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Some Fundamental Aspects of Fault-Tree and Digraph-Matrix Relationships for a Systems- Interaction Evaluation Procedure

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Abstract

Recent events, such as Three Mile Island-2, Brown's Ferry-3, and Crystal River-3, have demonstrated that complex accidents can occur as a result of dependent (common-cause/mode) failures. These events are now being called Systems Interactions. A procedure for the identification and evaluation of Systems Interactions is being developed by the NRC. Several national laboratories and utilities have contributed preliminary procedures. As a result, there are several important views of the Systems Interaction problem.

One view is that an enhanced Probabilistic Risk Assessment (PRA) with expanded boundary conditions and more emphasis on dependency analysis can solve the Systems Interaction problem. Alternative approaches include generic analysis, minimum-cut-set, common-cause/mode analysis, and digraph-fault tree analysis.

A second view is that techniques mathematically equivalent to the fault-oriented risk analysis of PRA may offer more complete or more efficient analytical or computational handling of dependent failures. This view seeks to explore the utility of a digraph-matrix, *success-oriented approach*.

A third point of view suggests the possibility of either heuristic or simplistic graph-based techniques. Heuristic techniques assign to a safety system a score based on some system characteristics.

This report reviews some fundamental mathematical background of both fault-oriented and success-oriented risk analyses in order to bring out the advantages and disadvantages of each. In addition, it outlines several fault-oriented/dependency analysis approaches and several success-oriented/digraph-matrix approaches. The objective is to obtain a broad perspective of present options for solving the Systems Interaction problem.

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Some Fundamental Aspects of Fault-Tree and Digraph-Matrix Relationships for a Systems Interaction Evaluation Procedure

1.0 Introduction

1.1 Background

Originally, nuclear reactor design was completed without a formal risk analysis. Instead, design basis accident scenarios were postulated, and the reactor systems were designed to safeguard against release of radioactive materials under these accident conditions. Of course, scenarios could always be devised under which the reactor would release radiation. The need to balance the likelihood of a scenario with its consequences eventually led to the landmark Reactor Safety Study in 1975.³⁰ Subsequent risk analyses took on a standard format of Probabilistic Risk Assessment (PRA). Event trees were used to describe the sequence of events that led to core damage. The resulting accident sequences identified critical reactor systems, normally the reactor safety systems and the main feedwater system.

With this approach, fault trees are used to determine the failure probability for each system in the accident sequence. The dependence among safety systems is captured in event trees, while the dependence of two systems upon the same support system is captured in the fault trees. However, the crucial assumption in risk analysis has always been that basic events, the last events in the fault tree, are independent, i.e., $P(A \wedge B) = P(A) \cdot P(B)$.

1.2 Statement of the Problem

Recent events such as Three Mile Island-2,⁴ Brown's Ferry-3,^{11,17,26,29,31,32} and Crystal River-3,¹⁸ have demonstrated that complex accidents can occur as a result of dependent (common-cause/mode) failures. These events are now being called Systems Interactions. These events have led us to the realization that the assumption of independence can, in some cases, be a serious error. This requires us to consider

$$|P(A \wedge B) - P(A) \cdot P(B)| = \epsilon$$

In other words, the difference between the actual dependency of basic events and the simple product approximation of independence leads to an error, ϵ . One way to resolve ϵ is to extend the fault tree analysis to a level of resolution where the new basic events are actually independent. This procedure would lead to an enhanced PRA. Alternatively, a statistical correlation analysis of the dependency, along with a sensitivity analysis,⁶ could be used to find out when $P(A) \cdot P(B)$ is a good approximation to $P(A \wedge B)$.

In an attempt to reduce ϵ , recent risk analyses, such as the Interim Reliability Evaluation Program (IREP), have included more of the instrumentation and control and support system dependence in fault trees; nevertheless, limited computational capability, lack of failure data (on support systems and human error), and limited modeling resources have necessitated assumptions that omit some dependence from each analysis. The elimination of these restrictive assumption is the essence of the Systems Interaction problem.

1.3 Our Approach to the Problem

In this report we explore some of the fundamental mathematical aspects of fault-oriented and success-oriented risk analysis in order to find mathematically equivalent techniques. In addition, we outline several fault-oriented and digraph-matrix approaches. Our objective is to offer insight into the advantages and disadvantages of viable Systems Interaction approaches.

We present the equivalence relationship between success-oriented (path-sets) expressions and failure-oriented (cut-set) expressions for a uniquely specified graph model. Then we explain that when a fault tree is constructed for a system with a specified top box that considers only a limited number of all possible failure modes, that fault-tree analysis represents a subset of a success-oriented analysis.

Furthermore, we discuss and illustrate the analytic difficulties of minimizing a Boolean expression, where such difficulties arise from either a fault-tree or a success-oriented analysis. We state that even using third-generation computers (such as CDC 7600) and Nelson's algorithm (double complementation) to find the complete set of prime implicants, the size of expression that the computer can minimize is limited. This limitation has always been the reason for preferring fault-oriented analysis for systems which are large and complex. Indeed, it was this limitation which caused the restriction of PRA to the safety-system level, to the exclusion of support systems, shared environmental conditions, and dynamic human error.

Unfortunately, recent reactor accidents have demonstrated that this level of modeling is inadequate. In Section 3.2, we discuss risk assessment alterations that expand and enhance the PRA approach without exceeding available computer limitations.

While such approaches seem adequate to redress the Systems Interaction problem, they have been criticized for lack of completeness. That is, the fault-tree analysis requires an analyst to flawlessly use deductive reasoning as he works down from the top box to the basic events. There is no explicit mathematical algorithm that will result in a uniquely correct fault tree. Indeed, there is often significant variation between analysts modeling the same system. For this reason, a success-oriented method would be preferable. In Section 2.5 we explain how, once a representative digraph of the system is constructed, a unique path-set solution (and therefore unique cut-set solution) is found by using adjacency matrices, reachability matrices, and Boolean minimization. Nevertheless, we realize that even with fourth-generation computers, such as CRAY-1, the combinatorics of a success-oriented analysis would face severe computer limitation in the Boolean minimization of the model.

There is, however, an alternative. If we could partition the system digraph model into independent subgraphs, then the matrix and Boolean manipulations could be solved separately. Correctly partitioning the digraph while saving computations puts a greater burden on the analyst. Subgraph independence also becomes a problem.

1.4 Organization of the Report

In Section 2 we review some fundamental mathematical aspects of fault-tree and digraph-matrix methods, including the algebra of logic, the algebra of sets, Boolean algebra, and graph theory.

In Section 2.6 we show the interrelationship between fault-oriented and success-oriented (reliability) analysis in order to demonstrate the relationship of cut sets to path sets. This is an important relationship to appreciate, since it provides comparisons of the various approaches.

In Section 3 we present preliminary outlines of Systems Interaction identification procedures for both fault-tree and digraph-matrix approaches. Fault-oriented approaches, including PRA, generic analysis, minimum cut-set common-cause/mode analysis, and digraph fault-tree analysis, are presented in Section 3.2. Section 3.3 introduces success-oriented, digraph-matrix approaches.

2.0 Fundamental Mathematical Background

2.1 Overview

Specified tasks are usually accomplished using several branches of mathematics. Fault-tree analysis, for example, requires the use of Boolean algebra (which is related to the algebras of logic and sets), graph theory, and computer science (for Boolean reduction and manipulation). In this section, we present some fundamental mathematical aspects of these disciplines. This background will illustrate how these disciplines can be brought together to provide either fault-tree analysis or a mathematically equivalent success-oriented digraph-matrix analysis. The objective is to determine if analyst effort and/or computer analysis can be made more efficient in addressing the Systems Interaction problem. For readers familiar with Boolean algebra and the relationship between path-sets and cut-sets, a summary is given in Section 2.6.6.

2.2 Review of Logic

There are many ways of operating on propositions to form new propositions. An operation is said to be truth-functional if the truth value (truth or falsity) of the resulting proposition is determined by the truth values of the propositions from which it is constructed. The investigation of truth-functional operations is called the algebra of logic, although its subject matter forms only one branch of modern mathematical logic.^{16,16,25}

Negation is the simplest common example of a truth-functional operation. If A is a proposition, then its denial, not A , is true when A is false and false when A is true. We use $\neg$ to stand for negation.

Another truth-functional operation is conjunction. We shall use $A \& B$ to stand for the conjunction (A and B). The truth table for $\&$ is

A	B	A & B
T	T	T
F	T	F
T	F	F
F	F	F

There are four possible assignments of truth values to A and B . The only row in which $A \& B$ has the value T is the first row, where each of A and B is true. " $A \vee B$ " stands for " A or B or both". In its truth table, the only case where $A \vee B$ is false is the case where both A and B are false. The expression $A \vee B$ is called a disjunction (of A and B).

A	B	A $\vee$ B
T	T	T
F	T	T
T	F	T
F	F	F

For any truth-functional operation, i.e., any truth table, we can introduce a special symbol, called a connective, to indicate that operation. Thus the symbols $\neg$, $\&$, $\vee$, $\rightarrow$, $\leftrightarrow$ are connectives. These five connectives suffice for most practical purposes (where $\rightarrow$, $\leftrightarrow$ are conditional and biconditional connectives, respectively).

By a statement form (in the connectives $\neg$, $\&$, $\vee$, $\rightarrow$, $\leftrightarrow$) we mean any expression built up from the statement letters $A, B, C, \dots, A_1, B_1, C_1, \dots$ by a finite number of applications of the connectives $\neg$, $\&$, $\vee$, $\rightarrow$, $\leftrightarrow$.

A statement form A logically implies a statement form B if, and only if, every assignment of truth values making A true also makes B true. Examples:

- A logically implies A .
- A logically implies $A \vee B$. For whenever A is true, $A \vee B$ also must be true.
- $A \& B$ logically implies A .

Literals are the statement letters A, B, C , and the corresponding denials of statement letters $\neg A, \neg B, \neg C$. By a fundamental conjunction we mean either a literal or a conjunction of two or more literals no two of which involve the same statement letter. For instance, $\neg B, A \& B, \neg A$, and $A \& C$ are fundamental conjunctions, while $\neg \neg A, A \& B \& A, B \& A \& C \& \neg B$ are not fundamental conjunctions.

One fundamental conjunction A is said to be included in another B if all the literals of A are also literals of B . For example, $A \& B$ is included in $A \& B, B$ is included in $A \& B$, and $\neg C \& A$ is included in $A \& B \& \neg C$, but B is not included in $A \& \neg B$.

A statement form A is said to be in disjunctive normal form (dnf) if either A is a fundamental conjunction or A is a disjunction of two or more fundamental conjunctions, of which none is included in another. For example, the following statement forms are in dnf.

- B
- $\neg C \vee C$
- $A \vee (\neg B \& C)$
- $(A \& \neg B) \vee (\neg A \& \neg B \& C)$

Every statement form which is not a contradiction is logically equivalent to a statement form in disjunctive normal form.

We can summarize the algebra of logic as a set of statements A and connectives $\neg, \&, \vee$, as $(A, \neg, \&, \vee)$.

2.3 Review of Sets

A set is any collection of objects. In most cases, sets are defined by means of a characteristic property of the objects belonging to the set.^{10, 11, 25} Following are some of the basic principles used.

Notation: for a given property $P(x)$, $\{x: P(x)\}$ denotes the set of all objects x such that $P(x)$ is true.

The sets A and B are equal if, and only if, A and B have the same members.

Whenever $P(x)$ is a property satisfied by no objects at all, then $\{x: P(x)\}$ is a set having no members. A set with no members is called the null set or empty set.

Given sets A and B , their union $A \cup B$ consists of all elements of A or B or both. Thus $A \cup B = \{x: x \in A \vee x \in B\}$. Recall from Section 2.2 that $\vee$ stands for the inclusive "or".

If we represent the elements of A and B by points within two circles (a Venn diagram), then their union consists of all points lying within either of the two circles.

Given sets A and B , their intersection $A \cap B$ consists of all objects which are in both A and B . Thus,

$$A \cap B = \{x: x \in A \& x \in B\}.$$

Important relations between unions and intersections are given by the distributive laws¹⁶:

$$\text{First Distributive Law: } A \cap (B \cup C) = (A \cap B) \cup (A \cap C).$$

$$\text{Second Distributive Law: } A \cup (B \cap C) = (A \cup B) \cap (A \cup C).$$

The complete fixed set of concern, X , is called the universal set.

We use $\bar{A}$ to define the negation of A .

The union and intersection of subsets of X are again subsets of X . When we restrict ourselves to subsets of X , there is still another operation which can be introduced. If $A \subseteq X$, then the complement $\bar{A}$ of A is defined to be $\bar{A} = \{x; x \in X \text{ \& } x \notin A\}$.

We can summarize the algebra of sets as the set of sets S and operators $-, \cap, \cup$ as $(S, -, \cap, \cup)$.

In general, a uniform way of determining the set-theoretic operation corresponding to a given logical truth function is to express the latter in terms of $\neg, \&, \vee$, and then replace $\neg, \&, \vee$ by $-, \cap, \cup$ respectively. The statement letters need not be replaced, since they can serve as set variables in the new expression. More explicitly, an isomorphism exists between $(A, \neg, \&, \vee)$ and $(S, -, \cap, \cup)$.

2.4 Review of Boolean Algebra

Basics

By a Boolean algebra we mean a set B together with two binary operations $\wedge$ and $\vee$ on B , a unary operation $'$ on B , and two specific elements 0 and 1 of B such that the following axioms hold¹⁰:

- For any x and y in B , $x \vee y = y \vee x$ and $x \wedge y = y \wedge x$. Commutative Laws
- For any x and y in B , $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$ and $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$. Distributive Laws
- For any x in B , $x \vee 0 = x$ and $x \wedge 1 = x$.
- For any x in B , $x \vee x' = 1$ and $x \wedge x' = 0$.
- $0 \neq 1$.

A Boolean algebra can be designated by a sextuple $(B, \wedge, \vee, ', 0, 1)$.^{10, 11, 21} Terminology

- $x \wedge y$ is called the meet of x and y .
- $x \vee y$ is called the join of x and y .
- x' is called the complement of x .
- 0 is called the zero element.
- 1 is called the unit element.

By a Boolean expression we mean any expression built up from the variables $x, y, z, x_1, x_2, y_1, y_2, z_1, \dots$ by applying the operations $\wedge, \vee, '$ a finite number of times. In other words, if all variables are Boolean expressions, and if τ and σ are Boolean expressions, so are $(\tau \wedge \sigma)$, $(\tau \vee \sigma)$ and (τ') .

A statement form A of the algebra of logic and a Boolean expression τ are said to correspond if τ arises from A by replacing $\neg, \&, \vee$ by $', \wedge, \vee$ (respectively) and by replacing the statement letters $A, B, C, A_1, B_1, C_1, \dots$ by $x, y, z, x_1, y_1, z_1, \dots$ (respectively).

Given a Boolean algebra and a Boolean expression built up of variables $(x_1, \dots, x_n)$ by applying operators $(\wedge, \vee, ')$, we can determine a corresponding Boolean function $f(x_1, \dots, x_n)$ or $f(x)$. For each $(x_1, \dots, x_n)$, assigning values $(v_1, \dots, v_n)$ respectively, and interpreting the Boolean expression, we find $f(v_1, \dots, v_n)$. If $(v_1, \dots, v_n)$ are into $(0, 1)$, then $f(v_1, \dots, v_n)$ is also into $(0, 1)$.

It is well known^{1, 4, 19} that system fault-oriented logic diagrams such as fault trees and success-oriented reliability diagrams can be mathematically represented by Boolean functions $f(x)$, where $x = (x_1, \dots, x_n)$ are the binary variables representing the system's components. When $f(x)$ is an increasing function and each component is relevant, we call $f(x)$ a coherent structure function.^{1, 4} Once we have modeled a system and found its equivalent Boolean representation, we wish to eliminate irrelevant Boolean symbols through minimizing the Boolean expression.

The minimization problem consists of determining methods for finding a simplest equivalent expression (see Table 1). Since all that matters about the given expression is the truth function that it determines, the minimization solution amounts to finding one simplest expression (or all of them) defining a given truth function.

To achieve this, recall that:

- literal $\equiv$ statement letters $A, B, C, \dots$ and their denials $A', B', C', \dots$
- fundamental conjunction (f.c.) $\equiv$ a literal or a conjunct $(\cap)$ of two or more literals, no two of which involve the same statement letter

Table 1. Boolean algebra theorems.

Or (+)	And (·)	Theorem
$x + 0 = 0 + x = x$	$x \cdot 1 = 1 \cdot x = x$	Identity
$x + 1 = 1 + x = 1$	$x \cdot 0 = 0 \cdot x = 0$	Null elements
$x + x = x$	$x \cdot x = x$	Idempotent
$x + x' = 1$	$x \cdot x' = 0$	Complement
$x'' = x$	$x'' = x$	Involution
$x + x' = 1$	$x \cdot x' = 0$	Commutative
$x + (y \cdot z) = (x + y) \cdot (x + z)$	$x \cdot (y + z) = (x \cdot y) + (x \cdot z)$	Associative
$x + (y \cdot z) = (x + y) \cdot (x + z)$	$x \cdot (y + z) = (x \cdot y) + (x \cdot z)$	Distributive
$x + (x \cdot y) = x$	$x \cdot (x + y) = x$	Absorption
$(x + y) \cdot (x + z) \cdot (x + y \cdot z) = (x + y) \cdot (x + z)$	$(x \cdot y) \cdot (x \cdot z) \cdot (x + y \cdot z) = (x \cdot y) \cdot (x \cdot z)$	Consensus
$(x + y) \cdot (x + z) = x + y \cdot z$	$(x \cdot y) \cdot (x \cdot z) = x \cdot y \cdot z$	DeMorgan's

- Φ is a disjunctive normal form (dnf) if either:
 - Statement Φ is a fundamental conjunct or
 - Φ is a disjunction of two or more f.c. of which none is included in another.
- Let $\Phi = AB'C \vee ABD \vee ABC'D' \vee AB'C'D$
 - then $\ell\phi$ = number of literals = 14
 - $d\phi$ = number of disjuncts = 4
 - for Φ_1 and Φ_2 , Φ_1 is simpler if ($\ell\phi_1 \leq \ell\phi_2$ and $d\phi_1 \leq d\phi_2$ and at least one of these relations is "strictly less than").
- A dnf Φ is minimal (mdnf) for a statement form A iff Φ is logically equivalent to A and no other dnf is simpler than Φ .

The stage is now set to minimize the Boolean expression. The next subsections will demonstrate how to accomplish this.

Prime Implicants

To minimize a Boolean expression by finding its prime implicants:

- Let A be a statement form.
- A fundamental conjunction Ψ is a prime implicant of A if Ψ implies A, but A is not logically implied by any other fundamental conjunct included in Ψ . (Note: whenever space limitations require we will replace $\wedge$ by juxtaposition where it is not confusing.) For example:

$$\begin{aligned} \text{Let } A &= AB \vee AB'C \vee A'B'C \\ &= AB \vee (A \vee A')B'C \\ &= AB \vee B'C \text{ by consensus law (see Table 1).} \end{aligned}$$

$A = AB \vee B'C \vee AC$. Thus AB, B'C, AC are prime implicants. However, at this point, there is no way of knowing if these are all the prime implicants.

- A mdnf of A need not be a disjunction of all the prime implicants of A
- If Ψ is a f.c. and ϕ is a dnf, then Ψ is superfluous in $(\Psi \vee \phi)$ whenever $\phi = \phi \vee \Psi$ (means ϕ subsumes Ψ).
- A dnf is irredundant if it contains no superfluous terms.
- An irredundant dnf is a disjunct of prime implicants.
- Any minimal dnf must be irredundant, but an irredundant dnf need not be an mdnf.

The Consensus Method for Finding Prime Implicants

The consensus method for finding the prime implicants is:

- Let f_1, f_2 be two fundamental conjunctions. If there is only one letter A which occurs negated in f_1 and unnegated in f_2 , then $f_1 \wedge f_2$ is formed by deleting A and A' and omitting repetition of any other literal. This gives the consensus of f_1 and f_2 . For example:

$$\begin{aligned} f_1 &= AB'C, f_2 = ABD, \\ f_1 \wedge f_2 &= ACD, \\ \text{then } f_1, f_2 &\rightarrow AB'C \vee ABD \vee ACD \end{aligned}$$

- The consensus method consists of:
 - (1) eliminating any disjunct which includes another,
 - (2) adding the consensus of any two disjuncts as a disjunct, and
 - (3) repeating (1) and (2) until no longer applicable.

As an example, we can write $AD \cup ED' = AD \cup ED' \cup AE$. The following also illustrate the procedure:

$$\begin{aligned}
 \Phi &= ABCD \cup ABC'D' \cup A'BC'D \cup A'B'C' \cup AC'D' \\
 \Phi &= ABCD \cup A'BC'D \cup A'B'C' \cup AC'D' \quad (\text{absorption law}) \\
 \Phi &= ABCD \cup A'BC'D \cup A'B'C' \cup AC'D' \cup AC'D' \quad (\text{consensus of } ABC'D \cup A'B'C' \cup AC'D' \text{ is added}) \\
 \Phi &= ABCD \cup A'B'C' \cup AC'D' \cup A'C'D \quad (\text{absorption}) \\
 \Phi &= ABCD \cup A'B'C' \cup AC'D' \cup A'C'D \cup B'C'D' \quad (\text{consensus of } ABC'D \cup AC'D' \cup B'C'D' \text{ is added}) \\
 \text{Hence, prime implicants} &= (ABCD, A'B'C', AC'D', A'C'D, B'C'D')
 \end{aligned}$$

The Consensus Method For Finding the Minimal dnf

Once all the prime implicants are found, it might seem advisable to find the minimal dnf. Some methodologies first expand the prime implicants to the full disjunctive form. For it is possible to avoid this by using irredundant dnfs, then comparing the irredundant dnfs and picking out the minimal one. As an example, we can continue the illustration above:

$$\Phi = \sum \Pi = ABCD \cup A'B'C' \cup AC'D' \cup A'C'D \cup B'C'D'$$

Eliminate superfluous disjuncts. Check to see if f is logically equivalent to $\Phi \cup f$. This is true if $\Phi \rightarrow f$, for each literal in Φ , replace it in f by true and the negation of that literal by false (see Table 2).

From Table 2 we see that rows 2 and 5 form a tautology and are therefore superfluous. Next we find the irredundant dnf and select the minimum. Irredundant dnf equals:

$$\begin{aligned}
 &\sigma_1, \sigma_4, \sigma_1 (\sigma_2 \cup \sigma_1, \sigma_3) (\sigma_3 \cup \sigma_2, \sigma_4) \\
 &\sigma_1, \sigma_2, \sigma_2, \sigma_3 \cup \sigma_1, \sigma_2, \sigma_1, \sigma_4 \cup \sigma_1, \sigma_1, \sigma_2, \sigma_5 \\
 &\sigma_1, \sigma_2, \sigma_2, \sigma_4 \cup \sigma_1, \sigma_1, \sigma_4, \sigma_2 \quad (\text{absorption}).
 \end{aligned}$$

so $\sigma_1, \sigma_2, \sigma_1, \sigma_4$ and $\sigma_1, \sigma_3, \sigma_2, \sigma_2$ are the two irredundant dnf; since they are the same size either can be selected as the mdnf. Choosing the first one gives:

$$\text{mdnf} = ABCD \cup A'B'C' \cup AC'D' \cup A'C'D$$

From these last sections we realize that the analytic reduction of large Boolean equations requires the computer capabilities of complex codes such as the Set Equation Transformation System (SETS). This is a program for manipulating Boolean equations which is both general and flexible. It can accommodate complemented events and special gates. SETS can solve the Boolean equation of a fault tree for the complete set of prime implicants.

One of the most serious limitations of fault-tree analysis, however, is the computational space limits of present-day computers. Even the most efficient computers, such as CRAY, are limited in the size of the Boolean equation it can efficiently reduce. For this reason, merely drawing larger and larger fault trees to

Table 2. The replacement of literals in Φ . Rows 2, 5 form $D \cup D' = 1$ tautology and $A \cup A' = 1$ tautology.

	σ_1 ABCD	σ_2 A'B'C'	σ_3 AC'D	σ_4 A'C'D	σ_5 B'C'D'
1. ABCD	1	1	F	F	f
2. A'B'C'	F	1	F	D	D'
3. AC'D	F	F	1	F	B
4. A'C'D	F	B'	F	1	F
5. B'C'D'	F	A'	A	F	1

respond to the System Interaction problem is not a completely satisfactory solution. Efficient ways of handling the dependency analysis must be found.

2.5 Basic Graph Theory

Graph theory is one of the simplest and most elegant subjects of modern mathematics, with a wide variety of applications. Based on the simple idea of points connected by lines, graph theory combines these ingredients into a rich assortment of forms with flexible properties, making the subject a useful tool for studying many kinds of systems.

The term "network" is frequently used instead of "graph," especially when quantitative characteristics are imparted to the points and lines. We can speak, for example, of electrical networks, project networks, and flow networks, in which quantitative measures of energy, effort, and flow, respectively, are associated with the edges.

In this section the concept of a graph is introduced, first as a geometric structure consisting of points (vertices) in space connected by a system of curves (edges), and then in abstract terms.

Every graph of interest is abstractly identical (with respect to those properties of interest in graph theory) with an appropriate geometric graph.

A geometric graph, G , is a set $V = \{v_i\}$ of points and a set $E = \{e_j\}$ of simple curves satisfying the following conditions:

- Every closed curve in E contains precisely one point of V .
- Every open curve in E contains precisely two points of V , and these agree with its end points.
- The curves in E have no common points, except for points of V .

Thus graph $G = (V, E)$.

A geometric graph is simply a geometric configuration or structure consisting of a set of points connected by a set of nonintersecting continuous curves.

Many familiar structures may (when idealized) be regarded as geometric graphs. A system of highways, for example, can be idealized as a graph by ignoring the width of roads and considering junctions to be points.

A graph is said to be a tree if it is connected and has no circuits. More generally, a graph which is free of circuits and consists of k connected components is sometimes referred to as a forest of k trees. The concept of a tree plays a central role in many aspects of graph theory. A graph is a tree if and only if every pair of distinct vertices is joined by precisely one chain. Connectivity implies the existence of at least one chain, while the absence of circuits implies the existence of at most one chain.

Where the emphasis is on ways of separating sets of vertices from one another, the concept of a cut-set becomes important. For example, in the study of flows through networks, one looks at cross sections or cut-sets of the network to separate flow origins from destinations in order to find a restrictive cross section which constitutes a bottleneck.

In many situations it is necessary to associate an orientation or direction with each edge of a graph. In the case of a geometric graph this can be interpreted as a direction of traversal of the edge, whereas in the case of an abstract graph it means merely that the end points of each edge are distinguished from each other by ordering them. Thus the only structural differences between an undirected graph and a directed graph (also called digraph) is that the end points of an edge constitute an ordered rather than an unordered pair of vertices.

An arc of length n is a sequence of (not necessarily distinct) arcs, $a_1, a_2, \dots, a_n$, such that for an appropriate sequence of $n + 1$ vertices, $v_0, v_1, \dots, v_n$, we have $a_i \approx (v_{i-1}, v_i)$ for $i = 1, 2, \dots, n$. For example, in the figure the sequence a_1, a_2, a_5, a_3 forms an arc progression of length 4 with v_0, v_2, v_3, v_1 as the corresponding sequence of vertices.

An open (closed) arc progression in a directed graph determines a corresponding open (closed) edge progression in the associated undirected graph. The converse is not generally true, however. For example, in the figure above, the sequence a_1, a_2, a_3 determines an open edge progression joining v_2 and v_4 , but it fails to constitute an arc progression for lack of consistent directions.

An arc progression in which no arc is repeated is called a path progression or cycle progression, depending on whether it is open or closed. The corresponding set of arcs, without regard to sequencing, is called a path or cycle, respectively. If the vertices $v_0, v_1, \dots, v_n$ are all distinct (in which case the arcs are also distinct) the path progression or cycle progression, as well as the corresponding path or cycle, is said to be simple. A directed graph is said to be cyclic if it contains at least one cycle, and acyclic otherwise. (Note that a loop is a special instance of a cycle.)

Various matrices may represent incidence relations between the vertices and edges of a graph and, more generally, represent incidence relations between circuits, cut-sets, chains, and the edges associated with them. These algebraic representations apply to both directed and undirected graphs. Whether one manipulates these matrices or not, they are a convenient way to record the structure of the graph. There is a close relationship between the incidence, circuit, and cut-set matrices whereby it is possible to obtain a basis for a certain vector space associated with one in terms of a basis associated with the space for another. This, for example, facilitates the task of determining all the cut sets of a graph once its circuits are known. Properties of the cut-sets are closely related to the maximal flow through a network. The notions of a tree, its branches, and its chords play an important role in the formulation of basic matrices for circuits and for cut-sets.

In the case of undirected graphs the incidence matrices have entries of 0 and 1. The addition of numbers is always reduced mod 2. Thus $1 + 1 = 0$ (carry 1), $1 + 0 = 0 + 1 = 1$, $0 + 0 = 0$. This choice of entries indicates whether a property between two elements holds (with entry 1) or does not (with entry 0). Matrices may be added and multiplied as usual, and then the entries of the resulting matrix are recorded mod 2. For directed graphs, we may use 1, -1, 0, since an arc, for example, may be incident with a vertex and directed away from the vertex, directed towards it, or not incident with it at all. The mod 2 reductions are no longer adequate in this and we must consider matrices with integer-valued entries. In fact, they are more cumbersome to deal with than the mod 2 matrices. However, basically the same kind of theory is obtained for the matrices of a directed graph as is obtained in the undirected case. With the idea of a matrix, we associate the concept of a vector space and related theory involving bases and linear combinations, of which we assume the reader has a working knowledge.

Let G be a graph having n vertices and m edges. The incidence matrix associated with G is the $n \times m$ matrix whose rows and columns correspond to the vertices and edges, respectively, where each entry a_{ij} is 1 or 0 depending on whether the j th edge is incident with the i th vertex or not. For a loop, however, every column entry is 0. For example, the 2-component graph shown below has the following incidence matrix:

Some interesting properties of a graph are observed in its incidence matrix. For example, since an edge of the graph is incident with exactly two vertices, every column of its incidence matrix contains two unit entries. A loop, since it is (twice) incident with a single vertex, is the only exception. Hence its column (e_i) has zero entries. Thus the incidence matrix gives no indication of the existence of loops, because one does not know whether a zero corresponds to a loop or not.

The adjacency matrix^{5,21,22} can be viewed as describing if there can be flow from node i to node j . One-way flow in a network from node to node is represented by following the above definition; that is, for a pair of nodes (i, j) for which flow is allowed from i to j but not from j to i :

$$a_{ij} = 1 \\ a_{ji} = 0$$

A reachability matrix^{21,22,27} contains information on all possible connections between all pairs of nodes. The determination of whether any node is reachable from any other node can be made via Boolean manipulation of the adjacency matrix. The reachability matrix can be derived from the following property: connection from k to n element $\sim a_{kn}$, connection from n to l element $\sim a_{nl}$, hence element a_{kl} is composed of two terms, a_{kn} and a_{nl} , both of which must be nonzero, that is $a_{kl} = a_{kn} \cdot a_{nl}$.

Using the Boolean product operation (Boolean AND function)

$$a_{kl} = a_{kn} \cdot a_{nl} \quad \text{where} \quad 1 \cdot 1 = 1 \\ 1 \cdot 0 = 0 \\ 0 \cdot 1 = 0 \\ 0 \cdot 0 = 0$$

In matrix notation,²⁷ for a network containing n nodes

$$R_2 = [A] \cdot [A] = \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix} \cdot \begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} \end{bmatrix}$$

where R_2 represents the reachability matrix for all those paths which require two steps for all pairs of nodes.

For n steps the reachability matrix becomes

$$R_n = [A]^n$$

For the case of reachability for all sets of path lengths²⁷ ($n = 1$ to M)

$$R = \sum_{n=1}^M [A]^n \quad (\text{Note: there are more efficient algorithms for performing this calculation.})$$

where the summation represents the Boolean sum (logical OR) operation.

$$1 + 0 = 1 \\ 0 + 1 = 1 \\ 1 + 1 = 1 \\ 0 + 0 = 0$$

and R converges to a steady state value.

We now write down the proper cut-sets of the graph below, therefrom forming the cut-set matrix, each row of which characterizes one proper cut-set. The minimum cut-sets by inspection are

$$\begin{aligned} K_1 &= \{e_2\} \\ K_2 &= \{e_1, e_7\} \\ K_3 &= \{e_2, e_3\} \\ K_4 &= \{e_1, e_4, e_6\} \\ K_5 &= \{e_1, e_3, e_6\} \\ K_6 &= \{e_1, e_6, e_7\} \\ K_7 &= \{e_3, e_6, e_7\} \end{aligned}$$

and the cut-set matrix is given by

$$K = \begin{matrix} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 \\ \begin{matrix} K_1 \\ K_2 \\ K_3 \\ K_4 \\ K_5 \\ K_6 \\ K_7 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \end{matrix}$$

Ways of finding the cut-set matrix is an ongoing research effort in mathematics.

In a connected graph with indexed vertices, a path (properly chain) matrix, P , is obtained by making its rows, P_i , the paths from the first to the last vertex, and for its columns the edges of the graph. Hence the entries of the matrix are 1 or 0 depending on whether an edge belongs to a given path or not. For example, the graph above has the matrix of paths between v_1 and v_3 given by

$$P = \begin{matrix} & e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 \\ \begin{matrix} P_1 \\ P_2 \\ P_3 \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

The graphs manipulated by the reachability technique can be either binary (0 or 1 on edges) or weighted (number or symbol on edges). In the case of the weighted graph, the reachability matrix contains the identification of the path through the graph. This is shown²⁷ below where the paths from Node 1 to Node 3 are $AB + CD$.

$$A = \begin{matrix} & v_1 & v_2 & v_3 & v_4 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{bmatrix} 0 & A & 0 & C \\ 0 & 0 & B & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & D & 0 \end{bmatrix} \end{matrix}$$

$$R = \begin{matrix} & v_1 & v_2 & v_3 & v_4 \\ \begin{matrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{matrix} & \begin{bmatrix} 0 & A & AB + CD & C \\ 0 & 0 & B & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & D & 0 \end{bmatrix} \end{matrix}$$

The logical AND operation can be represented in a relatively straightforward fashion. The figure below shows the logical AND symbol along with the Boolean equation which it represents.

This can be rewritten in adjacency matrix format as:

	A	B	C
A			B
B			A
C			

$C = AB + BA + AB$

where the matrix is read in the same from-to manner as before; that is, to get to C from A requires B.

The logical OR operation is shown below in adjacency matrix format as:

	A	B	C
A			1
B			1
C			

The figure below²⁷ is a logic network (fault-oriented digraph or fault tree) composed of two AND gates and one OR gate

The individual component logic adjacency matrices for this network are given below.

	A	B	F
A			B
B			A
F			

	F	G	E
F			G
G			F
E			

	C	D	G
C			1
D			1
G			

These matrices can be combined into a single adjacency matrix which represents the entire network as:

	A	B	C	D	E	F	G
A						B	
B						A	
C							1
D							1
E							
F					G		
G					F		

The reachability procedure can be applied to this adjacency matrix to yield the Boolean logic equation for the network.

Can we reach the node E from any combination of input nodes alone? To answer this question, the only input nodes which will be considered are A, B, C, and D, with E as the output node.

From the $A^1 \rightarrow A^2$ matrices:

$$E = [BG] A + [AG] B + [F] C + [F] D$$

However, the equation contains variables other than input variables. Substitution for the variables G and F from the reachability matrix yields

$$G = C + D$$

$$F = AB$$

$$E = [B \cdot C] A + [A \cdot C] B + ABC + ABD$$

$$E = ABC + ABD = AB[C + D]$$

which is the logic expression for the network.

Notice that once again we are confronted with a Boolean manipulation problem that grows dramatically as the size of the digraph grows. While the mathematics of these techniques has been available for many years, practical code development is still in the research stage and requires development of the state-of-the-art algorithms.

2.6 Understanding Fault Tree and Reliability Relationships

2.6.1 Overview

Fault-tree analysis provides a quantitative and qualitative assessment of the failure characteristics of safety systems. The system is modeled as a logic diagram composed of a directed graph with a superimposed Boolean algebra. The objective of fault-tree construction is to model system conditions that result in an undesired top event. This restriction reduces the size of the problem.

Fault trees have notably been used to analyze physical systems where improved performance of individual components does not degrade the performance of the system as a whole. Such systems are called coherent structures. Non-coherence occurs whenever NOT gates are introduced into a fault-tree structure. This is most prevalent whenever human decisions play a role in instrument functioning or, more generally, whenever subsystems are found to conflict with one another. To minimize a Boolean expression of a coherent system only the minimal disjunctive normal form is necessary (see Section 2.4). To minimize a Boolean expression of a non-coherent system one must find the complete set of prime implicants.

The objective of this section is to establish a basis for analyzing coherent two-state systems. This will provide a perspective of the relationship between fault-oriented and success-oriented analyses.

2.6.2 Coherent Structure Theory

2.6.2.1 Introduction. This section examines the deterministic structural relationships between a two state system and its components. In particular, this section is concerned with systems whose reliability performance improves when the reliability of any component improves. Such systems are called *coherent*. Subsections look at structural relationships, such as path-sets, cut-sets, reliability, and importance, in order to analyze the reliability of a system.

2.6.2.2 System Representation^{1, 4, 19, 28}. A binary variable x is a variable that may assume values of zero and one only. Such a variable is used to indicate the functioning of a system's components. The state of the i th component is given as

$$x_i = \begin{cases} 1 & \text{if the } i\text{th component is functioning} \\ 0 & \text{otherwise.} \end{cases}$$

The vector x , denoted $\mathbf{x}$, represents the state of n components as

$$\mathbf{x} = (x_1, \dots, x_i, \dots, x_n) \quad .$$

The state of the system is indicated by the binary variable ϕ as

$$\phi = \begin{cases} 1 & \text{if the system is functioning} \\ 0 & \text{otherwise.} \end{cases}$$

If the components may be considered to determine the system, then

$$\phi = \phi(\mathbf{x}),$$

which is called the structure function. ϕ is allowed to assume values zero or one.

In a series system (Fig. 1a), the structure function is

$$\phi(\mathbf{x}) = \prod_{i=1}^n x_i = \min(x_1, \dots, x_n) \quad .$$

A parallel system (Fig. 1b) yields

$$\phi(\mathbf{x}) = \sum_{i=1}^n x_i = \max(x_1, \dots, x_n) \quad .$$

The series and parallel characteristics are related through

$$\prod_{i=1}^n x_i = 1 - \prod_{i=1}^n (1 - x_i) \quad .$$

When the value of $\phi(\mathbf{x})$ is independent of the i th component, that component is called irrelevant. Stated mathematically, component i is irrelevant if

$$\phi(x_1, \dots, 1_i, \dots, x_n) = \phi(x_1, \dots, 0_i, \dots, x_n) \quad .$$

or

$$\phi(1_i, \mathbf{x}) = \phi(0_i, \mathbf{x})$$

for all possible 2^{n-1} realizations of $\mathbf{x}$.

Figure 1. Structure functions for serial and parallel components.

A system of components is a coherent structure if $\phi(x)$ is a nondecreasing function and each component is relevant.

Any coherent structure is bounded by

$$\prod_{i=1}^n x_i \leq \phi(x) \leq \sum_{i=1}^n x_i$$

which shows that performance of the system is bounded below by a series system and above by a parallel system.

A useful concept for analyzing systems is that of a dual space. For $\phi(x)$, a dual structure function $\phi^d(x)$ is defined:

$$\phi^d(x) = 1 - \phi(1 - x)$$

where $1 - x = (1 - x_1, \dots, 1 - x_n)$.

2.6.2.3 Path-Sets and Cut-Sets. Coherent structures may be represented in either path-set or cut-set form. A path-set is defined as a set of components whose functioning ensures the functioning of the system. Thus, a path vector x yields $\phi(x) = 1$. A minimum path-set is a path-set which cannot be reduced and be a path-set; i.e., there is no subset of a minimum path-set that is a path-set. The binary function $\rho_j(x)$ is associated with the j th minimal path-set P_j as

$$\rho_j(x) = \prod_{i \in P_j} x_i$$

This takes the value one, if all the components in the j th minimal path-set function; if not, the value is zero. The system will function only if at least one minimal path-set is functioning:

$$\phi(\mathbf{x}) = \prod_{j=1}^{N_p} p_j(\mathbf{x}) = \prod_{j=1}^{N_p} \prod_{i \in P_j} x_i \quad ,$$

where N_p equals the number of path-sets.

A cut-set is a set of components whose failure results in the failure of the system. A cut vector $\mathbf{x}$ gives $\phi(\mathbf{x}) = 0$. The binary function $k_j(\mathbf{x})$ is associated with the j th minimal cut-set K_j :

$$k_j(\mathbf{x}) = \prod_{i \in K_j} x_i \quad ,$$

which has the value zero if all the components of the cut-set fail, and one otherwise. When at least one complete minimal cut-set fails, $\phi(\mathbf{x})$ equals zero. This situation is represented as

$$\phi(\mathbf{x}) = \prod_{j=1}^{N_k} k_j(\mathbf{x}) = \prod_{j=1}^{N_k} \prod_{i \in K_j} x_i \quad ,$$

where N_k equals the number of cut-sets.

2.6.3 Fault Tree and Reliability Spaces and Their Duals

While reliability analysis is concerned with the functioning of the system, fault-tree analysis is concerned with system failure. Their correspondence is given by

$$1 \leq i \leq n \quad x_i = \begin{cases} 1 & \text{if the } i\text{th component is functioning} \\ 0 & \text{otherwise.} \end{cases}$$

The fault tree top event is

$$\Phi(1 - \mathbf{x}) = 1 \quad \phi(\mathbf{x}) = \begin{cases} 1 & \text{if the system fails} \\ 0 & \text{otherwise.} \end{cases}$$

The dual reliability function is

$$\phi^*(\mathbf{x}) = 1 - \phi(1 - \mathbf{x}) \quad ,$$

the dual fault tree representation

$$\Phi^*(1 - \mathbf{x}) = 1 - \Phi(1 - (1 - \mathbf{x}))$$

Simplified as

$$\Phi^*(1 - \mathbf{x}) = 1 - \Phi(\mathbf{x}) = 1 - (1 - \phi(1 - \mathbf{x})) \quad ,$$

then

$$\Phi^*(1 - \mathbf{x}) = 1 - \phi^*(\mathbf{x}) \quad .$$

Figure 2 demonstrates these relationships.

Figure 2. Relationships between fault trees and reliability spaces and their duals.

Dual space representations are often useful for mathematical manipulation. To demonstrate this relationship for fault trees, consider Fig. 3. The fault tree of this directed graph is given in Fig. 4a. Construction of the dual fault tree is accomplished by exchanging AND and OR gates to arrive at Fig. 4b.

Figure 2 also reveals the relationship between success-oriented reliability and fault-oriented fault tree spaces. An application of DeMorgan's Law on the path-sets of reliability space yields the cut-sets of fault tree space.

To illustrate, write the sum-of-the-products (SOP) Boolean expression for the complete set of minimal path-sets of Fig. 3.

$$\prod_{i=1}^2 P_i = x_1 \cdot x_2 \cdot \vee x_3 \cdot x_4$$

Figure 3. Finding the path-sets of a simple directed graph.

Figure 4a. Cut vectors in fault-tree space $K_i(x_1, x_2, x_3, x_4)$.

Figure 4b. Path vectors in dual fault-tree space $P_i(x_1, x_2, x_3, x_4)$.

By application of DeMorgan's Law,

$$\text{DeMorgan} \left\{ \bigcap_{i=1}^2 P_i \right\} = \{ (1-x_1) \vee (1-x_3) \vee (1-x_4) \} .$$

Then

$$\text{DeMorgan} \bigcap_{i=1}^2 P_i = (1-x_1)(1-x_3) \vee (1-x_1)(1-x_4) \vee (1-x_2)(1-x_3) \vee (1-x_2)(1-x_4) ,$$

which is equal to the cut-sets of Fig. 4a. Thus,

$$\prod_{i=1}^{N_c} \prod_{n \in K_i} (1-x_n) = \text{DeMorgan} \prod_{i=1}^{N_p} \prod_{n \in P_i} x_n .$$

The following similarity transform can be formed:

The following theorem defines a structure group.

Theorem. (Structure group) The set $\theta = \{h, \Delta, *, \bullet\}$ of operators form the Abelian group $\{h(x), \bullet\}$ over any structure function, $f(x)$.

While this is an abstract concept, it is important in illustrating the fundamental mathematical connection between success-oriented and fault oriented analysis. It will become important later, in following papers, when relationships between cut-sets and path-sets and their duals are used in algorithms of various procedures.

2.6.4 Probability

In establishing a probability analysis of a coherent system, we first assume that the components are statistically independent. Then the probability of state x_i of the i th component is

$$p(X_i = 1) = EX_i = p_i \text{ for } i=1, \dots, n,$$

where EX_i is the expected value of the random variable X_i , and p_i is the reliability of the i th component.

The reliability of the system is

$$h = p(\phi(X) = 1) = E\phi(X).$$

By assuming that the components are statistically independent, the system reliability function is given as a function of components' reliability:

$$h = h(p),$$

where

$$p = (p_1, \dots, p_n).$$

Then

$$h(p) = E\phi(X) = E \prod_{i=1}^{N_0} \prod_{j=1}^{n_i} X_j = E \prod_{a \in K_1} \prod_{b \in K_2} X_b$$

As an example, consider Fig. 3:

$$h(p) = E[X_1 X_2 \vee X_3 X_4]$$

$$h(p) = EX_1 \cdot EX_2 + EX_3 \cdot EX_4 - EX_1 \cdot EX_2 \cdot EX_3 \cdot EX_4,$$

or

$$h(p) = p_1 \cdot p_2 + p_3 \cdot p_4 - p_1 \cdot p_2 \cdot p_3 \cdot p_4.$$

In general, a basic event may appear in more than one minimal path-set (or cut-set); if so, the probability of each path-set (cut-set) is not statistically independent. The following boundary criterion is then used:

$$\prod_{i=1}^{N_p} \prod_{k=1}^{N_k} p(x_i) \leq h(p) \leq \prod_{i=1}^{N_p} \prod_{k=1}^{N_k} p(x_i)$$

The right-hand side of the above equation may be further simplified by a first-order expression of the full expression, yielding

$$h(p) \leq \sum_{i=1}^{N_p} \prod_{k=1}^{N_k} p(x_i)$$

2.6.5 Importance Analysis

The arrangement of components within a system determines the importance of certain components. For example, a component in series can be expected to play a more important role than one placed in parallel. When the state of the i th component determines if the system functions or fails, it is called a critical component, described by

$$\phi(1, \mathbf{x}) - \phi(0, \mathbf{x}) = 1$$

On the other hand, when the i th component is given by

$$\phi(1, \mathbf{x}) = \phi(0, \mathbf{x})$$

then it has no effect on the system.

A critical path-set, $c_i(1, \mathbf{x})$, is designated to account for critical components. The total number of critical path-vectors for component (i) is

$$N(i) = \sum_{k=1}^{N_c} |\phi(1, \mathbf{x}) - \phi(0, \mathbf{x})|$$

A measure for the structural importance of the i th component is

$$I_s(i) = \frac{1}{2^n - 1} N_s(i)$$

By finding and comparing $I_s(i)$ for each i , it is possible to develop an ordering of the importance of components.

Reliability importance is another measure which takes into account component reliability in addition to system structure. This importance of a component can be measured by the rate at which its reliability increases the reliability of the system. Then

$$I_h(i) = \frac{\partial h(p)}{\partial p_i} = h(1, p) - h(0, p)$$

where $I_h(i)$ is the reliability importance of the i th component, and

$$I_h(i) = E[\phi(1, \mathbf{x}) - \phi(0, \mathbf{x})]$$

The reliability importance evaluates the effect of an improvement in component reliability on system reliability by

$$\frac{dh}{dt} = \sum_{i=1}^n \frac{\partial h}{\partial p_i} \cdot \frac{\partial p_i}{\partial t}$$

where t is time

Similarly, for fault trees,

$$i_i(t) = \frac{\partial h}{\partial p_i}$$

and

$$\frac{dh}{dt} = \sum_{i=1}^n i_i \frac{\partial p_i}{\partial t}$$

holds.

2.6.6 Summary

The conclusion to be drawn from this section is the mathematical equivalence relationship of fault-oriented analysis and success oriented analysis when both are carried out on the same graph (or system model). However, "success" tends to be associated with the efficiency of a system, the amount of output, the degree of usefulness, and production and marketing features. These characteristics are describable by continuous variables which are not easily modeled in terms of simple discrete events, such as "valve does not open," which characterizes the failure space. (Partial failures, e.g., a valve opens partially, are also difficult events to model because of their continuous possibilities.) Thus, the event "failure" is generally easy to define, whereas the event "success" may be much more difficult to tie down. Therefore, the use of failure space in analysis is much more valuable than the use of success space.

Thus, purely from a practical point of view, the size of the population in failure space is less than the size of the population in success space. In analysis, therefore, it is generally more efficient to make calculations on the basis of failure space. Actually, all that is necessary is to show that the analyst who considers failure space can get his job done. This has, indeed, been shown many times in the past. The drawing of tree diagrams for a complex system is an expensive and time-consuming operation. When failures are considered, it may be necessary to construct only one or two system models, such as fault trees, which cover all the significant failure modes. When successes are considered, it may become necessary to construct several hundred system models covering various definitions of success. A good example of the parsimony of events characteristic of failure space is the analysis for the Minuteman missile. Only three fault trees were drawn, corresponding to the three undesired events— inadvertent programmed launch, accidental motor ignition, and fault launch. It was found that careful analysis of just these three events involved a complete overview of the whole complex system.¹³

It is important to understand that a fault tree is not a model for all possible system failures of all possible causes of system failure. A fault tree is tailored to its top event, which corresponds to some particular system failure mode. The fault tree thus includes only those faults that contribute to this top event. Moreover, these faults are not the only possible ones—they cover only the most credible faults as assessed by the analyst.

3.0 Outline of Systems Interaction Identification Approaches

3.1 Overview

Often several branches of mathematics can be brought together in various ways to achieve a specified goal. The sketch of several mathematical branches presented in Section 2 will now be used as a background reference for several different approaches toward the identification of systems interactions.

First, Section 3.2 presents well-established variations of event-tree fault-tree methods. Then Section 3.3 provides two preliminary outlines of viable approaches that identify systems interactions through the use of success-oriented digraph-matrix techniques.

3.2 Fault Tree Approaches

3.2.1 Reactor Safety Study Approach

Fault-tree analysis is a deductive failure analysis which focuses on one particular undesired event and which provides a method for determining causes of that event.

For a given accident sequence in the event tree, each system failure requires a fault tree. Event trees identify system interrelationships involving various combinations of system success and failure, control system interrelationships, etc. The use of such event trees permits the fault trees to be constructed with greater attention to the applicability of the tree to its planned use and to the adequate treatment of potential common cause failures.²⁴

An event tree begins with an initiating event, and as the tree is developed it defines the possible outcomes of this event. These outcomes are determined by all the physically possible permutations encompassed by the successful operation or failure of all the applicable nuclear power plant systems that can cope with the effects of the initiating event. The event tree restricts the common-cause analysis to those combinations of subsystems pertinent to system failure.

However, an event tree containing all mathematically possible permutations would be far too large to use in conducting an effective common-cause analysis. Therefore, the reduction of event trees is of great importance in: the handling of common-cause failures and in ensuring the ability of the methodology to logically decrease the scope of the analysis to a tractable size.²⁴

Qualitative reduction involves the elimination of physically meaningless sequences from the mathematically complete trees. For example, in those sequences involving core melt, choices concerning the success or failure of the reactor vessel are logically eliminated, since it is assumed that the vessel will surely fail.

Because each initiating event produces an event tree with many accident sequences, a large number of sequences survives the reduction process. These sequences can be grouped with each sequence in a group receiving identical common-cause analysis treatment.

In the Reactor Safety Study (RSS),²⁵ common-cause failures were incorporated into the fault-tree construction in five ways:

- (1) The fault trees were constructed to meet the criteria of the systems in the event trees; the fault trees were therefore conditional. For example, if an accident sequence indicated a severe environment, the system fault tree included the results presented by that environment.

- (2) The fault trees identified components that were common to multiple systems figuring in an accident sequence. The detection of commonality was possible because the consistent "component type" and "component identifier" make-up a unique code for each component.

- (3) Each fault tree was developed to an extremely detailed component level in order to locate single component failures and potential common-cause failures deep within the system.

- (4) Certain human failures were explicitly included in the fault trees.

- (5) Test and maintenance considerations were included in the fault tree.

The study resulted in the following observations:

- (1) The analyses should be developed to a level of resolution of components where failure data are generally available. In some cases, deeper analysis was required solely to determine the logical relationship or to ensure independence between various inputs to a component.

(2) When a subtree was found to be independent of other parts of the overall fault tree it was treated as a type of component.

To analyze sequences, the fault trees of all the system failures in the sequence were combined ("ANDed" together). The Boolean analysis of the combined fault trees then extracted the common components and common events. This process identified single failures that could cause failure of multiple systems.

The ground work for this step was laid earlier in the construction of the fault trees. When an event occurred, it was given the same code name (eight-character event identification) so that commonalities would be obvious. When he was in doubt about the independence of given events, or about whether the events were identical, the analyst coded the events identically. If later Boolean analysis indicated that these events were significant, further investigation was conducted.

To summarize, systems interaction analysis could be an expansion of probabilistic risk assessment analysis, using the same tools but with more emphasis on dependent failures, including more support system analysis, shared environmental conditions, and dynamic human error.

3.2.2 Generic-cause Approach

This approach (see Ref. 24) differs from most common-cause analysis methods because it deals directly with minimal cut-sets instead of adding secondary failures to the logic model. Thus, only component failures, which result in system failure are considered. Partial system degradation is excluded from the analysis. This exclusion significantly reduces the complexity of the analysis. Two additional important aspects of the technique are the use of generic causes, and the handling of component location and barrier information.

Integral parts of the method are the tables of generic causes. An example of a generic cause is "impact". Organization of these generic causes into general categories also increases the efficiency of the analyst and use of the computer.

Each basic event is analyzed for susceptibility to generic causes resulting from environment, manufacturer, installation, etc. When a susceptibility to a given generic cause is common to all minimal cut-set members, and the location of the minimal cut-set components offers no protection from the common cause noted, the minimal cut-set is called a Common Cause Candidate (CCC). If the cut set fulfills the common cause and location criteria, it "shares a common location."

3.2.3 Minimum Cut-set Approach

Very often, common-cause failure analysis is implemented by adding secondary failures to the logic model. Thus complex logic models, which are already taxing the capacity of present fault-tree computer programs, become more complex. To avoid this complexity, only the fault-tree minimal cut-sets are examined in this approach; secondary events are not added to the fault tree. It is, after all, the occurrence of the events in a minimal cut-set that causes the TOP event to occur.

By limiting the study to the fault-tree minimal cut-sets, the analysis for common causes becomes tractable because no additional basic events need to be added to the logic model, no additional minimal cut-sets result, analysis for common causes becomes an option that can be exercised, without preplanning, after other types of analyses are complete, and because computer-aided analysis can be used advantageously.²⁴

The number of possible sources of secondary failure of a component is indefinite, limited only by the analyst's knowledge and ability to conceive undesirable situations.

Other more complete descriptions of various other dependency analyses for systems interaction identification and evaluation can be found in Refs. 6, 7, 14, 15, and 24.

3.2.4 Digraph/Fault-tree Approach

The digraph/fault-tree method is based on a directed graph (digraph) and construction of a fault tree. This improved version of fault tree construction is particularly useful for control and instrumentation systems.

The first step in the procedure is the formulation of a general schematic for system modeling. Information from piping and instrumentation diagrams, the physical plant layout, and related procedures are used to formulate the schematic. The general schematic delineates the unit model digraphs needed to

model the system and the overall system interactions. The unit models include models of utilities, environmental conditions, human actions, process equipment, and procedures.

In the second step, unit model digraphs, the basic building blocks of the procedure, are generated. Digraphs are functional cause-and-effect models that describe the relationships among various system variables and the conditions that are necessary for these relationships to exist. In addition, digraphs can show events, such as accident initiator actions, that may nullify or change the relationships among variables. Digraphs are useful because they are multivalued network models and can readily model the dynamics of the relationships among variables. The advantage of generating unit model digraphs is that separate analyses can be performed on system components without performing an entire systems analysis. These unit models are analogous to minifault trees.

The third step is the generation of the system digraph, which is constructed from the unit model digraphs, for a selected top event variable. (The top event is the event being modeled.) The system digraph is obtained by following in detail the cause-and-effect information flow outlined in the general system schematic. The system is modeled as a control system designed to counter adverse actions. All potential ways in which the system can respond to prevent an accident initiator are modeled in terms of "cancellation loops" on the system digraph. These loops are similar to the negative feedback and negative feedforward loops designed to cancel disturbances in process variables.

In the fourth step the system fault tree is generated from the system digraph by means of a transformation algorithm. The top event in the fault tree corresponds to a disturbance in the top event variable of the system digraph. The top event variable for assessment is TOP, defined by

$$TOP = \begin{cases} -1 & \text{if failure of system occurs,} \\ 0 & \text{otherwise.} \end{cases}$$

A zero value for a variable on the system digraph corresponds to a true or expected value. Hence, any other value corresponds to a deviation or disturbance. The top event in the system fault tree for the assessment is $TOP = +1$. For a disturbance in the top event variable to exist (that is, $TOP = +1$) all loops in the system digraph that model the corrective actions of the system must fail. Thus, in order for system failure to occur, all protective function loops must fail.

To find the combinations of cancellation loops that must fail, we generate a fault tree from the system digraph by means of a transformation algorithm. The algorithm creates an AND logic gate in the fault tree for each cancellation loop that can fail.

Control loops in the system digraph are classified according to their response range and dynamics prior to application of the transformation algorithm. The corrective actions of loops that are too weak or too slow results in the automatic failure of these loops. The advantage of this prior loop classification is efficiency. One need not consider all the combinations of events listed above that can fail these loops.

H. Lambert¹¹ discusses the details of the digraph fault-tree and methodology.

The qualitative analysis of the fault tree provides much valuable information without using numerical data. It includes performing Boolean manipulations of the basic events, generating the adversary event sets, structurally ranking the basic events, determining the collusion requirements, and evaluating the effect of power loss on the material control system.

A structural ranking of the basic events in the event sets helps to identify important basic events for further analysis. This type of ranking is a function of the number of event sets in which a basic event appears and the relative length of those event sets.

The computer codes Fault Tree Analysis Program (FTAP) and the Set Equation Transformation System (SETS), designed to generate and handle numerous, high-order minimal cut sets, are used to perform the qualitative analysis.

3.3 Digraph-Matrix Approaches

3.3.1 Path-set Approach

While the mathematics of using path-set approaches has been available for many years, practical code development is still in the research stage and the following approaches require development of the state-of-the-art algorithms.

The first digraph-matrix approach that we present could be called the path-set approach. It is intended to represent a more mathematically complete examination of all system failures than the usual event-tree/fault-tree methods. The completeness relies less on the analyst and more on computational abilities. Its very completeness, however, leads to computational limitations restricting the size of the system it can address. This makes it more useful as an NRC review tool.

Basically, this approach (see Table 3) would start with a traditional event tree to identify accident sequences. Then, instead of fault-tree analysis of the systems in the accident sequence, a success-oriented analysis of the accident sequence is conducted. This step can be characterized as modeling the system in a reliability logic diagram form (see Section 2.6) consisting of a directed graph with a superimposed Boolean logic. A special effort would be made to include necessary support systems and shared environmental conditions as part of the digraph. In addition, human actions would be included as nodes in the digraph. The intention is to generate a Boolean expression modeling the system's successful performance.

The raw data of the system information comes in the form of adjacency matrices (see Section 2.5) representing the physical connectivity of the digraph (including the Boolean connections). A reachability code (PATH or CLAMOR) then generates all paths necessary for system functioning. Next, a Boolean manipulation code, such as the fault tree code (FTAP or SETS, see Section 2.4), generates the set of minimum path-sets (see Section 2.6). The minimum path-sets represent a Boolean expression in reduced "sum of products" form. Now the complete set of minimum cut-sets of the system can be found by application of DeMorgan's Law to the set of minimum path-sets. (This was demonstrated in Section 2.6.)

Next it is necessary to discriminate between minimum cut-sets that lead to complete system failure from those that lead to partial failure. The minimum cut-sets corresponding to complete system failure are then used in the usual accident sequence evaluation methods. This evaluation would include a qualitative importance ranking, a quantitative probability evaluation, and perhaps a sensitivity analysis.

3.3.2 Partitioned Digraph-Matrix Approach

In Section 2.6.3, we presented the equivalence relationship between success-oriented (path-set) expressions and failure oriented (cut-set) expressions for a uniquely specified graph which modeled a system. (See Ref. 2 for detailed isomorphic relationships.) However, in Section 2.6.6 we explained that a fault tree constructed for a system with a specified top box considers only a limited number of all possible failure modes, and that then the fault-tree analysis represents a subset of a success-oriented analysis.

In Section 2.4.2, we discussed and illustrated the analytic complexities of minimizing a Boolean expression that could result from either a fault tree or a success-oriented analysis. We stated that even a third-generation computer (such as CDC 7600) using Nelson's algorithm (double complementation) to find the complete set of prime implicants (minimum cut-sets of a non-coherent system) was limited in the size of expression that it could minimize. This limitation has always been the reason for preferring fault-oriented analysis for large and complexly-modeled systems. Indeed, it was this limitation which restricted PRA to construct fault trees only to the safety-grade system level and not to include support systems, shared environmental conditions, and dynamic human error.

Unfortunately, recent reactor accidents have demonstrated that the safety-grade system level of modeling does not adequately represent the risk involved. In Section 3.2, we discussed risk assessment alterations that expand and enhance the PRA approach without exceeding available computer limitations.

Table 3. Overview of path-set approach.

Step 1.	Traditional event-tree analysis. Output: accident sequence of systems.
Step 2.	Digraph model of each accident sequence, including Boolean connectivity between nodes. Digraph includes success-oriented nodes.
Step 3.	Construct adjacency matrices including Boolean connectivity from the digraph model.
Step 4.	Run reachability code (PATH or SQUEAK or CLAMOR) to generate all success path-sets.
Step 5.	Run Boolean manipulation code (FTAP, SETS, SQUEAK, etc.) to find complete set of minimum path-sets.
Step 6.	Apply DeMorgan's Law to find complete set of minimum cut-sets.
Step 7.	Evaluate which minimum cut-sets lead to complete failure of system as opposed to system degradation.
Step 8.	Continue with traditional accident sequence evaluation methods.

While such approaches seem adequate to redress the systems interaction problem, they have been criticized for incompleteness. That is, the fault tree analysis requires an analyst to flawlessly use deductive reasoning as he works backwards from the top box to the basic events. There is not an explicit mathematical algorithm that will result in a uniquely correct fault tree. Indeed, there is often significant variation between analysts over a system. For this reason, it would be preferable to use a success-oriented method. In Section 2.5, we demonstrated how once a representative digraph of the system was constructed, a unique minimum path-set solution (and therefore an unique minimum cut-set solution) is found by using adjacency matrices, reachability matrices and Boolean minimization. Nevertheless, in Section 3.3.1, when we outlined this approach, we realized that even with present day fourth-generation computers (such as CRAY-1) the combinatorics of a success-oriented analysis would face severe computer limitation in the Boolean minimization of the model.

There is an alternative however. If we could partition the system digraph model into separable (independent) subgraphs*, then the matrix and Boolean manipulations could be solved piecemeal. Correctly partitioning the digraph, or using unit models, while offering computational savings, puts a greater burden on the analyst. Deciding on the independence of the subgraph also becomes an issue.

The Partitioned Digraph-Matrix Approach starts (see Table 4) with a traditional event tree to identify accident sequences. Then a success-oriented digraph model of the accident sequence is constructed. Special effort is to be made to include necessary support system, environmental conditions, and human actions within the digraph model. Next, the digraph is partitioned into independent subgraphs and the analyst is required to store information on the subgraphs' interconnectivity (see Section 2.5). The procedure now follows the path-set approach, but analyzes each subgraph rather than each digraph.

Adjacency matrices (see Section 2.5) of the subgraphs are constructed. The Boolean relationships are included within the subgraph. Step 4 (see Table 4), then requires running a reachability code, such as CLAMOR or PATH, to generate all success path-sets for each subgraph. A Boolean manipulation code, such as FTAP or SETS, is used next to find the complete set of minimum path-sets of each subgraph. Finally, an application of DeMorgan's Law provides the complete set of minimum cut-sets of each subgraph.

The burden of composing the minimum cut-sets of all subgraphs into the minimum cut-sets of the original digraph now rests with the analyst.

The last two steps are repeats of the path-set approach which evaluate the minimum cut-sets and the accident sequences. This includes discriminating between minimum cut-sets that lead to complete and partial failure, and finally using qualitative and quantitative evaluation of the minimum cut-sets.

*A connected graph G is separable if there exists a subgraph g in G such that g' (the complement of g in G) and g have only one vertex in common.

Table 4. Overview of partitioned digraph-matrix approach.

Step 1.	Traditional event-tree analysis. Output: accident sequences of systems.
Step 2.	Digraph model of each accident sequence, including Boolean connectivity between nodes. Digraph includes success-oriented nodes. Digraph includes support systems, shared environmental conditions, and human error.
Step 3a.	Decompose digraph into independent subgraphs and require analyst to keep track of connectivity between subgraphs.
Step 3b.	Construct subgraph adjacency matrices including Boolean connectivity from subgraph model.
Step 4.	Run reachability code (PATH OR SQUEAK or CLAMOR) to generate all success path-sets for each subgraph.
Step 5.	Run Boolean manipulation code (FTAP, SETS, SQUEAK etc.) to find complete set of minimum path-sets of subgraphs.
Step 6.	Apply DeMorgan's Law to find complete set of minimum cut-sets of subgraphs.
Step 7.	Analyst compose subgraph minimum cut-sets into complete digraph minimum cut-sets.
Step 8.	Evaluate which minimum cut-sets lead to complete failure of system as opposed to system degradation.
Step 9.	Continue with traditional accident sequence evaluation, using IMPORTANCE for qualitative analysis and probability for quantitative analysis.

4.0 Conclusion

Recent events such as Three Mile Island-2, Brown's Ferry-3 and Crystal River-3 have demonstrated that complex accidents can occur as a result of dependent (common-cause/mode) failures. Events of that type are now being called Systems Interactions. NRC is developing a procedure for identifying and evaluating systems interactions. Several national laboratories and utilities have contributed preliminary procedures. Several different viewpoints have emerged.

In this report, we have reviewed some fundamental mathematical background of both fault-oriented and success-oriented risk analyses. This review has provided insight into some significant trade-offs of each type of analysis. For example, there are computer limitations in minimizing the Boolean expression which represents the system model. Such limitation exist for both fault-oriented and success-oriented analyses. However, in the past, the success-oriented combinatorical considerations have led to more severe computer limitations, and therefore restricted it to analysis of problems smaller than those that fault-oriented analysis can deal with. Fault-oriented analysis now has a large number of trained analysts and readily available supporting computer codes, adding to its attractiveness.

On the other hand, fault-oriented analysis requires the analyst to flawlessly use deductive reasoning as he works down from the top event of a fault tree without an explicitly unique algorithm. The success-oriented analysis is, therefore, more complete. In addition, it is able to address system operations in a continuous rather than two-state fashion, which makes it particularly valuable to analyze control systems. (An exception to this is the digraph fault-tree approach discussed in Section 3.2.4.) With the availability of fourth-generation computers (such as CRAY-1) and the possibility of partitioning digraphs into dependent subgraphs (or unit models) the computer limitations of success-oriented approaches can become comparable to fault-oriented computer limitations. We summarize the comparison of these approaches in Table 5.

Table 5. Comparison of fault-tree and digraph-matrix approaches.

	Advantages	Disadvantages
Fault-tree approaches	<ul style="list-style-type: none"> • Restricts the size of the analysis to be manageable amount by focusing on only the faults of most concern. • Computer codes and trained personnel available in industry. • Can use importance analysis to find qualitative information about critical dependencies. 	<ul style="list-style-type: none"> • Requires analyst to work deductively without a unique algorithm. • Analyzes only selected top-box fault. • Size of fault tree is limited to computer abilities.
Digraph-matrix approaches	<ul style="list-style-type: none"> • Analyzes all modes of system success and is in this sense more complete than fault-tree analysis, and is more flexible to address control systems. • Once a logic diagram model of the system is complete, a unique algorithm is used to find all cut-sets. • Can use culling techniques to find qualitative information about critical dependencies. 	<ul style="list-style-type: none"> • Requires state-of-the-art algorithm development. • Few computer codes and few trained personnel. • Size of matrix is limited to computer abilities.

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