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THE NEUTRON CAPTURE CROSS SECTION OF ^{88}Y

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THE USE OF GAMMA-RAY STRENGTH FUNCTIONS FOR PREDICTING*

THE NEUTRON CAPTURE CROSS SECTION OF ^{88}Y

by

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I. INTRODUCTION

According to the usual statistical model formulation, the neutron capture cross section in the region of overlapping resonances may be expressed as

$$\sigma_{\text{cap}} = \pi^2 \chi^2 \sum_{J\pi} \frac{(2J+1)}{(2I+1)} \frac{\langle \Gamma_N \rangle^{J\pi}}{D^{J\pi}} \frac{\langle \Gamma_{\text{cap}} \rangle^{J\pi}}{\langle \Gamma \rangle^{J\pi}} S^{J\pi}, \quad (1)$$

where I is the spin of the ground state of the target, $\langle \Gamma \rangle^{J\pi}$ is the average total width of the compound states with spin J and parity π , and $D^{J\pi}$ is the average spacing of such states. The average width for neutron emission is $\langle \Gamma_N \rangle^{J\pi}$, while $\langle \Gamma_{\text{cap}} \rangle^{J\pi}$ and $S^{J\pi}$ are the average width for neutron capture and the width fluctuation correction factor, respectively.

We distinguish here the neutron capture width $\langle \Gamma_{\text{cap}} \rangle^{J\pi}$ from the average radiation width $\langle \Gamma_\gamma \rangle^{J\pi}$ to account for the possibility that the gamma-ray cascade in the daughter nucleus may be interrupted by particle emission or fission. Thus $\langle \Gamma_{\text{cap}} \rangle^{J\pi} = \langle \Gamma_\gamma \rangle^{J\pi} - \langle \Gamma_{\gamma N} \rangle^{J\pi} - \langle \Gamma_{\gamma p} \rangle^{J\pi}$ etc. As a first approximation, $\langle \Gamma_{\text{cap}} \rangle^{J\pi}$ may be computed by assuming that all of the primary gamma-ray transitions from the initial $J\pi$ compound states that reach levels lying above the neutron separation energy will be terminated by neutron emission, and should be subtracted from $\langle \Gamma_\gamma \rangle^{J\pi}$ to yield $\langle \Gamma_{\text{cap}} \rangle^{J\pi}$.

In Eq. (1) the neutron width $\langle \Gamma_N \rangle^{J\pi}$ or the neutron strength function $\langle \Gamma_N \rangle^{J\pi}/D^{J\pi}$ may be computed from the optical model. For neutron energies below 1 or 2 MeV we use Moldauer's ¹⁾ global

parameters. For the average s-wave radiation width $\bar{\Gamma}_\gamma$ we have used the Stolovy-Harvey semi-empirical relation

$$\bar{\Gamma}_\gamma = K A^{2/3} U^\alpha [\text{DPS}(S_N)]^\beta, \quad (2)$$

where U is the neutron separation energy S_N corrected for pairing effects, $\text{DPS}(S_N)$ is the level spacing per spin state at the neutron separation energy, and K , α , and β are constants. Average values of these constants have been reported²⁾.

The energy dependence of the radiation width is taken to be the Brink-Axel E1 giant dipole formulation. In terms of the gamma-ray transmission coefficient $T_\gamma^{E1}(E_\gamma)$, we have

$$T_\gamma^{E1}(E_\gamma) = \frac{2}{\pi} \cdot \frac{1}{(\hbar c)^2} \cdot \frac{0.013A}{\Gamma_R} \cdot \frac{E_\gamma^4 \Gamma_R^2}{(E_\gamma^2 - E_R^2)^2 + (E_\gamma \Gamma_R)^2}. \quad (3)$$

Typical values for the giant dipole resonance energy and width are:

$$\begin{aligned} E_R &= 80A^{-1/3} \text{ MeV} \\ \Gamma_R &= 5 \text{ MeV} \end{aligned} \quad (4)$$

An expression may be written for M1 transitions, based on the single particle model of Blatt and Weisskopf³⁾:

$$T_\gamma^{M1}(E_\gamma) = \frac{15\pi}{20} \left(\frac{e^2}{\hbar c} \right) \left(\frac{1}{M^2 c^4} \right) E_\gamma^3. \quad (5)$$

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Recalling the relation between transmission coefficients and widths:

$$T_\gamma = \frac{2\pi\Gamma_\gamma}{D}, \quad (6)$$

the total radiation width $\Gamma_\gamma(S_N)$ at the separation energy is obtained by integrating the transmission coefficient over the density of final states.

Suppressing J and π we have:

$$\Gamma_\gamma(S_N) = \frac{1}{2\pi\rho(S_N)} \int_0^{S_N} (T_\gamma^{E1} + T_\gamma^{M1}) \rho(S_N - E_\gamma) dE_\gamma. \quad (7)$$

Alternatively, the E1 gamma-ray transmission coefficient may be expressed in terms of the E1 gamma-ray strength function. The definition used here is given by Bartholomew⁴⁾:

$$f_{i\lambda}^{J\pi} (E_\gamma) = \frac{\Gamma^{J\pi} \gamma_{i\lambda} E1}{E_\gamma^3} \rho_{J\pi}(E_\lambda). \quad (8)$$

The transition at an energy E_γ is from an upper state at E_λ to a lower state at E_i . The gamma-ray partial width for E1 transitions is averaged over states λ of a given J and π . The average density of such states is $\rho_{J\pi}(E_\lambda)$. It is customary to divide by E_γ^3 , and thus if the E1 transition is single-particle in character, the strength function is independent of energy. The Brink-Axel energy dependence yields an increasing strength function with gamma-ray energy below the peak of the giant resonance.

The strength function can be related to the photoabsorption cross section, $\bar{\sigma}_{\gamma a}$ in mb, by

$$f_{E1}(E_\gamma) = 26 \times 10^{-8} \frac{\bar{\sigma}_{\gamma a}^J(E_\gamma)}{g_J E_\gamma} (\text{MeV})^{-3}, \quad (9)$$

where g_J is the statistical factor $\frac{2J+1}{2I+1}$, and $\bar{\sigma}_{\gamma a}^J(E_\gamma)$ is the absorption cross section for the nucleus in ground state I to produce levels with spin J at $E_\lambda = E_\gamma$. For E1 transitions one assumes⁵⁾ that

$$\bar{\sigma}_{\gamma a}^J = \frac{g_J}{3} \bar{\sigma}_{\gamma a E1}. \quad (10)$$

It is common to assume that the shape of the giant dipole resonance can be described by the sum

of one or two Lorentz curves⁶⁾

$$\bar{\sigma}_{\gamma a}(E_\gamma) = \sum_{i=1}^2 \frac{\sigma_{\gamma a i} E_\gamma^2 \Gamma_{Ri}^2}{(E_\gamma \Gamma_{Ri})^2 + (E_\gamma^2 - E_{Ri}^2)^2}. \quad (11)$$

The strength of the giant dipole resonance, σ_0 , is the area under the curve

$$\sigma_0 = \int \bar{\sigma}_{\gamma a}(E_\gamma) dE_\gamma = \sum_{i=1}^2 \frac{\pi}{2} \sigma_{\gamma a i} \Gamma_{Ri} \quad (12)$$

and is given by the classical dipole sum rule

$$\sigma_0 = 60 \frac{NZ}{A} \text{ MeV} \cdot \text{mb}. \quad (13)$$

Equations 9-13 may be combined to give

$$f_{E1}(E_\gamma) = 3.3 \times 10^{-6} \frac{NZ}{A} \frac{E_\gamma \Gamma_R}{(E_\gamma \Gamma_R)^2 + (E_\gamma^2 - E_R^2)^2}. \quad (14)$$

A similar expression may be derived from the gamma-ray transmission coefficient equation fitted by Holmes, et al.⁷⁾ to experimental data.

II. THE RADIATION WIDTH

The recipe for Γ_γ that is built into our statistical model codes was determined about seven years ago²⁾ by fitting the Γ_γ and level spacing data available at that time. We thought that we would refit the Stolovy-Harvey expression in a mass region around $A = 90$ with the data now available in the Livermore Library⁸⁾. We also chose to use the level density parameters and pairing energies of Cook⁹⁾ because they also are built into our codes. We considered 33 nuclei in the range $31 \leq Z \leq 44$, and arrived at the following parameters for Eq. 2:

$$K = 2.65 \times 10^{-3}, a = 1.185, \beta = 0.1113. \quad (15)$$

Here Γ_γ is in eV units while U and DPS are in MeV units.

Figure 1 shows the ratio of Γ_γ (calculated) to Γ_γ (experiment) versus the mass of the target. While the fit has been improved, the scatter is still about $\pm 50\%$. For the compound nucleus ⁸⁹Y

our present Γ_γ value is 0.298 eV as opposed to our previous value of 0.622 eV. Assuming the Cook level density parameters are adequate we now compute a value of

$$\frac{2\pi\bar{\Gamma}_\gamma}{D} = 0.038 \text{ as opposed to a previous value of 0.075.}$$

For comparison, and to show the wide divergence of values available in the literature, we cite the results of Musgrove¹⁰⁾ who com-

putes $\Gamma_\gamma = 0.11$ eV and $\frac{2\pi\bar{\Gamma}_\gamma}{D} = 0.011$ for ^{89}Y .

Fig. 1. Ratio of calculated experimental radiation width values as a function of mass number A.

III. THE LEVEL DENSITY

Since the target nucleus ^{88}Y is not stable, the level density in the compound nucleus must be calculated from some level density expression. We use the parameters of Cook⁹⁾ and the Gilbert-Cameron level density form¹¹⁾ when experimental information is unavailable. The reliability of this set is put to a severe test around the $A = 90$ region because of the number of closed shells and subshells that occur here. A feeling for the errors involved may be obtained from Fig. 2. Here we show the ratio of the calculated to experimental⁸⁾ values for the level spacing per spin state as a function of mass. A scatter of $\pm 50\%$ is

common, while discrepancies of factors of 3 to 6 are observed. The figure does not include the estimated experimental errors, and these can be quite large in many cases, as is evidenced when the "experimental" values in BNL-325, Vol. 1, 1973 are compared with those in Ref. 8. Other level density sets, such as those of Schmittroth¹²⁾, Baba¹³⁾, or Dilg¹⁴⁾ do not seem to offer much improvement over the Cook set, and expectations of calculated level spacing accuracies of much better than 50% may not be reasonable. In many cases a 10% change in the level density parameter -a- can result in a factor of 2 change in the level density.

Fig. 2. Ratio of calculated to experimental level spacing values as a function of mass number A.

IV. THE GAMMA-RAY STRENGTH FUNCTION

From a consideration of the typical errors involved in estimating Γ_γ and the level spacing, it appears that it is difficult to compute $2\pi\bar{\Gamma}_\gamma/D$ to better than a factor of 2. This might correspond to a 50% change in the capture cross section at low energies, and an increasingly larger change as the incident neutron energy goes up. On the other hand, the gamma-ray strength function is expected to change rather slowly with Z and A , and at low neu-

tron energies no additional level density information is required. It would appear that trying to estimate the strength function rather than its parts would be the most dependable approach to the problem of calculating low energy neutron capture cross sections.

In a previous report¹⁵⁾ we discussed the method of extracting an average gamma-ray strength function from cross section and gamma-ray spectral data, using the Uhl¹⁶⁾ and Commuc¹⁷⁾ codes. Now we have used the codes to analyze the capture cross sections for ⁸⁷Sr, ⁸⁹Y, and ⁹⁰Zr to obtain the strength functions in the compound nuclei ⁸⁸Sr, ⁹⁰Y, and ⁹¹Zr. These are shown in Fig. 3, along with three calculated curves. The upper curve was computed using Eq. 14 and the resonance parameters of Berman⁶⁾. These are $\Gamma_R = 3.69$ MeV and $E_R = 16.8$ MeV, and were determined from photonuclear data with gamma rays in the 14-19 MeV range. As has frequently been observed in the past, the extrapolation of the tail of the giant dipole resonance to low energies often exceeds the experimental values.

Fig. 3. Gamma-ray strength functions extracted in this work (Δ , $\circ$, $\square$), compared with theoretical values (---, —, - - -).

The two lower curves were calculated from the gamma-ray transmission coefficient expression in Ref. 7. They are for ⁹⁰Y and ⁸⁸Sr, and show the same trend of a lower ⁸⁸Sr curve that the experimental data show. The expression in Ref. 7 has an arbitrary normalization factor, which if increased by 30% or so would reproduce the data very well.

We have chosen to use for ⁸⁹Y the average of the experimental strength functions for ⁹¹Zr and ⁹⁰Y, primarily because the cross section data for the ⁸⁷Sr target is so sparse. This may well be too high, and the lower curve may be more appropriate; we will consider the problem more carefully in the near future. The resulting $2\pi\Gamma_\gamma/D$ for ⁸⁸Y turned out to be 0.034, as compared with the value of 0.038 we determined from the new Γ_γ fit. The agreement is unexpectedly good, and is probably fortuitous.

V. THE ⁸⁸Y CAPTURE CROSS SECTION

In Fig. 4 we show the history of the Livermore estimate of the ⁸⁸Y capture cross section. The 1971 curve was computed using the ABACUS-NEARREX code¹⁸⁾, and the value for $2\pi\Gamma_\gamma/D$ was obtained from the unpublished estimate of D. Gardner and C. Gatrouris for Γ_γ and the level density parameters of Gilbert and Cameron¹¹⁾. The two 1977 sets of results are about the same and came from the Γ_γ and strength function studies previously described. For comparison we also show the published estimate of Holmes⁷⁾, the LASL estimate of Don Barr, and the results computed with the COMMUC code if the Musgrove¹⁰⁾ Γ_γ and level density parameters are used. The Livermore results remain about a factor of 3 greater than the LASL (Don Barr) estimate.

Fig. 4. Livermore values for the $^{88}\text{Y}(n,\text{capture})$ cross section, compared with LASL and with the literature.

The history of the LASL estimate is given in Fig. 5, along with the Livermore and the Holmes⁷⁾ estimates. The two curves marked February 1977 and April 1977 were calculated by us using the COMNUC code and two estimates of $2\pi I_\gamma/D$ made by Ed Arthur of LASL. These were 0.0214 and 0.0115, respectively. The latter value is indistinguishable from the value of Musgrove¹⁰⁾. The April results essentially split the difference between the Livermore and the LASL (Don Barr) estimates.

Fig. 5. LASL values for the $^{88}\text{Y}(n,\text{capture})$ cross section, compared with Livermore and with the literature.

One feature that is common to all of the cross-section estimates discussed here is that none of them consider the effect of isolated resonances. The statistical model calculations apply to the case of overlapping resonances. Eventually at low enough energy the first unbound resonance will be reached. Below this point the statistical model calculation will be grossly in error. In Figs. 6 and 7 we illustrate this for the case of ^{89}Y as the target. The thermal neutron capture measurement has been extrapolated according to $1/v$ until the first resonance is reached. It is then abruptly increased and joined to the COMNUC calculation at two places, one on either side of the resonance. The resonance integral was calculated for each case, and both agree with experiment within the error of the data. The abrupt step corresponds to a factor of about 35, and the statistical model extrapolation would have seriously over estimated the reaction if the neutron fluence contained many neutrons below 1 keV.

Fig. 6. Low energy extrapolation of the $^{89}\text{Y}(n,\text{capture})$ cross section.

Fig. 7. Statistical model calculation of the $^{89}\text{Y}(n,\text{capture})$ cross section.

A similar situation will occur when ^{88}Y is the target, although we expect the first resonance to occur at a lower energy. In Fig. 8 we show the statistical model calculation connected to a $1/v$ line at about 50 eV. From level density considerations we might expect the resonance to occur below 100 eV. If so, the step will not seriously affect nuclear explosion calculations.

Fig. 8. Predicted $^{88}\text{Y}(n,\text{capture})$ cross section.

VI. PROPOSED EXPERIMENTAL MEASUREMENTS

Livermore expects to undertake two experimental measurements in the next few months that bear on the ^{88}Y question. One will be a photoneutron measurement to find the first unbound state in ^{89}Y . The second will be a (p,γ) measurement on a ^{88}Sr target, which will produce the ^{89}Y compound state of interest. Hopefully, we will be able to extract the gamma-ray strength function for ^{89}Y from the latter work.

VII. CONCLUSIONS

1) The present study indicates that the estimation of the gamma-ray strength function is the approach least subject to error when unmeasured capture cross sections are to be computed.

2) Although our 1971 estimate of the ^{88}Y cross section appears to be 20-30% too high, we are still faced with a factor of at least 3 discrepancy with the LASL (Don Barr) estimate and a factor of 2 discrepancy with the latest LASL (Ed Arthur) estimate.

3) Experimental measurements can contribute to the better understanding of the problem. Photoneutron measurements can locate the first resonance and perhaps get the level spacing in ^{89}Y , while the $^{88}\text{Sr}(p,\gamma)$ measurement should yield the gamma-ray strength function of interest.

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