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ABSTRACT

The nonlinear evolution of collisionless drift instabilities in a shear-free magnetic field has been studied by means of gyrokinetic particle simulation as well as numerical integration of model mode-coupling equations. The purpose of the investigation is to identify relevant nonlinear mechanisms responsible for the steady-state drift wave fluctuations. It is found that the saturation of the instability is mainly caused by the nonlinear $\mathbf{E} \times \mathbf{B}$ convection of the resonant electrons and their associated velocity space nonlinearity. The latter also induces energy exchange between the competing modes, which, in turn, gives rise to enhanced diffusion. The nonlinear $\mathbf{E} \times \mathbf{B}$ convection of the ions, which contributes to the nonlinear frequency shift, is also an important ingredient for the saturation.

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I. INTRODUCTION

It has been widely recognized that the investigation of steady-state collisionless drift wave fluctuations produced by the spatial inhomogeneity in a magnetized plasma is of considerable relevance for the understanding of confinement properties of tokamaks. Since the fluctuations in such devices are generally considered to be driven by a constant free energy source (the density and temperature profiles are quasistatic), the usual quasilinear relaxation of the background inhomogeneity is precluded as a means of nonlinear saturation. Thus, the identification of the nonlinear effects which are responsible for the steady-state fluctuations has been an area of much interest in recent years. Inspired in part by the experimental measurements,¹ a number of authors have proposed the wave-wave coupling model, based on the $\underline{E} \times \underline{B}$ nonlinearity for the cold ions and an approximate linear response for the electrons, to describe the fluctuations.^{2,3} Steady states within such a model can indeed be established through the energy transfer between the linearly unstable and linearly damped modes.³⁻⁵ Most importantly, these calculations have reproduced some of the salient features of drift wave turbulence observed in the experiment. However, they are not fully self-consistent calculations in that the potentially important nonlinear effects associated with the resonant particles are not considered. For example, as demonstrated earlier, nonlinear wave-particle interaction can also give rise to the steady state either by quenching the linear instability directly or by enhancing the damping processes.⁶ The relative importance of these nonlinear effects as well as the others can only be ascertained by treating them on the same footing in the calculation. Obviously, this is extremely difficult analytically, and is also nontrivial computationally. However, such a requirement can now be fulfilled due to the recent advance of gyrokinetic

particle simulation,⁷ which enables us to develop a scheme⁸ suitable for studying steady-state drift wave turbulence.

The use of particle simulation to study drift wave turbulence has long been considered as essential for gaining insight into the anomalous transport processes in confined plasmas. In the past, drift wave simulations have produced some highly interesting results such as the generation of ambipolar drifts⁹ and the excitation of convective cells.¹⁰ However, steady-state fluctuations could not be maintained in these earlier simulations because of the rapid quasilinear relaxation of the background inhomogeneity. Since experimentally the background is quasistatic, it was impossible to compare those simulations directly with the experimental results. Attempts have been made to maintain the steady state by recycling the simulation particles.¹¹ However, the uncertain role played by the recycled particles in the saturated state greatly complicates the interpretation of the results. The new steady-state gyrokinetic particle simulation scheme completely eliminates this troublesome problem.

Gyrokinetic particle simulation consists of utilizing the existing particle simulation techniques to solve a set of equations corresponding to a reduced Vlasov-Poisson system based on the gyrokinetic ordering. The basic formalism in slab geometry has been derived by Lee⁷ and, more rigorously, by Dubin et al.¹² The resulting nonlinear gyrokinetic Vlasov equation conserves phase space volume along its characteristics and, together with the gyrokinetic Poisson equation, also conserves energy. These properties are the essential ingredients necessary for the successful implementation of the particle simulation techniques. (Thus, the nonlinear gyrokinetic formalisms presented in Refs. 13 and 14 are not useful for our purpose.) In the reduced system, the particle gyromotion is eliminated explicitly; however, the all-

important finite gyroradius effects remain. Hence, without sacrificing the physics involved, we can accomplish particle pushing on the time scale of the frequency of interest and with a considerable reduction in numerical noise.^{7,8} Analytically, this set of equations gives a more expeditious route for obtaining the statistical descriptions of drift wave turbulence,¹² and can be further reduced, in the appropriate limits, to the model equations currently in use.^{2,3,13} The steady-state gyrokinetic particle simulation scheme used in the present paper has been obtained by performing a multiple-scale expansion on the gyrokinetic Vlasov equation, in which the equilibrium spatial scale lengths, after being separated from the perturbed scale lengths, are held constant.⁸ In this scheme, quasilinear relaxation in velocity space is still possible. However, its effect on the steady-state fluctuations is usually negligibly small for tokamak parameters.

As the first application of the newly developed gyrokinetic particle code in two dimensions,⁸ we have studied the collisionless drift wave instabilities driven by the density gradient in a shear-free magnetic field. The purposes of this investigation are several: we wish (1) to identify the dominant nonlinear effects responsible for the steady-state fluctuations, (2) to understand the mechanisms which give rise to the associated particle transport, and (3) to explore the strengths and weaknesses of the new gyrokinetic simulation scheme in the simplest, most understandable geometry. We believe knowledge of this kind is of utmost importance for future simulation efforts involving more realistic geometry.

It is found that in the simulation, after the initial stage of linear growth, a steady state can indeed be achieved, in spite of the absence of any quasilinear relaxation of the background density gradient. The fluctuations are characterized by a broad frequency spectrum with $\Delta\omega_k \sim \omega_k$ and a broad wavenumber spectrum in the range of $0 < k_{\perp}\rho_s \sim 1$, where $\rho_s = \rho_i(T_e/T_i)^{1/2}$ and

ρ_i is the ion gyroradius. In addition, there exists in the saturated state a continuous exchange of energy between modes. However, no evidence of ion heating has been detected regardless of the parameters used. As such, nonlinear wave-wave coupling to stable modes³⁻⁵ can be ruled out as the cause of saturation, since in this case ion Landau damping would be needed to balance the linear growth due to the electrons. Similarly, lack of ion heating also indicates that the nonlinear wave-particle interaction due to resonance broadening or induced scattering of the ions^{6,15} is not important. Thus, the observed saturation must be related to the nonlinear behavior of the electrons. To further understand the physics involved, we have studied a simpler system in which the longest wavelength modes in the system ($k_{\perp} \rho_s \sim 1$) also have the largest linear growth rates, so an inverse energy cascade is inhibited. Again, a steady state exists in the simulation. However, both the frequency and the wavenumber spectra are now sharply peaked indicating a more coherent state. As such, we can study this simple system with an initial-value mode coupling code based on the reduced Vlasov-Poisson system. The code assumes that $T_{\parallel,i} = 0$, but takes into account the self-consistent linear response of the electrons, as well as the terms corresponding to the nonlinear $\vec{E} \times \vec{B}$ convection for both species, the electron velocity space nonlinearity, and the ion nonlinear polarization density.¹² The advantage of this approach is that we can study various aspects of the instability by turning on and off different nonlinearities in the code. We have found that saturation can take place due to the presence of the nonlinear electron $\vec{E} \times \vec{B}$ convection alone, whereas the nonlinear ion $\vec{E} \times \vec{B}$ convection merely causes a frequency shift. However, when the instability is strong, the electron velocity space nonlinearity, which induces energy exchange between the competing modes, also plays an important role in determining the amplitude of the fluctuations in

the nonlinear state. The $\mathbf{E} \times \mathbf{B}$ particle orbits resulting from the energy exchange give rise to enhanced diffusion, the rate of which is quite different from the quasilinear estimate. The nonlinear ion polarization density has a rather minor effect on the results described here. Finally, by using a perturbation analysis for the case where only the nonlinear electron $\mathbf{E} \times \mathbf{B}$ convection is considered, we are able to ascertain that it is the convection of the resonant particles which shuts off the linear growth — and, moreover, that this occurs by changing the relative phase between the waves and the particles, rather than by direct modification of the wave-particle resonance in velocity space. The saturation criterion can be written as

$$\frac{e\phi}{T_e} \approx - \frac{\gamma_l / \Omega_i}{(k_L \rho_S)^2}$$

where ϕ is the electrostatic potential, γ_l is the linear growth rate and $\Omega_i \equiv eB/m_i c$. A more detailed investigation of this few-mode problem is given in Ref. 16.

The present paper is organized as follows. Section II gives the basic formulation for the simulation scheme as well as the simulation results. It is followed in Sec. III by the discussions of the initial-value mode coupling calculations and their results. The derivation of the saturation criterion and its physical interpretation is presented in Sec. IV. In Sec. V, anomalous diffusion is briefly discussed. Concluding remarks and recommendations for future work are in Sec. VI.

II. GYROKINETIC PARTICLE SIMULATION OF DRIFT INSTABILITIES

The nonlinear gyrokinetic equations, on which the present simulation scheme is based, have been obtained from the Vlasov-Poisson system by

averaging away the fast gyromotion through the use of the well-known gyrokinetic ordering.^{17,18} The ordering assumes that ω/Ω , ρ/L , $k_{\parallel}/k_{\perp}$, and $e\phi/T$ are of order ϵ , where ω is the frequency of interest, Ω is the gyrofrequency, ρ is the gyroradius, L is the equilibrium scale length, $k_{\parallel}$ and $k_{\perp}$ are the perturbed wavenumbers parallel and perpendicular to the magnetic field, respectively, and ϕ is the electrostatic potential. The averaging procedure has been described in detail in Refs. 7 and 12. Because of the nature of the averaging processes, the reduced system must be expressed in two different coordinate systems, with the gyrokinetic Vlasov equation in the guiding center variable space and the gyrokinetic Poisson equation in the original particle coordinates. To circumvent the numerical difficulties involving the transformation between the two coordinate systems, the simulation scheme described in Ref. 7 used the analytically transformed gyrokinetic Vlasov equation in the particle variable space with the assumption that, nonlinearly, $T_{\perp,i} = 0$. In the present paper, we eliminate this approximation by using an improved scheme,⁸ in which the transformation is accomplished by calculating the gyrophase-averaged quantities numerically.

The reduced Vlasov-Poisson system in slab geometry valid to order ϵ^2 can be written as^{7,12}

$$\frac{\partial F}{\partial t} + \left(v_{\parallel} - \frac{q}{m} \frac{1}{\Omega} \frac{\partial \psi}{\partial R} \times \hat{b} \right) \cdot \frac{\partial F}{\partial \tilde{R}} - \frac{q}{m} \frac{\partial \psi}{\partial \tilde{R}} \cdot \hat{b} \frac{\partial F}{\partial v_{\parallel}} = 0 \quad (1)$$

where

$$\psi \equiv \bar{\phi} - \frac{1}{2} \frac{q}{T} \frac{v_{\perp}^2}{\Omega} \left| \frac{\partial \phi}{\partial R} \right|_{\tilde{R}}^2 \quad (2)$$

and

$$\nabla^2 \phi - \tau k_{De}^2 (\phi - \Phi) + (k_{De}^2 \rho_s^2 / n_0) \nabla_{\perp} \cdot [(n_i - n_0) \nabla_{\perp} \phi] = -4\pi e (\bar{n}_i - n_e) \quad (3)$$

Here $F(\tilde{R}, \mu, v_{\parallel}, t)$ is the gyrophase-independent distribution function. $\tilde{R} \equiv \underline{x} - \underline{\rho}$, $\underline{\rho} \equiv -\underline{v}_{\perp} \times \hat{\underline{b}} / \Omega$, $\Omega \equiv qB/mc$, $\hat{\underline{b}} \equiv \underline{B}/B$, $\underline{B}$ is the external field, $\mu \equiv v_{\perp}^2/2$, $v_{\parallel} = \underline{v} \cdot \hat{\underline{b}}$, q is the signed charge, $v_t \equiv (T/m)^{1/2}$, $\phi(\underline{x})$ is the electrostatic potential, $\nabla \equiv \partial/\partial \underline{x}$, $k_{De}^2 \equiv 1/\lambda_{De}^2 \equiv 4\pi n_0 e^2 / T_e$, $\rho_s \equiv \rho_i (T_0/T_i)^{1/2}$, $\rho_i \equiv (T_i/m_i)^{1/2} / \Omega_i$, $\tau \equiv T_e/T_i$, and subscripts e and i denote species. In terms of Fourier transforms with

$$\phi(\underline{x}) = \int_{\underline{k}} \phi(\underline{k}) \exp(i \underline{k} \cdot \underline{x})$$

and

$$F(\tilde{R}) = \int_{\underline{k}} F(\underline{k}) \exp(i \underline{k} \cdot \tilde{R})$$

the coordinate-transformed quantities are

$$\bar{\phi}(\tilde{R}) = \int_{\underline{k}} \phi(\underline{k}) J_0\left(\frac{k_{\perp} v_{\perp}}{\Omega}\right) \exp(i \underline{k} \cdot \tilde{R}) \quad (4)$$

$$\bar{n}(\tilde{x}) = \int_{\underline{k}} \int F(\underline{k}) J_0\left(\frac{k_{\perp} v_{\perp}}{\Omega}\right) \exp(i \underline{k} \cdot \tilde{x}) d\mu dv_{\parallel} \quad (5)$$

and

$$\Phi(\tilde{x}) = \int_{\underline{k}} \phi(\underline{k}) \Gamma_0(k_{\perp}^2 \rho_i^2) \exp(i \underline{k} \cdot \tilde{x}) \quad (6)$$

where J_0 is the ordinary Bessel function, $\Gamma_0(b) \equiv I_0(b) \exp(-b)$, $b \equiv k_{\perp}^2 \rho_i^2$, and

I_0 is the modified Bessel function, $\phi(\underline{R})$ and $n(\underline{x})$ correspond, respectively, to $\bar{\phi}(\underline{R})$ and $\bar{n}(\underline{x})$ with $J_0 = 1$, and $n_0 \equiv \langle n(\underline{x}) \rangle_{\underline{x}}$ is the spatially averaged number density. Equation (1) is valid for both species. However, for the electrons, the finite gyroradius effects are negligible and Eq. (1) reduces to the usual drift-kinetic equation. In Eq. (3), $\rho_e = 0$ is assumed, and the additional terms on the left-hand side of the equation are the density response due to the ion polarization drift. Finally, in Eqs. (2) and (3), we have only kept the leading terms from the small gyroradius expansion for $O(\epsilon^2)$ quantities.

As we have mentioned earlier, the existing particle simulation techniques can be applied for solving Eqs. (1) and (3), and efficient numerical schemes for calculating Eqs. (4) and (5) accurate for $k_{\perp} \rho_i \lesssim 1$ are also available.⁸ Since the explicit gyromotion is totally eliminated from the system, the use of a large time step restricted only by the highest frequency of the perturbations is now possible. The associated numerical noise is also considerably reduced. Moreover, Eq. (1), by virtue of being in the guiding center coordinates, allows a steady-state simulation in which the zeroth-order inhomogeneities are held constant. This has been accomplished by first separating the equilibrium scale lengths from the perturbed scale lengths for the $\partial F / \partial \underline{R}$ term in Eq. (1), and then using the resulting equation to deduce the equations of motion for the particle pushing. Interested readers should consult Ref. 8 for details.

The simulation was carried out with a two-dimensional gyrokinetic particle code in terms of the variables $(x, y/R_x, R_y; \mu, v_{\parallel})$ in slab geometry. The zeroth-order density inhomogeneity, characterized by a constant $\kappa (\equiv -n'/n)$, is in the x direction, and the external magnetic field is described by $\underline{B} \equiv B_0(\hat{z} + \theta \hat{y})$ in the y - z plane, where θ is a small constant. Both the particles and the waves are assumed to obey the simple periodic

been detected. The saturation amplitude for the associated electron density response, $n_e(1,\pm 1)/n_0$, is about 1%; for the ion density response, $\bar{n}_i(1,\pm 1)/n_0$ is about 2%. In addition, zero-frequency density fluctuations corresponding to $n_e(2,0)$ and $\bar{n}_i(2,0)$ have been observed. They are the consequence of nonlinear coupling between $\phi(1,1)$ and $\phi(1,-1)$ modes, and their saturation level is very low, less than 0.5% of n_0 .

Case 2: $\theta = 0.002$

The instability in this case is much stronger than the previous one. Although linear theory predicts that there should be in the system a large number of modes with quite comparable growth rates, the fluctuation spectrum is again dominated by $\phi(1,1)$ and $\phi(1,-1)$. The resulting saturated field energy is almost three orders of magnitude higher than the thermal level. However, unlike Case 1 several neighboring modes do become excited to a level several times higher than the initial fluctuation level. Figure 3 shows the time histories and the frequency spectra for $\phi(1,1)$ and $\phi(1,-1)$, where the theoretical frequencies and growth rates are indicated by solid straight lines. Since the signals in the simulation are very strong, no bandpass filter was used for the data. Here again, the two modes do not grow at the same time. Nevertheless, their frequencies and growth rates, deduced from the linear part of the evolution, agree well with the theoretically prescribed values. The measured frequencies are $\omega/\Omega_i = \pm 0.045$, indicating that the nonlinear frequency shift is somewhat more pronounced for this case. Figure 3 also shows that there is a rapid energy exchange between the modes in the nonlinear stage, and the saturation takes place at $|e\phi/T_e| \approx 8\%$. The density fluctuations for $n_e(1,\pm 1)/n_0$ and $\bar{n}_i(1,\pm 1)/n_0$ achieve a maximum amplitude of 10% and 20%, respectively, and the nonlinearly generated zero-frequency

density responses, $n_e(2,0)$ and $\bar{n}_i(2,0)$, saturate at around 10% of n_0 . These levels of fluctuations are an order of magnitude higher than those of Case 1, although there is only a factor of 2.5 difference between their linear growth rates. In addition, oscillations for the $n_e(0,2)$ and $\bar{n}_i(0,2)$ modes have also been observed in the nonlinear stage. Their maximum amplitude is about 10% of n_0 , and the frequencies are $\omega/Q_i \approx 0.09$. These particular density oscillations were probably masked by the numerical noise in the previous case, if they existed at all.

For the cases described here, the potential and density fluctuations in their nonlinear stage of the development are dominated by a handful of coherent modes. This important simplifying feature enables us to study the nonlinear evolution in more detail. This we do in Sec. III by using a set of model mode-coupling equations in $(k, v_{||})$ space, so that the dominant nonlinear mechanisms can be easily identified. We have also carried out similar simulations in a larger system with $L_x \times L_y = 32\Delta \times 128\Delta$, for which $(k_x \rho_s, k_y \rho_s) = (0.842\pi, 0.2105\pi)$. Here, the most unstable linear modes correspond to $(m,n) = (1,\pm 4)$. In the simulation, they are the first ones to reach saturation. Subsequently, we have observed that other modes in the system with $k_{||} \rho_s \lesssim 1$ have also been excited through the inverse energy cascade processes^{2,5} to levels comparable to those of Cases 1 and 2. The resulting k - and ω -spectra become broadly peaked, indicating a more turbulent state. Since the development of the instability can be roughly divided into two stages, we will concentrate our effort in the present paper on the first stage. Investigations for more general cases will be presented elsewhere.

III. MODE-COUPLING CALCULATIONS FOR DRIFT INSTABILITIES

The mode-coupling equations in $(k, \mu, v_{||})$ space for the drift instability

in a shearless slab can be easily derived from Eqs. (1) - (6). Let $F = (F_M + f)n_0$ and $\kappa = -(\partial F_M / \partial R_X) / F_M$, where F_M is the zeroth order distribution function, taken to be Maxwellian. The gyrokinetic Vlasov-Poisson system then becomes

$$\begin{aligned} \frac{\partial f_k^e}{\partial t} + i k_{\parallel} v_{\parallel} f_k^e + i(\omega_{*k} - k_{\parallel} v_{\parallel}) \phi_k F_M^e \\ + \sum_{k=k'+k''} (k' \times \hat{b} \cdot k'' + i \frac{m_i}{m_e} k_{\parallel}' \frac{\partial}{\partial v_{\parallel}}) \phi_{k'} f_{k''}^e = 0, \end{aligned} \quad (8)$$

$$\begin{aligned} \frac{\partial f_k^i}{\partial t} + i k_{\parallel} v_{\parallel} f_k^i + i(\omega_{*k} + \tau k_{\parallel} v_{\parallel}) \phi_k F_M^i \\ + \sum_{k=k'+k''} (k' \times \hat{b} \cdot k'' - i k_{\parallel}' \frac{\partial}{\partial v_{\parallel}}) \phi_{k'} f_{k''}^i = 0, \end{aligned} \quad (9)$$

where

$$\psi \equiv \phi_k J_0(k_{\perp} v_{\perp}) + \frac{1}{2} \sum_{k=k'+k''} k_{\perp}' \cdot k_{\perp}'' \phi_{k'} \phi_{k''}, \quad (10)$$

and

$$\tau[1 - J_0(b)] \phi_k + \sum_{k=k'+k''} (k_{\perp}''^2 + k_{\perp}' \cdot k_{\perp}'') \phi_{k'} n_{k''}^i = \bar{n}_k^i - n_k^e, \quad (11)$$

where

$$n_k^e = \int f_k^e d\mu dv_{\parallel}, \quad (12)$$

$$\bar{n}_k^i = \int f_k^i J_0(k_{\perp} v_{\perp}) d\mu dv_{\parallel}, \quad (13)$$

and $\bar{n}_k^i$ corresponds to $\bar{n}_k^i$ with $J_0 = 1$. Here time, length, potentials (ϕ and ψ), and number densities are in units of Q_i^{-1} , ρ_s , e/T_e , and n_0 , respectively, and the superscripts denote species. In addition, we have assumed that $k^2 \lambda_{De}^2 \ll 1$ in Eq. (11) and $F_M = \exp(-v^2/2v_e^2)/[(2\pi)^{1/2} v_e^3]$, where the spatial variation of the density is ignored since $|k_x| \gg |k|$. [Note that velocities are in units of c_s ($\equiv \rho_s Q_i$).] Numerical methods for solving Eqs. (8) and (9) fall into the general category of Vlasov code simulation.¹⁹ If successfully implemented, they should yield the same information as that given by the particle code simulation in Sec. II. Although numerical considerations usually favor the particle approach, direct integration of Eqs. (8) and (9) can provide us with additional physical insight. However, before we make the attempt to reproduce the previous results, let us first simplify this set of equations.

For the cases studied in Sec. II, we noticed that there are only a few dominant modes during the evolution of the drift instability, and there is no evidence of ion Landau damping. Accordingly, Eqs. (8) through (13) can be considerably simplified. Let k_x and k_y represent their $(m,n) = (1,1)$ values, $k_\perp^2 \equiv k_x^2 + k_y^2 = 2k_x k_y$, $k_\parallel = k_y \theta$, $\omega_* \equiv \omega_{*1}$, and the subscripts $\pm$, 0, and 2 denote modes $(1,\pm 1)$, $(2,0)$, and $(0,2)$, respectively. We obtain

$$\begin{aligned} \frac{\partial f_+^e}{\partial t} + i k_\parallel v_\parallel f_+^e + i(\omega_* - k_\parallel v_\parallel) \phi_+ F_M^e \\ + (k_\perp^2 + i \frac{m_i}{m_e} k_\parallel \frac{\partial}{\partial v_\parallel})(\phi_-^* f_0^e - \phi_- f_2^e) = 0, \\ \frac{\partial f_-^e}{\partial t} - i k_\parallel v_\parallel f_-^e - i(\omega_* - k_\parallel v_\parallel) \phi_- F_M^e \end{aligned} \quad (14)$$

$$-(k_{\perp}^2 + i \frac{m_i}{m_e} k_{\parallel} \frac{\partial}{\partial v_{\parallel}})(\phi_+^* f_0^e - \phi_+ f_2^{e*}) = 0 \quad , \quad (15)$$

$$\frac{\partial f_0^e}{\partial t} + (k_{\perp}^2 + i \frac{m_i}{m_e} k_{\parallel} \frac{\partial}{\partial v_{\parallel}})(\phi_+ f_-^e - \phi_- f_+^e) = 0 \quad , \quad (16)$$

and

$$\frac{\partial f_2^e}{\partial t} + 2ik_{\parallel} v_{\parallel} f_2^e + k_{\perp}^2(\phi_-^* f_+^e - \phi_+ f_-^{e*}) + i \frac{m_i}{m_e} k_{\parallel} \frac{\partial}{\partial v_{\parallel}}(\phi_-^* f_+^e + \phi_+ f_-^{e*}) = 0 \quad (17)$$

for the electrons, where $F_M^e \equiv \exp(-v_{\parallel}^2/v_t^2)/[(2\pi)^{1/2}v_t]$,

$$\frac{\partial \bar{n}_+^{-i}}{\partial t} + i \omega_* \phi_+ (1-b) + k_{\perp}^2(\phi_-^* \bar{n}_0^{-i} - \phi_- \bar{n}_2^{-i}) = 0 \quad , \quad (18)$$

$$\frac{\partial \bar{n}_-^{-i}}{\partial t} - i \omega_* \phi_- (1-b) - k_{\perp}^2(\phi_+^* \bar{n}_0^{-i} - \phi_+ \bar{n}_2^{i*}) = 0 \quad , \quad (19)$$

$$\frac{\partial \bar{n}_0^{-i}}{\partial t} + k_{\perp}^2(1-b)(\phi_+ \bar{n}_-^{-i} - \phi_- \bar{n}_+^{-i}) + k_{\perp}^4(\phi_+^2 \bar{n}_2^{i*} - \phi_-^2 \bar{n}_2^{-i})/(1-b) = 0 \quad , \quad (20)$$

and

$$\frac{\partial \bar{n}_2^{-i}}{\partial t} + k_{\perp}^2(1-b)(\phi_-^* \bar{n}_+^{-i} - \phi_+ \bar{n}_-^{i*}) + k_{\perp}^4(\phi_-^2 \bar{n}_0^{-i} - \phi_+^2 \bar{n}_0^{i*})/(1-b) = 0 \quad (21)$$

for the ions, where $T_{\parallel,i} = 0$ and $b \equiv k_{\perp}^2 \rho_i^2 \ll 1$ are assumed, and

$$k_{\perp}^2 \phi_+ = \bar{n}_+^{-i} - n_+^e \quad , \quad (22)$$

$$k_{\perp}^2 \phi_- = \bar{n}_-^{-i} - n_-^e \quad , \quad (23)$$

and

$$k_{\perp}^2 (\phi_+ \bar{n}_-^{i*} + \phi_- \bar{n}_+^{i*}) / (1 - b/2) = \bar{n}_2^i - n_2^e \quad (24)$$

for the Poisson equation, where

$$n^e = \int f^e dv_{\parallel} \quad (25)$$

Here, we have set $\phi_0 = \phi_2 = 0$ in accordance with our simulation model. In arriving at Eqs. (20), (21), and (24), we have also assumed that the ion distribution functions associated with the nonlinear terms are Maxwellian in $v_{\perp}$ in order to simplify the formulation. Since $b \ll 1$, the approximation is not expected to have any significant effect. The ordering in these equations necessitates a brief discussion. For Eqs. (14) through (21), if f^e and $\bar{n}^i$ are regarded as $O(1)$ quantities and F_M^e as an $O(1/\epsilon)$ quantity, where ϵ is the original gyrokinetic ordering parameter, one can show that all the terms are of $O(\epsilon)$ except for those nonlinear terms associated with $\partial/\partial v_{\parallel}$ for the electrons and ϕ^2 for the ions, which are of $O(\epsilon^2)$. Thus, the usual $\mathbf{E} \times \mathbf{B}$ nonlinearities, those proportional to $k_{\perp}^2$, become $O(\epsilon)$ in the present formalism. Moreover, consistent ordering of the gyrokinetic Vlasov and Poisson equations requires that n^e and $\bar{n}^i$ be considered as $O(\epsilon)$ in Eqs. (22) through (24). Thus, the nonlinear $\phi \bar{n}^i$ terms are of $O(\epsilon^2)$. Since ϕ_2 has been set to zero, there is a redundancy among Eqs. (17), (21), and (24), which we will discuss later. The simplified system conserves energy. It can be shown easily that

$$k_{\perp}^2 \frac{d}{dt} (|\phi_+|^2 + |\phi_-|^2) = 2k_{\parallel} \text{Im} \left(\phi_+ \int v_{\parallel} f_+^{e*} dv_{\parallel} - \phi_- \int v_{\parallel} f_-^{e*} dv_{\parallel} \right) \quad (26)$$

which leads to

$$\frac{d}{dt} \{ k_1^2 (|\phi_+|^2 + |\phi_-|^2) + \frac{1}{2} \frac{m_e}{m_i} \int v_{\parallel}^2 f_{00}^e dv_{\parallel} \} = 0, \quad (27)$$

where f_{00}^e is the distribution function for $k_{\perp} = 0$. Equation (27) simply states that the electron kinetic energy is balanced by the ion sloshing energy.¹²

In order to isolate the nonlinear effects arising from the usual $\mathbf{E} \times \mathbf{B}$ convection, Eqs. (14) through (25) can be further reduced by ignoring the $O(\epsilon^2)$ terms, without violating the energy conservation. Moreover, Eqs. (17) and (21) now indicate that f_2^e and $\bar{n}_2^i$ would remain zero if

$$\frac{\phi_+}{\phi_-} = \frac{f_+^e}{f_-^{e*}} = \frac{\bar{n}_+^i}{\bar{n}_-^{i*}}. \quad (28)$$

To satisfy this relation at all times, the reduced system requires that

$$\frac{\phi_+}{\phi_-} = c \text{ and } |c|^2 = 1, \quad (29)$$

where c is a complex constant independent of time. Therefore, when the terms of $O(\epsilon^2)$ are neglected, the $+$ and $-$ quantities will have equal amplitudes at all times, if their initial amplitudes are identical. Consequently, f_2^e and $\bar{n}_2^i$ will not be excited nonlinearly. This interesting feature will be explored in detail later.

An initial value mode coupling code has been written for solving Eqs. (14) through (25) using the predictor-corrector method. To facilitate the integration in time, Eqs. (14), (15), and (17) have been multiplied first by a factor of $\exp(\alpha i k_{\parallel} v_{\parallel} t)$, where $\alpha = 1, -1$, and 2 , respectively. Each f^e is represented by a 128 grid in the velocity space to cover the range of $-3v_t \leq$

$v_{\parallel} \leq 3v_{Te}$. The integration step is $\Delta t = 1$. The numerical differentiation in $v_{\parallel}$ is accomplished by using a six-point average scheme. It is interesting to note that the method utilizing the Fourier-transformed $v_{\parallel}$ ¹⁹ does not offer any appreciable improvement numerically for our problem. The results corresponding to the two cases in Sec. II are as follows.

Case 1: $\theta = 0.01$

Let us start with the most simplified form of the code in which we (1) ignore all the $O(\epsilon^2)$ terms, (2) ignore the nonlinear $\underline{E} \times \underline{B}$ terms for the ions, and (3) use the condition of $c = 1$ in Eq. (29) for the initial perturbation. Thus, the only nonlinear effect kept is the electron $\underline{E} \times \underline{B}$ convection. The time history and the frequency spectrum for ϕ_+ (or ϕ_-^*) given by the code under these assumptions are shown in Fig. 4. Because of the approximations for the ions, the linear frequency and the growth rate ($\omega/\Omega_i = 0.046 + 0.009i$) deviate slightly from the prescribed values indicated in Fig. 1. The evolution has a distinct linear stage and is followed by steady-state nonlinear oscillations with no appreciable frequency shift. Aside from the fact that there is no energy exchange between ϕ_+ and ϕ_- , the nonlinear part of the evolution ($\Omega_i t = 350 - 1000$) closely resembles the particle code results in Fig. 2, in terms of the frequency and the saturation amplitude. This is also true for the electron density responses, n_1^e and n_0^e , and the ion density response, $\bar{n}_1^i$. The inclusion of the ion $\underline{E} \times \underline{B}$ nonlinearity has no apparent effect except for the excitation of $\bar{n}_0^i$. We have also found that the velocity-space nonlinearity associated with $\partial/\partial v_{\parallel}$ terms produces energy exchange between ϕ_+ and ϕ_- . However, it does not have any appreciable effect on the other aspects of the instability. Thus, we can safely conclude that the nonlinear electron $\underline{E} \times \underline{B}$ convection is chiefly responsible for the saturation in this case. A

perturbation analysis for the set of equations including only this nonlinearity will be presented in Sec. IV.

Case 2: $\theta = 0.002$

We again start with the most simplified case by keeping only the electron $\underline{E} \times \underline{B}$ convection in the code with $c = 1$. The time history and the frequency spectrum for ϕ_+ (or ϕ_-^*) are shown in Fig. 5. Again, the linear frequency and growth rate ($\omega/\Omega_i = 0.027 + 0.02i$) deviate slightly from the exact values given in Sec. II because of the assumptions for the ions. However, as one can see, the nonlinear part of the evolution is quite different from the particle code results in Fig. 3. While the frequency shift is just as large, the saturation amplitude of 2.3% is considerably smaller than that of the particle code. The accompanying density responses, $n_{\pm}^e$, n_0^e , and $\bar{n}_{\pm}^i$, are also much smaller in amplitude. Although we again obtain steady-state oscillations with the electron $\underline{E} \times \underline{B}$ convection alone, some important nonlinear mechanisms in our mode-coupling equations have apparently been left out. To reintroduce them into the code, let us begin with the $O(\epsilon)$ terms arising from the ion $\underline{E} \times \underline{B}$ convection. With $c = 1$, the results for ϕ_+ (or ϕ_-^*) are shown in Fig. 6. Here, the saturation amplitude remains low. However, the frequency of the wave has undergone a substantial downward shift, and the frequency for the nonlinear amplitude oscillation has also decreased. The same is true for the density fluctuations, which now also include $\bar{n}_0^i$. The discrepancy between Fig. 3 and Fig. 6 suggests that we need the terms of $O(\epsilon^2)$ to reproduce the particle code results. This raises the question concerning the redundancy among Eqs. (17), (21), and (24). We have found that the only reasonable solution comes by using Eq. (24) for evaluating $\bar{n}_2^i$ instead of Eq. (21). This reflects the fact that the quasi-neutrality condition is well satisfied in the

particle simulation, probably due to the existence of a large number of low amplitude modes which have been ignored in the mode-coupling code. Figure 7 gives the results for such a calculation where $\phi_+ = \phi_-^*$ at $t = 0$ has been used, i.e., $c = 1$ initially. The first part of the evolution ($\Omega_1 t = 0 - 350$) is identical to that of Fig. 6, since the nonlinear density fluctuations of n_2^e and $\bar{n}_2^i$ have not yet been fully excited, while the evolution in the fully developed nonlinear stage ($\Omega_1 t = 350 - 1000$) is very similar to those obtained from the particle code shown in Fig. 3. The saturation amplitude and the resulting frequencies are in remarkable agreement, as are the processes of energy exchange between the two modes. The density responses are also in accord with the particle simulation results. By switching on and off different $O(\epsilon^2)$ terms, we have further established that the dominant $O(\epsilon^2)$ terms in the second half of the evolution are those associated with the electron velocity-space nonlinearity. A brief comment about the initial condition is now in order. Since the initial perturbations for + and - modes are roughly equal in amplitude in the particle code as seen from Fig. 3, it is not unreasonable to use $c = 1$ for our case. Calculations with c slightly displaced from unity have not produced any significantly different results. However, it is not well understood why the data from the particle code do not show the same initial growths as those of the mode-coupling calculation.

We have so far demonstrated that, when the instability is weak, the nonlinear electron $\underline{E} \times \underline{B}$ convection is the dominant mechanism for the saturation which involves only three modes. However, when the instability is strong, the $\underline{E} \times \underline{B}$ convection for both species as well as the electron velocity-space nonlinearity are responsible for the saturation, which now involves four modes. The case of three-mode coupling will be discussed further in the next section.

We have also examined the consequence of using the Maxwellian approximation for the ion density responses. It is found that they do deviate somewhat from Maxwellian, but with negligible effect on the results presented here.

IV. SATURATION DUE TO ELECTRON $\tilde{E} \times \tilde{B}$ CONVECTION

As we demonstrated numerically in Sec. III, the nonlinear electron $\tilde{E} \times \tilde{B}$ convection can cause the saturation of drift instability via three-mode coupling. To explore the underlying physics, let us study the problem using a perturbative approach. The governing equations, obtained from Eqs. (14) through (25), and Eqs. (28) and (29) with $c = 1$, are

$$\frac{\partial f_+^e}{\partial t} + ik_{\parallel} v_{\parallel} f_+^e + i(\omega_* - k_{\parallel} v_{\parallel}) \phi_+ f_M^e + k_{\perp}^2 \phi_+ f_0^e = 0, \quad (30)$$

$$\frac{\partial f_0^e}{\partial t} - 2i k_{\perp}^2 \text{Im}(\phi_+^* f_+^e) = 0, \quad (31)$$

$$\frac{\partial \tilde{n}_+^{-i}}{\partial t} + i \omega_* \phi_+ (1 - b) = 0, \quad (32)$$

$$k_{\perp}^2 \phi_+ = \tilde{n}_+^{-i} - n_+^e, \quad (33)$$

where

$$n_+^e = \int f_+^e dv_{\parallel}. \quad (34)$$

Using the ansatz of $\exp(-i\omega t)$ for f_+^e , $\tilde{n}_+^{-i}$, and ϕ_+ , where $\omega \equiv \omega_{\ell} + i\gamma$ and $|\omega_{\ell}| \gg |\gamma|$, we can reduce Eq. (30) to

$$f_+^e = [F_M^e - (\omega_* - \omega_l) i \pi \delta(k_{\parallel} v_{\parallel} - \omega_l) F_M^e + i \frac{k_{\perp}^2 f_0^e}{k_{\parallel} v_{\parallel} - \omega_l}] \phi_+ , \quad (35)$$

which, from Eq. (34), gives

$$n_+^e = [1 - i\alpha + ik_{\perp}^2 \int \frac{f_0^e}{k_{\parallel} v_{\parallel} - \omega_l} dv_{\parallel}] \phi_+ , \quad (36)$$

where

$$\alpha \equiv (\omega_* - \omega_l) \sqrt{\pi} / (\sqrt{2} k_{\parallel} v_{te}) . \quad (37)$$

Here, the nonlinear term contains only the contributions from the resonant particles. Finally, we obtain the dispersion relation from Eqs. (32) and (33) as

$$1 + k_{\perp}^2 - i\alpha - \frac{\omega_*}{\omega} (1 - b) + ik_{\perp}^2 \int \frac{f_0^e}{k_{\parallel} v_{\parallel} - \omega_l} dv_{\parallel} = 0 . \quad (38)$$

Without the last term, Eq. (38) simply reiterates the linear dispersion relation given by Eq. (7), and yields the familiar results of

$$\frac{\omega_l}{\omega_*} = \frac{1-b}{1+k_{\perp}^2}$$

and

(39)

$$\frac{\gamma_l}{\omega_l} = \frac{\alpha}{1+k_{\perp}^2} .$$

It is instructive to note here that the linear growth comes about because of the phase lag between the density response and the potential, as indicated by the linearized Eq. (36).

The nonlinear dispersion relation, Eq. (38), can be solved iteratively. Let us first use the linear part of f_+^e in Eq. (35) to evaluate f_0^e in Eq. (31). Equation (38) becomes

$$1 + k_l^2 - i\alpha' - \frac{\omega_*}{\omega} (1 - b) = 0, \quad (40)$$

where

$$\alpha' = \alpha \left(1 - \frac{k^4}{2\gamma_l} |\phi_+|^2 \right). \quad (41)$$

Recalling Eq. (39), this indicates that the real frequency remains the same and the growth rate is now

$$\gamma = \gamma_l \left(1 - \frac{k^4}{2\gamma_l} |\phi_+|^2 \right). \quad (42)$$

Thus, the growth of the instability decreases as the wave increases in amplitude. Accordingly, the density response in Eq. (36) takes the form of

$$n_+^e = (1 - i\alpha')\phi_+. \quad (43)$$

Letting $\gamma = 0$ in Eq. (42), we obtain the saturation criterion as

$$|\phi_+| = \frac{\gamma_l}{k_l^2}. \quad (44)$$

Since $\alpha' = 0$ at saturation, we find that $n_+^e = \phi_+$ from Eq. (43). Therefore, the nonlinear saturation of the drift instability can be viewed as the result of the $\mathbf{E} \times \mathbf{B}$ convection of the resonant electrons, which nullifies the phase difference between the density response and the potential. The phase

difference exists linearly because of the finite ion gyroradius effects.

Let us carry the iteration procedure further by evaluating the new f_+^e and f_0^e from Eqs. (35) and (31), respectively. By making use of Eq. (42) and assuming $\gamma_\ell/(\gamma + \gamma_\ell) \approx 1$, we arrive at the new dispersion relation which has the same form as Eq. (40) but with

$$\alpha' = \alpha \left[1 - \frac{k_\perp^4}{2\gamma_\ell} |\phi_+|^2 \left(1 + \frac{k_\perp^4}{\gamma\gamma_\ell} |\phi_s|^2 \right) \right] \quad (45)$$

where $|\phi_s|$ corresponds to the time-asymptotic saturation amplitude. Again, the real frequency remains unchanged, and the growth rate is given by

$$\gamma = \gamma_\ell (\alpha'/\alpha) \quad (46)$$

By replacing γ with d/dt in this equation, we have

$$\frac{d^2 |\phi_+|}{dt^2} - \gamma_\ell \left(1 - \frac{k_\perp^4}{2\gamma_\ell} |\phi_+|^2 \right) \frac{d |\phi_+|}{dt} + \frac{k_\perp^8}{2\gamma_\ell} |\phi_+|^2 |\phi_s|^2 (|\phi_+| - |\phi_s|) = 0 \quad (47)$$

We have added the extra term proportional to $|\phi_s|$ in order to ensure the proper behavior of the solution at $t = \infty$. Roughly, the equation delineates the evolution in two stages, i.e., the saturation of the linear instability and the ensuing nonlinear amplitude oscillation. The former is described by Eq. (42), while the latter has a frequency of

$$\omega_L \approx k_\perp^2 |\phi_s| \approx \gamma_\ell \quad (48)$$

The solutions for Eq. (47) are shown in Fig. 8. For case 1, $\gamma_\ell = 0.011$ and

$|\phi_S| = 0.0085$ were used, while for Case 2, $\gamma_L = 0.027$ and $|\phi_S| = 0.022$. Using values of $|\phi_S|$ smaller than those given here, Eq. (47) would produce nonphysical solutions at $t = \infty$. Comparisons of the time evolution of the instabilities in Fig. 8 with the results in Fig. 4 and Fig. 5 reveal remarkable agreement between the two approaches. They not only confirm the validity of the iterative scheme but also lend credence to the physical arguments presented here.

The problem of three-mode coupling is studied in greater detail in Ref. 16. The saturation mechanism can be further understood using the Hamiltonian constructed from the characteristics of the equations of motion in the wave frame.¹⁶ This shows that the resonant particles are the ones which are most susceptible to the $\vec{E} \times \vec{B}$ convection, which, in turn, realigns the phase of the density response to that of the potential through the particle motion on the surface of constant Hamiltonian.

V. DIFFUSION DUE TO DRIFT INSTABILITIES

The use of the steady-state particle simulation scheme greatly facilitates the measurement of anomalous diffusion due to drift instabilities. Since the zeroth-order density scale length has been separated from those of the perturbations, the total simulation volume can be viewed as an infinitesimal segment of the density profile. Accordingly, quantities calculated from the global averages of the simulation plasma are also the local values for the much larger system. This is radically different from the conventional particle codes, in which the simulation plasma actually represents the entire system. Such a representation is obviously inadequate, since it obscures the different scale lengths involved.

In order to calculate the particle diffusion in the simulation, let us

begin with the continuity equation which, for the electrons, can be obtained readily from Eq. (1) with $\rho = 0$ as

$$\frac{\partial n_e}{\partial t} + \nabla \cdot \tilde{\Gamma}_e = 0 \quad , \quad (49)$$

where, in the same units as in Sec. III,

$$\tilde{\Gamma}_e = n_e v_e^D = - n_e \nabla \phi \times \tilde{b} \quad . \quad (50)$$

Upon invoking the multiple spatial scale expansion for the background and perturbed quantities, we find that the diffusion coefficient in units of ρ_{Si}^2 becomes

$$D = \langle \Gamma_{ex} \rangle / \kappa \quad , \quad (51)$$

where $\langle \rangle \equiv \int dx dy / L_x L_y$ denotes spatial average, Γ_{ex} is the electron flux in the inhomogeneity (x) direction, and κ is the inverse of the density scale length. From the point of view of single particle motion, the average electron flux can be calculated by

$$\langle \Gamma_{ex} \rangle = \sum_{j=1}^N v_{ex}^D(x_j) / N \quad (52)$$

where x_j is the position of the j -th particle, v_{ex}^D is given by Eq. (50), and N is the total number of particles in the simulation. In terms of the Fourier modes, defined in Sec. III, the flux can also be represented by

$$\langle \Gamma_{ex} \rangle = \sum_{\tilde{k}} k_y \text{Im}(\phi_{\tilde{k}} n_{\tilde{k}}^{e*}) \quad . \quad (53)$$

Either of these two equations can be used in the simulation to measure the flux, and they should give identical results. For the simplified system described by Eqs. (14) - (17), Eq. (53) becomes

$$\langle \Gamma_{ex} \rangle = 2 k_y \text{Im}(\phi_+ n_+^{e*} - \phi_- n_-^{e*}) . \quad (54)$$

The diffusion coefficient can also be evaluated for test particles as

$$D_t = \lim_{t \rightarrow \infty} \frac{1}{t} \sum_{j=1}^N (\Delta x_j)^2 / 2Nt , \quad (55)$$

where Δx_j is the change in position for the j-th particle in time t and N is the number of test particles. In the conventional particle simulation, Eq. (55) is usually used for determining the diffusion coefficient.

The diffusion measurements for the two cases studied in Sec. II and III are presented as follows. Because of the boundary conditions imposed for the waves and the particles, the particle diffusion is expected to be ambipolar. Thus, only electron diffusion is discussed here.

Case 1: $\theta = 0.01$

The particle diffusion associated with the instability in this case is very small. The coefficient calculated from Eqs. (51) and (52) using the particle code results in Sec. II is $D < 5 \times 10^{-5}$. It is significantly smaller than the Bohm diffusion coefficient of $D_B = 0.0625$. This has also been confirmed by the test particle measurements using Eq. (55). The time-averaged diffusion from the mode-coupling calculation in Sec. III is also nearly zero, although the flux obtained from Eq. (54) with $c = 1$, i.e., $\phi_- n_-^{e*} = \phi_+ n_+^e$, is

actually oscillatory in time because of the relative phase variation between the potential and the density. Furthermore, the perturbative calculation in Sec. IV also gives negligible diffusion after the saturation. This can be seen by substituting Eqs. (43) and (45) into Eq. (54) with $c = 1$.

Case 2: $\theta = 0.002$

Particle diffusion is quite significant in this case. The coefficients calculated from the particle code results using Eqs. (51) and (52) are shown in Fig. 9. In the linear stage, D increases exponentially, then reaches the maximum value of 0.035. In the saturated state, the coefficient changes rapidly in time with a time-averaged value of $D = 0.01$. (It is more than two orders of magnitude larger than that of Case 1, while there is only a ten-fold increase in the saturation amplitude.) Obviously, the nature of the diffusion is quite different for the two stages in this case. While the former can be described by the quasilinear theory, the latter is a highly nonlinear process. Furthermore, since Fig. 9 can be reproduced almost exactly using Eq. (54), we are lead to conclude that the diffusion is mainly caused by the dominant $(1, \pm 1)$ modes in the system. The observation has been verified by the mode-coupling code. For the case corresponding to Fig. 7, the time-averaged diffusion coefficient is slightly smaller than, but within a factor of two of the particle code result. Again, when the $O(\epsilon^2)$ terms are ignored in the mode coupling code, the time-averaged diffusion goes to zero. Test particle diffusion from the particle code has also been examined. The results are shown in Fig. 10, where Eq. (55) has been used to calculate the diffusion coefficients for the two distinct stages of the evolution. Interestingly, they agree very well with those obtained directly from the particle flux in Fig. 9. The agreement between the two strongly suggests that the diffusion in

the nonlinear stage in this case is the result of the random walk process. Theoretically, it can be estimated by²⁰

$$D_r = (\Delta x)^2 / 2\Delta t = (\Delta\phi)_{rms}^2 / 2, \quad (56)$$

where $\Delta\phi$ can be taken as the deviation of the potential with respect to its mean value. To obtain the coefficients in Figs. 9 and 10, we need $(\Delta\phi)_{rms} = 0.02$, which is not an unreasonable number judging from the amplitude variation of the fluctuating potentials in the saturated state in Fig. 3. As we have pointed out previously, these fluctuations are mostly the result of the energy exchange between the two modes. The random walk process can be described with the aid of the equal potential plots taken from the particle simulation at two different times, as shown in Fig. 12(a) for $\phi(1,1) = \phi(1,-1)$ and in Fig. 12(b) for $\phi(1,1) < \phi(1,-1)$, where the arrow indicates the direction of the phase velocity. Since the orbits of the particles moving with the waves are confined by the $\phi = \text{constant}$ surfaces through the $\vec{E} \times \vec{B}$ motion, the diffusion will take place whenever the two modes are of unequal amplitude. The number of particles moving with the waves can be quite substantial due to the usual velocity-space trapping as well as the $\vec{E} \times \vec{B}$ trapping.¹⁶

VI. CONCLUSIONS

We have demonstrated in the present paper the usefulness and the versatility of gyrokinetic particle simulations in the investigation of drift-type instabilities. Its successful implementation gives us experience with, and confidence in this new scheme. These are very necessary to have before we plunge ahead to carry out more realistic simulations in toroidal geometry — which, of course, is our ultimate aim. The wealth of information and physical

insight we have gained here will undoubtedly help us in such an endeavor as well.

Meanwhile, the investigation of collisionless drift instability in a shear-free magnetic field has yielded results of considerable interest in their own right. The effect of the nonlinear $\vec{E} \times \vec{B}$ convection of the resonant electrons on the saturation of the instability has been explored, as was the role played by the electron velocity-space nonlinearity in relation to the resulting anomalous transport. Neither of these mechanisms have been widely discussed in the existing literature. Instead, for most of the current efforts,^{3-5,15} the emphasis has been on the nonlinear effects associated with the ions, where the linearized electron response was routinely assumed. Our approach here is quite different; in the particle simulation, all the nonlinear effects have been treated equally without making any prior assumptions.

Finally, one interesting aspect of our work here is the fact that the same set of equations has been studied via two completely different numerical methods. This was possible only because of the unique feature of the gyrokinetic formalism, which we discussed in detail in Sec. I. The ease with which the underlying physics has been unraveled in our work is primarily due to the complementary nature of the two approaches. We hope that this experience will provide the impetus for further development of the gyrokinetic formalism for the general Vlasov-Maxwell system.

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FIGURE CAPTIONS

- FIG. 1. Real frequencies and growth rates for the first 6×6 modes in the simulation plasma calculated from the linear dispersion relation, Eq. (7). $\theta = 0.01$.
- FIG. 2. Time evolution and frequency spectrum for $\phi(1,1)$ and $\phi(1,-1)$ from the particle simulation. Case 1.
- FIG. 3. Time evolution and frequency spectrum for $\phi(1,1)$ and $\phi(1,-1)$ from the particle simulation. Case 2.
- FIG. 4. Time evolution and frequency spectrum for $\phi(1,1)$ or $\phi^*(1,-1)$ from the mode-coupling code including only electron nonlinear $\tilde{E} \times \tilde{B}$ convection. Case 1.
- FIG. 5. Time evolution and frequency spectrum for $\phi(1,1)$ or $\phi^*(1,-1)$ from the mode-coupling code including only electron nonlinear $\tilde{E} \times \tilde{B}$ convection. Case 2.
- FIG. 6. Time evolution and frequency spectrum for $\phi(1,1)$ or $\phi^*(1,-1)$ from the mode-coupling code including only electron and ion nonlinear $\tilde{E} \times \tilde{B}$ convection. Case 2.
- FIG. 7. Time evolution and frequency spectrum for $\phi(1,1)$ or $\phi^*(1,-1)$ from the mode-coupling code including all nonlinearities. Case 2.

FIG. 8. Time evolution for $\phi(1,1)$ from the perturbative scheme including only electron nonlinear $\underline{E} \times \underline{B}$ convection.

FIG. 9. Diffusion coefficient evaluated from the electron particle flux in the particle simulation. Case 2.

FIG. 10. Electron test particle diffusion from the particle simulation. Case 2.

FIG. 11. Equal potential contour plots from the particle simulation for (a) $\phi(1,1) = \phi(1,-1)$ and (b) $\phi(1,1) < \phi(1,-1)$. Case 2.

Fig. 1

FIG. 2

Fig. 3

Fig. 4

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Fig. 5

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FIG. 6

Fig. 7

Fig. 3

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Fig. 9

Fig. 10

Fig. 11

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