cooperative observers. Furthermore, this approach does not account for the precipitation rate of hail at the ground.

The precipitation rate itself might be used as an index of storm severity. However, very little reliable data exist with which to take this approach. Furthermore, using the precipitation rate as an index does not account for storm duration.

The most accurate and easily characterized measure of storm severity is the total number of stones that falls per storm. The data upon which to base such an approach have been of more interest to previous researchers, and are therefore more readily available. Such data are available from two locations: Colorado (NHRE) and Illinois (DESH & METROMEX). In both studies, the total number of hailstones per hailpad per storm was recorded; these figures can be used to define a distribution of storm severity to be used in the risk model. In this study, only the Colorado NHRE data were evaluated. The acquisition, reduction and analysis of the data from the Illinois hail projects were beyond the present resources.

As will be seen in Section 3, the storm severity parameter used in the proposed risk model must be expressed as a probability density function. That is, storm severity must be expressed as the probability, $P_2(n)$, of encountering "n" number of hailstones in any storm in a given location.

The characterization of storm severity, as measured by total number of hailstones per storm, is discussed further in APPENDIX B. However, to avoid confusion, one further point needs attention. It will be noticed that the data sets from which a storm severity distribution is based are the same as those from which the size distribution is derived. Each of these parameters, however, is based on different information contained in the data. The size distribution uses the data classed by diameter interval and is normalized with respect to the actual number of hailstones and actual number of hailstorms. It represents the relative (or cumulative) frequency of all hailstone sizes, regardless of the number of hailstones or hailstorms sampled.* On the other hand, the storm severity distribution is expressed in terms of the total number of stones (regardless of size) that fall per storm.

e. Hailstorm Duration

Various authors have reported hailstorm durations of from 10 seconds to 45 minutes. Generally, storms last from 1 to 15 minutes, with a median figure of about 5 minutes [24]. A significant point to note is that no research has found any correlation between storm duration and other storm conditions, except that the duration of hailfall seems to vary with geographic location [24]. Specifically, no systematic relationship between hailstorm duration and hailstone size has been noted (this is discussed further in APPENDIX C).

The statistical model proposed in this report is not concerned with the duration of hailstorms. Rather, storm severity (of

*random sampling error would be expected in any sample taken, especially in smaller samples.

which duration might be a measure) has been quantified in this study by the total number of stones that fall per storm.

2. PREVIOUS WORK

Most hail work has been either basic theoretical research (e.g. use of hailfall characteristics to model thunderstorm development) or practical research associated with attempts at weather modification. Very little work has been done to evaluate the risk of hail damage in establishing design criteria.

Irving Gringorten, in a study for the Air Force, arrived at tentative figures for the risk of encountering damaging hailstones under various conditions [30]. For ground installations, Gringorten determined the probability of encountering a damaging hailstone in one year. First, by combining many observer reports and experimental data, he arrived at an empirical cumulative distribution of the probability of encountering hailstones of a given size in a randomly chosen storm. Then, using a Poisson distribution to describe the probability of encountering (x) storms, he arrived at the composite probability that the largest stone encountered in one year would be D cm in diameter (given any x)*

$$\sum_x P_1(\leq D)^x \cdot P_3(x)$$

Eqn. 2.1

where

$P_3(x)$ = the probability of encountering x storms in one year

$P_1(\leq D)$ = the probability that a size D hailstone will be the largest stone encountered in one year.

* $P_1(\leq D)$ in this report represents the cumulative probability distribution of hailstone size; $P_3(x)$ represents the yearly distribution of haildays. Gringorten used a different notation, $P(h/H)$ representing the former, and $p(h)$ representing the latter.

Gringorten's study did not investigate the use of other distributions to describe hailday data; nor did it account for a hailstorm severity variation in a given location; nor did it consider the possibility of location-dependent size distributions.

Gonzales, using Gringorten's model, included a term for what he called "areal density" [4] (which this report refers to as "storm severity"). He began by assuming that the "probability of being struck by a hailstone" is given by a Poisson distribution. Given this, he assumed that the average number of impacts in a storm, λ , of hailstones of diameter D on area A is a function of the "density" of hailfall

$$\lambda = A \cdot \text{XMD}(D)$$

where

A = the area susceptible to hail

XMD = the "areal density" of stones of diameter D (i.e. number of hailstones of size D per unit area).

λ was then taken as the mean of the Poisson distribution described above, the resulting function being the probability of being struck by a size D hailstone "m" times

$$f(m) = e^{-\frac{(A \cdot \text{XMD}(D)) \cdot (A \cdot \text{XMD}(D))^m}{m!}}$$

Gonzales then claimed that the probability of being hit at least once by a hailstone D cm in diameter is given as

$$P(A, \text{XMD}) = 1 - e^{-(A \cdot \text{XMD}(D))} \quad \text{Eqn. 2.2}$$

based on 1 minus the probability of no hits of a size D hailstone on area A (i.e. $1 - f(0)$). Gonzales then used a "piggy-back" approach, multiplying Eqn. 2.2 by a modified version of Gringorten's model, Eqn. 2.1, arriving at an "ultimate" probability of

encountering a hailstone D cm or larger on area A in K years,* given the input variable, XMD(D)

$$1 - \left(\sum_x P_1(<D)^x \cdot P_3(x) \right)^K \left(1 - e^{-(A \cdot XMD(D))} \right) \quad \text{Eqn. 2.3}$$

Using values for the mean and maximum areal density per storm, $\overline{XMD}$ and $XMD_{\max}$ (i.e. the input variable), Gonzales calculated ranges of hail damage risk. The reader is referred to [4] for further reference.

There are some drawbacks to this model that should be noted. One problem is the large variation in ultimate risk found when using the mean and maximum figures for areal density ($\overline{XMD}$ and $XMD_{\max}$); that is, the distribution of areal density in a given location is not taken into account.

It should be noted that in Gonzales' report, XMD(D) must have been meant to be XMD($\geq D$), since XMD(D) has no statistical meaning. The original size data from which this parameter was derived were organized into histogram cells of given width. XMD(D) can not be determined for any given "D", but only for $D_i \leq D \leq D_j$, or $D \leq D_i$, or $D \geq D_i$. Furthermore, since Gonzales' ultimate probability (calculated from the risk model) is that of a hailstone D cm or larger, the input parameter XMD(D) must be XMD($\geq D$) to be consistent with these results.

More basic questions arise when Eqn. 2.3 is applied to cases of possible hail damage. It can be seen in Eqn. 2.2 that as A and/or XMD($\geq D$) increase, the probability of encountering a hailstone $\geq D$ cm in diameter is effectively "1". This means that in Eqn. 2.3, the term represented by Eqn. 2.2 is effectively

*for the integration of this parameter into Gonzales' model, the reader is referred to Ref. [4].

"1". As a result, for larger values of A or $XMD(\geq D)$, Gonzales' risk model's predictions depend exclusively on the hailstone size distribution and the hailday distribution. In other words, as the area of exposure and the severity of hailfall increase, the model does not predict that large stones will be encountered more frequently. This is clearly not a valid prediction, since a larger surface area and more severe storms statistically serve to increase the number of trials with which to encounter a larger hailstone. It can only be concluded that the statistical model in Eqn. 2.3 does not accurately predict hail damage risk in the extremes of calculations.

Since in this study we are interested in accurately characterizing the occurrence of severe storms (and the correspondingly higher incidence of large hailstones) and since solar flat plate collectors are commonly $> 10 \text{ m}^2$ (that is, a large surface area) another approach must be taken.

Elsewhere, researchers involved in testing the hail resistance of commercial products have treated hail risk either by simply assuming "worst cases" of hailstone size and frequency [e.g. 1,2], or by slightly modifying Gringorten's initial research [e.g. 32].

3. PROPOSED MODEL

A viable hail prediction model should consider the three factors discussed in Section 1 which relate directly to the size of hail likely to be encountered. These are again, (1) the distribution of stone size; (2) the distribution of number of haildays per year and, (3) the distribution of storm severity (measured by hailfall density). The ultimate risk of encountering a large hailstone (which we define as the probability, as calculated by the risk model, of encountering a $> D$ cm hailstone) is also a function of the surface area exposed to hail and of the duration of exposure. These two parameters must also be taken into account.

A rather confusing, but unavoidable feature of the proposed model should be noted at the outset. The risk model is first derived to predict the probability of encountering only stones of diameter D or smaller (given that hail has fallen). The alternate probability, that of encountering at least one hailstone of size D or larger, is "one minus the probability of encountering stones smaller than D ".

Now, the cumulative probability that a randomly selected stone will be of diameter " D " or smaller can be expressed as

$$P_1(\leq D)$$

and the probability that " n " hailstones (per m^2) will fall in a single storm can be expressed as

$$P_2(n)$$

Given that a hailstorm occurs, the probability that n stones will fall (on $1 m^2$) and that all of the n stones will be of size D cm or smaller is thus

$$P_2(n) \cdot P_1(\leq D)^n$$

In other words, the probability, $P_1(\leq D)$, of randomly selecting a stone $\leq D$ in one trial is raised to the power of the number of trials, n ; the result is multiplied by the probability, P_2 , that a sample of n stones will fall.

Given a surface, $A \text{ m}^2$, exposed to a hailstorm, the probability that $n \cdot A$ stones will fall on that surface, and that all stones will be D cm or smaller is

$$P_1(\leq D)^{n \cdot A} \cdot P_2(n) \quad \text{Eqn. 3.1}$$

The composite probability, F , of encountering only stones smaller than or equal to D cm on area A in a randomly chosen storm (regardless of the number of hailstones that fall) is

$$\begin{aligned} F(\leq D) &= [P_2(1) \cdot P_1(\leq D)^{1 \cdot A}] + [P_2(2) \cdot P_1(\leq D)^{2 \cdot A}] \\ &\quad + [P_2(3) \cdot P_1(\leq D)^{3 \cdot A}] + \dots + [P_2(N) \cdot P_1(\leq D)^{N \cdot A}] \\ &= \sum_{n=1}^N P_2(n) \cdot P_1(\leq D)^{n \cdot A} \quad \text{Eqn. 3.2} \end{aligned}$$

where

n = the number of stones per storm on 1 m^2

N = the maximum number of stones per storm observed in any storm of the sample (that is, $P_2 = 0$ for $n > N$)

An assumption inherent in this approach is that the size distribution (expressed by $P_1(\leq D)$) and the number of stones per storm, n , are independent. If this is not true, $P_1(\leq D)$ will change with each n as Eqn. 3.2 is summed for all n . There is evidence that this is a valid assumption, although the available data are insufficient to offer a rigorous proof (see APPENDIX C).

$F(\leq D)$ defined above is the probability of encountering only stones of size D or smaller on area A in one hailstorm. The probability of experiencing a year in which x storms occur and and in which only stones of diameter D cm or smaller are encountered on area A is thus

$$P_3(x) \cdot F(\leq D)^x$$

where

$P_3(x)$ = the probability of encountering x hailstorms in one year.

That is, this is the probability of experiencing x storms multiplied by the probability of encountering only stones $\leq D$.

The probability of encountering stones only $\leq D$ in any year, regardless of the number of storms (any x) is*

$$G(\leq D) = \sum_{x=0}^X P_3(x) \cdot F(\leq D)^x \quad \text{Eqn. 3.3}$$

where

*Note that Eqn. 3.3 was derived in the same manner as was Eqn. 3.2. Given two events, Y and Z , where Z occurs only if Y occurs and each occurrence of Y represents a trial event for Z , the probability of encountering Z given Y_1 is

$$P_2(Y_1) \cdot P_1(Z)^{Y_1}$$

The probability of encountering Z , given any possible value of Y , is therefore given as

$$\sum_Y P_2(Y) \cdot P_1(Z)^Y$$

An assumption inherent in this approach is that Z and Y are independent events. That is, that the value of $P_1(Z)$ does not change with either Y or $P_2(Y)$.

This statistical operation can be considered a distribution in itself, and has been referred to as a "contagious" distribution [23].

X = the maximum number of storms observed in one year
(that is, $P_3 = 0$ for $x > X$; this is effectively the case
in most locations when $X = 20$)

This is the probability that all hailstones encountered on area A
in one year will be smaller than or equal to D cm.

The probability, therefore, of encountering only hailstones $\leq D$ on
area A in K years is given as

$$G(\leq D)^K$$

based on the probability of encountering hailstones of this size
in one year, raised to the power of the number of trials, i.e.
number of years. This operation is justified by the nature of
the hailday distribution, $P_3(x)$, which describes, by definition,
the probability of encountering x days of hail in any one year.
Also note that an inherent assumption is that the occurrence of
hail each year is an independent event. Since hail occurs
primarily during the summer, hailstorms are separated each year
by relatively long time periods; independence is likely a valid
assumption.

We now have an expression for the probability of encountering
only hailstones $\leq D$ cm on a given surface area in a given number
of years. Finally, the alternate probability, that of encounter-
ing at least one stone larger than size D in K years on area A is
simply

$$P(>D, A, K) = 1 - G(\leq D)^K$$

The proposed risk model, then, expressed in terms of the three
parameters, $P_1(\leq D)$ (distribution of hailstone size), $P_2(n)$
(distribution of hailstones/m² per storm -- storm severity) and
 $P_3(x)$ (distribution of annual number of haildays) is

$$P(>D, A, K) = 1 - \left[\sum_{x=0}^X P_3(x) \cdot \left(\sum_{n=1}^N P_1(\leq D)^n \cdot A \cdot P_2(n) \right)^x \right]^K \quad \text{Eqn. 3.4}$$

where

A = the surface area exposed to hail, m^2 .

K = the number of years of exposure to hail.

$P_1(\leq D)$ is discussed in Section 1.c. and in APPENDIX B.

$P_2(n)$ is discussed in Sections 1.d. and in APPENDIX B.

$P_3(x)$ is discussed in Sections 1.b. and in APPENDIX A.

The results of a sensitivity analysis, and some guides to using the risk model appear in Section 4.

APPENDIX A

DATA EVALUATION

Hailday Frequencies

The two distributions which have been found to describe yearly hailstorm frequencies are the Poisson and the negative binomial. With the Poisson, the mean, μ , is equal to the standard deviation, σ^2 . The negative binomial allows another degree of freedom, not requiring $\mu = \sigma^2$.

The two parameters, p and k , of the negative binomial are related to the mean and variance of the hailday distribution and, given a sample mean, $\bar{x}$, and sample variance, s^2 , can be approximated using the "method of moments"

$$p = \frac{s^2 - \bar{x}}{\bar{x}}$$

and

$$k = \frac{\bar{x}^2}{s^2 - \bar{x}}$$

However, the method of moments is not always a satisfactory estimate of the parameters. p and k can first be calculated using moments, and then tested with the following

$$C = (1 + 1/p)(k + 2)$$

If $C \geq 20$, then the moment estimates are satisfactory.

If $C < 20$, maximum likelihood estimate must be used.

The latter estimate consists of maximizing the likelihood equation

$$M = \prod_{i=1}^m f(x_i, p, k)$$

where

$f(x, p, k)$ = the negative binomial distribution.

m = number of years of hailday observations

The first condition for a maximum is obtained by differentiating the logarithm of M with respect to p, giving

$$L_1 = \frac{m\bar{x}}{p} - \frac{mk + m\bar{x}}{1 + p} = 0$$

Substituting and simplifying, the first likelihood equation becomes

$$\bar{x} = kp \quad \text{Eqn. A.1}$$

Setting the partial derivative of the logarithm of the likelihood function with respect to k equal to zero gives the second necessary condition for a maximum,

$$L_2 = \sum_{i=1}^m \psi(x_i + k) - m \psi(k) - m \log(1 + \bar{x}/k) = 0 \quad \text{Eqn. A.2}$$

where

$\psi(Q)$ represents a Psi function, which is the first derivative of the gamma function [i.e. $\psi(Q) = \Gamma'(Q)$]

$\psi'(Q)$ represents the second derivative of the gamma function [i.e. $\psi'(Q) = \Gamma''(Q)$]

To solve L_2 , a Newton approximation is used, which sets a Taylor expansion (truncated to two terms) of the function equal to zero. Taking the second partial of M with respect to k,

$$L_{22} = \sum_{i=1}^m \psi'(x_i + k) - m \psi'(k) + m \bar{x}/k(k + \bar{x})$$

set

$$L_2 + hL_{22} = 0$$

or

$$h = -L_2/L_{22}$$

The parameters p and k are first calculated using the method of moments to provide an initial approximation. The parameter k is then used in the maximum likelihood estimate to calculate h. A new approximation to k is then k_{new}

$$k_{\text{new}} = k + h$$

Eqn. A.1 is used to calculate p .

The approximation is repeated until h is sufficiently small (i.e. until L_2 in Eqn. A.2 converges). With parameters p and k in hand, the negative binomial can then be calculated for any x . For computer applications, the source code to calculate $\Gamma(Q)$, $\Psi(Q)$ and $\Psi'(Q)$ is included in APPENDIX D. However, evaluation of these functions can be done by hand; tables of values of Gamma and both Psi functions can be found in Ref. [36].

The Poisson distribution is more easily calculated than the negative binomial. It is defined simply by one parameter, λ , the mean of the distribution. One needs only a map or tables of the mean annual number of haildays (available, for example, in Refs. [7,37]) to calculate this probability density function for any location.

Because of this ease of calculation, previous authors have chosen to use the Poisson rather than the negative binomial to calculate hailday frequencies. However, the Poisson does not represent the stochastic processes of hail in all cases. As we shall see, when $\bar{x} \neq s^2$, illegitimate use of the Poisson can lead to large errors in ultimate risk estimates.

On the other hand, calculation of the negative binomial is more difficult; the values of the mean, sample variance, sample size, and maximum observed number of haildays are required. Unfortunately, no publications at present contain a listing of all these parameters for various locations. One must gather the raw data from numerous National Weather Service annual reports.

In utilizing hailday data in the risk model (Eqn. 3.4) the adequacy of the distribution chosen to represent the actual stochastic processes is an important factor in the accuracy of results. This study is concerned with large hailstones whose frequency of occurrence is small. The more accurately a function accounts for the "tails" of the hailday distribution, the more accurately the statistical risk model will account for the "tails" of large hail distribution.

In this study, the adequacy of the Poisson and the negative binomial to describe yearly hailday frequencies was examined. The objective was to determine under what circumstances each distribution is more applicable (and therefore most likely to give accurate results). With the data collected from Refs. [20] and [21], and the data from Ref. [5], Poisson and negative binomial fits to hailday data from 25 weather stations were calculated. A Chi Square Goodness of Fit test was applied to the results to determine the adequacy of the fit in each case. Generally, histogram cells with an expected frequency of five or less are combined to ameliorate the effect of random sampling error. However, cells with an expected frequency of as small as two have been shown to give valid results in Chi Square tests [23]. This latter procedure was followed.

In the first application of discrete distributions to hailday data, Thom found the mean annual number of haildays to be the most important factor in choosing a distribution [5]. Thom found that the Poisson was usually applicable when the mean was two or fewer haildays per year; when the mean number of haildays was greater than two, a negative binomial generally provided the better fit.

Changnon and Schickedanz [18] applied both distributions to data from Illinois and found that summer data were best fit by the negative binomial, annual data were best fit by the Poisson. This is reasonable since hail occurs almost exclusively during the summer. In this shorter time span, an equivalent number of hailstorms would be less likely to be independent events. However, findings for Illinois annual data are consistent with Thom's hypothesis--in almost all cases in the Illinois study, the mean annual number of haildays (at a given location) was fewer than two.

For our study, annual data were analyzed. In 15 percent of the samples, both distributions failed to yield good fits (with significance levels $< .05$). There seemed to be no correlation between geographic location and poorness of fit. The inadequacy of the two distributions might be due to the "roughness" of the

samples, which might in turn be due to limited sample size. The sample size for most stations was between 45 and 55 years. Is this sufficiently large? A Weather Bureau analysis [22] has reported that the mean annual number of haildays for a location becomes quite consistent with a sample size of 10 or larger. The sample variance may or may not stabilize at such a sample size, perhaps explaining the inadequacy of the negative binomial fit. But what of the Poisson?

It might be that these samples have been contaminated by a change in the Weather Station personnel that recorded haildays. Or, perhaps the parent population from which each sample is drawn is not adequately described by either distribution.

It is of interest to note that many of the samples (even some marginally fit by one or both distributions) suspiciously contained two maxima, suggesting possibly two parent populations. This might, again, be due to inconsistencies in sampling technique. However, the statistical analysis described above does not consider any periodicity in hailday occurrence; and, perhaps haildays occur in some long-term cyclical fashion. However, further analysis of this point is beyond the resources of this study.

An arithmetical problem arose in calculation of some of the samples. In 20 percent of the hailday data sets analyzed, the negative binomial distribution could not be calculated for the sample (due to a negative argument to the gamma function). For three of these, the Poisson distribution provided an adequate fit.

In roughly 30 percent of the samples, both distributions fit the data equally well (with differences in significance of $<.15$).

In the remaining cases (35 percent of the samples), the negative binomial provided a significantly better fit twice as often as did the Poisson. This is intuitively reasonable since the negative binomial is a more flexible distribution than is the Poisson, which is constrained to $\mu = \sigma^2$. In many of the samples s^2 was significantly greater than $\bar{x}$

Finally, it can be noted that if the negative binomial was found to be inadequate, the Poisson was also found to be inadequate. In cases where the Poisson provided the better fit, it was only a matter of degree of significance.

The distribution that provided the better fit to a given set of data did not depend just on the mean number of haildays, as postulated by Thom, but rather on a combination of factors, one of which was the ratio between the sample variance and the mean. For cases in which the sample variance was considerably larger (e.g. $s^2 > 1.5 \times \bar{x}$) than the mean, the negative binomial almost always provided the only fit, regardless of the magnitude of the mean. And, the smaller the difference between the mean and the sample variance, the better the fit of the Poisson. For samples where $\bar{x} \approx s^2$, the Poisson generally provided the better fit.

Four representative samples were chosen to perform a sensitivity analysis in the model. Some stations were eliminated on the basis of poor distribution fits. Some stations were eliminated because the size hail encountered is generally small (e.g. stations on the periphery of the "hail belt", such as Michigan [39]).

The samples finally chosen represent various mean annual numbers of haildays between 2.33 and 8.68. Unfortunately, no samples with $\bar{x}$ between 5 and 8 were adequately fit by either distribution, so there is a gap here in the sensitivity analysis described in Section 4. However, note that in Figure 1 there are few locations in North America that encounter a mean number of haildays of greater than 5.

APPENDIX B

DATA EVALUATION

Hailstone Size, Storm Severity

Hailstone Size

Of the two types of data available, observer reports and hailpad records, the latter were used to determine a distribution of hailstone size in this study. Though observer reports are more numerous than the experimental records, they are far more questionable [34,38].*

Furthermore, observers have generally recorded only the "most common" stone size and/or "maximum" stone size. That is, only the median and/or maximum of the sampled distribution for each storm have been recorded, and information on other hail sizes has been lost. For these reasons, this study has utilized only data from hailpad experiments.

As mentioned before, to determine the incidence of large hail across the country, one would ideally have a distribution of size for each geographic location under consideration. Since such detailed records do not exist, it is necessary to assume that the hailstone size distribution is not widely variant from location to location (or at least between locations where large hail is a problem).

Figure 2 was referred to in Section 1.c. to show that there is very little information to determine the incidence of large hail. Note that the frequency of stones > 2.0 cm is quite low and that there are no data on hail > 3 cm. For this reason, the frequency of large stones must be extrapolated from the information available on smaller hail. This can be done either graphically ("by eye") or by fitting an analytical function to the data

*Should the use of observer reports be desirable in spite of the difficulties, Changnon has developed a technique (not pursued in this report) for validating observer reports with hailpad records [34].

points. Both approaches have been used in this study, with the plotted size distributions shown in Figures 6 and 7.* The three are plotted not as relative frequencies, but as cumulative probabilities. The ordinate value corresponding to a size D_i hailstone is the parameter $P_1(\leq D_i)$ used in the model (Eqn. 3.2). The value of $P_1(\leq D_i)$ taken from Figure 6 or 7 is used directly in the model to calculate hail risk. The ultimate risk (i.e. final prediction from the risk model calculations) however, is not that of a stone of size D_i or smaller, but rather, of a stone D_i cm or larger $[1 - P_1(\leq D)]$. This is discussed further in Section 4.

Two of the cumulative size distributions (Alberta and Colorado) were collected just east of the Rocky Mountains, within the range of orographically-caused thunderstorms. The third distribution is from an area where storms are primarily frontal. Although some authors [e.g. 38] have postulated that mountain thunderstorms produce more severe hail than do the thermal or frontal type, that is not substantiated by the data, and, in fact, other authors have stated otherwise [e.g. 30].

Each of the three distributions in Figures 6 and 7 is based on a sample size of about 200,000 hailstones. The relative cumulative frequency of stones $D \leq 3.5$ cm is, from Figure 6 about .9999999 (using the Colorado distribution); in other words, roughly 1 in every 10,000,000 stones is 3.5 cm or larger. Clearly, a sample of <200,000 would most likely not contain such a hailstone. That stones of this size (and larger) do exist is well documented [e.g. 1,2,24,30] with two cases of 5 inch stones reported in the literature [24].

Over long time periods and for large surface areas exposed to the environment, large hailstone encounters are undoubtedly more likely. "How likely" is the question that is addressed by the proposed risk model. That is, what is the ultimate risk of encountering large hailstones over long time periods and on large surface areas? As noted above, the occurrence of a very large

*Note that the plots are on semilogarithmic paper. This is to distinguish between data points differing by small absolute values, but differing relatively by as much as an order of magnitude.

hailstone, (say, 3.5 cm) is a rare event; exactly how rare is not adequately measured by the hailstone samples so far gathered. So, the frequency of these larger stones must be extrapolated from the distribution of smaller hail. Two approaches to this problem are immediately evident. The plotted data might be graphically extrapolated ("by eye") to larger sizes, as was done in Refs. [4,30]. Or, an analytical function can be fit to the data points, and frequencies of larger stones directly computed.

The distribution most commonly fit to the size data has been an exponential [6,11,12,13]. An example of such a fit is shown in Figure 3.*

Note that the exponential does not correspond well with data points of stones greater than about 1.5 cm. The authors acknowledge this and explain that the plot on semilog paper exaggerates this effect, that the absolute difference between the data and the fit is no more significant in this region than at smaller stone sizes. (For example, an order of magnitude difference in frequency at these larger diameters, visually significant on semilog paper, may be the difference between .001 and .0001). However the "dropping-off" observed in the raw data appears in all of the NHRE data ('72, '73, '74) and also in the Illinois data, and appears to be a real characteristic of the parent populations from which the samples were taken. Unfortunately, the Alberta data are not published in a form that this trend can be evaluated.

The authors who examined the data from the NHRE were not concerned with the incidence of large stones. In choosing an analytical function to describe the data, they were interested in

*Two sets of data and two exponentials are plotted, one for "seeded" and one for "not seeded" haildays. These two forms relate to weather modification in the Colorado experiment. Since the authors of Ref. [6] found no significant difference between the size distributions of "seeded" and "not-seeded" days, the distinction between the samples is disregarded in this report.

FIGURE 3 -- Size distribution (published in Ref. [6]) showing fitted exponentials to 1972 NHRE data. Note that there is little difference between seeded and not-seeded data. Note also that the exponentials appear to overestimate the occurrence of large hail. These points are discussed in the text.

characterizing the distribution of smaller diameters (that is, characterizing the bulk of the data). In this study, we are interested in the occurrence of large hailstones, and are therefore interested in an analytical function that adequately describes their frequency of occurrence. That (in the four samples) an exponential always overestimates the occurrence of large stones casts doubt on its use to characterize this region of the size distribution. In fact, an empirical check of the plotted cumulative frequencies (one example is shown in Figure 4) suggests that there are two regimes of the hailstone size distribution. One extends to about 1.3 cm, at which point a transition seems to occur. Hailstones larger than this seem to follow a distribution of a different form.*

Such a transition has not previously been postulated. However, there is ample precedent for the hypothesis. For example, the behavior of fluid flow around an object can be observed to be a function primarily of flow speed and of the size of the object. At low speeds, the fluid flows evenly. At higher speeds, the flow becomes more turbulent behind the object and exhibits quite different behavior. The former is laminar flow, the latter turbulent flow. The speed at which the transition from one to the other occurs is a function of the characteristics of the fluid, and of the object itself. In this connection, it is interesting to note that most authors have treated hailstone growth primarily as a fluid flow problem. Hailstone fallspeeds abruptly increase when diameters of about 6.3 - 7.6 cm are exceeded; this is due to a transition from laminar to turbulent flow.

That this might result in a transition in size distribution at 6.3 - 7.6 cm would not be surprising (though it is unknown whether this actually occurs). A transition in distribution at about 1.5 cm is not at all inconceivable, though an underlying causal mechanism is not proposed here.

*(Though the transition occurs in 1972 at 1.3 cm, other NHRE years and the Illinois data seem to show a transition at about 1.5 to 1.8 cm).

FIGURE 4 -- Hailstone size distribution (1972 NHRE data) plotted as cumulative frequency. The straight line shows the exponential distribution fit by the authors of Ref. [6].

Unfortunately, there is much less information on stones larger than 1.5 cm than there is on smaller hail. The form of the distribution below this point is reasonably clear; the form above is ambiguous, making an extrapolation difficult. Even if there were conclusive evidence for the transition proposed above, it would be difficult to justify fitting two analytical distributions with so little data. There is simply not enough information on which to base a separate distribution of larger hailstones. The sample size of these stones is quite small. A separate function could not be fit with any assurance of accuracy; only one function can realistically be fit.

What functional form should be used? This is a critical point since the chosen function will be used to extrapolate the occurrence of stones of larger size, and since the data upon which the analytical function is based is quite limited.

A fitted exponential all but disregards points beyond the transition and might very likely not even approximate the occurrence of larger stones. Furthermore, an exponential distribution has no upper bound; using this function, there is a finite probability of encountering hailstones the size of basketballs. (Needless to say, hail this size has been neither observed nor theoretically proposed). The knowledge of a maximum possible stone size is potentially valuable information in fitting an analytical distribution to the data, adding, in a sense, an additional constraint to the curve (i.e. such that the function of cumulative probability intersects the point $(D_{\max}, 1)$). The maximum calculated theoretical size [44] fortunately agrees quite well with literature reports of the largest hail [24]. The maximum possible hailstone diameter can thus be assumed to be about 12 - 13 cm.

One last point needs to be mentioned. In Alberta and Colorado, few or no stones were recorded smaller than about .25 cm. Douglas postulated that the lack of these stones in his samples was due to melting before collection. In Colorado, it was postulated that the stones were too small to be recorded by the hailpads

used.* In Illinois, hailstones were sorted into histograms of 1/8" diameter intervals. The smallest stones were included in one cell with midpoint 1/8", so it cannot be determined to what extent small hailstones are missing.

A hailday is defined by the U.S. Weather Bureau as any day on which hail falls. The approach of this study can be seen as first determining the frequency of large hailstones per hailstorm (Eqn. 3.2), and then, determining with what frequency hailstorms can be expected (Eqn. 3.3). Inevitably included in the National Weather Service records are days on which very little hail fell; and, inevitably, on some of these days only small hail fell. It is important that the hailday distribution and the size distribution be based on the same sampling biases. For example, if the hailday distribution includes days on which only small hail fell, and the size distribution does not include the frequency of smaller stones, the risk model utilizing these two distributions will give inaccurate results. It might very well overestimate the occurrence of large hailstones. Any approach to determine (analytically or otherwise) the distribution of stone size should consider the occurrence of small hail not sampled by the experimental techniques presently in use.

In this study, the three sets of available data were evaluated separately. For Colorado, the raw data available in Refs. [6] were used; for Illinois, reduced data (available in Refs. [8,9,18,29, etc.]) were used. For Alberta, Douglas' publications contain only histograms of the reduced data, and the form of a fitted exponential. These can be found in Refs. [11,12,13].

o NHRE Data

From 1972 - 1974, a total of 1250 hailpads in a total of 37 storms were struck with hail, registering about 150,000 hailstone indentations.

*In both cases, authors recognized that small hail (<.25 cm) may not commonly fall, and that the lack of these stones may, in fact, not be an artifact of the method of collection. However, in both cases, this was not deemed likely.

Different diameter categories were used to tabulate the data in each of the three years, and a combined histogram for all three years would be cumbersome at best. Initially, separate histograms of the three years were compared graphically. The 1974 distribution departed substantially from the distributions of the other two years. This is understandable in light of the sampling problems discussed in the Final Report of NHRE [6]. In 1974 new equipment was used to record and measure hailstones. The new devices failed often, and the hailpads for that year were less sensitive to smaller stones than were the pads from the previous two years [6]. The graphs from '72 and '73, on the other hand, were quite similar, exhibiting similar "transition" points (as discussed above). Consequently, 1974 data were disregarded in this study (losing only about 10,000 hailstones from the sample).

A special case of the beta distribution having a probability density of the form

$$p(x) = (n + 1) (n + 2) (1 - x)^n x \quad \text{Eqn. B.1}$$

where

$$0 \leq x \leq 1$$

was chosen to represent the remaining data. The beta distribution is well known for representing natural phenomena with upper and lower bounds.

In fitting a continuous probability distribution to discrete data, the integral of the chosen function must be fit to the cumulative frequency of the observed data. Integrating Eqn. B.1, we have

$$P(x) = (n + 1)(1 - x)^{n+2} - (n + 2)(1 - x)^{n+1} + C \quad \text{Eqn. B.2}$$

The constant of integration must be $C = 1$ to give $P(0) = 0$ and $P(1) = 1$ as required by all cumulative distributions. The lower and upper bounds of hailstone size are 0 and $D_{\max}$, corresponding with $x = 0$ and $x = 1$ respectively, as required by the beta distribution.

Therefore,

$$x = \frac{D}{D_{\max}}$$

where

D = stone diameter

$D_{\max}$ = maximum observed or theoretically calculated stone size (~13 cm)

$P(x)$ was fit to the data by varying the parameters n and $D_{\max}$ in Eqn. B.2 and minimizing the Root Mean Square (RMS) error. Ideally, the best fit would be found when $D_{\max}$ = approx. 13 cm (5"). That a "best" fit was found when $D_{\max}$ = 10 cm is rather reassuring. With $D_{\max}$ = 10.0 cm and $n = 41.1$, the RMS error was, for 1972 data, < .2%, and for 1973 data, < 1%. Good agreement was found between the fitted function and the original data when Eqn. B.1 was fitted against the relative frequencies of both 1972 and 1973.

The solid curve in Figures 6 and 7 is the plotted function

$$P_{\text{cum}}(\leq D) = 42.1 (1 - D/D_{\max})^{43.1} - 43.1 (1 - D/D_{\max})^{42.1} + 1$$

where

$$D_{\max} = 10.0 \text{ cm}$$

It will be noted (in Figure 6) that the raw data points to about 1.75 cm lie quite close to the fitted curve. As mentioned before, since there is a consistent shift (transition in distribution) of the data at about 1.75 cm, all points (but one) for $D > 1.75$ cm lie below the curve. Using the RMS error technique to fit a cumulative probability distribution to data tends to weigh more heavily points with smaller ordinate values. This is because as cumulative frequency increases, an order of magnitude difference between two probabilities represents an

increasingly smaller absolute difference. Hence, the absolute errors are actually quite small in the high tail of the cumulative frequency plot. On semilog paper, therefore, the function fits best for data points of smaller diameters (of $D < 1.75$ cm). However, in Figure 6, points for larger diameters (after the transition) fall more or less equidistant from the function, indicating similar slope, and implying that the fitted function is quite similar to the distribution of larger hailstones.

o Illinois Data

The raw data from Illinois studies were not obtained for analysis. The data presented in the open literature are reported in histograms of only four cells. A curve was not fit to these and the data are simply plotted for comparison in Figures 6 and 7. The dashed line shows how the distribution might be extrapolated by eye; as can be seen, the extrapolation is based on only a small amount of information.

From these limited data, it appears that hail is larger in Illinois than it is in Colorado, but there is no reason to believe that the hailstone size distribution is fundamentally different.

o Alberta Data

The raw data from Alberta are not available. In the publications of Douglas [11,12,13], the figures are not expressed in a form from which either the relative frequencies, or the cumulative frequencies of stone size, can be determined. Unfortunately, not even the sample size is published. At least a few stones 5 and 6 cm large were collected, indicating either a relatively large sample or a size distribution skewed toward larger sizes.

Douglas fit an exponential to his Alberta data, to which the cumulative probability density function is given by

$$P_{\text{cum}}(D) = 1 - e^{(-3.09 D)}$$

This is plotted in Figures 6 and 7 for comparison with the other two distributions. It cannot be determined how accurately this exponential describes the distribution of larger stone sizes.

Storm Severity

As discussed in Section 1.d, storm severity could be characterized in a number of ways. However, the only viable approach, and that for which readily available and reliable data exist, is to use data on total number of stones per unit area per storm.

Of the three samples of hailstones gathered in North America (Alberta, Colorado, Illinois), only the Colorado data were used to quantify storm severity. The data from Alberta and Illinois are not published in useable form (and the Illinois raw data were not acquired).

The NHRE was conducted over a three year period. For the purpose of characterizing storm severity, any of the three years might have been chosen. However, the sample size would have been far too small. To attain a large enough sample, the data from all three years were combined in histogram form. A single cell width was chosen to minimize the irregularities of the sample. However, as can be seen in Figure 5, the data are still quite rough.

It will be remembered from Section 3 that storm severity must be expressed as a probability density ($P_2(n)$). Therefore, an analytical function must be fitted to the data in Figure 5. The question arises: what functional form should be chosen? Two constraints are immediately apparent: the function should be lower-bounded (in this case, zero), and the function should have

FIGURE 5 -- Distribution of total number of stones per storm (vs. number of occurrences) used to characterize storm severity (NHRE data). The curve shows fitted gamma distribution .

no upper bound, since the maximum number of stones per m^2 per storm is unknown. Schleusener et al. [19] have fit a gamma distribution to data on the total number of stones per storm with reasonable results. Since a gamma distribution satisfies the above requirements, it was chosen for this study. A special case of the function was used, of the form*

$$e^{a_1} \cdot x^{a_2} \cdot e^{(a_3 \cdot x)} \quad \text{Eqn. B.3}$$

The distribution parameters were determined by minimizing the sum of the squares of the deviations between the estimating function and the data. These deviations were defined by

$$\begin{aligned} R_i &= \ln \frac{e^{a_1} \cdot x_i^{a_2} \cdot e^{(a_3 \cdot x_i)}}{y_i / \Delta x_i} \\ &= a_1 + a_2 \ln x_i + a_3 x_i - \ln(y_i / \Delta x_i) \end{aligned} \quad \text{Eqn. B.4.}$$

where

x_i is the midpoint of a histogram cell of width ΔD_i and y_i is the number of occurrences of which the number of hailstones per square meter is between $x_i - \Delta x_i / 2$ and $x_i + \Delta x_i / 2$.

By minimizing

$$\sum_n \left[a_1 + a_2 \ln x_i + a_3 x_i - \ln (y_i / \Delta x_i) \right]^2 \quad \text{Eqn. B.5}$$

we effectively minimize the RMS percentage deviation of Eqn. B.3 from the data. It is important to minimize the percentage

*It will be noted that this is an unnormalized distribution. Since $P_2(n)$ in the risk model must be expressed as a probability density the FORTRAN source code in APPENDIX D (SUBROUTINE SIZEAD) contains a term for normalizing the curve with respect to number of stones.

deviation rather than the absolute deviation because the relatively few storms with many hailstones are as important in calculating the occurrence of large hailstones (using the risk model) as the many storms with fewer number of hailstones. Minimizing Eqn. B.5 with the data grouped into histogram cells according to Figure 5 gives the distribution parameters

$$a_1 = 6.6512$$

$$a_2 = -.3123$$

$$a_3 = -.0005012$$

with a Root Mean Square error of .32. This corresponds to a percent error of 32%. The magnitude of this error can be explained with a cursory look at Figure 5. The "roughness" of these data might be due to a number of factors. As has been previously discussed, there was inconsistency in sampling technique from year to year in the NHRE. Furthermore, the local spatial variability of hailfall has not been examined (within the area of NHRE study); nor has the short-term temporal variability of hail (NHRE 1972 - '74). The extent to which these (and other factors) contribute to the "bumpiness" of the data is not examined here. Since no other data were analyzed (only one other set is available--through the Illinois State Water Survey), the inaccuracy in the risk model's predictions cannot be quantified beyond the effect of $\pm .32$ fitting error in the storm severity distribution. It is our opinion that this variance will have minimal effect on the risk model's predictions. As will be seen in Section 4, the ultimate predictions from the risk model are little affected by variations of similar magnitude in the other parameters. For example, a change of a factor of 2 in either the mean annual number of haildays or in the exposure index results in a change in hailstone size ultimately predicted by the model of only about .1 cm.

However, until further work is begun (as discussed in Section 6), the effect of this discrepancy between the function and the data cannot be more accurately quantified.

FIGURE 6 -- Three hailstone cumulative size distributions generated from three data sets (Colorado, Illinois, Alberta).

FIGURE 7 -- A larger view of the three cumulative size distributions shown in Figure 6. Arrow for the Colorado distribution, and the dashed line for the Illinois distribution, show from what point the data have been extrapolated to larger hailstone size.

4. CONCLUSIONS

This section is divided into three.

The first section, 4.a., is offered as a brief guide to utilizing the information in Figures 8 - 12; that is, in interpreting the results of extensive calculations of the risk model.

4.b. contains a rather detailed discussion of a sensitivity analysis of the model and the conclusions drawn from the results.

4.c. is intended as a guide to using this report.

4.a. Interpreting the results

In Figures 8 through 12 are graphed extensive calculations from the risk model. Note that in the figures (and in the remainder of this report), the output from the model is expressed as ultimate risk. This is the probability that at least one stone larger than D cm in diameter will strike the exposed surface, A, in K years, given P_1 , P_2 , and P_3 . Ultimate risk is plotted on the abscissa in Figures 8 - 11.

It will be noted that the ordinate axis in these figures does not represent hailstone size, D. This means that the figures cannot be read directly to determine the ultimate risk of encountering larger than a D cm hailstone. The ordinate value, $[1 - P_1(\leq D)^A]^*$ must be first adjusted by a power of "A" (area of exposure) of interest and then correlated with hailstone size via Figure 7.

More specifically, two approaches can be employed in using the information in these figures:

- 1) Suppose we wish to know the ultimate risk of encountering a D cm stone on an exposed surface area, A, in K years in a given location.

First, we would determine $P_1(\leq D)$ from Figure 7. Note that three distributions to represent $P_1(\leq D)$ are shown. Ideally, we would use the distribution that best represents the geographic location of interest. However, certain assumptions have been made in extrapolating the raw size data to larger hailstone sizes, and the beta distribution based on Colorado data is perhaps the best representation of a parent population of hailstones, regardless of location.

*The ordinate value is not $P_1(\leq D)^A$, but $[1 - P_1(\leq D)^A]$. This is because $P_1(\leq D)$ represents the occurrence of all stones D cm or smaller, whereas we are concerned with the alternate probability, the occurrence of stones $> D$ cm, which is $[1 - P_1(\leq D)]$.

Once $P_1(\leq D)$ has been determined, we are ready to make some calculations concerning the area, A , exposed to hail. As background, recall from Eqn. 3.2 that " A " is incorporated in the risk model as follows:

$$\sum_n P_1(\leq D)^{n \cdot A} \cdot P_2(n)$$

Note, however, that this is equivalent to

$$\sum_n [P_1(\leq D)^A]^n \cdot P_2(n)$$

So, the computer code of APPENDIX D was written so that the entire term, $P_1(\leq D)^A$, is an input parameter of the model. The results plotted in Figures 8 - 12 were obtained, not for specific values of $P_1(\leq D)$ and of A , but for a plausible range of the term $P_1(\leq D)^A$. This approach leads to greater flexibility in applying the results of fewer calculations.

For example, suppose we are interested in the risk of encountering a 3 cm hailstone on either of two flat plate collector installations, one 10 m^2 and one 100 m^2 . From Figure 7, $P_1(\leq 3 \text{ cm}) = .999994$ (using the Colorado size distribution).

For the smaller of the two installations, $A = 10 \text{ m}^2$, $P_1(\leq 3)^{10} = .99994$, and $[1 - P_1(\leq 3)^{10}] = .00006$.

For the larger installation, $A = 100 \text{ m}^2$, $P_1(\leq 3)^{100} = .9994$, $[1 - P_1(\leq 3)^{100}] = .0006$.

The ultimate risk of encountering a 3 cm stone in both cases can now be read from one of Figures 8 - 11. Suppose we are interested in the risk of encountering such a hailstone in Kansas City, Missouri; suppose also that both collector installations will be operating for 5 years ($K = 5$).

For the smaller installation, $[1 - P_1(\leq 3)^{10}] = .00006$, so ultimate risk = .82.

For the larger installation, $[1 - P_1(\leq 3)^{100}] = .0006$, so ultimate risk $\gg .99$.

Following this procedure, one can determine the risk of encountering larger than a D cm hailstone on area A in K years for many combinations of parameters, with one exception. The parameter K in Figures 8 - 11 has been calculated only to K = 10. For larger values, a sensitivity analysis of the risk model demonstrated that ultimate risk increased only marginally with large increases in K (this is discussed in more detail Section 4.b.). However, for utilizing the model for longer time periods, Figure 12 has been plotted to show the hailstone size predicted at an ultimate risk = .50 for values of K as large as 210 years.

- 2) Suppose that we do not know the hailstone size that will cause damage to our ground installation, or we are interested in determining "hail-proof" design specifications. We might be interested in the size we can "expect" to encounter at a given level of risk (ultimate risk).

In this case, we would go from the abscissa value in Figures 8 - 11 to the corresponding ordinate value of $[1 - P_1(\leq D)^A]$ (for a given value of K). One minus this value gives us $P_1(\leq D)^A$. Before this can be correlated with hailstone size via Figure 7, we must take the A^{th} root. That is,

$$[P_1(\leq D)^A]^{1/A} = P_1(\leq D)$$

For example, suppose we want to know the hailstone size that is likely to be encountered with an ultimate risk of, say, .30 in Kansas City in 10 years. Using curve "D" from Figure 10, we find that $[1 - P_1(\leq D)^A] = .000008$. So, $P_1(\leq D)^A = .999992$. Suppose, again, that we have two collector installations, one 10 m^2 and one 100 m^2 .

Where $A = 10 \text{ m}^2$,

$$P_1(\leq D)^{10} = .999 \ 992$$

$$P_1(\leq D) = .999 \ 999 \ 2$$

From Figure 7, $D = 3.3 \text{ cm}$.

Where $A = 100 \text{ m}^2$,

$$P_1(\leq D)^{100} = .999 \ 992$$

$$P_1(\leq D) = .999 \ 999 \ 92$$

From Figure 7, $D = 3.75 \text{ cm}$.

Following this procedure, values of $[1 - P_1(\leq D)^A]$ at an ultimate risk = .50 have been plotted against the number of years of exposure, K , in Figure 12. The hailstone size that is predicted by the risk model at an ultimate risk = .50 might be viewed as the "probable maximum hailstone size" that will be encountered in a given location on area A in K years, and as such, is perhaps of most widespread interest.

TABLE 3 shows the probable maximum hailstone size that is predicted in the four locations analyzed (see TABLE 1) for various values of A and K .

Figure 8 -- Predictions of the risk model for Albuquerque, New Mexico.

Figure 9 -- Predictions of the risk model for Abilene, Texas.

Figure 10 -- Predictions of the risk model for Kansas City, Missouri.

Figure 11 -- Predictions of the risk model for Cheyenne, Wyoming.

Figure 12 -- Predictions of the risk model for values of $K \leq 210$. All four locations shown in TABLE 1 are plotted at an ultimate risk = .50.

4.b. Sensitivity analysis results

An analysis of the risk model was performed to determine the model's sensitivity to changes in the various parameters. Specifically, the following questions were investigated:

- 1) To what extent are the final predictions of the risk model affected by changes in the nature of $P_3(x)$? That is, to what extent are the results of the model affected by changes in the mean annual number of haildays (and of the sample variance of the hailday data sample)?
- 2) Does the choice of distribution (e.g. Poisson or negative binomial) to describe $P_3(x)$ have significant effect on the risk model's predictions?
- 3) Three approaches have been used to derive hailstone size distributions, $P_1(\leq D)$ (for Colorado, Alberta, and Illinois). All three extrapolate the frequency of large hailstones from the frequencies of smaller diameters. What size hailstone does the model predict when each of the three distributions is used? And which approach to extrapolate $P_1(\leq D)$ to larger sizes appears to be most viable?
- 4) What is the probability of encountering large hail over long time periods and on large surface areas? For example, how much does expected maximum stone size increase when the number of years or area of exposure, say, doubles?

Four representative locations were chosen to test the model's sensitivity to changes in hailday data (see TABLE 1). These locations represent the most common range of characteristics of the 25 hailday data samples analyzed in this investigation (i.e. similarity of $\bar{x}$, s^2). The number of hailstorms in Cheyenne, Wyoming, can be considered a "worst case", with the highest value of mean number of haildays found in the literature, and probably one of the most "severe" distributions of haildays, with a sample variance of $s^2 = 2.5 \bar{x}$. The dispersion of hailday distributions has not been given adequate attention by previous authors,

and as will be seen, is a significant factor in predicting the incidence of large hail.

As can be seen in TABLE 1, only one of these four data sets was fit better by a Poisson probability density function than by a negative binomial. This is typical of the 25 hailday data samples.

Of the hailday data analyzed, there were unfortunately no distributions with a mean annual number of haildays of between 5 and 8. The resultant gap in our sensitivity analysis of the risk model is, however, not significant. From Figure 1, it can be seen that a mean annual number of haildays of greater than 5 is a localized high (i.e. our hailday samples account for most cases). Furthermore, as will be discussed, it appears that the mean annual number of haildays is not the most significant parameter of the hailday distribution. From the sensitivity analysis, it was found that the sample variance of the distribution has a far greater effect on the risk model's predictions.

In Figures 8 through 11, the predictions of the model are plotted for a wide range of the parameter $[1 - P_1(\leq D)^A]$ and for four values of K (years of exposure).^{*} The solid line represents the results when the negative binomial was used to describe hailday frequencies, the dashed line when the Poisson was used. It can be seen in Figures 8 - 10 (not in Figure 11) that in the "mid-range" of ultimate risk, (with values between .2 - .8) there is not much difference between the results when either distribution is used. However, as the number of years of exposure increases, the negative binomial becomes increasingly divergent from the Poisson distribution. This trend becomes evident by plot "D" in Figures 8 - 10. Results for larger values of K show that the two distributions give increasingly divergent results.

^{*}Note that these plots show the effect of increasing the number of years of exposure, K, but do not illuminate the effects when the area of exposure, A, is increased. The effect of increasing A is simply correlated with the A^{th} power of $P_1(\leq D)$ and can be seen in TABLE 3 or by calculating a few examples as per the methodology outlined in Section 4.a.

WEATHER STATION SAMPLES TESTED

weather station	mean number haildays	s^2	sample size, years of record	Goodness of Fit:	
				Pois.	neg. binom.
Albuquerque, New Mexico	2.33	4.5057	30	<.005	>.4
Abilene, Texas	3.58	5.2466	64	<.02	>.5
Kansas City, Missouri	4.72	4.8865	55	.65	.5
Cheyenne, Wyoming	8.68	20.7752	54	<.001	.4

TABLE 1 -- Poisson and negative binomial fits to hailday data from Refs [20] and [21] are shown.

It is interesting to note that the greatest differences between the two distributions are found for Cheyenne, Wyoming (Figure 11). As can be seen, for $K = 10$, the two distributions give results that hardly seem to apply to the same geographic location. Clearly, the distribution used to describe $P_3(x)$ in the risk model is an important criterion in accurately predicting the occurrence of large hailstones, especially over long time periods.

The risk model's predictions at this point can be checked for consistency with the raw data. Not considering the model's extrapolation to larger hailstone sizes, do the results agree with the data? Does the hailstone size predicted from Figures 8 - 11 for one year on 1 m^2 agree with the raw size data for 1 year on 1 m^2 ? The model's predictions are shown in TABLE 2.

TABLE 2 Probable maximum hailstone size predicted on 1 m^2 in 1 year (that is, ultimate risk = .50)

weather station	Size Distribution used:		
	Colorado	Alberta	Illinois
Albuquerque	2.1	2.5	3.0
Abilene	2.25	2.7	3.2
Kansas City	2.35	2.8	3.3
Cheyenne	2.4	3.0	3.5

These values are consistent with extreme stone sizes reported for Colorado and Alberta, and are well within the range of raw data (i.e. the values were not obtained via extrapolation of the size distribution). The sizes predicted using the Illinois distribution appear too high. But perhaps stones this large do occur in Illinois as often as "every other year", or on "every other

hailpad".

Note that the size predicted in Abilene (which has a hailday distribution almost identical to that of Denver) at an ultimate risk of .50 is 2.25 cm. This means that of a group of hailpads left in Abilene (or in Denver) for one year, one would expect half to be struck by a 2.25 cm stone or larger. It is reassuring that this prediction agrees quite well with the data gathered near Denver in the NHRE. We can see that at the very least, the model's predictions agree well with the original data.

At least on 1 m^2 for 1 year, the risk model gives acceptable results. We can only assume that it also applies for larger values of A and K, since there are insufficient data to show otherwise.

On this basis, the model was tested for sensitivity to increases in the value of K. It has already been noted that for each location shown in TABLE 1, the Poisson and negative binomial distributions (to represent $P_3(x)$) led to increasingly different values of ultimate risk as K increased from 1 to 10. For a common base of comparison between locations, only one distribution was chosen to describe $P_3(x)$ as the model was tested for sensitivity to increasing K; the negative binomial was used because it provided an adequate fit of the raw hailday data in all four cases (TABLE 1), and the Poisson provided a better fit (and marginally so) in only one case. In Figure 12, K is plotted against $[1 - P_1(\leq D)^A]$. For each plot, the ultimate risk is .50. That is, each plot represents the size hailstone (correlated to $[1 - P_1(\leq D)^A]$) that can be expected, "on the average", in K years. This might be considered the "probable maximum hailstone size".

In Figure 12, note that there is little difference between values of $[1 - P_1(\leq D)^A]$ predicted for Albuquerque, Abilene and Kansas City. This means that the hailstone size predicted by the model (i.e. the size that corresponds with the frequency $[1 - P_1(\leq D)^A]$) is nearly the same in these locations. TABLE 3 shows actual sizes predicted. One would expect that the larger the value of the mean annual number of haildays, the larger the hailstone

	Size distribution used	Surface area exposed, A m ²	Number of years of exposure (K)				
			5	10	20	50	100
Albuquerque, New Mexico	COLORADO	10	2.95	3.12	3.17	3.35	3.48
		50	3.2	3.4	3.5	3.58	3.7
		100	3.4	3.5	3.55	3.75	3.8
	ALBERTA	10	4.05	4.25	4.35	4.75	4.95
		50	4.4	4.8	5.0	4.15	5.47
		100	4.8	5.0	5.15	5.5	5.7
	ILLINOIS	10	4.45	4.6	4.7	5.05	5.25
		50	4.75	5.15	5.35	5.45	5.7
		100	5.15	5.35	5.4	5.75	5.95
Abilene, Texas	COLORADO	10	3.1	3.14	3.25	3.4	3.5
		50	3.35	3.47	3.55	3.63	3.75
		100	3.48	3.54	3.63	3.78	3.83
	ALBERTA	10	4.23	4.34	4.46	4.83	5.0
		50	4.72	4.95	5.06	4.2	5.5
		100	4.96	5.10	5.2	5.6	5.75
	ILLINOIS	10	4.58	4.7	4.8	5.15	5.3
		50	5.05	5.3	5.4	5.5	5.8
		100	5.3	5.4	5.5	5.83	6.0
Kansas City, Missouri	COLORADO	10	3.15	3.17	3.3	3.45	3.55
		50	3.4	3.5	3.6	3.7	3.8
		100	3.5	3.57	3.66	3.81	3.85
	ALBERTA	10	4.28	4.37	4.6	4.9	5.05
		50	4.83	5.0	5.12	5.33	5.6
		100	5.05	5.15	5.3	5.7	5.77
	ILLINOIS	10	5.63	4.72	4.92	5.21	5.32
		50	5.15	5.35	5.5	5.6	5.85
		100	5.35	5.45	5.6	5.9	6.05
Cheyenne, Wyoming	COLORADO	10	3.25	3.5	3.82	3.92	4.2
		50	3.55	3.73	4.08	4.2	4.45
		100	3.63	3.82	4.2	4.25	4.53
	ALBERTA	10	4.45	4.95	5.7	5.8	6.45
		50	5.05	5.5	6.25	6.45	6.97
		100	5.2	5.7	6.45	6.57	7.15
	ILLINOIS	10	4.8	5.27	5.95	6.08	5.95
		50	5.4	5.7	6.45	6.6	7.15
		100	5.5	5.95	6.63	6.75	7.3

TABLE 3 -- Probable maximum hailstone size (ultimate risk = .50) as predicted by the risk model.

predicted by the risk model. That this is not the case indicates that there is little statistically significant difference between the hailday distributions from these three locations, in spite of the differences in mean annual number of haildays.

The results for Cheyenne, Wyoming seem to depart radically from those of the other three locations. This can be seen in Figures 11 and 12. For example, the most significant departure of the Poisson from the negative binomial is found for this hailday distribution. And clearly, the results for large values of K are quite deviant from those of the other locations tested. In Figure 12, the results for Cheyenne are outside the range of the graph at a value of $K = 35$, at which point a 4 cm stone is predicted (using the Colorado size distribution).

The risk model was tested for Cheyenne for larger values of K. A plot (as per Figure 11) using a value of $K = 50$ indicated that while $[1 - P_1(\leq D)^A]$ steadily decreased, the ultimate risk decreased by a smaller and smaller increment. In fact, with decreasing $[1 - P_1(\leq D)^A]$ (i.e. increasing stone size), the plot asymptotically approached an ultimate risk of .828 (similar results were found for $K > 30$). That is, the model predicted an infinitely large stone with an ultimate risk of not less than .828. It should be noted that the value of $[1 - P_1(\leq D)^A]$ was limited by computational constraints to values of less than 15 significant digits. The trend noted above might have changed with values of even more significant figures. However, the correlation of such numbers to stone size is purely speculative in any case. It can only be concluded at this point that the model cannot be calculated in Cheyenne for many years of exposure ($K > 30$).

The hailday distribution for Cheyenne probably represents an extreme for North America. The probability of encountering as many as 20 days of hail in one year, for example, is about .01. For other hailday distributions evaluated, the probability of encountering this many days of hail is at least an order of magnitude smaller. The effect of this high probability in Cheyenne is magnified many times over many years.

What effect do extreme variances in hailday distributions such as Cheyenne's have on the risk model's predictions? Hypothetical hailday distributions were used to represent $P_3(x)$ in the model to answer this question. These "cities" had equivalent $\bar{x}$'s, but smaller s^2 's (i.e. less dispersed distributions). The hypotheticals used were

Case 1: $\bar{x} = 8.68$, $s^2 = 2.0 \bar{x}$

Case 2: $\bar{x} = 8.68$, $s^2 = 1.5 \bar{x}$

It will be remembered that the parameters of the hailday distribution for Cheyenne are: $\bar{x} = 8.68$, $s^2 = 2.5 \bar{x}$.

The results of this experiment showed that the bizarre predictions made by the risk model (noted above) were reduced with corresponding reduction in sample variance. That is, values of $[1 - P_1(\leq D)^A]$ predicted with an ultimate risk of .50 at various values of K were increased with corresponding reduction in sample variance. It will be remembered that the smaller the value of $[1 - P_1(\leq D)^A]$, the larger the hailstone predicted by the model. In other words, hypothetical reduction in the sample variance of the distribution (while the mean remained the same) led to a reduction in the size hailstone predicted by the model, such that the size predicted in each location was

Cheyenne > Case 1 > Case 2 > Kansas City

It can be noted that the less dispersed the hailday distribution (i.e. the smaller the ratio of the sample variance to the mean), the smaller the hailstone predicted by the model.

It is unfortunate that the risk model cannot be evaluated in Cheyenne for large values of K. However, since the results using values of K = 1 (A = 1) are at least consistent with the raw data (see TABLE 2) hail risk may be extrapolated from the plots of K < 30 (these values can be calculated by the model). The points using values of K between 10 and 30 lie in a straight line in Figure 12. It can be seen that there is an order of magnitude decrease in $[1 - P_1(\leq D)^A]$ for every 10 years of increase in K.

This relationship can be extrapolated to larger values of K. The size stones to which these values correspond are shown in TABLE 3.

It will be seen in this TABLE that the hailstone sizes predicted using each of the three size distributions differ significantly (for all combinations of parameters used in the model). Recall that the distribution of larger diameters was extrapolated from data on smaller diameters for each of the three size distributions in Figures 6 and 7. In correlating $[1 - P_1(\leq D)^A]$ with larger hailstones (using Figure 7) we are dealing with an unquantified uncertainty. Each of the three distributions has been extrapolated in a different manner, and we are now in a position to evaluate the validity of the model's predictions based on the extrapolations used.

It has already been noted that hailfall parameters vary with geographic location. More specifically, the hailstone size distribution of frontal thunderstorms is most likely composed of larger stones relative to the size distribution of orographic storms. Ideally, one would use the size distribution derived from the data gathered at the exact location of interest. However, such detailed data are not available.* As a best estimate, one would use the size distribution based on data gathered from a similar location (e.g. the Illinois size distribution to represent hail sizes in Kansas City, Missouri). However, this latter approach is not necessarily the most viable when using this report.

*There is, however, limited (primarily qualitative) information concerning regional variation in the hailstone size distribution. Some idea of the variation can be gained by comparing the size data contained in this report with data available elsewhere -- especially those data available from the Illinois State Water Survey. Some idea can also be gained from unpublished information (e.g. local meteorologists). For example, Robert Beebe of the Cheyenne weather station reports that the size of hail encountered in Cheyenne is somewhat smaller than that encountered in the Denver area [55].

Predictions of hail size obtained using the Illinois size distribution for one year on 1 m^2 (TABLE 2) are questionable. As previously discussed, the predictions of the model using this size distribution are inconsistent with the raw data even for one year on 1 m^2 . Predictions of the risk model using the Illinois distribution are therefore quite dubious for larger values of A and K (and therefore for larger hailstone sizes).

From TABLE 2, it was determined that the risk model's predictions using both the Alberta and Colorado fitted size distributions are reasonable (at least for 1 year on 1 m^2). As A and K increase, however, each distribution for $P_1(\leq D)$ leads to increasingly divergent results (see TABLE 3). Both size distributions are fitted functions (an exponential for Alberta, a beta distribution for Colorado). As was discussed in APPENDIX B, both functions are used to extrapolate the incidence of large hailstones from data on smaller diameters. The use of an exponential for this purpose is questionable, since it has no upper bound. The use of an unbounded function will, at some large value of A and K, result in the prediction of a supernatural hailstone (i.e. one not within the realm of physical reality). At what value of exposure an exponentially fitted size distribution becomes unacceptable is unknown. Nobody has previously used an exponential size distribution for the purpose of predicting hail damage risk, and there is no way at present of testing the values found in TABLE 3. For values of $K < 200$, both the exponential and beta fits appear to predict realistic hailstone sizes. Perhaps the best value for hailstone size can be taken as somewhere between the beta and exponential predictions. (Without acquisition and analysis of the Illinois raw data, we recommend against the use of the Illinois size distribution presented in this report). Using this approach, the hailstone predicted in Kansas City in 20 years on 100 m^2 is between 3.3 and 4.6 cm. As discussed in APPENDIX B, it is our opinion that the more accurate prediction is closer to 3.3 cm. This is, however, a debatable point.

4.c. Utilizing the results

What do the results mean?

A number of approaches can be taken in utilizing this report, and the choice of these will depend on the degree of accuracy required in the predictions and on the resources available.

Hailfall is a geographically variable event. The yearly occurrence of hail, the size distribution, and the distribution of storm severity all vary from location to location. Of these three parameters, the yearly occurrence of hail is the most thoroughly characterized. There are but limited data regarding these latter two parameters, and so predictions of hail risk in a given location must generally be based on the data available from other locations. There is a degree of bias in such predictions which can only be characterized by the analysis of additional data.

The simplest application of this report begins on page 9. From Figure 1, the mean annual number of haildays for a given location can be determined. Unfortunately, the sample variance of the hailday distribution cannot be determined from this figure. As discussed in Section 4.b., the sample variance has a far greater effect on the predictions of the risk model than does the mean of this distribution. Although characteristic hailday distributions have been used to plot Figures 8 - 12 and to calculate the values found in TABLE 3, some locations of interest will have more (or less) dispersed hailday distributions (larger sample variances).

One might simply use the mean annual number of haildays (determined from Figure 1) with Figures 8 - 12 (or TABLE 3) to determine the hail risk in a given location. Using this approach, however, one must recognize the results as ballpark approximations.

Another approach can be taken that leads to more accurate predictions with a minimal expenditure of resources. The hailday data available in Refs [20] and [21] can be obtained and the mean

and sample variance of the hailday sample determined for the location of interest. $\bar{x}$ and s^2 can then be compared with the data in TABLE 1. If the values are similar to any of those in the table, then Figures 8 - 12 and TABLE 3 can be used to determine hail risk. If not, then the risk model of Section 3 should be employed with the new $\bar{x}$ and s^2 as input parameters. The source code contained in APPENDIX D is included for such a purpose. The results might be plotted as per Figures 8 - 12 and correlated with the size distributions of Figure 7.

Finally, if even greater accuracy is needed in predicting hail risk, the raw data from the Illinois State Water Survey should be obtained and analyzed. As has been discussed, this report most accurately predicts hail risk on the lee side of the Rockies. The results reported here are based primarily on data from Colorado, where orographic (mountain-effect) thunderstorms are the most prevalent. Size and storm severity distributions (P_1 and P_2 , derived as per APPENDIX B) based on Illinois data would provide much more accurate predictions of hail risk on the Plains than are presented in this report. This is, however, another study in itself and is discussed in more detail in Section 6.

5. HAILSTONE TERMINAL VELOCITY AND KINETIC ENERGY

A fully formed hailstone falling from a thunderhead will accelerate until the aerodynamic drag limits further acceleration. At this point, the hailstone's velocity is its terminal velocity and the weight of the hailstone and the drag on its surface are equal.

Using the usual definition of drag [45,46], we have

$$d = (C_d \cdot A \cdot \rho_a \cdot V^2)/2 \quad \text{Eqn. 5.1}$$

where

C_d = a dimensionless "drag coefficient".

A = the frontal area of the hailstone, cm^2 .

ρ_a = the density of air, g/cm^3 .

V = the hailstone's velocity relative to that of the airstream, cm/sec .

The weight of a falling hailstone is given as

$$W = \rho_H \cdot Q \cdot g \quad \text{Eqn. 5.2}$$

where

ρ_H = the density of the hailstone, g/cm^3 .**

Q = the volume of the hailstone, cm^3 .

g = the acceleration due to gravity, cm/sec^2 .

*It will be noted that this differs by a factor of ~ 2 from many expressions for hailstone terminal velocity appearing in the literature [e.g. 44]. This is due to a difference in definition of drag coefficient by a factor of 2. However, values obtained using either expression are equivalent.

**Literature values for ρ_H are highly variable, ranging from $\sim .6 \text{ gm/cm}^3$ [44,48] to $.83 - .87 \text{ gm/cm}^3$ [30]. A median figure has been reported as $\sim .7 \text{ gm/cm}^3$ [51]. This latter value has been used in our calculations.

Taking $A = \pi \cdot D^2$ and $Q = \pi \cdot D^3/6$, equating drag with weight, and simplifying, we have*

$$V = \left(\frac{4 \rho_H D g}{3 C_d \rho_a} \right)^{1/2} \quad \text{Eqn. 5.3}$$

The drag coefficient, C_d , for a sphere, is a function only of Reynolds number, Re , where

$$Re = V \cdot D / \nu$$

and

ν = the kinematic viscosity of the fluid, air, g/cm sec.

D = the diameter of the hailstone, cm.

Over the range of Reynolds numbers corresponding to the range of hailstone sizes observed in nature, the relation between Re and C_d cannot be determined theoretically. Various workers have reported experimentally determined values of C_d vs. Re . for spheres by using wind tunnels, radar observations, and by towing models behind an airplane.

A plot (from Ref. [45]) of drag coefficient vs. Reynolds number is shown in Figure 13. Note that there is a sudden drop in C_d at $Re \approx 2 \times 10^5$. This is due to a transition in the boundary layer from laminar to turbulent flow. (That is, the "wake" begins to separate from the hailstone further toward the leading face. This results in turbulence, hence less viscous friction and less drag). This drop in C_d corresponds to the "critical Reynolds number".

The curve shown in Figure 13, however, shows the relationship between C_d and Re for a smooth sphere. In nature, hailstones are not perfect smooth spheres, and the drag coefficient for a given Reynolds number varies with hailstone shape, surface roughness, surface depressions, and protuberances. For example, irregular shapes (e.g. ellipsoidal and "conical") tend to have larger

Figure 13 -- Re vs. C_D for smooth spheres, taken from Ref [45].

values of drag coefficient (i.e. slower fallspeeds) for a given Reynolds number.

The relative occurrence of various hailstone shapes has been investigated by some researchers [49,50]. Others have attempted to quantify the effect of irregular shape on the drag coefficient [52,53]. In the former studies, however, very few locations have been evaluated (leaving possible geographic variability uncharacterized) and samples have been small (leaving unanswered questions regarding the representation of a parent population). Furthermore, experiments (to date) to determine drag coefficients of irregularly shaped hailstones have been far from rigorous, and literature values differ considerably. Since reliable data do exist for C_d vs Re for spheres [45,46], we have assumed in our calculations that hailstones are spherical.

Hailstone surface characteristics and their effects on drag coefficient (and hence, fallspeed) have been examined by a few researchers [52,53]. Generally, surface roughness, surface depressions, and protuberances cause an increase in drag coefficient and hence, a decrease in fallspeed. As with hailstone shape, however, good data do not exist to characterize the relative occurrence of different surface characteristics. Furthermore, only crude measurements of drag coefficient as a function of surface characteristics have been obtained. Hence, we are forced to assume hailstones to be not only spherical, but smooth. Since smooth spheres most likely have the fastest fallspeeds, this represents a "worst case" approach.*

*Willis, Browning and Atlas [52] have noted that large dry hailstones behave as if rough, resulting in a boundary layer transition to turbulence at lower Reynolds numbers than for smooth stones. (That is, the critical Reynolds number is observed for smaller hailstones). However, as discussed further on, it is difficult to envision natural hailstones with surfaces sufficiently dry (and therefore rough) to observe this phenomenon in nature.

Figure 14 -- Hailstone terminal velocity vs. size, for three elevations.

It will be noted in Figure 14 that there are two values of terminal velocity for hailstones between 6 and 8.5 cm diameter. For hailstones this size, the terminal Reynolds number is near the critical value of about 2×10^5 (see Figure 13). For a slight increase in Reynolds number, there is about a factor of two decrease in C_d . At just under $Re = 2 \times 10^5$, a stone falling through still air would normally have a laminar boundary layer, hence the lower of the two terminal velocities. However, some authors have proposed that a real hailstone might actually attain the faster of these fallspeeds by attaining a $Re > 2 \times 10^5$.

Bilham & Relf [44] have proposed that if a sufficiently large hailstone entered a powerful updraft, turbulence in the boundary layer would result, and the stone would fall at the faster of the two terminal velocities. If the updraft ceased, the stone's new fallspeed would maintain turbulence in the boundary layer, thus maintaining the faster fallspeed. Or, the same effect might be found if the stone were caught in a powerful downdraft. In effect, a stone of a diameter such that its terminal velocity is in the double valued region will tend to remain at the faster velocity once a disturbance has caused the boundary layer transition.

Willis, Browning and Atlas [52] have found that hailstones with critical Reynolds numbers (see Figure 13) might attain the faster terminal velocity if their surface is dry. Large dry hailstones behave as if their surfaces were rough, and rather than act to increase aerodynamic drag, the roughness serves to induce turbulence in the boundary layer. Hence, dry hailstones with critical Reynolds numbers might attain the faster terminal velocity. However, Willis et al. also observed that when these large hailstones become wet, they behave as if smooth, and fall at the lower terminal velocity. The authors concluded that large dry hailstones behave as if their surfaces were rough, wet hailstones as if the surfaces were smooth.

However, hailstones falling during a thunderstorm (that is, in vivo, not hailstones dropped from a balloon on a clear day) are most likely wet. Below the freezing level in a cloud, falling stones are likely to accrete condensation and are also likely to

experience some surface melting. It is therefore difficult to envision that the mechanism proposed by Willis et al. operates in nature, and in fact, the authors themselves reported their observation with reservation, not proposing that it represents a real world phenomenon.

A third mechanism by which hailstones might attain the faster terminal velocity has been debated in the literature. Various authors have proposed that turbulence in the boundary layer might be induced by small scale external turbulence. Some believe that small scale turbulence in the updraft itself may induce a transition from laminar to turbulent flow. For example, W.C. Macklin has proposed that wakes from other falling stones may be sufficient to induce such turbulence [52], and therefore cause stones with critical Reynolds numbers to fall at the faster terminal velocity.

At present, none of the mechanisms discussed above has been either proven or completely disproven. As will be seen below, however, the double-valued region of terminal velocity leads to a double-valued region of "terminal kinetic energy". It will also be seen that there is a factor of 2 difference between the lower and higher terminal kinetic energy. It is not presently known whether or not natural hailstones attain the faster fallspeed in the double-valued region of terminal velocity; however, given the present paucity of knowledge, a "worst case" approach is advisable.

The terminal kinetic energy, E_k , of a moving body is given by the simple relation

$$E_k = 1/2 \cdot m \cdot v^2 \quad \text{Eqn. 5.4}$$

where

m = mass of the body

Using this relation and Figure 14, the terminal kinetic energies of hailstones have been calculated and plotted in Figure 15. It

Figure 15 -- Hailstone terminal kinetic energy vs. size, for three elevations.

can be seen by equating Eqns. 5.3 and 5.4 and taking $m = Q \cdot \rho_H$ that the kinetic energy of a falling hailstone varies approximately as the 4th power of the diameter. That is,

$$E_k \cong k \cdot D^4$$

Eqn. 5.5

With small increases in stone size, one finds large increases in the terminal kinetic energy. For this reason, a hail risk model must accurately predict the maximum probable hailstone size, D . The predictions of hail risk reported in Section 4 of this report are as accurate as possible given the data and locations evaluated. However, there are data available that have not been examined. Acquisition and analysis of these data would require relatively minimal work and would validate and extend the predictions of Section 4 to many more locations. Because of the critical importance of hailstone diameter shown in Eqn. 5.5, the recommendations outlined in Section 6 would significantly refine the accuracy of the results presented in this report.

6. RECOMMENDATIONS

At least three parameters must be quantified in developing a statistical model to predict hail damage risk: the distributions, P_1 , P_2 , and P_3 . And, to the extent possible (given the data available), the geographic variability of each must be quantified. Not all of the existing data have been evaluated in this study; more work can be done at present in more accurately defining the risk of hail damage, and this is discussed below. However, this is not the most serious problem. Quite simply, hail has not been measured in enough locations to determine accurately geographic variability in hailstone size distribution and hailfall density distribution.

- 1) P_2 -- Storm severity. Geographic variation in hailfall density (P_2) has not been quantified in this study. Although two sets of data are presently available (Illinois and Colorado), only one was obtained for analysis (Colorado). As a result, the risk model's predictions reflect hailfall density characteristics in only one location. For example, the predictions are likely to be quite accurate for locations where storm severity is similar to that in Colorado's "hail alley." For other locations, the degree of deviation from these Colorado data add a degree of inaccuracy to the model's predictions. Since no other data were analyzed, the magnitude of this deviation is unknown.

For this reason, DESH & METROMEX data should be evaluated. An analytical distribution should be fit to the raw data (i.e. a function to describe P_2) and the model tested using this alternate function (as described in APPENDIX B). It would be illuminating to examine the differences in distribution of hailfall density (per storm) between Colorado and Illinois. Is hailfall more severe (i.e. more dense) in frontal thunderstorms or in orographic storms? Furthermore, once the risk model is calculated using a distribution for P_2 derived from Illinois data, a somewhat better picture of geographic variability in this parameter will arise.

However, even at this point, only two locations will have been examined. A much larger sample is clearly needed.

- 2) P_1 -- Hailstone size distribution. In this study, three size distributions were generated using three separate data sets. It should be noted that the same raw data are used to define P_1 as are used to define P_2 . P_2 represents the distribution of total number of stones that falls per storm (not classed by diameter intervals), and P_1 represents the distribution of stone size (normalized with respect to total number of stones/storm and total number of storms). In the raw data, total number of stones per storm are classed by size interval. Since both parameters are derived from the same data, the same problems are faced. However, hailstone size distributions are more widely studied than are hailfall density distributions, and are therefore more readily available. Condensed tables and histograms of the former are published for the three sources of raw data (Colorado, Alberta, and Illinois). These condensed forms are normalized with respect to total number of stones per storm, and hence, cannot be used to derive P_2 .

As mentioned in (1) above, this study has not exhausted all possible information. The raw data from Illinois (DESH and METROMEX) were not obtained, and condensed data were used to plot $P_1(\leq D)$. With such reduced information, an analytical function could not be fit, and the size distribution was extrapolated to larger diameters "by eye" (see Section 1.c and APPENDIX B). For future work, it would be instructive to fit an analytical function to the raw data and compare the result with the beta distribution that was fit to the Colorado NHRE data. Although a detailed picture of geographic variability of the size distribution would still not be obtained, this work would at least answer some important questions. For example, does the size distribution tend to larger sizes on the Plains and in the midwest (where hail is formed primarily in frontal thunderstorms), or on the lee side of the Rockies (where most storms are orographic)? Furthermore, a size distribution generated from Illinois data would allow more accurate predictions of hail risk on the

Plains and in the midwest than were obtained in this study (which used $P_1(\leq D)$ derived from condensed data).

Even more useful than further analysis of the data now available would be further measurements of hail size and hailfall density (P_1 and P_2). Hailpads should be set up in a handful of locations across the hail belt, with ongoing hail monitoring. These data could be used to determine quite accurately the geographic variability of both parameters, P_1 and P_2 . Also, as mentioned in APPENDIX B, the sampling techniques used in at least two of the three major hail research projects failed to monitor the occurrence of small hailstones. It is important that all of the parameters in a statistical model contain the same sampling biases. The hailday data contain some days on which very little hail fell (and some days on which only small hail fell). The sampling methods used in Colorado and in Alberta might not have recorded these hailstones. In other words, there is at least some discrepancy between these two parameters. An attempt was made in this study to account for this problem; the beta distribution fit to NHRE data (APPENDIX B) was not fit to the smallest histogram cell of the size data. The function was then extrapolated to zero. However, this makeshift approach could be eliminated by better sampling technique. For future data collection, we recommend the use of hailpads more sensitive to small hailstones.

These new data would be of interest not only to workers developing design criteria to withstand hail (and it appears there are many) but would have even more general application. The limited data now available suggest a transition in the size distribution at about 1.8 cm (see Section 1.c and APPENDIX B). Meteorologists might correlate this transition with physical cloud processes given sufficient data.

- 3) P_3 -- Hailday frequencies. In APPENDIX A, it was noted that the predictions from the risk model are more affected by changes in the magnitude of the sample variance of the hailday sample than by changes in the mean annual number of haildays. The larger the sample variance (in relation to the

mean), the larger the hailstone predicted. The model's predictions were not significantly altered by changes in the mean annual number of haildays.

25 hailday data sets were evaluated in this report. About another 200 locations remain in Refs. [20] and [21]; these should be fit to both a Poisson and negative binomial and the trend noted above tested in the risk model. To quantify variation in the hailday distribution from location to location, perhaps a single index of hailday variability could be devised, based on the mean and the magnitude of the sample variance. The data are available, and the time and resources required would be relatively minor. The results might be a map or table of locations across the country showing the "severity" of the hailday distribution. Then, the risk model outlined in this report could be applied to representative hailday distributions to quantify how the geographic variance of P_3 (e.g. $\bar{x}$, s^2) affects the size of hail predicted. The results of such work should also prove most interesting to meteorologists. The nature of the distribution in each location could be correlated with topologic features, as has been done in Ref. [7] with the mean annual number of haildays. However, while a correlation of topologic features with the mean has been quite revealing [7], a correlation of such features with the mean and variance would be even more so, since ultimate risk of large hail is so sensitive to this parameter.

It was noted in APPENDIX A that some weather station samples were not fit well by either a Poisson or negative binomial. Why? The work described above would help clarify this point. Were the fits poor because of limited sample size or error in sampling technique? Or because yet another function is required to adequately describe the hailday distribution?

It should be noted that in this study, long-term periodicity in hailday occurrence was not examined. As with all weather, the possibility of a hail cycle should be kept in mind. Such a cycle would have implications not only in hail damage risk research, but also in basic meteorological research.

The approach used in this study has been to develop a workable model based on the available data to determine the occurrence of large, possibly damaging hailstones. At least one other approach could be taken.

If careful records of property damage were kept, the incidence of hail damage to structures of various strength could be determined, bypassing the study of hailfall characteristics altogether. Perhaps a location-specific index of the "mean time between damaging hail" could be calculated for structures with a wide spectrum of resistance to hail.

Unfortunately, the insurance companies--who are the most likely candidates to do this work--do not keep such records. The gain to researchers and to the insurance companies themselves* would surely be worth the extra manpower this undertaking would require. For example, when a high-ranking official in an insurance advisory group was asked how his company calculated hail insurance premiums, he responded that, "We take a 'best guess' at the problem."

Many insurance companies have reported that the individual claims for hail damage are on file. Compilation of these, though time consuming, would be quite valuable. Very recently, some insurance companies have begun compiling data on the incidence of insured collector breakage [40]. At least two companies have established Solar Advisory Groups. However, the data from these efforts, though quite valuable, will not be adequate for another five years to predict damage risk [40].

In developing design criteria to withstand any phenomenon, two separate studies must be done: (1) the risk of encountering the phenomenon must be evaluated, and (2) damage thresholds of the structure in question must be ascertained. In other words, one wants to know the force that is likely to be exerted on the structure, and also the force that the structure can withstand.

*not to mention consumers.

Knowing this, one will be presented with one of three possibilities: (1) the structure will be deemed sufficiently sturdy to withstand the phenomenon (at a certain level of risk), (2) some modification of the structure will be required, or (3) the resources necessary to protect against the potentially damaging phenomenon will outweigh the benefits accrued; in this case, the risk of damage simply does not warrant the expense accrued in protection.

Although the risk model in this report has been developed to assess possible damage to solar flat plate collectors, it can be used with complementary studies of any ground installation (as described above). First, the size hail expected at a given level of ultimate risk is determined using the model; then, the fallspeed and kinetic energy of this size stone are determined from Section 5. This is the incident likely to act on a given ground installation.

In the case of solar flat plate collectors, a parallel study to determine the damage thresholds of various glazings is currently underway at Altas Corporation. The results of this work, with recommendations for hail protection devices, should be complete by October 1979.

APPENDIX C

INDEPENDENCE OF SIZE DISTRIBUTION AND STORM SEVERITY

Using a network of farmers with wire-mesh baskets, Douglas [11,12,13] collected 67 samples of hailstones from Alberta storms. The stones were separated into size intervals, the mass of each size category was measured, and the "total ice content aloft," M , was calculated.*

$$M = \sum_D \frac{(M_{DO})}{(A \cdot t \cdot v_D)} \quad \text{Eqn. D.1}$$

where

M_{DO} = the observed mass of stones in the size category with midpoint, D .

v_D = the terminal velocity of stone size D .

A = the area open for hail collection.

t = the duration of the storm, measured by the hail-collecting farmers.

For each of the 67 samples, the hail precipitation rate R , was calculated:

$$R = \sum_D \frac{M_{DO}}{A \cdot t} \quad \text{Eqn. D.2}$$

*This is the terminology used by the original author. M , as defined by Douglas, can be viewed as the "average" mass of hail aloft per cubic meter of air at any instant during the storm.

M and R were compared for each sample, and a proportional relationship between the two was found--with an increase in one, there was a proportional increase in the other.

By definition, M represents the mass of hail aloft at any instant; R represents the rate at which hail strikes the ground. Intuitively, it can be seen that of two samples with equal mass, M, the one composed of fewer, larger stones, will have the higher precipitation rate, R. This is true because the larger stones have a faster fallspeed. If two samples with the same mean mass aloft, M, fell at the same precipitation rate, R, it is implied that the size distribution was the same in both. In other words, if M increases proportionally with R, the fallspeed of each sample M must remain constant. If the fallspeed of a group of samples of M remains constant, the size distribution of each sample must remain constant--because a change in size distribution would almost certainly lead to a change in fallspeed of the sample.

Each of the 67 samples can be considered to have a "mean mass aloft," M, falling at a mean terminal velocity, $\bar{v}$. More rigorously, then, M can be related to R by

$$R = k \cdot \bar{v} \cdot M$$

Eqn. D.3

If Eqn. D.3 is found to be linear, then $\bar{v}$ must be constant, regardless of the values of M or R. A change in the size distribution would almost certainly result in a change in the mean fallspeed, $\bar{v}$, of the sample (since larger stones simply fall faster).

The linear relationship observed between M and R implies a constant $\bar{v}$, and therefore a constant size distribution. This implies that the size distribution was in each case a sample of a fixed parent population of hailstone sizes in Alberta.

Douglas also divided his 67 samples into three categories: low, medium and high M. He plotted the size distributions for each of

these and for the combined histogram of all 67 samples, and found all plots to represent the same distribution. He fitted an exponential to these data, yielding*

$$N_D = 10 \exp (-2.93 D)$$

How is this analysis correlated with storm severity as defined in this report? Douglas demonstrated that the parent hailstone size distribution remains the same from storm to storm regardless of M, which can be seen as the mean hail mass aloft in one storm. The total mass of hail, M_T , collected in a storm is therefore the denormalized quantity

$$M_T = t \cdot M$$

Assuming that the size distribution does in fact remain constant regardless of M, the quantity "n" defined in this report (storm severity) can be related to M_T by

$$n = \frac{M_T}{V}$$

where

V = the volume of the mean diameter of the size distribution.

We now have

$$n = \frac{t \cdot M}{V}$$

Eqn. D.4

We want to know if the hailstone size distribution remains constant with changes in storm severity, n, as defined in this

*The category of lowest M had a poorer fit, possibly because of the inherently smaller sample size.

study. This can be shown if the size distribution remains constant with changes in each parameter on the right side of Eqn. D.4. Clearly, $\bar{V}$ itself is a parameter of the size distribution and is independent of changing M (as per Douglas). All that remains is to show that the size distribution does not change with changing values of " t "; that is, that the hailstone size distribution is independent of the duration of the storm in which the sample is collected. Gokhale, in a summary of the literature [24] reports that there is no correlation between these two hailfall parameters. Therefore, the hailstone size distribution is independent of changes in storm severity, n . Such independence is required in calculating Eqn. 3.1.

While the foregoing analysis is not rigorous proof of independence in Alberta, or of independence in thunderstorms in all locations, it is at present the best evidence available. Therefore, in this study, it has been assumed that the stone size distribution is not correlated with the total number of stones per storm.

APPENDIX D

List of Programs

- ¹NEWMOD (main)
- ²SIZEAD (subroutine)
- ³GAMMA (subroutine)
- ⁴PSI1 (function)
- ⁵PSI2 (function)
- ⁶POISMOD (main)
- ⁷XFAC (function)

¹NEWMOD calculates: $1 - \sum_x \left[P_3(x) \cdot \text{SIZEAD}^x \right]^K$
using a negative binomial for $P_3(x)$

²SIZEAD calculates: $\sum_n P_2(n) \cdot P_1(\leq D)^{n \cdot A}$

³GAMMA calculates: $\Gamma(Q)$ for use in NEWMOD

⁴PSI1 calculates: $\Psi(Q)$ for use in NEWMOD

⁵PSI2 calculates: $\Psi'(Q)$ for use in NEWMOD

⁶POISMOD calculates the same equation as NEWMOD (above), using a Poisson distribution for $P_3(x)$

⁷XFAC calculates: $X!$ for use in POISMOD

```

PROGRAM NEWMOD(NS,ANS,D,TAPE6=NS,TAPE7=ANS,TAPE1=D)
C-----
C
C THIS PROGRAM CALCULATES THE PROBABILITY OF ENCOUNTERING A
C HAILSTONE OF A GIVEN SIZE (XPROB CORRESPONDS TO THE
C CUMULATIVE FREQUENCY OF THAT SIZE STONE OR SMALLER)
C
C PARAMETERS USED ARE:
C     NEGATIVE BINOMIAL DISTRIBUTION FOR HAILDAY DATA,
C     SIZE DISTRIBUTION FOR HAILSTONE SIZES
C     GAMMA DISTRIBUTION FOR STORM SEVERITY
C     AREA SUSCEPTIBLE TO HAIL
C     NUMBER OF YEARS SUSCEPTIBLE TO HAIL
C
C REQUIRES SUBROUTINES GAMMA(X), PSI1(Q), PSI2(Z), AND
C     SIZEAD(XPROB,SUMSUB,AREA)
C-----
    DIMENSION X(25)
    REAL METERS
    DATA LGT,LGT2,GAMANS/1,1,1.00000/, JTEST/0/
    SUMSUB = 0
    20 READ (6,1000) XPROB, AREA, K, XMEAN, XVAR, N
1000 FORMAT (F21.18,F6.2,I3,2F10.5,I3)
    IF (XPROB.EQ.0.5) GO TO 21
    IF (XPROB.LE.0.45) GO TO 30
C-----
C
C SUBROUTINE SIZEAD CALLED, CALCULATES "SUM:  $XPROB^n \cdot A \cdot P_2(n)$ "
C
C-----
    CALL SIZEAD(XPROB,SUMSUB,AREA)
C-----
C NEGATIVE BINOMIAL PARAMETERS CALCULATED
C-----
    DO 500 I = 1,N
    X(I) = I - 1
    500 CONTINUE
C
C p* AND k* CALCULATED, METHOD OF MOMENTS
C
    YP = (XVAR - XMEAN)/XMEAN
    YK = (XMEAN ** 2)/(XVAR - XMEAN)
    YCHECK = (1 + 1/YP) * (YK + 2)
    IF(YCHECK.GT.20) GO TO 10
C
C METHOD OF MAXIMUM LIKLIHOOD
C
    11 SUM1 = 0
    DO 501 LK = 1,N
    Z = X(LK) + YK
    TERM1 = PSI1(Z)
    SUM1 = SUM1 + TERM1
    501 CONTINUE
C
C (continued)
C

```

```

TERM2 = PSI1(YK)
YH2 = TERM2 * N
Z = 0
YBLOOP = 1 + (XMEAN/YK)
YCALC = ALOG(YBLOOP)
YH3 = N * YCALC
SUM2 = 0
DO 502 IK = 1,N
Q = X(IK) + YK
TERM4 = PSI2(Q)
SUM2 = SUM2 + TERM4
502 CONTINUE
TERM5 = PSI2(YK)
YH5 = TERM5 * N
YH6 = (N * XMEAN)/(YK * (YK + XMEAN))
ZH = - ((SUM1 - YH2 - YH3)/(SUM2 - YH5 + YH6))
YK = YK + ZH
YP = XMEAN/YK
IF (ZH.LT.0.01.AND.ZH.GT.-0.01) GO TO 10
GO TO 11
C-----
C
C GIVEN PARAMETERS, "NEG BINOMIAL(x) x SIZEADx" FOLLOWS
C-----
C
C TEST SECTION FOR GAMMA RESULTS (IF X.GE.33, GAMMA(X) RETURNED
C AS ALOG(GAMMA(X)) )
C-----
C
10 CALL GAMMA(YK,GAMANS,LGT,LGT2)
ASSIGN 70 TO LEPT
IF(LGT - 10) 50, 59, 51
70 ATEST = YK + 21
CALL GAMMA(ATEST,GAMANS,LGT,LGT2)
ASSIGN 71 TO LEPT
IF(LGT - 10) 50, 59, 51
71 IF(JTEST.EQ.1) GO TO 80
GO TO 81
50 GO TO LEPT, (70,71)
51 JTEST = 1
GO TO LEPT, (70,71)
C
59 WRITE (1,2011)
2011 FORMAT (1X,35H ERROR AT "59" FROM "IF" STATEMENT )
GO TO 99
C
C IF AT LEAST 1 GAMMA IS RETURNED AS ALOG, ALL GAMMA TRANS-
C FORMED INTO ALOG FOR CALCULATIONS
C
80 ZSUM = 0
T = YP + 1
C
C (continued)
C

```

```

DO 504 M = 1,21
MA = M - 1
R = YK + MA
IS = MA + 1
CALL GAMMA(YK,GAMANS,LGT,LGT2)
ASSIGN 90 TO LEEP
IF(LGT2 - 12) 52, 59, 53
90 YGYK2 = GAMNEW
CALL GAMMA(R,GAMANS,LGT,LGT2)
ASSIGN 91 TO LEEP
IF(LGT2 - 12) 52, 59, 53
91 YGR2 = GAMNEW
S = FLOAT(IS)
CALL GAMMA(S,GAMANS,LGT,LGT2)
ASSIGN 92 TO LEEP
IF(LGT2 - 12) 52, 59, 53
52 GAMNEW = ALOG(GAMANS)
GO TO LEEP, (90,91,92)
53 GAMNEW = GAMANS
GO TO LEEP, (90,91,92)
92 YGS2 = GAMNEW
TERM1 = ALOG(YP)
TERM2 = ALOG(T)
ZIT2 = (MA * TERM1) - (R * TERM2)
ZINTER = ZIT2 + YGR2 - (YGS2 + YGYK2)
ZNB = EXP(ZINTER)
YM2 = FLOAT(MA)
ZFIN = ZNB * (SUMSUB ** YM2)
ZSUM = ZSUM + ZFIN
504 CONTINUE
GO TO 60

```

C

C NO GAMMA RETURNED AS ALOG(GAMMA)

C

```

81 ZSUM = 0
T = YP + 1
CALL GAMMA(YK,GAMANS,LGT,LGT2)
YGYK = GAMANS
DO 503 M = 1,21
MA = M - 1
R = YK + MA
IS = MA + 1
CALL GAMMA(R,GAMANS,LGT,LGT2)
YGR = GAMANS
S = FLOAT(IS)
CALL GAMMA(S,GAMANS,LGT,LGT2)
YGS = GAMANS
ZIT = (YP ** MA)/(T ** R)
ZNB = (YGR/(YGS * YGYK)) * ZIT
YM = FLOAT(MA)
ZFIN = ZNB * (SUMSUB ** YM)
ZSUM = ZSUM + ZFIN
503 CONTINUE

```

C

C (continued)

C

SUBROUTINE SIZEAD(XPROB,SUMSUB,AREA)

```
C-----
C THIS SUBROUTINE, WHEN CALLED BY THE MAIN PROGRAM, "NEWMOD",
C CALCULATES:
C
C SUM:  $XPROB^{n \cdot A} \cdot P_2(n)$ 
C
C WHERE
C XPROB REPRESENTS THE CUMULATIVE FREQUENCY OF STONES OF LESS
C THAN OR EQUAL TO DIAMETER "D"
C n REPRESENTS A GIVEN NUMBER OF HAILSTONES PER STORM
C  $P_2(n)$  REPRESENTS THE RELATIVE FREQUENCY OF n STONES PER STORM
C-----
C DIMENSION WIDTH(3), A(3)
C REAL NORMAL
C DATA SUMP2, A(1), A(2), A(3)/.188910E6, 5.213, -.3123, -.05012/,
C 1WIDTH(1), WIDTH(2), WIDTH(3)/1., 10., 50./
C XPROB2 = ALOG(XPROB)
C NORMAL = ALOG(SUMP2)
C CALC = ALOG(100.0)
C TERM5 = A(2) * CALC
C SUMSUB = 0
C DO 501 I = 1,2500
C LOOP = 1
C YMID = (0.5 + (I-1)) * AREA
C WHAT = AREA * 201.
C IF(YMID.GE.WHAT) GO TO 10
C TEST = YMID * XPROB2
C TERM2 = ALOG(WIDTH(LOOP))
C CALC2 = ALOG(YMID)
C BLOOP = TERM2 + A(1) + (A(2) * CALC2) - TERM5
C BLOOP2 = (A(3) * (YMID/100.0)) - NORMAL
C BLEEP = EXP(TEST + BLOOP + BLOOP2)
C SUMSUB = SUMSUB + BLEEP
501 CONTINUE
10 YNEW = YMID - 0.5
C DO 502 K = 1,2500
C LOOP = 2
C YMID = (YNEW + 5. + ((K-1) * 10.)) * AREA
C WHAT = AREA * 1001.
C IF(YMID.GE.WHAT) GO TO 11
C TEST = YMID * XPROB2
C TERM2 = ALOG(WIDTH(LOOP))
C CALC2 = ALOG(YMID)
C BLOOP = TERM2 + A(1) + (A(2) * CALC2) - TERM5
C BLOOP2 = (A(3) * (YMID/100.0)) - NORMAL
C BLEEP = EXP(TEST + BLOOP + BLOOP2)
C SUMSUB = SUMSUB + BLEEP
502 CONTINUE
C
C (continued)
C
```



```

11 YNEW2 = YMID - 5.
DO 503 K2 = 1,2500
  LOOP = 3
  YMID = (YNEW2 + 25. + ((K2-1) * 50.)) * AREA
  TEST = YMID * XPROB2
  WHAT = AREA * 15000.
  IF(YMID.GE.WHAT) GO TO 21
  TERM2 = ALOG( WIDTH(LOOP) )
  CALC2 = ALOG( YMID )
  BLOOP = TERM2 + A(1) + (A(2) * CALC2) - TERM5
  BLOOP2= (A(3) * (YMID/100.0)) - NORMAL
  BLEEP = EXP(TEST + BLOOP + BLOOP2)
  IF (BLEEP.LE.-0.1E-35) GO TO 21
  SUMSUB = SUMSUB + BLEEP
503 CONTINUE
  WRITE (1,5001) YMID
5001 FORMAT(1X,20H ERROR MESSAGE **** ,F20.5)
21 RETURN
END

```

```

      SUBROUTINE GAMMA(X,GAMANS,LGT,LGT2)
C-----
C THIS SUBROUTINE CALCULATES GAMMA(X)
C WHERE GAMMA(x) = (x - 1)! FOR INTEGER AND NONINTEGER
C VALUES OF x
C
C GAMMA CAN BE FOUND IN REF [48].
C-----
      DOUBLE PRECISION Q
      DIMENSION Q(8)
      DATA Q/ -0.57719165D0, 0.98820589D0, -0.89705694D0, 0.91820686D0,
1-0.75670408D0, 0.48219939D0, -0.19352782D0, 0.03586834D0/,
1ADDON/.9189385/
C-----
C DETERMINATION OF PROCEDURE FOR GIVEN x
C-----
      LEAP = 2
      XNEW = X - 1
      IF (X) 1000, 1001, 1002
C-----
C x NEGATIVE
C-----
1000 LEAP = 1
      N = IFIX(XNEW)
      XN = FLOAT(N)
      F = XNEW - XN
      SYA = F
      GO TO 2001
2000 PROD = 1
      IJ = N - 2
      J = + (IJ)
      DO 13 K = 1,J
      L = K - 1
      DENOM = F - L
      PROD = PROD * DENOM
13 CONTINUE
      GAMANS = BLIP/PROD
      LGT = 9
      LGT2 = 10
      RETURN
C-----
C x = 0
C-----
1001 WRITE (1,101)
      101 FORMAT (1X,16H GAMMA(X), X<0? )
      STOP
C
C (continued)
C

```

```

C-----
C x POSITIVE
C-----
1002 XNEW = X - 1.
      IF (X.LE.2) GO TO 1003
      IF (X.GE.33)GO TO 1005
      IDIV = IFIX(XNEW)
      IR = IDIV - 1
      XR = FLOAT(IR)
      S = XNEW - XR
C-----
C COMPUTE S FOR USE IN RECURSION FORMULA
C-----
      SY = S - 1.
      SSUM = 1
      DO 12 I = 1,8
      SVALU = Q(I) * (SY ** I)
      SSUM = SSUM + SVALU
12 CONTINUE
      PROD = 1.
      IM = IR + 1
      DO 14 J = 1,IM
      INT = J - 1
      TERM = IDIV + SY - INT
      PROD = PROD * TERM
14 CONTINUE
      GAMANS = PROD * SSUM
      LGT = 9
      LGT2 = 10
      RETURN
C-----
C 1.LT.X.LE.2
C-----
1003 IF (X.LT.1.AND.X.GE.0.000) GO TO 1004
      IDIW = IFIX(X)
      XDIW = FLOAT(IDIW)
      SYA = X - IDIW
2001 SUM = 0
      DO 11 I = 1,8
      VALU = Q(I) * (SYA**I)
      SUM = SUM + VALU
11 CONTINUE
C
C (continued)
C

```

```
BLIP = SUM + 1
IF (LEAP.EQ.1) GO TO 2000
GAMANS = SUM + 1
LGT = 9
LGT2 = 10
RETURN
```

```
C-----
C 0.LE.X.LE.1
```

```
C-----
1004 WRITE (1,102)
102 FORMAT (1X,17H GAMMA(X), 0<X<1 )
STOP
```

```
C-----
C x GREATER THAN OR EQUAL TO 33, STIRLING'S FORMULA USED
C-----
```

```
1005 TERM1 = ALOG(XNEW)
GAMANS = ( (XNEW + 0.5) * TERM1) - XNEW + ADDON
LGT = 11
LGT2 = 13
RETURN
END
```

```

      FUNCTION PSI1(X)
C-----
C THIS FUNCTION CALCULATES PSI(X),
C THE FIRST DERIVATIVE OF THE GAMMA FUNCTION
C
C FOR USE IN NEGATIVE BINOMIAL
C METHOD OF MAXIMUM LIKLIHOOD CALCULATION
C-----
      SUM = 0
      DO 500 L = 1,151
      I = L - 1
      Y = X + I
      YBLOOP = ((2 * Y) + 1)/(2 * Y * (Y + 1))
      YCALC = ALOG (1 + (1/Y))
      SUM = (YBLOOP - YCALC) + SUM
500 CONTINUE
      YINTER = ALOG(X)
      PSI1 = YINTER - (1/(2 * X)) - SUM
      RETURN
      END

```

```

      FUNCTION PSI2(X)
C-----
C THIS FUNCTION CALCULATES PSI2(X),
C THE SECOND DERIVATIVE OF THE GAMMA FUNCTION
C
C FOR USE IN NEGATIVE BINOMIAL
C METHOD OF MAXIMUM LIKLIHOOD CALCULATION
C-----
      DIMENSION B(10), XNUM(10), XDEN(10)
C
C DATA FOR NUMERATOR AND DENOMINATOR OF BERNOULLI NUMBERS
C
      DATA XNUM/1.,1.,1.,1.,5.,691.,7.,3617.,53867.,174611./,
1      XDEN/6.,30.,42.,30.,66.,2730.,6.,510.,798.,330./
      DO 520 N = 1,10
      B(N) = XNUM(N) / XDEN(N)
520 CONTINUE
      IF (X - 2.5) 10, 11, 11
10 SUM1 = 0
      XNEW = X - 1
      DO 510 K = 1,10
      L = (2 * K) + 1
      TERM = (1/ (XNEW ** L)) - (1/ ((XNEW + 1) ** L))
      TERM2= ((-1) ** K) * B(K) * (2 * K)
      SUM1 = SUM1 + (TERM * TERM2)
510 CONTINUE
      TERM3 = (-.5) * ((1/ (XNEW ** 2)) + (1/ ((XNEW + 1) ** 2)))
      PSI2 = TERM3 - SUM1
      RETURN
11 SUM = 0
      J = 1
      DO 500 I = 3,21,2
      J = J + 1
      ICE = I - J
      SERIES = (( (-1) ** (J - 1)) * I * B(ICE)) / (X ** (I+1))
      SUM = SUM + SERIES
500 CONTINUE
      PSI2 = (1/ (X ** 2)) - (1/ (X ** 3)) - SUM
      RETURN
      END

```

PROGRAM POISMOD

```

C-----
C
C      USES POISSON DISTRIBUTION FOR HAILDAY DATA,
C      EMPIRICAL SIZE DISTRIBUTION FOR HAILSTONE SIZES
C      GAMMA DISTRIBUTION FOR AREAL DENSITY
C      AREA SUSCEPTIBLE TO HAIL
C      NUMBER OF YEARS SUSCEPTIBLE TO HAIL
C
C THIS PROGRAM REQUIRES THE FUNCTION XFAC
C      AND SUBPROGRAM SIZEAD(XPROB,SUMSUB,AREA)
C-----
      DIMENSION X(15)
      REAL METERS
      CALL OPEN (6,'POISDAT',2)
20  READ (6,1000) XPROB, AREA, K, XLEMNA
1000 FORMAT (F10.8,F6.2,I3,F10.5)
      IF (XPROB.EQ.0.5) GO TO 21
      IF (XPROB.LE.0.45) GO TO 30
      SUMSUB = 0
      CALL SIZEAD(XPROB,SUMSUB,AREA)
      ZSUM = 0
      TERM1 = EXP(-XLEMNA)
      DO 500 I = 0,20
      DENOM = XFAC(I)
      XI = FLOAT(I)
      POIS = (TERM1 * (XLEMNA ** XI))/DENOM
      CONTAG = POIS * (SUMSUB ** XI)
      ZSUM = ZSUM + CONTAG
500  CONTINUE
      FLT = FLOAT(K)
      YEARS = ZSUM ** FLT
      FINAL = 1 - YEARS
      CALL OPEN (7,'POISANS',2)
      WRITE (7,2009) XPROB, AREA, K, XLEMNA
2009  FORMAT (1X,F10.8,2X,F6.2,2X,I3,2X,F10.5)
      WRITE (7,2010) ZSUM, YEARS, FINAL
2010  FORMAT( 3(3X,E30.25/)/)
      GO TO 20
      30  WRITE (7,2012)
2012  FORMAT ( ' ----- '/')
      GO TO 20
      21  WRITE (1,2011)
2011  FORMAT ( ' PROGRAM DONE, RESULTS ON "B:POISANS" ')
99  STOP
      END

```

```

      FUNCTION XFAC(I)
C-----
C THIS PROGRAM CALCULATES I!
C-----
      XFAC = 1
      1 IF (I.LE.1) GO TO 2
      DO 500 J = 2,I
      XFAC = XFAC * J
500 CONTINUE
      RETURN
      2 IF (I.EQ.1.OR.I.EQ.0) GO TO 3
      WRITE (1,2000)
2000 FORMAT ( ' XFAC CALLED,  "I.LE.1" ')
      STOP
      3 XFAC = 1
      RETURN
      END

```



```

C-----
C  YEARS ENTERED IN CALCULATIONS
C-----
  60 FLT = FLOAT(K)
    YEARS = ZSUM ** FLT ,
    FINAL = 1 - YEARS
    WRITE (7,2009) XPROB, AREA, K, XMEAN, XVAR, N
2009 FORMAT (1X,F21.18,2X,F6.2,2X,I3,2X,F10.5,2X,F10.5,2X,I3)
    WRITE (7,2010) ZSUM, FINAL
2010 FORMAT(1X,E30.20/,1X,E28.20/)
    GO TO 20
    30 WRITE (7,2012)
2012 FORMAT (1X,36H ----- /)
    GO TO 20
    21 WRITE (1,2019)
2019 FORMAT (1X,34H PROGRAM DONE, RESULTS ON "B:ANS" )
  99 STOP
    END

```

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