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In Vivo Detection, Localization and
Measurement of Radionuclides in Man:
A Detection System for the Localization
and Measurement of Small Amounts of
Photon Emitters

Progress Report: DE-AS02-78-EV04957
for Period March 1, 1981 - October 31, 1981

Principal Investigator: Gerard R. Laurer, Ph.D.

Co-Investigators: Merril Eisenbud, Sc.D.
Norman Cohen, Ph.D.

There is no objection from the patent
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12/16 1981 By Ull

New York University Medical Center
New York, N.Y. 10016

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Comprehensive Summary

The purpose of this summary is to present a comprehensive overview of the activities of the contract from the time of its initiation, in a format directed at fulfilling several specific reporting requirements of the Department of Energy for this year's report. The itemized report requirements are:

1. The main research accomplishments, with special reference to originally stated objectives.
2. Plans for the continuation of present objectives and possible new objectives in consideration of past results.
3. Graduate students trained, degrees granted, and post-doctoral tenures completed.
4. Bibliography, with titles of publications associated with this contract.
5. Our opinion as to the present state of knowledge in this area of research, its significance in the field of environmental matters, and needed future investigations.
6. The present division of federal support for our overall research program.

These will be addressed in the above order as part of this summary (with parenthetical reference to sections of the main body of the progress report for detail on a particular item if desired).

1) The original objective of this work was, and remains, the design, construction and testing of a photon detection system capable of yielding, simultaneously, information proportional to both the magnitude and locations of sources of radioactivity. Having its genesis (Main Report COO4957-3, pp. 1) in a health physics related application, the number of possible applications of such a device has since increased as the work continued; these will be discussed subsequently.

The operating principle of the device is "active collimation" wherein the detector itself, i.e., the NaI/CsI crystals that comprise the detector, act as the collimator. By utilizing some number, n , of these crystals in an array and placing them in a recessed/promontory geometry relative to each other, a high degree of collimation is achieved without significant loss of geometrical efficiency in the recessed crystals and no loss of efficiency in the promontory detectors. Mathematical processing of the pulses from the recessed crystals is used to develop the image of the radioactive deposition, while the sum of the pulses from the recessed and promontory crystals provides a non-collimated, large area signal for the determination of activity. Because the imaging is provided with a significantly reduced degree of collimation than other currently available imaging devices, much less radioactivity is necessary.

To avoid the expense of constructing such an n -crystal detector during the design and optimization stages, the response was synthesized instead using the experimental response

of a single detector element of the n-elemental array as a function of the motion of a point source (^{109}Cd or ^{241}Am). From this, the point source response of any number of elements arranged in an array could be obtained and by iteratively summing their responses, the response to any number of point sources arranged to simulate any desired areal deposition could be obtained.

Two approaches to the geometry of the single element have been tested in this manner:

- 1) a five crystal system in a square geometry in which 5 separately delineable signals, one for each crystal, are obtained from a single P.M. tube utilizing the different characteristics of the the three crystal materials used - NaI(Tl), CsI(Tl) and CsI(Na);

- 2) a single NaI crystal/phototube detector element.

The former geometry was thus tested in two configurations, one with two recessed, three promontory crystals (Figs. 2 through 8, Appendix I) and the other with 3 recessed, two promontory (Figs. 1 and 2, Appendix II) to synthesize an array of them (Appendix II, Figure 5) in which the position of each is displaced 90° with respect to its neighbor thus providing X and Y axis position information simultaneously.

The latter single, cylindrical NaI geometry was tested using a brass/Pb collimator to simulate the effect of a recessed detector surrounded by promontory crystals (Fig. 1, pp. 65, C004957-3). Similar to the square, 5 crystal element, an array of such detectors would have every other crystal

recessed. Unlike the 5-crystal element, however, the latter geometry would provide for a variable amount of recess and hence a variable degree of collimation and thus allow for greater or lesser resolution in the image depending on the amount of radioactivity present (see Figs. 14a through d, pp. 78-81, COO4957-3). Also, this geometrical arrangement would provide simultaneous diagonal axis position information in addition to X and Y axis information.

Mathematical processing of the pulses to obtain positional information initially was in the form of ratios of the counts in the recessed to counts in the promontory crystals of the two recessed/three promontory, 5-crystal element. This was discarded soon in favor of the better behaved products of these counts. It was found also at this point that, due to the relatively slowly varying nature of the promontory crystals' response to source motion, the best positional information was obtained from the product of the counts from the recessed crystals. This led subsequently to development of the multiple-product response of the recessed crystals and thus the design of the Mod II detector element with 3 recessed and 2 promontory crystals. Using this design, it was shown (Figs. 20 through 24, Appendix I) that the development of the multiple-product-response (MPR) leads not only to considerably enhanced spacial resolution compared to a single crystal (Fig. 31, pp. 98, COO4957-3) but also to a response with an extremely steep slope due to the appearance or disappearance of the illuminating photons as one or the other of the recessed crystals moves into or out of the shadow of its promontory

neighbor with respect to an area of radioactive (Figs. 7 through 10, Appendix II) deposit.

The use of the output of the recessed crystals to develop product responses of enhanced resolution thus led to the design of the Mod III detector element (Fig. I, pp. 65, COO4957-3) employing the single crystal/phototube combination in an array using the recessed/promontory geometry. This arrangement, in addition to yielding a 5 crystal MPR to more sharply delineate the edge of the active deposition, also allows the reversal of recessed/promontory aspect of the various crystals in the array. Since the recessed crystals' outputs produce the MPR, the ability to reverse them in a programmed manner, producing a scan-like motion in the MPR that can serve as a vernier in precisely delineating the outline of the object being imaged, emerges as one of the more promising aspects of this geometry as opposed to the fixed, 5-crystal design.

Data from the point source scan using the Mod II detector element was used to synthesize the response of a 3x3 array of such elements with axes displaced 90° (Appendix II, Fig. 5). Curves of the MPR versus array motion (Appendix II, Figs. 7 through 10) for various simulated source configurations served to indicate the acute sensitivity of the MPR to very small changes in source position as well as its usefulness in narrowing the resolution limits over the response of the individual crystals.

The inherent potential of the recessed/promontory geometry in conjunction with the multi-crystal, multiple-product development is most dramatically presented in current periods results, however (COO4957-3, pp. 34), in which the output of a 9x9, 81 crystal array is synthesized from the experimental, point-source data obtained with the collimated $\frac{1}{2}$ in. diameter NaI detector. Results for various source configurations are presented in the form of 3-dimensional plots (COO4957-3, Figs. 17 through 30) of either the total response of the entire array of individual detectors or as the multiple-product-response. Five MPR's have been tested thus far using the responses of three adjacent detectors for their development on the horizontal, vertical, two diagonal axes, and the average of these.

Displaying the results in 3 dimensions, in which the Z axis is a "gray" scale gradation proportional to the individual detector or multiple-product-response, has added enormously to the visualization and understanding of the response of the array and the MPR as a sensitive parameter in the imaging potential of this geometric arrangement of crystals. The MPR, shown previously to be very sensitive to source motion, is shown this period to respond to a motion of 0.025 in. ($\approx$ 0.9 mm). The imaging potential of the array is perhaps best illustrated in Figs. 24, 25 and 26 (COO4957, pp. 91-93) showing the synthesized response of three more practical source configurations, a line source, a large, circular configuration,

and a simulation of a tracheo-bronchial deposit in which the activity was distributed non-symmetrically in the trachea and right and left branches of the bronchi at the first bifurcation. In each of these, the position and distribution of the source deposition is clearly defined in the total response, followed by an immediately observable sharpening of the resolution in the visualization of the MPR.

It should be emphasized at this point that these responses were developed using a detector-plane to source-plane distance of 4 in. and a collimator depth of 0.63 in. These parameters represent a very conservative test of the array's capability since in many imaging situations, e.g., the tracheo-bronchial, the detector could be brought significantly closer to the organ and allow the use of a deeper recess with better collimation and thus improved resolution. The trade-off between sensitivity and sensitivity inherent in such a device is recognized and curves have been developed (COO4957-3, Figs. 5 through 15) to indicate the kind of latitude available in the control of experimental parameters with the use of the variable recess/promontory crystal positioning.

In terms of the original objectives of this contract, therefore, the results of the work completed thus far, although still at the synthesis level, are believed to leave no doubt as to the potential of the recessed/promontory crystal array - especially with the added ability to vary the depth of recession - to act both as a self-collimated, imaging device with significantly higher sensitivity than other

currently available systems, and as a very sensitive, large area detector for whole body and/or organ radioisotope burden determinations, simultaneously.

2) Plans for the continuation of present objectives include continuing the developing and study of various multiple-product responses and their use in refining the images thus far developed. Effort will be especially directed toward the multi-product response slope, and the development of a correlation between that very rapidly changing function at the approach of the edge of an activity deposit, and the image outline produced. Results obtained thus far indicate that this may be an excellent parameter for accurately defining the radioactive deposit. Work will also be directed towards examining the response of the array at several other heights and collimator depths to begin to establish the family of responses necessary to define the operating characteristics of such a variable parameter device.

The results as presented this period in a 3-dimensional format have shown one of the unique aspects of the rapidly expanding use of computer-graphics - the ability to display data in ways which allow an immediate visual grasp of the interplay of many separate but related functions involved in a dynamic process such as radioisotope imaging. Work will, therefore, continue on the refinement of the image presentation, both CRT and hardcopy, directed toward maximizing their potential in obtaining all the visual information possible.

Several other new tasks have been defined and may be seen in detail in the proposal accompanying this report. Briefly, these are:

1) Embed point and other source configurations in tissue equivalent absorbing/scattering medium to obtain array responses closer to what is expected in actual use of such a device.

2) Define the size and configuration of an optimized array and obtain estimates of its sensitivity as an imaging device and as a large area detector.

3) Obtain and test a dual-crystal, CsI/NaI version of the single $\frac{1}{2}$ in. diam. NaI crystal used to synthesize Mod III results.

4) Construct a nine crystal, Mod III detector configuration using crystals already on hand. Obtain and build pre-amplifier-amplifier-single channel pulse-height/pulse-shape analyzer from miniature, hybrid circuit elements, to test the feasibility of the use of such circuitry on a full-sized prototype array, using the above nine-crystal Mod III element.

The possible new objectives for this study are many, not so much in terms of the device itself, which with the coming year's work should be well defined and ready for prototype assembly, but more in terms of its possible uses. As was stated, the original objective was to define the location and size of accidentally ingested radioactivity so as to more accurately determine the body burden through a more accurate

phantom. It is now believed that the imaging capabilities of such a device with its much increased sensitivity may find use in the field of nuclear medicine in imaging tasks where the use of large amounts of radioactivity would not be desirable.

Additionally, as has been shown, the array and the MPR development allow two modes of investigation - imaging of a static deposit as in the health physics application and, more interestingly, the imaging of dynamic processes in which imaging the movement of radioactivity through a system is desired. The sensitivity of the array response to source motion makes such a device an ideal tool for such studies and opens many possible avenues of investigation, e.g., radiobiological applications to study the uptake and movement of materials such as the trans-uranics in animal experimentation; toxicological experimentation in which the motion and deposition of radioisotope-tagged, toxic environmental agents are studied in animal models; deposition and clearance studies of tagged aerosols; study of the motion of organs such as the lungs or heart through which radioactive material is moving.

Other possible industrial application can be envisioned - e.g., study of fluid flow in pipe runs, or, considering the efforts currently going into increasing the efficiency of automobile engines, its use as a diagnostic tool in imaging the flow of oil in such a system.

3) Thus far two graduate students have worked on the project. One has left with an M.S. degree, the second now

has an M.S. and is working toward the doctorate with the expectation that his thesis will evolve from his work on the definition and use of the prototype array.

We would also like to acknowledge here the addition to our staff for a two-year term, Mr. Zunsu Hu, a visiting scientist from the North China Institute of Radiation Protection. His interest in this area of instrument development for low level radiation measurement in people and the skills he brings with him have already added considerably to the project.

4) See COO4957-3.

5) As suggested in section (2) above, it is my belief that the successful development of a device such as the one this project defines, falling as it does in the intermediate area between the gross counting sensitivity of our present large crystal detectors with no imaging capabilities, and the current imaging devices which demand the use of large amounts of radioactivity to produce an image, could have wide-ranging significance in the field of environmental matters where its use may be related to the study of materials in the environment and their effect on man, either directly through exposure and uptake or indirectly with study of materials in industrial and other processes which affect the environment.

While significant amounts of work have been done on the non-imaging type of instrumentation, and even more on the medical imaging devices, very little has been done in the intermediate area on an instrument that would offer a dual capability.

6) Presently, the salaries of the principal and co-investigators on the project are contributed by NIEHS and NCI Center Grant Funds. Staff and secretarial support as well as capital equipment, supplies, travel to meetings, etc., i.e., all other costs, are obtained through the DOE contract.

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Principal Investigator: Gerard R. Laurer, Ph.D.

Co-Investigators: Merrill Eisenbud, Sc.D.
Norman Cohen, Ph.D.

Staff: Zunsu Hu*
Theodore Rahon, M.S.

Institute of Environmental Medicine
New York University Medical Center

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Prepared for:

U.S. Department of Energy
Division of Pollution Characterization
and Safety Research

*Visiting Scientist, North China Institute of Radiation
Protection, Shanxi, China.

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*Appendix IV is a Preprint of a paper to be submitted for publication in the journal "Health Physics."

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SYNOPSIS OF PROJECT

Summary

The objectives of this research include the design, construction and testing of a photon detection system which will yield, simultaneously, information proportional to both the magnitude and location of sources of radioactivity. The operating principle of the detector system, "active collimation", allows spatial resolution by utilizing the crystal elements themselves as the collimating device; hence a greater counting efficiency is attainable than with other currently available systems. Two approaches have been tested thus far: in the first, the differences in characteristics of various crystals are utilized as the means of separating events occurring in each of five crystals viewed by a single phototube; in the second, smaller, single crystal/phototube elements are used in the array. Mathematical processing of the counts from the individual crystal elements is then employed in conjunction with their geometric relationship to develop both positional data and to quantify the amount of radioactivity.

The results of experimentation to date have led to promising new approaches. By utilizing crystal elements in a "active collimation" configuration and incorporating them in an array which permits these crystals to be alternately advanced and retracted in a programmed, ordered manner, a high degree of spatial resolution is believed achievable without unduly compromising system sensitivity. A series of experiments is

being planned to provide data as to the ultimate performance that can be achieved with an N-element detector array. The experiments include development of the optimal crystal configuration and the evolving of data processing techniques to translate detector response into precise source position information for localization or imaging purposes.

Significance

There is wide-ranging potential importance for a system capable of accurately defining the location of low level radiation sources. One application would be the localization of inadvertently ingested or inhaled radionuclides within the body for dosimetric purposes. Moreover, because of the considerably improved resolution-to-activity factor believed achievable, there is promise for this system in nuclear medicine as well. Good imaging should be obtainable with much lower radiation exposure levels than is possible with present methods.

In areas of research involving experimental animals - toxicology, radiation biology, etc., such a system could add another dimension in studies involving uptake, distribution, and retention of radionuclide labeled compounds because of its ability to actively track, in vivo, the distribution of these compounds. In addition, it should be possible to study, dynamically, the motion of organs, such as the heart or lungs, through which these compounds move.

Yet another use of this approach would involve surveillance or monitoring of the environment, namely the detection and localization of radioactive plumes or clouds in the atmosphere. N-element detector arrays could be constructed in a manner similar to the unit developed here that would define the outline and ascertain the location of radionuclide-bearing air masses in the atmosphere, as might result from accidental emission by a nuclear facility.

Extra-galactic X-ray emanation sources might also be located by suitable N-element detector arrays which would function in a manner analogous to a radio telescope.

In Vivo Detection, Localization, and Measurement of
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A. Introduction

1. Objective:

The purpose of this research is the design, construction and testing of a photon detection system which will yield, simultaneously, information proportional to both the magnitude and the site(s) of deposition of radioactivity accidentally inhaled or ingested or otherwise deposited in the body. The operating principle of the detector system, "active collimation," allows spatial resolution without the use of separate collimating devices and, hence, with a greater counting efficiency than other currently available systems.

2. Background:

Whole body in vivo detection systems for the quantitative assessment of body burdens or organ burdens of radionuclides and subsequent radiation dose determination depend on knowledge of the distribution of the nuclide in the body. In the case of more hazardous nuclides like the transuranics, treatment of the contaminated individual may rest on the accuracy of the estimate. At the present time, in vivo measurements of lung burdens of the nuclides, ^{238}Pu , ^{239}Pu and/or ^{241}Am in man are accomplished by detection and measurement of the low

energy photons which they emit using either large area gas proportional counters or NaI-CsI, dual-crystal scintillation detectors positioned anterior to the thorax. Calibration factors are usually determined by measuring the activity of a tissue-equivalent torso surrogate containing a known quantity of the radionuclide, usually distributed uniformly throughout mock lung material. It is generally conceded, however, that this may not be the way this material is actually distributed and that it does, in fact, concentrate with time in certain tissues such as the lymph nodes, depending on factors such as chemical form, solubility, particle size, etc. In any case, the distribution would probably be complex and, since bioassay measurements may be even more uncertain in terms of arriving at a body or lung burden, it is essential to refine the in vivo photon counting technique. To obtain the best calibration factor relating external counting rate to internal burden, experimental knowledge of isotope distribution is necessary. It is essential, therefore, to design and construct a system which can adequately determine not only the magnitude but also the sites of deposition of internally-deposited material.

At present, sites of internally-deposited radioactive materials, such as those necessitated by prescribed medical diagnostic procedures, are delineated with the use of scintillation cameras or other devices which may remain stationary or scan the whole body or organ of interest in a rectilinear fashion (1,2,3). These devices provide an image of the site

of deposition of the isotope with the use of NaI crystals and external collimators of various configurations, i.e., pinhole, multichannel (parallel or focusing), or slit, made of lead or some other high Z material. The requirement of a collimator to form an image of the photon emitter and to obtain good resolution between an area of deposition and an adjacent area of no deposition restricts the geometric counting efficiency of these devices. This restriction requires that relatively large amounts of radioactivity be given to the subject in the diagnostic procedures and limits the usefulness of these devices for the location of accidentally ingested or inhaled radioactive nuclides. Low-level amounts of these nuclides that may present a serious, long-term health hazard to the individual might not have sufficient activity to yield a statistically reliable signal in terms of spatial resolution.

Recently, the use of ultra-pure germanium diodes in imaging systems (4,5) has indicated the possibility of improved resolution over the scintillation crystal-phototube system because of the greater energy resolution of the Ge diodes. This improved energy resolution allows better discrimination against Compton events which occur in the detector assembly as well as tissue. Instead of using a phototube to signal where an interaction has taken place, as with a NaI crystal, the Ge devices use parallel grooves cut into the n and p faces, with the n face grooves oriented 90° with respect to the p face grooves, thus forming an orthogonal

system from which signals proportional to the location of an event in the detector are obtained. As with the gamma camera, however, a collimator is still necessary to locate and form an image of a source of photons in space. The operating principle of the proposed device, "active collimation", allows the construction of a detector capable of resolving the shape and location in space of a source of photons, but without the geometrical restrictions imposed by the need for pinhole or other collimating devices made of lead or some other inactive material. Thus, the instrument may have potential usefulness in nuclear medicine as an imaging device which will function at lower radiation doses to the patient as a result of the elimination of the collimator, as well as in the health physics application of determining the magnitude and site of deposition of accidentally deposited radioactivity.

3. Rationale:

Dual crystal systems of NaI-CsI, utilizing pulse-shape discrimination, accomplish a reduction in low energy background due to Compton scattered events as well as a reduction in room background, making possible the measurement in vivo of small amounts of radionuclides which emit photons in the energy range below 100 kev. Prior to the advent of the dual crystal technique, low level in vivo determination of nuclides emitting photons in this energy range (uranium, transuranic nuclides, ^{210}Pb , etc.) had been difficult and statistically unreliable because of the high background count rates obtained.

Many laboratories now use dual-crystal systems in their whole-body counters on a routine basis to screen personnel handling transuranic material. The design, construction, and the first use of these systems for in vivo measurements of low-energy emitters was accomplished at this laboratory several years ago (6,7,8,9,10). In these systems, pulse-shape discrimination, based on the difference in photomultiplier output pulse shapes of the light from the NaI and CsI crystal materials, is used to gate anti-coincidence circuitry and eliminate from pulse height analysis those Compton interactions in the thin, primary NaI detector which result in forward scattered photons that also interact in the secondary CsI crystal. Figure 1 shows a cross-section of the dual crystal detector as well as the output pulse shape and a block diagram of the detection and measurement system used.

The active collimation principle also utilizes pulse shape discrimination. For this application, however, the crystals are oriented differently with respect to each other and with respect to the photomultiplier tube. The principle of active collimation and the basic operating details of the detection system may be seen in Figures 2, 3, 4 and 5.

Figure 2 shows the plan and elevation views of a single detector element out of an N-element array. Each detector element consists of five interspersed crystals of NaI(Tl) and CsI(Tl and Na), each L inches long and W inches wide (W_{Na} , W_{Cs}) such that the five crystals make up a square configuration. The thicknesses (T_{Na} , t_{Cs}), while not related in any fixed manner,

may be used with the parameters W, w , to optimize the detector dimensions for a specific photon energy range, resolution and/or degree of collimation.

Figure 3 is a top and side view of the detector element shown without PM tube or light pipe for simplification. A point source of photons is centered beneath it. With the source at the center of the crystal array as shown, interactions will occur in the NaI(Tl), the CsI(Tl), and the CsI(Na) individually, and as a result of Compton scattering interactions, in adjacent crystals of NaI and CsI simultaneously. With the use of pulse shape discrimination,* however, it is possible to obtain an output signal from the detector for the NaI interactions alone, $S_{\text{NaI(Tl)}}^{**}$, the CsI alone, $S_{\text{CsI(Tl)}}^{**}$ and $S_{\text{CsI(Na)}}^{**}$ and reject those that occur simultaneously in any two or more crystal elements. Thus, it is possible to establish three ratios, $S_{\text{NaI(Tl)}}/S_{\text{CsI(Tl)}}$, $S_{\text{NaI(Tl)}}/S_{\text{CsI(Na)}}$, and $S_{\text{CsI(Na)}}/S_{\text{CsI(Tl)}}$, that describe source position.*** Moreover, as shown in Figure 4, movement of the detector element along the X axis will cause the ratios $S_{\text{NaI(Tl)}}(X)/S_{\text{CsI(Tl)}}(X)$, $S_{\text{NaI(Tl)}}(X)/S_{\text{CsI(Na)}}(X)$, and $S_{\text{CsI(Na)}}(X)/S_{\text{CsI(Tl)}}(X)$

*To keep the electronic system as simple as possible for the experiments, pulse "shape" discrimination has thus far not been necessary. Rather, pulse "height" has been used, and is possible because of the different light output of the three crystal materials for any given energy deposition.

**In subsequent discussion, $S_{\text{NaI(Tl)}}$ will be referred to as R_3 or R_4 , depending on which NaI crystal, i.e., central or outer, is involved. Similarly, $S_{\text{CsI(Tl)}}$ is designated as R_1 , $S_{\text{CsI(Na)}}$ as R_2 .

***Other functions besides a ratio are also possible, and as will be seen subsequently, the product, e.g., $S_{\text{NaI(Tl)}} \cdot S_{\text{CsI(Tl)}}$ is a better behaved parameter for defining position.

to change. The magnitude of the change will be different for each ratio and will depend on the direction of motion and the total distance moved.

It follows that these ratios may be obtained in the Y axis direction also by turning the detector element through 90° , establishing the magnitude of change as the detector moves along that axis. Thus, using a single detector element and moving it incrementally along the X and Y axes, it is possible to resolve the position of a point source in the X-Z and Y-Z planes or to trace the outline of an organ or the area in which deposition of radioactivity, accidental or deliberate, has occurred. The detector element, therefore, would be behaving in the same manner as a single crystal detector with a slit collimator, but with a significant difference, i.e., the total amount of signal available. By pulse shape discrimination, the ratio S_{NaI}/S_{CsI} is obtained; however, the sum $S_{NaI} + S_{CsI}$ is also obtained and, in terms of the signal available for assessing how much total radioactivity is present at the site of deposition, the detector element acts as a single crystal without a collimator. Hence, with the use of "active collimation," it will be possible to construct a photon detector which will operate in two modes simultaneously, as a detector with a collimator for spatial resolution, and as a detector without a collimator for optimum detection efficiency.

N-Element Array: Although the single detector element could be moved along the X-axis and the Y-axis as described above, the same data may be obtained by positioning several

detector elements in an array as indicated in Figure 5.

In the array, the detector elements are placed side by side, but with their axes turned 90° with respect to each other.

In this manner, the signals from detector elements a, b, c... etc. would define Y-axis position for that row, while elements 1, 2, 3...etc. would define X-axis position for the same row.

In a like manner, X-axis oriented detector elements would

define Y-axis position along columns a, b, c...etc., and

Y-axis oriented elements would define X-axis position along

columns 1, 2, 3...etc. In terms of its signal output, the

array of N elements may thus be pictured as a single detector element being moved into each of the individual positions in

the array, with the signal ratios $S_{\text{NaI(Tl)}}(X)/S_{\text{CsI(Tl)}}(X)$,

$S_{\text{NaI(Tl)}}(X)/S_{\text{CsI(Na)}}(X)$, $S_{\text{CsI(Na)}}(X)/S_{\text{CsI(Tl)}}(X)$, and $S_{\text{NaI(Tl)}}(Y)/$

$S_{\text{CsI(Tl)}}(Y)$, $S_{\text{NaI(Tl)}}(Y)/S_{\text{CsI(Na)}}(Y)$, $S_{\text{CsI(Na)}}(Y)/S_{\text{CsI(Tl)}}(Y)$,

changing at each position. In terms of the array, the output

is a gradation in magnitude of the X-axis and Y-axis ratios

along the respective rows and columns of elements -- with the

gradation in turn dependent on the geometry of the radioactive

deposition, i.e., point source, line source, area source, etc.

The array will thus provide 3·n signals (3 for each element)

to define source position in the X-Z and Y-Z planes, and at

the same time give $S_{\text{NaI}} + S_{\text{CsI}}$ for magnitude determination using

the total counts obtained from all elements.

B. Summary of Work to Date

This project has been underway since September 1, 1978, with contract periods from September 1, 1978 through August 31, 1979. September 1, 1979 through February 28, 1981 and March 1, 1981 through February 28, 1982. The work of the first two periods will be summarized below with details available in the progress reports (COO-4957-1 and COO-4957-2) appended to this report. This will be followed by the summary and detailed report for the current period.

1. Period September 1, 1978 Through August 31, 1979:

Work throughout this period was performed using the prototype detector element (Figure 7, Appendix A) and a 1 mm diameter source of ^{109}Cd (88 keV, 22 keV X-ray filtered out). Although misconstrued initially with two CsI(Tl) crystals rather than one CsI(Tl) and one CsI(Na) this was eventually rectified. Some initial experiments were performed with this detector before modification to test the validity of the rationale. The results of some of these are shown in Figures 8 through 12, indicating how movement of the source across the face of the detector produces two peaks, from the NaI and CsI crystals, respectively, and how the ratio of the two peak heights changes with source location. Throughout these tests, data was acquired with a TN-1700, computer-assisted MCA and electronic submodules, i.e., pre-amplifier, amplifier, H.V. supply, etc.

After return from the manufacturer with the desired CsI(Na) crystal, work began to define the primary detector characteristics; spacial and intrinsic resolution of each crystal and response to

positional changes of the ^{109}Cd point source. These measurements were performed by securely supporting the detector element on a solid base and securing the source to one of two rack and pinion assemblies mounted perpendicularly to each other, thus facilitating X and Y positioning in front of the detector at various detector-to-source distances.

Initial tests were done with a collimated source to determine the spacial FWHM (mm) and the intrinsic fwhm (%) of each crystal as well as checking the center-line to center-line distances between them. The FWHM response of each crystal was found to be approximately 7.5 mm (Fig. 13) as expected with a collimated source since the crystal dimension is nominally 0.3 in. or 7.62 mm wide. Spectral responses of the individual crystals (Figure 15) yielded fwhm values of 47%, 27%, and 15% for the CsI(Tl), CsI(Na) and NaI(Tl), respectively. The same source directed at the 15 cm diameter by 3 mm thick NaI crystal of a dual crystal assembly and a 20 cm x 10 cm thick NaI crystal, yielded peaks of 18% fwhm by comparison.

The response of the detector to the uncollimated source positioned over its center line (Figure 14) produced an unexpected fourth peak not seen in the first prototype because of its construction without reflecting material between the various crystals. When returned for reconstruction with the CsI(Na), reflector was included and, as a result, the end NaI detectors, whose faces are covered by less P.M. photocathode area, have their peak height positions shifted downward with respect to

the center NaI whose face is fully covered by the photocathode. As will be subsequently discussed, this result eventually led to a modified detector element which yielded five peaks, one for each crystal.

Intrinsic spatial resolution, as a function of crystal width, is an important parameter because of the role this dimension plays in the ultimate spatial resolution of the detector element. To determine if there was a variation of this parameter, a NaI crystal was obtained of the same length and height as the prototype, 38 mm x 38 mm but with a width of 2.5 mm - one-third of the prototype 7.6 mm. The results (Figures 16 through 19) indicate a significant increase in fwhm to 40% as compared to the 15% obtained with the 7.6 mm wide NaI in the prototype. Additionally, the pulse height changes as a function of the point of photon interaction along the length of the crystal. Although the intrinsic resolution did not change significantly with the point of interaction, the result of varying pulse height with distance along the crystal is a large peak - broadening effect when the crystal is exposed by an uncollimated source as seen in Figure 18. The spectrum with the inner half of the crystal shielded is analogous to the response expected from a promontory crystal in the detector assembly when the inner half is shielded by a recessed crystal. Although not as serious as when the entire crystal is exposed from the side, the loss of resolution by the narrower 2.5 mm crystal is an important factor in determining the design limit of this parameter.

To ascertain whether changes in intrinsic resolution with crystal thickness were gradual or sudden, two additional crystals were obtained, with thicknesses of 6.5 and 4.5 mm, respectively. Although measurements made on these crystals represent part of the work from the next contract period (September 1, 1979 through February 28, 1981) they are presented in Table I for the sake of completeness at this point. As seen, the intrinsic resolution with the collimated source directed at the crystal faces was 18% for the 6.5 and 19% for the 4.5 mm width, respectively. Similar to the 2.5 mm width, the resolution does not vary significantly with distance along the detector. Thus, intrinsic resolution is found to be a relatively slowly changing function with thickness, at least between 7.5 and 4.5 mm. Between 4.5 and 2.5 mm, the change is much faster, going from 19% to 40%. The thickness originally selected, 7.6 mm, is thus found to be in the optimum thickness region, in terms of intrinsic resolution. Whether this would change significantly with increased crystal height or as a function of energy has not been determined.

In the next phase of work for this first contract period, data were obtained for various positions of the uncollimated point source in a plane 10 cm from the detector face using the rack and pinion rig described previously. An X-coordinate range of 7 cm either side of the detector center line and a Y-coordinate range of 3 cm above and below detector center were scanned in 1 cm steps. The relatively large steps were taken initially to provide

an immediate overview of detector behavior and spacial resolution. As will be discussed subsequently, facilities were developed which allowed more precise measurements, over much smaller increments.

Data from the planar array of points after suitable background correction was used to generate ratio and product terms from the net cps values derived from the region of interest around the peak obtained from each crystal. Routines were programmed to develop all six ratios of interest, in addition to a like number of normalized terms in which each ratio is referred to a reference ratio obtained when the source is at a central position. Figure 20 shows plots of R_1 and R_2 , the response of the two recessed CsI crystals, R_4 , the central promontory NaI, and R_3 , the response of the two outer promontory NaI crystals -- all as a function of source (uncollimated) displacement from the detector center line. Also shown are plots of the ratios, R_2/R_1 and R_3/R_2 . The recessed crystals produced broad peaks (FWHM $\approx$ 6 cm) while the promontory crystals produce archlike, bi-modal responses with less pronounced peaks. Only two of the six possible ratio terms are shown. Of the remainder, i.e., R_3/R_1 , R_4/R_2 , R_4/R_1 and R_4/R_3 , the ratio R_4/R_3 may be discarded because of the broadness of the R_4 and R_3 responses. R_3/R_1 would appear the same as R_3/R_2 but with its node symmetrically displaced on the opposite side of the detector center-line. R_4/R_2 and R_4/R_1 are much broader than R_3/R_1 ,

R_3/R_2 , with nodes at the same positions, since this aspect of the ratio with R_4 or R_3 is dictated by the peak locations of R_2 and R_1 . Due to the broad and rather constant response of the promontory crystals, the ratios are dominated by the response of the recessed crystals. The plots shown are from data which does not include an improved K-corrected response of the individual crystals. The K-corrected counts of the recessed crystals now go to zero at the points where they are effectively shielded from the source by the promontory crystals. Because of the broad peaks produced by ratios and the awkward ratio values that resulted as the divisor response approached zero, alternate approaches to data reduction were explored.

This resulted in development of product and multiproduct exponentialized terms which produces peaks of enhanced sharpness and symmetry, thereby improving the spatial resolution characteristic beyond that obtained from untreated, contributing crystal data.

Data processed in product form produces the responses depicted in Figure 21. As with the ratios, the peaks of the product responses of R_1 and R_2 with R_3 and R_4 are dictated by the responses of the recessed crystals, and displaced symmetrically about the detector center line. Product $R_1 R_2$ peaks at the center line of the detector, although the individual responses, R_1 and R_2 , peak on either side of the center line. The response (Figure 22) of the multiple product, $R_1 \cdot R_3 \times R_2 \cdot R_3$, is also shown to peak at the detector center line while the individual

products, $R_1 \cdot R_3$ and $R_2 \cdot R_3$ (Figure 21) peak on either side. The multiple product response is thus seen to be the product of $R_1 \cdot R_2 \times R_3^2$, i.e., the individual product response $R_1 \cdot R_2$ multiplied by a more or less constant term R_3 , which has a broad, less pronounced bimodal response. There is an enhancement of spatial resolution because the product $R_1 \cdot R_2$ is narrower than either R_1 or R_2 . The resolution of the product $R_1 \cdot R_2$ is approximately 3.5 cm as compared to the individual resolutions of about 6 cm each. Because R_3 is approximately constant, the successive multiple product development does not affect resolution any further. It can, however, have a most substantial effect on the steepness of the response, and when the multiple product development includes additional recessed crystal products, whose slopes are changing with source position, still further improvement of the resultant peak is realized.

To test this, the responses of the individual crystals were used to develop the multiple product response of a detector element (Figure 23) which has 3 recessed crystals (R_1 , R_3 and R_5) and two promontory crystals (R_2 and R_4), the reverse configuration of the prototype element. From the above discussion and the product development of Figures 20, 21 and 22, it follows that the products $R_1 \cdot R_3$ and $R_3 \cdot R_5$ will produce peaks centered over detectors 2 and 4, respectively, and the multiple product $(R_1 \cdot R_3)(R_3 \cdot R_5)$ will produce a peak centered over detector 3. In this case, however, R_3 is not constant and its inclusion in the multiple product does enhance resolution. The FWHM of $R_1 \cdot R_3$

is about 3.5 cm (Figure 24). The multiple product $(R_1 \cdot R_3)$ $(R_3 \cdot R_5)$, however, now has a resolution of approximately 2.2 cm. A third product may now be factored in, viz., $R_1 \cdot R_5$ which, because of the zero points of the individual responses, will have its peak also centered over crystal 3 but be lower in magnitude than $(R_1 \cdot R_3) (R_3 \cdot R_5)$. The third curve is the response of $(R_1 \cdot R_3) (R_3 \cdot R_5) (R_1 \cdot R_5)$ or $R_1^2 \cdot R_3^2 \cdot R_5^2$. The FWHM has not improved since $R_1 \cdot R_5$ has about the same resolution as $(R_1 \cdot R_3) (R_3 \cdot R_5)$. Note, however, the change in slope as measured between 2 and 3 cm from the center line. Although not shown, each individual crystal has a count change ratio of 1.3 (in that region of its response) whereas the product of two of these individual responses increases this change ratio as shown to 3. The double product further increases it to 6 and finally, the triple product brings the count change ratio to 18. Product development, therefore, can result in enhancement of resolution and, in addition, can produce at least a log-order increase in the slope of the multiple product response curve. This suggests that imaging resolution may be improved by utilizing the edge of the response as the sensing parameter, rather than the peak.

With the above indication of improved resolution with the inclusion of the third recessed crystal, a modified prototype having the same dimensions as the original but with three recessed NaI(Tl) and two promontory CsI(Tl) crystals, as in Figure 23, was proposed for more refined measurements during the next contract period. Tests on this detector element will be discussed subsequently.

Paralleling the above tests of detector performance with the rack and pinion positioning fixture, a means was sought for making precise source position measurements on a much finer coordinate grid system, to be used with the proposed modified detector element. A decommissioned Searle Pho-Dot scanner was obtained on loan and put into operating condition. This unit, incorporating a precision, sturdy, motorized detector-positioning system, was subsequently modified to automatically sweep a 20 cm by 20 cm field size in steps of 5 mm under external program control utilizing the TN-1700 analyzer system.

2. Contract Period September 1, 1979 Through February 28, 1981

During this contract period, the tests on thin, individual, NaI crystals to determine the variation of intrinsic resolution with crystal width were completed. The results have been discussed above and presented in Table I. Work otherwise was concentrated on obtaining the response characteristics of the modified prototype (Mod. II) detector element and the use of this data to develop the synthesized response of a 3x3 array of similar detectors to point sources arranged in various configurations.

The physical aspects of the Mod. II detector are shown in Figure 1, Appendix II. As indicated previously, it consists of three recessed NaI(Tl) and two promontory CsI(Tl) crystals. By masking the light output of two of the crystals as indicated, it was possible to obtain 5 peaks from interactions of the 88 keV,

^{109}Cd photons; one for each of the crystals (Fig. 2). Fortunately, the peaks of interest for the analysis of spacial response are those of the NaI units, because upon return of the detector from the manufacturer for repair of a defect, it was found impossible to separate the two CsI peaks by a similar amount and they were subsequently summed as a single peak. For purposes of discussion, therefore, the response of CsI crystals 2 and 4 become R_1 and NaI crystals 1, 3 and 5 responses are identified as R_3 , R_5 , and R_4 because of the location of their respective peaks in the spectrum.

Response of the Mod. II detector at defined locations about the point source was obtained by mounting the shielded detector element on the movable arm of the modified Pho-Dot scanner. The TN-1700 MCA, with its dedicated Nova 2 computer, provided a trigger, under control of a stored sequence, to a position control unit which incremented the arm in 5 mm steps. The program control unit (PCU), designed and built here, completed the logical loop between the Nova 2/MCA and the Pho-Dot scanner. After each repositioning of the detector, the PCU indicated the location of the detector by a mark struck electronically on a precisely located sheet of graph paper on the Pho-Dot table. These marks are measured from a zero reference starting point which accurately coincided with the position of the detector centered over the source at the starting position. The source, mounted in a cross-hair cursor, was placed on a similar piece of graph paper secured to the laboratory bench.

The stored program sequence, after a 300 sec. count time at each location, then provided on-line, typed output consisting of location number, individual region counts corrected for background, and the multiple products resulting from left and right side crystal counts. The size of the field was dictated by the need to cover an area the size of a 3x3 detector array (Fig. 5) or about 114 mm square. At the end of the abscissa distance, preset by adjustable microswitches, the arm incremented 5 mm along the ordinate and returned in 5 mm steps along the abscissa in the reverse direction until the microswitches in this direction were encountered, whereupon the motion was again reversed. This automatic capability enabled a large number of incremental values to be obtained without manual intervention over counting intervals sufficiently long to provide the desired level of statistical significance.

Plots of the region responses versus detector displacement over the point source at a distance of 10 cm from the detector face are shown in Figure 4. If the detector was rotated by 90° , a similar response would be obtained but with a longer plateau, related to the longer longitudinal dimension of the crystal, 38.1 mm, versus the 7.6 mm dimension as for the plotted case. As the rotated detector is moved downfield a function is obtained analogous to the X-function plot but with a different shape. Thus, when the response data was plotted as a function of X, as in Figure 4, for a series of 2.5 cm increments in the Y direction, a family of response curves was obtained from which the Y

function could be approximated linearly and incorporated in a linear regression routine, as had been done with the X values.

Linear regression functions were thus derived for the responses of the three regions R_3 , R_4 , and R_5 for use in the response-approximation expressions of a program written to synthesize the response of each of the detectors in a 9-detector, 3x3 array. This program developed, as needed, the region responses for both vertically and horizontally-aligned crystals, summing their responses to any number of point sources at any given location in the field of the array. In addition, the program allows the array center to be located at any desired set of coordinates in the 133 cm-square field and successively view the one or more sources located in the coordinate system. For each position of the array, the respective regional counts for each detector element, in turn, are indicated both separately and as a running cumulative total for each point source that is viewed. In this manner, the response of a single detector, as well as the array, to any simple or complex source configuration was ascertained by an iterative, step-by-step, integration process.

When the last point has been viewed by a given detector, the multiproduct terms are printed out for the accumulated region responses as $(R_{3T} \times R_{4T}) (R_{3T} \times R_{5T}) (R_{4T} \times R_{5T})$, etc. and the next detector in the array is similarly synthesized (Fig. 6). When the last detector (3,3) response has been completed, the program "moves" the detector array by a pre-selected increment along the abscissa and the process repeats

itself for the entire array. In this manner the response of a horizontally-scanned array is synthesized for any source configuration.

As may be inferred from this brief description, the logic involved in developing the synthesis program, primarily due to the shape and configuration within the array of each detector element, can become complex. The program, with its ancillary routines for array orientations, multiproduct development, and trace printout utilized all available locations of a 24,000 word memory. A listing of the final version of the program is provided in Appendix A of COO-4957-2 (Appendix II of this report).

Tests of array response were made using the synthesis program with several source configurations; a single point centered at the origin, a 2.5 cm long horizontal line segment, 2.5 cm long line segments inclined 45° and 22.5° , respectively, and a monolithic, circular source comprised of two concentric circles of radius 2.5 cm and 1.25 cm, with 4 radii, each represented by two point sources.

The point source results confirmed expectations that the detector is very responsive to small positional changes. When scanned by an array moving along the abscissa, the multiproduct factor of the upper right detector (1,3) changed from background to an 8th order-of-magnitude value within an incremental distance of less than 2 mm (Fig. 7). Each vertically-oriented detector in the array (1,3),* (3,3), (1,1), (3,1) and (2,2)

*Bracketed (i,j) values refer to an individual detector at row i, column j in the 3x3 detector array, not coordinate values.

displays an analogous response, but reflects that detector's relative array location in terms of actual distance from the source. Thus, response of detector (1,3) decreases and falls to zero as the array center approaches the source location, while the response of (1,1) (the vertically-oriented detector at the upper left) begins to increase when the array center passes over the source. The shape of the two responses is identical except for the displacement and the fact that one represents decreasing (1,3) detector output while the other shows an ascending response (1,1). As expected, (3,1) and (3,3) show the same product response as (1,1) and (1,3), respectively, due to their symmetrical placement with respect to a source at 0,0. Were the point not at the Y-axis center with respect to the array, product responses of (3,1) and (3,3) would not mirror (1,1) and (1,3) and the degree of dissimilarity.

The midpoint of the array always has the same coordinates as detector (2,2); hence, the response curve of this detector is symmetrical about zero. The other detectors in this column, (1,2) and (3,2), are horizontally aligned and, since the point source is 38.1 mm from their respective centers, the promontory CsI(Tl) crystals completely shield the middle and distal edge NaI(Tl) crystals (corresponding to Regions 5 and 3, respectively) from the low-incidence-angle photons). Therefore, only R_4 , associated with the proximal edge NaI(Tl) crystal is illuminated and, consequently, these detectors have no multiple product output.

Detectors (2,1) and (2,3) exhibit a different response characteristic to the point source as the array moves along the abscissa and the horizontally-aligned detector geometry permits illumination of all crystals, the response increasing with larger x displacement. The two responses intersect at a point that coincides with the source center, as will be elaborated upon later.

Central to source position determination of this detector design is the extremely steep slope of the response rise and drop-off of vertically-aligned detectors 1,1 and 1,3.

The enhancement of the response slope results through development of the multiple product term, equivalent to $R_3^2 R_4^2 R_5^2$. It can readily be seen, from Fig. 7, that the slope of the fundamental R_3 , R_4 , and R_5 responses is not large and, moreover, that the product $(R_4 \cdot R_5)$ and $(R_3 \cdot R_5)$ produces a combined response showing accentuation of slope along with narrowing of the peaks. However, when the triple product $(R_3 \cdot R_4) \cdot (R_3 \cdot R_5) (R_4 \cdot R_5)$ is derived, the resulting combined response for a detector rises and falls suddenly when the crystals pass over the point source. This acutely sensitive response is elicited by the sudden illumination or

extinction of a remote (distal) edge crystal (R_3 or R_4) as the detector approaches and then passes away from the source. A miniscule movement of the detector allows one of these edge crystals to either pass out of the shadow of its associated promontory CsI crystal or be shielded by it. Thus, a very small X displacement sends the product from zero to some very large value, even when the edge crystal output is an extremely small value. Edge crystals, therefore, function in a manner analogous to the base excitation trigger in a transistor circuit, a small signal controlling a very large output. As the detector comes upon the source, the edge crystal that is closest to the source, as well as R_5 , the center crystal, can indicate very large values which would be effectively suppressed by the zero output of the remote edge crystal, until the modulating effect results in multi-product output.

As can be noted from Figure 7 the responses of (1,3/3,3) and (1,1/3,1) are not tangential to each other at 0,0, as would be expected in this case from the foregoing analysis. This system error of approximately 6 mm is ascribed to the fact the linear ramp approximations, adopted to simplify the synthesis program in the interests of limited available computer memory, results in an abscissa intersection which is approximately 6 mm less than the experimental data would

indicate. The portion of crystal response near the abscissa is curvilinear, and would require a quadratic or perhaps a spline curve fit to accurately represent the function involved. Inasmuch as the present simplified linear approximation utilizes all available memory, further refinement of the synthesis program was not attempted.

Extending the point source to become a 3 point, 45° diagonal line (Fig. 8a), the responses of detectors (1,3) and (3,3) as well as (1,1) and (3,1) no longer coincide. The decline of detector (3,3)'s response will fall below that of (1,3), while the ascending response of detector (1,1) will be below that of detector (3,1). The reason for this becomes clear from Figure 8b, which depicts the array geometry in terms of the 3-point line configuration. When the array is so positioned that the responses of detectors (1,3) and (3,3) are equal and, therefore, their response curves intersect, corresponding to $X \approx -35$, the three point sources occupy positions A-B-C relative to (1,3) and (3,3). Due to the mirror-image, 3-point symmetry viewed by (1,3) and (3,3), their respective responses are equal.

Before this, the edge of detector (1,3) was slightly to the left of point C (whose relative position was at C') the other points residing relative to the detector at B' and A', respectively. Since point A' was in line and very close to an edge crystal of (3,3), while B' was equidistant between (3,3) and (1,3), and C' was closest to (1,3) but not in line

was at C') the other points residing relative to the detector at B' and A', respectively. Since point A' was in line and very close to an edge crystal of (3,3), while B' was equidistant between (3,3) and (1,3), and C' was closest to (1,3) but not in line with its edge crystal (3,3) should show a slightly greater output than (1,3). The dotted-line response in Figure 8a confirms this analysis.

Similarly, when the array has travelled sufficiently to the right in Figure 8b so that the line point sources are in relative positions A"-B"-C", the situation above is reversed. A" is no longer in line with any of the crystals of detector 3,3, while C" is closest to (1,3) and offers head-on illumination of its crystals. As theorized, the response of detector (1,3) is indeed slightly greater than that of (3,3) for these positions of the array (Fig. 8a). As the array proceeds to the right, the 3-points shift further left, points A and C becoming more and more remote from the right edge crystals of detector (1,3) and (3,3) as the interposing edges of the promontory CsI(Tl) crystals causes both detectors' multiproduct outputs to fall and begin to approach each other. When points A and C become completely obscured by the promontory crystals, the multiproduct terms have become equal and fall to or rise from zero at an abscissa value, again 6 mm less than the X coordinate of the end points of the line. The 6 mm discrepancy has already been explained, and consistently appears in the

synthesized responses. The foregoing description applies equally to detectors (1,1) and (3,1) which trail their counterparts (1,3/3,3) owing to their position in the 3x3 array.

The intersection of responses for horizontally-aligned detectors (2,1) and (2,3) at the origin occurs as it did for the point source and analysis of this behavior suggests that these curves should intersect at an abscissa value corresponding to the center point of the sources viewed by these detectors. Thus, the X-coordinate of response intersection indicates the middle of the source configuration as it appears to the array scanning along the abscissa. Horizontally-aligned detectors (1,2) and (3,2) are similar but provide slowly-varying responses that yield similar symmetry information, as can be seen. Referring to Figure 8a and 7 when the array is situated to the left of the line points, point B is equidistant from detectors (1,2) and (3,2) but point A is relatively closer to detector (3,2) while C is farther from detector (1,2). Detector (3,2) therefore has more counts than (1,2) as the array approaches the line source. When the array center is located symmetrically over the sources, both detectors have equal responses, corresponding to the intersection point. As it proceeds further to the right, the relative position of detectors (1,2) and (3,2) is such that the illumination picture becomes reversed, (1,2) being illuminated more strongly by point C than is the case with (3,2) by point A which has passed to

the left. Hence, the response of detector (1,2) increases, while (3,2) declines.

Further insight as to the behavior of the 3x3 array may be gleaned from review of Figure 8(c), which depicts the background-corrected responses of each recessed crystal in a 9-detector array when illuminated by variously positioned single or multiple point sources. Thus, a characteristic pattern of individual crystal responses exists for the single point source (C-1) and its approximate location could, in fact, be derived from the crystal responses. The highest, almost equal counts, 131, 130, and 126 are observed in the 3 crystals whose confluence is approximately over the point. The marked falloff of counts in the other crystals, and their pattern of change serves to locate the approximate position of the point source under the array.

Similarly, the crystal responses in 8(C-2) indicate the approximate source location, i.e., the intersection of crystals having the greatest, almost equal counts, 132 and 128. Thus, the recessed crystal responses alone, without benefit of multiproduct development, may also provide meaningful data.

A 3-point line segment (Fig. 8C-3) results in a more complex distribution of counts among the crystals. Nevertheless, a consistent pattern emerges which approximately locates the position of the line segment under the array. The very low counts observed in crystals along the top and

middle left and right sectors of the array, along with the highest counts being noted in the edge crystal of (2,2) and those of (3,2) effectively brackets the general location of the line segment.

Similarly, the position of the 3-point line segment in Fig. 8(C-4) becomes apparent in considering the distribution of counts among the recessed crystals. Crystals having the highest counts again are clustered around the configuration of point sources.

It seems possible that such primary crystal data might also contribute significantly to the process of decoding responses of a multi-detector array into a mosaic of constituent individual points that define the shape and features of the sources.

A 3-point line inclined 22.5° (Fig. 9), reveals response patterns similar to Figure 8a (45°). Responses (1,3) and (3,3) are displaced, as before, but the gap is reduced. This can be expected since the deviation of these responses stems from the inclined configuration of the sources, such that the end points of one or the other line will favor detector (1,3) or (3,3) depending upon array position. Therefore, as the configuration approaches the horizontal, this illumination imbalance condition is lessened. Similarly, as the angle of inclination increases beyond 45° , the illumination received by detectors (1,3) and (3,3) becomes more equivalent until, at 90° , it is equal for both and the gap again disappears.

The extent of the separation between the two responses could therefore be an indication of the angularity of the source edge. This is based on the premise that activity distributed beyond, as would be encountered with an organ, for instance, will affect the magnitude of the responses but not their geometric relationships.

Studying the responses for detectors (1,2) and (3,2), it will be noted that both the separation and the degree of curvature are reduced as the line approaches the horizontal. At a critical angle, θ_c , which positions the end points sufficiently far from the remote edge crystals of (1,2) and (3,2) their illumination will cease due to promontory crystal eclipse and the two response curves will abruptly disappear. In the instance depicted, this extinction angle will be approximately 17.7° .

As the slope of the line sources increases beyond 45° , the extent of the separation between the two detectors' response curves again decreases but, in this case, the degree of curvature becomes more pronounced and greater in magnitude. This circumstance is analogous to the detector (1,2/3,2) response obtained with the circular source (Fig. 10).

If the responses for detectors (2,1) and (2,3) are compared for the 45° and 22.5° inclined lines, it is evident that the maximi are different. The end points of the 45° line are more displaced from these detectors than is the case with 22.5° , hence the latter circumstance elicits

responses of greater magnitude. The two inclined lines produce slight differences in the response of detector (2,2) at the center of the array.

The 45° inclined line places the end points of the line closer to the center line of detector (2,2) in comparison with the 22.5° line. Therefore, these points will better illuminate vertically-aligned detector (2,2) and its peak will therefore be higher. Moreover, the width of the response will be narrower, reflecting the smaller displacement of the end points from the center. By contrast, the less steep 22.5° line with its greater end-point displacement along the abscissa will produce a smaller, more broadened response.

A circular source 5 cm in diameter was represented by 2 concentric circles of 5 and 2.5 cm dia., the larger had source points at 0° , 30° , 90° , etc. in 30° intervals; the smaller at 0° , 15° , 45° , 75° , 105° , etc. This avoided coincidence of points on the inner and outer circles except on the major axes which had points at 5 and 2.5 cm, for a total of 28 sources.

The detector response patterns are shown in Figure 10. As was the case with the previous array syntheses, the responses of detectors (1,3/3,1) and (1,3/3,3) define the edges of the course, subject to the 6 mm systematic error already described. In an interval of 2 mm their responses change from zero (background) to approximately 5.6×10^{11} . The responses of detectors (1,3) and (3,3) or (1,1) and (3,1) are not displaced since the circle symmetry does not favor

an upper or lower detector at any array location, as was the case with the inclined line sources. The responses of detectors (2,3) and (2,1) again intersect at an abscissa value corresponding to the center of the source configuration, or 0,0. As has been mentioned, the responses of detectors (1,2) and (3,2) coalesce and a more peaked curve is obtained which is symmetrical over the source center. Responses of detector (2,2) demonstrate the symmetry of the source configuration, and falls to zero at equally distant points from the outside of the source. It should be noted that detector (2,2) behaves identically to any other vertically-aligned detector, and the sudden increase in its response from background to a very large value could be used to locate the edge of a source in the same manner as with detectors (1,1/3,1) and (1,3/3,3).

There are interesting possibilities that intersections between the response curves may provide information regarding the shape and location of the source. For example, it can be seen that the intersections of detectors (2,3) with (1,1/3,1) and (2,1) with (1,3/3,3) correspond to the two edges of the source less 6 mm, as indicated by the responses of detectors (1,1) and (1,3) dropping to zero.

When these intersections are examined for the 45° inclined line, response of detectors (1,3) with (2,1) corresponds to one end of the line, while (2,3) with (3,1) is aligned with the other end. In the instance of the 22.5° line, it is the intersection of detector (2,1) response with that of (3,3) that matches the one end, and (2,3) with (1,1) that is aligned with the other. These correspondences do not seem to be coincidences but additional analyses in terms of differently dimensioned simulated sources, other configurations, and varied asymmetries, will be needed before the full significance of these intersections can be assessed.

Summing up, the various syntheses of point, line and circular sources demonstrate the high sensitivity of the detector to changes in source position, in terms of the ascending or descending edge of the response curve. It would also appear the individual detector response characteristics individually and through their intersections can also yield considerable information regarding source shape and symmetry about the abscissa and ordinate axes. Although depictions thus far have been confined to displacement along the abscissa, array movement along the ordinate will provide analogous information and thus fully define a plane figure.

C. Current Contract Year: Progress Report for Period
March 1, 1981 Through October 1, 1981

Work this year has been directed toward obtaining the response characteristics of the Mod III detector as discussed in our proposal. This detector, unlike Mod II which contained 3 recessed NaI and 2 promontory CsI crystals viewed by a single PMT, is made up of 9 NaI crystals, each with its own PMT. As discussed in previous reports, the development of the multiple-product response (MPR) suggests that the gradient of the count change involved in the MPR as a function of detector motion could be more effective as an imaging parameter than the peak of the response itself is, be it from a single crystal or that of the multi-crystal MPR. To take advantage of these facts, the 9 crystal detector element outlined in the upper lefthand corner of Figure 1 was created. As shown, R_1 through R_5 are the recessed crystals (as were the 3 NaI crystals in Mod II) and P_1 through P_4 are the promontory crystals (like the 2 CsI crystals of Mod II). As implied in the drawing, the 9 crystal detector becomes only a portion of a larger array of recessed and promontory crystals which may, within limits, be any size. Having their own PMT's, the crystals of this array are separate entities, which substantially facilitate data acquisition and reduction.

By analogy to the previous MPR developed using three recessed crystals, this assembly would allow the following products: $(R_1 \cdot R_2)$ $(R_1 \cdot R_3)$ $(R_1 \cdot R_5)$ $(R_1 \cdot R_4)$ $(R_2 \cdot R_3)$ $(R_2 \cdot R_4)$

$(R_2 \cdot R_5) (R_3 \cdot R_4) (R_3 \cdot R_5) (R_4 \cdot R_5)$; or $R_1^4 \cdot R_2^4 \cdot R_3^4 \cdot R_4^4 \cdot R_5^4$, which produces a very steep slope at any position off the central axis, i.e., R_3 . Also, because of its symmetry, MPR data is provided simultaneously for both the X and Y axes and the two 45° off-axis directions. There are, of course, other MPR's that may be developed from this type of crystal arrangement that add to the versatility of such an array, and some of these will be discussed subsequently with the development of the response.

Operationally, there are several unique aspects to this crystal arrangement not possible with the Mod II design:

a) In terms of response, the output of the larger array can provide a series of ordered MPR's, i.e., $R_1^4 \cdot R_2^4 \cdot R_3^4 \cdot R_4^4 \cdot R_5^4$, by selecting the individual response (R-values) in an ordered, programmed fashion. For example, MPR_1 could be the 5 recessed crystals outlined in solid black in Figure 1, MPR_2 could then be the 5 recessed crystals within the heavier of the dashed lines, i.e., the previous R_3 would become R_1 , R_5 would become R_3 , etc. MPR_3 is then calculated from the recessed 5 crystal's responses outlined in the lighter dashed lines; MPR_4 would begin with R_2 , etc. In this manner a series of MPR's could be generated, governed by the position of the contributing crystals in the array, whose individual responses are in turn determined by their relationship to the source, i.e., its geometry and position beneath the array.

b) Since they are now separate, it is possible to provide a completely variable depth for the recessed crystal, thus allowing selection of individual response sharpness and hence MPR resolution for widely-varying imaging applications.

c) Separate crystals also allow for the reversal of recessed and promontory crystal position. Thus, in terms of the output of the array discussed above, R_1 is now derived from the position previously occupied by crystal P_1 , R_3 from position P_4 , etc. and a second series of MPR's is developed, in effect producing a scanlike motion of the entire array over the dimension of the contributing crystals. By again reversing the recessed/promontory position of the crystals, the array's spatial response is shifted back to its original center. Recalling that the unique aspect of the recessed/promontory geometry is the production of the MPR and, more importantly, the development of the very steep slope as one of the recessed crystals moves just into the shadow of its neighboring promontory crystal, this ability to change the recessed/promontory position could thus produce a wave-like motion in the MPR that can serve as a vernier in precisely delineating the outline of the object being imaged. Since this is the overall objective of this study, this ability to reverse the recessed/promontory crystal geometry in some programmed manner for fine scanning emerges as one of the more promising attributes of the new detector element's design.

a) Summary of This Period's Work:

1) Similar to Mod II, the response of the Mod III detector element has been synthesized by obtaining, experimentally, the response of a single one-half inch diameter, collimated, NaI(Tl) crystal to a point source as a function of-motion on the axis that crosses the crystal's face at its center line. Data were obtained with the source attached to the arm of a digital plotter which then moved in precisely programmed steps beneath the crystal at two different source-to-detector heights and several different collimator depths.

2) Concurrently, the response of the detector to a point source was obtained analytically from geometrical principles and a program written to produce the response curves to several source-detector distances using several collimator depths. The analytical solutions are compared to the experimental responses using the same parameters and are shown to be in excellent agreement. The calculated response to a variety of collimator depths and crystal heights has been obtained.

3) The experimental data are used, as with Mod II, to synthesize not only the response of the single 9 crystal detector element, but the response of a group of 81 crystals in a 9x9 array.

The response of the one-half inch NaI(Tl) detector by driving the source on a digital plotter and the subsequent

calculations and comparisons of point source responses was accomplished with our TN-1700 analyzer utilizing the interactive basic language of the Nova 2 computer to program the various tasks.

The synthesis of the array response was accomplished using the T.I. (Texas Instruments) 90/4 computer obtained in May of this period. Utilizing its associated color monitor and an DEC LA-120 printer, it has been programmed to produce 3-dimensional views of the response of the array to clusters of point sources in various configurations.

The ability to present the response of the array in 3 dimensions adds enormously to the visualization of the multi-product approach and brings into much sharper focus the possibilities envisioned in the reversal of the recessed/promontory crystal positions discussed earlier.

b) Operational Details.

Rather than attempt to obtain the spacial response with a 9 crystal assembly that makes up the Mod III detector element as originally planned, it was decided, because of the symmetry of the individual crystals obtained for the purpose, i.e., one-half inch diameter cylindrically-shaped NaI(Tl), to accurately determine the spacial response of one of them by moving a point source directly across the center line of the detector. Since it is symmetrical about its center line, this single response would be sufficient to obtain the response of the other crystals, not only in the 9 crystal arrangement making up Mod III, but any similarly shaped crystal anywhere

in the larger array, to a point source situated anywhere on the experimental source plane.

During the last contract period, as discussed earlier, the detector element was mounted in the arm of a Pho-Dot scanning mechanism which had been modified to allow movement of the detector along the X and Y axis in 5 mm steps over the fixed source. For the current contract period, the step interval has been refined considerably with the acquisition of a digital plotter*. The one-half inch diameter detector, housed in a collimator to simulate the response of the detector as if it were recessed and thus surrounded by promontory crystals, is securely mounted to the Pho-Dot arm which now remains stationary (except for lowering or raising the height) above the source plane. The source is ^{241}Am (10 μCi obtained as a spot of approximately 1 mm in diameter affixed in the center of an aluminum ring) and is mounted directly to the vernier arm of the digital plotter. The collimator is a brass cylinder of approximately 3/4 in. I.D. with an outer sheath of Pb of sufficient thickness to eliminate penetration of the wall by the 60 keV, ^{241}Am photons. The detector, in a steel housing just under 3/4 in. O.D. fits snugly in the collimator. Two more cylindrical collimators were fashioned, each of which fit exactly into the brass collimator, and which had one-half inch holes

*Houston Instrument HiPlot DMP-2.

drilled. Both approximately one-quarter inch deep, one was affixed permanently and flush with the brass tube, the other to the face of the detector. In this manner, the response of the one-half inch diameter crystal could be obtained at various easily changed collimator depths. A schematic representation of the experimental set-up for this portion of the study is shown in Figure 2.

Under interactive program control of the Nova 2-based TN-1700 analyzer/computer, the digital plotter was moved beneath the detector in precise increments of 0.05 in. (1.27 mm; 0.005 in. steps can also be obtained). Data was collected at each point for 1000 sec. The plotter, programmed to increment 160 times across the centerline of the detector, yielded a total path length of $0.05" \times 160 = 8$ in. As seen in Figure 3, this was sufficient to totally define the spacial response of the crystal at a face height of 4 in. with a measured collimator depth of 0.75 in. (Actual depth = 0.02 in. The significance of the dotted and solid curves will be discussed subsequently.) These conditions ($H=4$ in.; $D=0.75$ in.) were selected to match the parameters of the Mod II tests in which the 3 NaI crystals are recessed 0.75 in. with respect to the 2 CsI crystals. The 4 in. height was selected originally, based on the "standard man" model with an average body thickness of 20 cm (8 in.) and the conservative assumption that the activity to be imaged would appear on

the mid-plane at a depth of 4 in. The program to drive the plotter, written in the interactive basic language of the Nova 2 appears in Appendix III, along with a sample data printout.

As expected, the response of the one-half inch diameter cylindrical detector (Figure 3) is similar to that of the 0.3 inch wide detector (R_5 , Figure 13, Appendix II) with the same flattened peak region and a FWHM of approximately 2.4 in., compared to the FWHM of about 2 in. for the narrower rectangular crystal. As was found subsequently, however, the actual depth of the collimator is 0.07 in. deeper than measured due to the finite window thickness and a small layer of reflector material on the face of the crystal. Therefore, the resolution quoted above is somewhat better than would be obtained, since the one-half inch crystal was actually at a collimation depth of 0.82 in. rather than 0.75 as measured.

The distance from the face of the crystal container to the surface of the crystal was obtained from the X-ray photographs taken of the crystal during the next phase of the study, which was to obtain the response of the crystal analytically, utilizing geometrical principles, and compare this calculated response to the experimental data. The X-ray photos appear in Figure 4. As seen, the distance between the container face and the crystal face is easily measurable and, when added to the externally measured depth, accounted for the differences in peak height and slope between the calculated and experimental data.

Results of this phase of the study are presented in Figure 5 through 14. The analytical methodology used to solve for the theoretical response and the program to generate the calculated results are presented in Appendix IV. Figures 5 through 9, in which the experimental data, shown dotted, is compared to the calculated solid line, indicates the excellent agreement achieved between the model and the experiment. Note, however, the tilt in the peak points of the experimental data as compared to the calculated curve. Close examination of the two X-ray photos (the crystal was rotated in one) clearly shows that the axis of the detector is in fact tilted with respect to the axis of the container. Thus, as the source moved under the detector a portion of the incident photons are passing between the crystal and the edge of the one-half inch collimator affixed to the face of the container, not interacting as the source moves into the shadow of the edge opposite the opening and causing the observed tilt in the data. The data of Figure 9, showing the comparison between experiment and calculation at a collimator height of 3 in. and a depth of 1.07 in. were taken after the crystal had been rotated approximately 90° to place the tilt along the axis of source motion rather than across it. The peak of the experimental points is now noticeably flatter, corresponding even closer to the calculated result. Figure 10 shows the calculated response at a collimator height of 4 in. but with no depth, i.e., the response expected of an uncollimated promontory crystal. The calculated response at the 4 in. height with a depth of 2.5 in.

is given in Figure 11. These curves may be contrasted with Figure 12, in which the calculated response as a function of collimator height is plotted at a constant depth of 1 in. The curves of FWHM versus collimator depth at $H=4$ in. and versus height at $D=1$ in. are further reduced in Figure 13, where a plot of these functions shows the FWHM to be slowly changing with height of the crystal at a depth of 1 in., passing through the ordinate at about $\frac{1}{2}$ in., the diameter of the collimator, at 0 height as would be expected. The change in FWHM with depth is more pronounced, with the greatest increase in resolution occurring in the earlier portion of the curve. Thus, from 0 collimator depth to 1.5 in., the FWHM changes from about 6 to about 1.5, a factor of 4. Considering the loss in sensitivity which must be accepted as the crystal is recessed behind its neighbor to obtain the spacial information, data of this nature is necessary to help determine the amount of trade-off between the two, i.e., insensitivity and resolution, at any given activity level. There is no doubt there will be a loss of sensitivity in obtaining an image; the question is how much? Thus, at a height of 4 in., there would be no reason for increasing collimator depth beyond 1.5 in. and as will be seen, reasonable information should be possible at a depth of 0.5 in.

Data such as that developed in Figure 13 is presented for several different collimator heights and depths in Figures 14a, b, c and d in which families of response curves versus source position are shown. The effect of collimator height and depth is now easily visualized.

To complete this phase of the study, the calculation of the detector response was extended to include a correction for penetration of the incoming photons through the edges of the collimator and corners of the crystal as a function of energy. A plot of the response at two energies is shown in Figure 15, in which it is seen that the correction is hardly noticeable at 60 keV, but of some consequence at 200 keV. This portion of the calculation has not been verified experimentally as yet.

In the next phase of activity, work was started to synthesize the response of an entire array of $\frac{1}{2}$ " detectors to n point sources distributed randomly on the source plane. This portion of the study is being done with a Texas Instrument 99/4 computer with 16K memory, a CRT monitor, RS232 interface to a DEC LA-120 printer, and a single $5\frac{1}{4}$ in. floppy disk mass storage device.

Using the T.I. BASIC language, a program has been written* which synthesizes the response of a 9x9, 81 crystal array to any randomly distributed set of up to 30 point sources.

Since the detectors involved are symmetrical, this task was somewhat less involved than that required for the Mod II rectangular crystals. Similar to that program, the current effort develops the response of all the detectors to the one

*Shown in Appendix V.

or more selectably positioned point sources - using as original data the experimental points obtained with the digital plotter as described above - one source at a time, sums the responses to each source until all sources are accounted for, then develops the multiple-product responses.

Several MPR's have been developed and more are anticipated as the study proceeds and more insight into the response is obtained. Figure 16, in which the 9x9 array configuration is diagrammed, will aid in explaining the MPR development. The crystals are shown as squares for simplicity. The size of the array for these initial studies has been based for convenience on detectors with a 1 in. separation between their centerlines on the X and Y axis. Since the experimental response is developed from a $\frac{1}{2}$ in. diameter detector, the crystals could and will be grouped together more tightly for future tests.

As seen, the crystal coordinate positions have been established using a graphing-type convention, i.e., crystal 1 is located at coordinate (0,0) in the lower left corner, and crystal 81 at coordinate (8,8) in the upper righthand corner; the usual row, column convention is followed to establish the other positions in the array. From this, the response of the detector in the center of the array, (4,4), is labelled R_{44} . If it is a recessed or promontory crystal, the labelling becomes (4,4)r; (4,4);, or R_{44r} ; R_{44p} , respectively.

For the development of the MPR's, the response of the detectors (0,0) through (8,8) is first developed by calculating the distance of each from the source as a function of position on the source plane which uses the same coordinate system as the detector plane. Then, based on this distance, interpolating where necessary, their response is obtained by use of the experimental, point source reference data. The reference data for the initial tests to be presented below, was taken from the point source data collected at H=4 in., as discussed earlier. The response of each of the detectors, summed if more than one source is used, is thus developed as if all were recessed. From this data, the MPR's may be developed to reflect any pattern of recessed/promontory crystal arraignment desired, by choosing which of the crystal responses will be used in the MPR development. Since the program logic develops the detector responses consecutively, row by row and column by column, starting at (0,0), as if all are recessed, the initial product development follows that same pattern, i.e., they have been developed as if all detectors are recessed. Thus far, six MPR's have been developed, called ProdH, ProdV, ProdR, ProdL, Prodivg, and Prod5. ProdH is a horizontal MPR and is calculated as, e.g., $R_{00r}^2 \cdot R_{01r}^2 \cdot R_{02r}^2$. Recalling how the MPR develops as a function of motion, i.e., the result of the product is a narrower peak than the response of the individual crystals involved and is centered over the center crystal, the result of the above calculation would be assigned to coordinate (0,1). The program then continues, calculating the next horizontal product

$R_{01r}^2 \cdot R_{02r}^2 \cdot R_{03r}^2$ and putting the result in (0,2), etc..
 ProdV, meaning vertical product, ProdR and ProdL, meaning
 diagonal (45°) products to the right and left respectively,
 are calculated in a similar manner. Prodivg is the average
 of ProdH, ProdV, ProdR and ProdL. Prod5 is product originally
 described in the proposal for the Mod III detector element,
 i.e., the product of the fourth power of five crystal responses,
 or, in terms of the coordinate system above, $\text{Prod5} = R_{00r}^4 \cdot$
 $R_{02r}^4 \cdot R_{11r}^4 \cdot R_{20r}^4 \cdot R_{22r}^4$, with the response assigned to (1,1).

In terms of program logic, the above process is halted
 momentarily after the response of all the detectors to the
 one or more sources has been completed, at which point a
 selection is made about which of the above calculations is
 to be output. In addition to enabling output of the product
 results, another output may be called, i.e., Resp, which is
 simply the crystal responses to the one or more sources
 used. When this, or one of the others is selected, this
 data is displayed on the color monitor as a group of coordinate
 squares in the same manner as in Figure 16 (see Figure 28),
 within which the individual response or product data, which-
 ever the case, is presented in the form of a character, of
 which there are 12, forming a "gray" scale which goes from
 all white for the coordinate of a crystal which has 0 counts
 (after background subtraction) to all black for the coordinate
 square of the crystal having the maximum number of counts.
 The result is a display of the response of the entire array

in X and Y space above the source plane with the counts from each detector or the MPR development presented in each coordinate square as a gradation of white to black, depending on the magnitude of each.

The results for the Mod II tests it will be recalled, were presented as a series of curves depicting the change in the multiple product response of the various detector elements as a function of array motion along the X-axis. These curves were able to indicate the significant and sudden changes in the slope of the MPR as opposed to the individual response curves as a function of motion and, as a result, it was possible to imply what the improvement in resolution would be to any given source configuration. Due to the limitations of the output equipment at the time and the length and complexity of the program just to get X-axis information, much less simultaneous Y-axis information, it was not possible to show even with simple graphs what the actual response might be. It is now possible, with the upgrading of the equipment at hand, to present the entire response and, even at this early stage in the development of the program and its output parameters, the results as compared to the last contract period are much more dramatic in their ability to display the characteristics of the multiple-product response and the possibilities we feel are inherent in this approach.

After presenting the display, another selection is indicated; either to print the results or return to the beginning and select the coordinates for another source configuration. If print is elected, the results which have been displayed on the CRT monitor are output via the RS232 port to the DEC, LA-120 printer. The program now selects, using ASCII coding, characters to be printed, such that they appear as a 12-level gray scale, grouped again in the 9x9 array configuration as in the monitor display, with the lowest count or product having no character in its square, the next highest having a pattern of 3 rows of 5 vertical dots in the center of the square, etc. up to the maximum density which is shown in a square with 15 (3x5) #'s. Printed for this display also is the kind of response, i.e., Resp, ProdH, etc., as well as the coordinate position of the source(s).

The results of the initial tests will be presented for the most part as the printed material because of the ease of reproducing these as xerox copies. Only a few, because of the expense of reproduction, will be presented as photographs of the color monitor display to illustrate the possibilities inherent in this medium in terms of immediate visual grasp of information for pattern recognition.

The results of preliminary tests appear in Figures 17 through 26. Figure 17a through d illustrates the simplest response of the array, that to a point source centered at

(4,4). 17a is the plot of the unaltered response of the individual detectors. Note the gradual, but grouped gradation in response from the maximum above the source, represented by the 15 "#" symbols, to the least response above background represented by the 9 "." symbols. The gradation follows the position of the individual detectors with respect to the position of the source and the response tends to be "squared" in appearance. Even so, it is not difficult to see the circular symmetry of the total response pattern by observing the symbol pattern. Thus, the least responsive pattern is seen at the corners - (6,2)/(6,6)/(2,6)/(2,2) - where the crystals are farthest from the source, followed by the 15 "." pattern on either side of each corner at (5,2)/(6,3)/(6,5)/(5,6)/(3,6)/(2,5) and (2,3)/(3,2), etc. This also represents our current level of sophistication in choice of symbols and their positions to represent the response gradations. Figure 17b, 17c and 17d represent the horizontal, right diagonal and average MPR's for the point and illustrate two points; first, the sharp increase in resolution as evidenced by the much more rapid fall off to zero from the (4,4) peak response, and second the fact that when the source is symmetrically oriented with respect to the detectors, all of the products yield the same response pattern, as seen with the point source, but the gradation may be different from one to another. Thus, the pattern for the point source is seen to be symmetrical about the center but the diagonal

products (ProdL not shown) show a more rapid gradation than do the horizontal and vertical products (ProdV not shown). As will be shown, the average MPR, generally contains the best spacial information of these initial tests.

Figure 18, a through d, represents the individual, horizontal, right diagonal and average product responses to 3 sources placed as indicated, at (2,2), (3,6), and (6,3). Again the sharp increase in resolution is noted between the total array response and the MPR's. Now, however, a definite difference is seen between the various MPR's. ProdH shows the sources in the same density and position as the total response for (2,2) and (3,6) and the correct position but a slightly lesser density at (6,3). The ProdV response (not shown), shows the same positions but the less dense position is transferred to (2,2). The diagonal products are not symmetrically related however. ProdR, though similar in shape to the others, shows the most dense response at (5,4) and (4,5), while placing less density at the source positions. ProdV (not shown), is more similar to ProdV and ProdH in its symmetry. Nevertheless, the average product is very symmetrical, clearly representing the positions of the sources. Some source configurations are best defined by one or the other of the MPR's, i.e., ProdH or ProdV or one of the diagonal products, as will be shown subsequently. For the configurations and products developed thus far, however, the Prodivg development appears to give the

best definition regardless of shape and placement. In any event, as will be seen throughout these tests, the array response will always signal which of the MPR's would be more advantageous to use in addition to the Prodvavg.

Figures 19 and 20, a through d, illustrate the results of a small motion of the crystal array. Figure 19a and 19b shows the array response and Prodvavg response to two sources at (4,4) and (5,5); note the symmetry of the array response gradation about the sources and the tightness and symmetry of the Prodvavg response. Moving the array diagonally downward to the left by 0.1414 in. results in the plots shown in 19c and 19d. The change in both the array and Prodvavg. response is immediately apparent. The array gradation has filled in around the upper right quadrant and become less dense (note the "*" taking the place of "#" at 4,4) in the lower right quadrant. The effect of the movement is even more apparent in the Prodvavg response where the "#" have changed to "1" at (4,4) and the two array positions in the upper right quadrants now show a response as opposed to no response before motion, i.e., as in 19b.

It is this abrupt change in response/no response, in addition to the symmetry aspects that we wish to take advantage of as a means of defining more precisely the outline of the activity deposition, through investigations of the rapid change in slope of the various MPR's.

Figures 20, a and b, again shows the result of moving the array over the two sources defined in Figure 19a, this time with a 0.1 in movement to the left along the X axis. Again the movement is reflected immediately in the symmetry of the gradation in both responses. Note the appearance of the response pattern in (5,6) in 20b, as well as the way (4,4), (5,4) and (4,5) have changed to reflect the small increment in the X direction. Comparison of 20a and b to 19a and b, and 19c and d indicates the possibilities inherent in the analysis of MPR.

One of the questions that always arises when discussing the use of this array in imaging is resolution. The answer to that question is as yet undetermined and depends ultimately on the outcome of our investigations into the use of the MPR slope and the changes therein to define the edge of "deposit/no deposit", or, the transition region of "little activity" to "much activity". Towards that definition, however, one criterion to use might be the smallest movement of a point source that is detectable by the array. Shown in Figure 20, c and d are the total array and Proavg. responses to movement of a point source from its position at (4,4) diagonally to (4.025, 4.025), a distance of approximately 0.035 in. ($\approx$ 0.9 mm). Comparison of Figure 20 c with 17a shows no discernible change in the array response, however, the change in the Proavg response as seen by comparing Figures 20d and 17d is unquestionable and

moreover, by its symmetry, defines the direction of that small motion. In Figure 21, a and b, the array and Prodavg responses to the further motion of the source to (4.1, 4.1) are plotted and, as seen, show further gradation changes, again symmetrically, as the source is moved. The usefulness of this type of array in the *in vivo* study of organ clearance of radionuclides or even of *in vivo* organ motion is thus clearly indicated by these initial motion/response tests.

Another parameter of use as a preliminary indication of resolution is the closest possible approach of two point sources before they become indistinguishable. Figure 21, c and d, show the plot of an initial attempt at such a definition. The two sources have been positioned at (4.35, 4.35) and (4.65, 4.65), i.e., at a distance of approximately 0.71 in. (1.8 cm) apart. 21c shows the array response, which from its symmetry would indicate a single source as (4.5, 4.5). Figure 21d indicates that the MPR, with its greater resolution, is able to define them as two sources. Moreover, comparison of 21d and 19b also shows that the difference in distances between the two sources, i.e., 1.414 in. (19b) and 0.7 in. (20d) is reflected in the change in density in the diagonal, off-axis gradation between the sources. Thus, as our ability to define gradation in finer amounts improves, we also expect to improve our ability to define not only small changes in distance, but also closer absolute distances.

Continuing the tests, the source configuration was now expanded to an 8 source arrangement to simulate the response of the array to a circular source configuration centered at (4,4). The results are given in Figure 22, a through d, where the increased diameter or area of the source configuration is clearly reflected in the increased area and gradation shown in 22a, the total array response. The Proavg (22b) again reduces the size of the plot to the area of the configuration. Note that this plot is the same as that of Figure 17d, i.e., the Proavg response to a single point source, the result of the fact that the distance between the sources of this configuration and the center of the circle (4,4), is less than the "resolvability" of two sources. This is not so, however, for the Proavg response shown in 22d when compared to the Proavg response of 21b. After the array is moved in both instances by a left diagonal increment of 0.1414 in., the response in the case of the larger source configuration is clearly of a different gradation, reflecting the source size. Note the gradation symmetry also in both the array and Proavg responses (22c and 22d).

The effect of increasing the source size is illustrated further in Figure 23, a through d, in which another circular source configuration centered on (4,4) is used but now with the distance between sources and center (4,4) increased to 0.7, approximately the "resolvable" distance. The total array

response (23a) change in gradation is readily apparent, as is also the change in the tighter, Prodavg MPR (23b) gradation, again showing the sensitivity of this response.

Imbalance in the source configuration symmetry produces another response however. Shown in Figure 23c and d are the plots of the array and Prodavg response to a smaller configuration in terms of distance between sources than that of Figure 22. Nevertheless, because of the slight asymmetry of the configuration caused by the sources in (2,3), the Prodavg response is clearly different than the point-like response of 22b and what is more, the asymmetry of the configuration even by this small amount, is reflected in the asymmetry of the response. Thus, the net effect of an asymmetrical source distribution appears to be an increase in resolution of the array. This effect will be further explored.

The response of the synthesized array to several larger, more practical, kinds of distribution was also tested. Figure 24, a through c, is the plot of the total array and two product responses to 6 sources along a diagonal straight line. The symmetry of the array response is obvious, as are the product responses. Note in this case, however, the difference between the Prodavg and ProdR responses. The ProdR, taken along the

axis of the sources is more definitive of the relative activity across the length of the source row than is Proda_{avg} and illustrates the point alluded to earlier, that the total array response may often indicate which of the MPR's would result in the most useful information.

Figure 25, a and b, illustrates the attempt to configure the 13 indicated sources as if deposited along the tracheo-bronchial tree, with the bifurcation at (4,4). The sources were distributed such that more activity was deposited in the right bronchial leg than the left or the trachea, the left having less than either. The shape of the deposit is clearly defined in the total array response (25a). This shape is obviously more clearly defined by the multiple product (25b) which also reflects, in its response gradation, the more active right bronchial region. The response does leave the two end sources, one in the left bronchial leg, the other at the top of the trachea, undefined, however, and indicates a possible direction in exploring the MPR capability - that of optimizing the final image by some unique combination of each of the MPR's, in a way more definitive than the Proda_{avg}. Nonetheless, the responses achieved in these first tests certainly leave no doubt as to the sensitivity of the MPR's.

Figure 26, a and b, represent the array and product responses to 20 sources arranged in the configuration of a large circle. The responses, from discussions up until now, is as would be expected. Again the symmetry is to be noted, along with the tightness of the MPR and its ability to more clearly define

the activity-free area in the center of the configuration.

Figure 27, a and b, show the array response and multi-product response once again to a point source centered at (4,4). Now, however, the multi-product response is tightened the maximum possible, given the present presentation format, to a single all black response at that coordinate position. This is the response of Prod5, the five product development and shows what happens as the steepness of the multiple-product response curve is increased. This is the only test of this MPR as yet. Testing of this and other MPR's is continuing at present.

Photographic reproductions of the CRT monitor output of the latter 3 configurations (Fig. 24, 25 and 26) are shown in Figures 28, 29 and 30. As was stated earlier, the responses are represented here, as in the hardcopy output, as a "gray" scale having 12 gradations. Nevertheless, the effect of the color, even though only two are represented (the multi-color effect is the result of "bleeding" in the dot pattern), is somehow more dramatic than the hardcopy. Anticipated for the future, in addition to refinement of the presentation as our level of sophistication increases, is the use of the CRT monitor to take "snapshots" of itself, by disk storage of the display at the discretion of the experimenter based on his judgment of the results of whatever imaging task is being pursued. Thus, as was suggested earlier, it would be possible to actively track in vivo, the route of radioactively-labeled compounds in an experimental animal - more than one label at a

time if synergistic effects are sought, e.g., the labeling of more than one size particle using radioisotopes of differing photon energies in an inhalation, deposition and retention format. Another possibility, as previously suggested, is the ability of such an array to observe, in real time, the active functioning, i.e., motion of an organ or organ system, such as the heart or lungs, through which labeled compounds are moving.

Once these images are stored they may be recalled at any time, either to be re-viewed on the CRT or output as hardcopy.

As an aid to visualizing and thus to our intuition concerning the response of the array, both as individual crystals and multiple product development, another program has been written which transforms the response of a single detector and its output as a 2-dimensional plot of response versus point source position (individual crystal and multiple product as seen in Figure 31) into a 3-dimensional, X,Y (position), Z (response) plot in which the axes may be tilted into the forward direction only or presented as an orthogonal projection.

The results of the first tests of this additional dimension are shown in Figures 32 through 38. Figure 32 is the 3-dimensional projection of the response of a single crystal over a point source, shown with the axes tilted forward. Note the flat top, the 3-dimensional representation of the straight

top in the 2-dimensional plot of Figure 31. Next is Figure 33, showing the effect of multiple-product response of the three crystals with a separation of 1 in. in the horizontal direction. This is followed by the 3-dimensional MPR of these same crystals, the same distance apart, along the right diagonal shown in Figure 34. The next three Figures, 35, 36 and 37 are 3-dimensional plots of the crystals' response to seven sources, plotted on 3 different size X,Y bases of 24, 16, and 8 in. respectively, to illustrate the effect of closing the sources' center-line to center-line distance. Finally, to make a comparison between the two methods of presentation and effectiveness of each in offering a different view of the same phenomenon, Figure 38, a and b, shows the total array response and Prodavg responses of the array to sources placed in the same configuration as shown in Figure 37. As stated earlier, it is believed the comparisons of this nature will greatly assist our intuitive understanding of the responses involved and aid in the final stages of development of the optimal detector array.

In summarizing these early tests of the synthesized array and the results shown, it is believed that these initial results clearly indicate the effectiveness of the recessed/promontory array of crystals and most especially the power of the multiple-product-response development as an aid in resolving the location and areal size of a deposition of radioactivity

within the body - or anywhere else for that matter. While the outline of the deposition is now only roughly defined, it is most certainly better than no outline and has brought us much closer to the original purpose of this detector's development - that of visualizing where and how large a given accidental deposition of radioactivity is, to enable the construction of a more accurate phantom structure for a more refined determination of radionuclide burden. It is believed that further investigation into response functions utilizing the MPR's will achieve more definitive resolutions.

It should also be borne in mind that these initial tests were performed at a collimator height of 4 in. and a depth of 0.63 in., i.e., the smallest amount of collimation. These data thus represent the most conservative array synthesis in terms of collimation while, for the parameters defined thus far, yielding the highest sensitivity for the recessed crystals at that height. Considering organs such as the trachea, however, it will be possible to bring the recessed detectors to within approximately two inches of the center-line of that tissue and still be recessed to a depth of about 1 in. Recalling the families of curves (Fig. 14, a through d) showing resolution and response versus collimator height and depth, this would allow a considerable gain in sensitivity and improvement in resolution as well, thus demonstrating the versatility and utility of the recessed/

promontory detector array design. The ability to vary the depth of recessed crystals will allow the investigator to select the best compromise between sensitivity and resolution for the given experimental situation.

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Figure 1

Mod. III Detector Element

R_n = recessed

P_n = promontory

Figure 2
Data Acquisition System

Data Acquisition System

Figure 3

Point Source Response

H = 4 in.

D = 0.75 in. (measured)

= 0.82 in. (actual)

..... = experimental

—— = calculated

Figure 4

X-ray Photograph of NaI Detector

Figure 5

Point Source Response

H = 4 in.

D = 0.63 in.

..... = experimental

—— = calculated

Figure 6

Point Source Response

H = 4 in.

D = 0.82 in.

..... = experimental

—— = calculated

Figure 7

Point Source Response

H = 4 in.

D = 1.07 in.

..... = experimental

—— = calculated

Figure 8

Point Source Response

H = 4 in.

D = 1.32 in.

.... = experimental

— = calculated

Figure 9

Point Source Response

H = 3 in.

D = 1.07 in.

..... = experimental

———— = calculated

Figure 10

Point Source Response, Calculated

$$H = 4 \text{ in.}$$

$$D = 0$$

Figure 11

Point Source Response, Calculated

$$H = 4 \text{ in.}$$

$$D = 2.57 \text{ in.}$$

Figure 12

Point Source Response, Calculated

H = 1,2,3,4,5 in.

D = 1.07 in.

Figure 13

Spacial Resolution (FWHM) vs.
Collimator Depth at H = 4 in.
Collimator Height at D = 1 in.

Figure 14a

Crystal Response vs. Point Source

Motion for Various Collimator

Depths at $H = 4$ in.

Figure 14b

Crystal Response vs. Point Source

Motion for Various Collimator

Depths at $H = 3$ in.

Figure 14c

Crystal Response vs. Point Source

Motion for Various Collimator

Depths at $H = 2$ in.

Figure 14d
Crystal Response vs. Point Source
Motion for Various Collimator
Depths at $H = 1$ in.

Figure 15
Effect of Photon Penetration
vs. Energy on Crystal Response.

Figure 16
9x9/81 Crystal Array

8								8,7	8,8
7									7,8
6									
5									
4									
3	3,0								
2	2,0	2,1	2,2						
1	1,0	1,1	1,2	1,3					
0	0,0	0,1	0,2	0,3	0,4				
	0	1	2	3	4	5	6	7	8

9×9 Crystal Array

Figure 17

Simulated Array Response/Point Source

a) Array	b) ProdH
c) ProdR	d) Prodivg

SQARRAY: RESP

1 SOURCES ARE LOCATED AT:

4 , 4

8 a

7

6

5

4

3

2

1

0

0 1 2 3 4 5 6 7 8

SQARRAY: PRODR

1 SOURCES ARE LOCATED AT:

4 , 4

8 c

7

6

5

4

3

2

1

0

0 1 2 3 4 5 6 7 8

SQARRAY: PRODN

1 SOURCES ARE LOCATED AT:

4 , 4

8 b

7

6

5

4

3

2

1

0

0 1 2 3 4 5 6 7

SQARRAY: PRODAVB

1 SOURCES ARE LOCATED AT:

4 , 4

8 d

7

6

5

4

3

2

1

0

0 1 2 3 4 5 6 7

Figure 18

Simulated Array Response/3 Point Source

a) Array	b) ProdH
c) ProdR	d) Prodavg

SQARRAY: RESP

3 SOURCES ARE LOCATED AT:
3, 6 6, 3

2, 2

0 1 2 3 4 5 6 7 8

SQARRAY: PRODR

3 SOURCES ARE LOCATED AT:
3, 6 6, 3

2, 2

0 1 2 3 4 5 6 7 8

SQARRAY: PRODH

3 SOURCES ARE LOCATED AT:
3, 6 6, 3

2, 2

0 1 2 3 4 5 6 7 8

SQARRAY: PRODAVG

3 SOURCES ARE LOCATED AT:
3, 6 6, 3

2, 2

0 1 2 3 4 5 6 7 8

Figure 19

Simulated Array Response/2 Point Source

a) Array	b) Prodavg (4,4) (5,5)
c) Array	d) Prodavg (4.1,4.1) (5.1,5.1)
- diagonal motion

SQARRAY: RESP

2 SOURCES ARE LOCATED AT:
4.4 5.5

8 a

SQARRAY: PRODAVG

2 SOURCES ARE LOCATED AT:
4.4 5.5

8 b

SQARRAY: RESP

2 SOURCES ARE LOCATED AT:
4.1, 4.1 5.1, 5.1

8 c

SQARRAY: PRODAVG

2 SOURCES ARE LOCATED AT:
4.1, 4.1 5.1, 5.1

8 d

Figure 20

Simulated Array Response

2 Point Source, Horizontal Motion	a) Array	b) Prodavg
1 Point Source, Diagonal Motion	c) Array	d) Prodavg

SDARRAY: RESP

2 SOURCES ARE LOCATED AT:
4.1 , 4 5.1 , 5

8 a

0 1 2 3 4 5 6 7 8

SDARRAY: RESP

1 SOURCE ARE LOCATED AT:
4.025 , 4.025

8 c

0 1 2 3 4 5 6 7 8

SDARRAY: PRODAVG

2 SOURCES ARE LOCATED AT:
4.1 , 4 5.1 , 5

8 b

0 1 2 3 4 5 6 7 8

SDARRAY: PRODAVG

1 SOURCE ARE LOCATED AT:
4.025 , 4.025

8 d

0 1 2 3 4 5 6 7 8

Figure 21

Simulated Array Response

1 Point Source, Diagonal Motion	a) Array	b) Proavg
2 Point Source, Minimum Separation	c) Array	d) Proavg

SQARRAY: RESP

1 SOURCES ARE LOCATED AT:

4.1 , 4.1

SQARRAY: PRODAVG

1 SOURCES ARE LOCATED AT:

4.1 , 4.1

SQARRAY: PRODAVG

2 SOURCES ARE LOCATED AT:

4.35 , 4.35

4.65 , 4.65

SQARRAY: RESP

2 SOURCES ARE LOCATED AT:

4.35 , 4.35

4.65 , 4.65

Figure 22

Simulated Array Response, 8 Point Circle/Motion

- a) Array b) Prodavg before move
- c) Array d) Prodavg after move

SDARRAY: RESP

8 SOURCES ARE LOCATED AT:
 4.3, 3.3 4.5, 3.5 4.7, 4 4.5, 4.5 4, 4.7 3.5, 4.5
 3.3, 4 3.5, 3.5

SDARRAY: RESP

8 SOURCES ARE LOCATED AT:
 4.1, 3.4 4.6, 3.6 4.8, 4.1 4.6, 4.6 4.1, 4.8 3.6,
 4.6 3.4, 4.1 3.6, 3.6

SDARRAY: PRODAVO

8 SOURCES ARE LOCATED AT:
 4.3, 3.3 4.5, 3.5 4.7, 4 4.5, 4.5 4, 4.7 3.5, 4.5
 3.3, 4 3.5, 3.5

SDARRAY: PRODAVO

8 SOURCES ARE LOCATED AT:
 4.1, 3.4 4.6, 3.6 4.8, 4.1 4.6, 4.6 4.1, 4.8 3.6,
 4.6 3.4, 4.1 3.6, 3.6

Figure 23

Simulated Array Response, 8 Point Circle/
9 Point Non-symmetrical Shape

- a) Array b) Proavg - circle
c) Array d) Proavg - non-symmetrical shape

SGARRAY: RESP

8 SOURCES ARE LOCATED AT:
 4, 3 4, 7 3, 3 5, 4 4, 7 4, 7 4, 5 3, 3 4, 7 3, 3 3, 3

a

SGARRAY: RESP

9 SOURCES ARE LOCATED AT:
 2, 1.5 2, 35 1, 45 2, 45 1, 85 2, 45 2, 15 2, 35 2, 35
 2, 2.5 1, 65 2, 35 1, 5 2 1, 45 1, 45

c

SGARRAY: PRODAVG

8 SOURCES ARE LOCATED AT:
 4, 3 4, 7 3, 3 5, 4 4, 7 4, 7 4, 5 3, 3 4, 7 3, 3 3, 3

b

SGARRAY: PRODAVG

9 SOURCES ARE LOCATED AT:
 2, 1.5 2, 35 1, 45 2, 45 1, 85 2, 45 2, 15 2, 35 2, 35
 2, 2.5 1, 65 2, 35 1, 5 2 1, 45 1, 45

d

Figure 24

Simulated Array Response, Line Source

a) Array b) Prodavg c) ProdR

SOARRAY: RESP

2.2 4 SOURCES ARE LOCATED AT:
3.3 4.4 5.5 6.6 7.7

SOARRAY: PRODAVG

2.2 6 SOURCES ARE LOCATED AT:
3.3 4.4 5.5 6.6 7.7

SOARRAY: PRODR

2.2 4 SOURCES ARE LOCATED AT:
3.3 4.4 5.5 6.6 7.7

Figure 25

Simulated Array Response, Tracheo-Bronchial Deposit

a) Array b) Prodavg

SOARRAY: RESP

13 SOURCES ARE LOCATED AT:
 4, 8 4, 7 4, 6 4, 5 4, 4 4.5, 3.5 5.5, 2.5
 6.5, 1.5 7, 1 7.5, .5 3, 3 2, 2 1, 1

0 1 2 3 4 5 6 7 8

SOARRAY: PRODAVG

13 SOURCES ARE LOCATED AT:
 4, 8 4, 7 4, 6 4, 5 4, 4 4.5, 3.5 5.5, 2.5
 6.5, 1.5 7, 1 7.5, .5 3, 3 2, 2 1, 1

0 1 2 3 4 5 6 7 8

Figure 26

Simulated Array Response, Large Circular Source

a) Array b) Proavg

SGARRAY1 RESP

20 SOURCES ARE LOCATED AT:

4, .5 5, .75 6, 1.25 6.75, 2 7.25, 3 7.5, 4
7.25, 5 6.75, 6 6, 6.75 5, 7.25 4, 7.5 3, 7.25 2
6.75 1.25, 6 .75, 5 .5, 4 .75, 3 1.25, 2 2, 1.25
3, .75

0 1 2 3 4 5 6 7 8

SGARRAY1 PRODAVG

20 SOURCES ARE LOCATED AT:

4, .5 5, .75 6, 1.25 6.75, 2 7.25, 3 7.5, 4
7.25, 5 6.75, 6 6, 6.75 5, 7.25 4, 7.5 3, 7.25 2
6.75 1.25, 6 .75, 5 .5, 4 .75, 3 1.25, 2 2, 1.25
3, .75

0 1 2 3 4 5 6 7 8

Figure 27
Simulated Array Response, Point Source
a) Array b) Prod5

SQARRAY: RESP

1 SOURCES ARE LOCATED AT:

4 , 4

SQARRAY, PROD5

8

7

6

5

4

3

2

1

0

0

1

2

3

4

5

6

7

8

8

7

6

5

4

3

2

1

0

0

1

2

3

4

5

6

7

8

#####

Figure 28

Simulated Array Response, Line Source, CRT Output

Upper = Array Lower = Proavg

Figure 29

Simulated Array Response, Tracheo-Bronchial, CRT Output

Upper = Array Lower = Proavg

Figure 30

Simulated Array Response, Large Circular Source, CRT Output

Upper = Array Lower = Proavg

Figure 31
Single Detector Response and Three
Detector, Multiple-Product-Response
vs. Point Source Motion

Figure 32
3 Dimensional, Tilted Plot of
Single Detector Response

X, Y, Z, tilted plot of
single detector response
to point source motion -
 $R=0.25$ in, $H=4$ in, $D=0.63$ in.

Figure 33
3 Dimensional, Tilted Plot of
Three Detector, Multiple Product Response

X, Y, Z tilted plot of
horizontal, triple-product
response to source motion -
 $R=0.25\text{ in.}$, $H=4\text{ in.}$, $D=0.63\text{ in.}$

Figure 34

3 Dimensional, Tilted Plot of
Diagonal, Multiple Product Response

x, y, z tilted plot of
right-diagonal multi-product
response to source motion -
 $R=0.25\text{ in.}$, $H=4\text{ in.}$, $D=0.63\text{ in.}$

Figure 35
3 Dimensional, Tilted Plot of
Single Detector Response
to 7 Source on 6 in. Centers

X, Y, Z tilted plot of
single detector scan
response of 7 sources -
 $R=0.25$, $H=4$ in., $D=0.63$ in.

Figure 36
3 Dimensional, Tilted Plot of
Single Detector Response
to 7 Sources on 4 in. Centers

x, y, z tilted plot of
single detector scan
response of 7 sources -
 $R = 0.25 \text{ in.}$, $H = 4 \text{ in.}$, $D = 0.63 \text{ in.}$

Figure 37

3 Dimensional, Tilted Plot of
Single Detector Response
to 7 Sources on 2 in. Centers

x, y, z tilted plot of
single detector scan
response of 7 sources -

$R = 0.25 \text{ in.}$, $H = 4 \text{ in.}$, $D = 0.63 \text{ in.}$

Figure 38

Simulated Array Response, 7 Sources on 2 in. Centers

a) Array b) Prodavg

SQARRAY: RESP

7 SOURCES ARE LOCATED AT:

4 , 4
2.268

2 , 4

6 , 4

5 , 5.732

3 , 5.732

5 , 2.268

3 ,

Total Array Response

Produg. Response

APPENDICES

APPENDIX III

Program to Drive Digital Plotter
for Acquisition of Point Source
Response of Collimated NaI Detector

```

) PRINT "CRYSTAL LOCATION: X, Y=";
.5 INPUT X,Y
16 PRINT
20 PRINT "COLLIMATOR HEIGHT & DEPTH: H,Z=";
25 INPUT H,Z
30 PRINT
40 PRINT "FIELD SIZE (X1 TO X1+P, Y1 TO Y1+Q), X1,Y1,P,Q=";
50 INPUT X1,Y1,P,Q
60 PRINT
70 PRINT "SOURCE INCREMENT (IN.) D=";
80 INPUT D
90 PRINT
100 PRINT "LIVE TIME T=";
110 INPUT T
120 PRINT
130 PRINT "DO YOU WANT A MARK AFTER EVERY SOURCE INCREMENT"
131 PRINT "(IF YOU WANT,TYPE 1, OTHERWISE TYPE 0)";
132 INPUT N
133 PRINT
135 IF N=1 GOTO 150
140 GOSUB 600
150 CALL 1,106,T
160 CALL 10,30,0
170 LET V= INT (D/.005+.001)
180 LET P= INT (P/D/5+.001)
185 LET Q= INT (Q/D+.001)
190 LET D= INT (P/8)
200 LET W= INT (P-8*D+.001)
201 LET S= LCN (645)- LCN (644)+ LCN (643)- LCN (642)+2
202 LET R= LCN (641)- LCN (640)+1
205 DIM G(7,33)
210 GOTO 800
230 FOR J=1 TO D
240   FOR I=1 TO 8
250     GOSUB 400
255     GOSUB 470
260   NEXT I
265   IF N=1 GOTO 271
270   GOSUB 600
271   PRINT 1000+K*100+J;"DATA";
273   FOR I=1 TO 7
275     PRINT M(I);", ";
277   NEXT I
278   PRINT M(8)
280 NEXT J
282 LET J=D+1
285 RETURN
290 FOR I=1 TO W
300   GOSUB 400
305   GOSUB 470
310 NEXT I
11 LET I=W+1
12 GOSUB 400

```

Source Motion Program

```

313 PRINT 1000+K*100+D+1;"DATA";
314 FOR I=1 TO W
316   PRINT M[I];", ";
318 NEXT I
319 PRINT M[W+1]
320 RETURN
321 FOR C=1 TO 20
330   LET A= LOG 3: LOG 5:6
340 NEXT C
350 CALL 9,2, 0,8,119
370 CALL 10,1,121
375 CALL 10,P*V,118
380 CALL 10,V,116
384 IF N=1 GOTO 390
385 CALL 10,5,122
390 LET A= LOG 3: LOG 5:6
392 CALL 9,2, 0,8, 0
394 RETURN
395 CALL 10,30, 0
398 END
400 CALL 4,4
410 CALL 5
420 CALL 9, 0,2,49, 0
430 CALL 1,96,15& LCN (97)
440 CALL 5
450 IF 1& DIB (49)=1 GOTO 440
460 CALL 5
462 LET M[I]= INT ( R0I ( 0 )-(( R0I (1)+ R0I (2))/S)*R)
463 LET G[K,8*(J-1)+I-1]=M[I]
464 RETURN
470 CALL 9,2, 0,8,119
472 IF N= 0 GOTO 480
474 CALL 10,5,122
480 CALL 10,1,121
490 CALL 10,V,114
494 CALL 10,10,127
495 PRINT
500 CALL 10,1, 0
505 LET A= LOG 3: LOG 5:8
510 CALL 9,2, 0,8, 0
530 RETURN
600 FOR C=1 TO 10
610   LET A= LOG 3: LOG 4:5
620 NEXT C
630 CALL 9,2, 0,8,119
640 CALL 10,5,122
645 CALL 10,1,121
647 PRINT
650 LET A= LOG 3: LOG 4:5
660 CALL 9,2, 0,8, 0
670 RETURN

```

```

700 FOR C=1 TO 20
701   LET A= LOG 3+ LOG 5+7
702 NEXT C
705 CALL 9,2, 0,8,119
710 CALL 10,V,112
720 PRINT
730 LET A= LOG 3+ LOG 5+6
740 CALL 9,2, 0,8, 0
750 RETURN
760 FOR C=1 TO 20
765   LET A= LOG 3+5
770 NEXT C
775 CALL 9,2, 0,8,119
780 CALL 10,5,122
785 CALL 10,1,121
787 PRINT
788 CALL 10,1, 0
790 LET A= LOG 3+ LOG 5+7
792 CALL 9,2, 0,8, 0
795 RETURN
800 FOR K= 0 TO 6
810   GOSUB 230
820   IF K=1 GOTO 1000
830   IF K=2 GOTO 1020
840   IF K=3 GOTO 1050
850   IF K=4 GOTO 1070
860   IF K=6 GOTO 1100
870 NEXT K
1000 GOSUB 700
1010 GOTO 870
1020 GOSUB 290
1025 GOSUB 760
1030 GOSUB 321
1040 GOTO 870
1050 GOSUB 321
1060 GOTO 870
1070 GOSUB 290
1075 GOSUB 760
1080 GOSUB 700
1090 GOTO 870
1100 GOSUB 290
1105 GOSUB 760
1110 GOTO 395
1200 CALL 10,30, 0

```

```

1205  FOR K= 0 TO 6
1210      FOR J= 0 TO D-1
1220          PRINT 1000+K*100+J+1;"DATA";
1230          FOR I= 0 TO 6
1240              PRINT G[K,8*J+I];",,";
1250          NEXT I
1260          PRINT G[K,8*J+7]
1270      NEXT J
1272      IF K= 0 GOTO 1330
1273      IF K=1 GOTO 1330
1274      IF K=3 GOTO 1330
1275      IF K=5 GOTO 1330
1280      PRINT 1000+K*100+D+1;"DATA";
1290      FOR I=1 TO W
1300          PRINT G[K,8*D+I-1];",,";
1310      NEXT I
1320      PRINT G[K,8*D+W]
1330  NEXT K
1340  CALL 10,30, 0
1350  END

```

*

APPENDIX IV

Analytical Methodology and Computer
Program to Generate Theoretical
Response of Collimated NaI Detector
to Motion of a Point Source*

*This appendix is a preprint of a paper to be submitted for publication in the journal "Health Physics."

THE RESPONSE OF A COLLIMATED CYLINDRICAL NaI DETECTOR TO A POINT GAMMA-RAY SOURCE

Zunsu Hu*

North China Institute of Radiation Protection
Taiyuan, Shanxi Province, China

Gerard R. Laurer

Institute of Environmental Medicine
New York University Medical Center
New York, N.Y.

The response of a collimated cylindrical 0.5 in. diameter NaI detector to a point gamma-ray source was obtained theoretically and experimentally. The calculated data fit well to the experiments. Though the simplified method is accurate enough for calculating the response to a low-energy source, a more accurate method is also given to correct the penetrance of the collimator by gamma-rays with higher energy.

INTRODUCTION

This work is part of a program dedicated to the development of an active collimator. For the sake of locating gamma-emitting nuclides in the human body while measuring their whole-body burden, a matrix of detectors is used with a single cylindrical collimator attached to each detector. In calculating the matrix response to a theoretical source shape and boundary, produced by summing the output of n point sources, the response of each detector to a single point source on a given plane perpendicular to the cylinder is needed. Because of the symmetry of a cylinder, only the response of those points which are located on the plane along a straight line passing through the center of the projection of the cylinder is necessary. The purpose of the present work is to determine the position response of a

* Currently visiting scientist, New York University Medical Center, Institute of Environmental Medicine.

0.5 x 0.8 in. NaI(Tl) scintillation counter, to which a 0.5 in. diameter cylindrical collimator is attached. The depth of the collimator and the distance from the collimator to the source are adjustable. Data have been acquired both experimentally and theoretically. Though the calculation was performed for a special series of collimators, it is generally applicable to any kind of cylindrical geometry.

DATA ACQUISITION SYSTEM

The schematic diagram of the system is given in Fig. 1. A Tracor Northern Model TN-1700 computerized pulse height analyzer is used for data acquisition and for process control. The built-in NOVA-2 computer with 32 K memory is sufficient for this purpose. Both the detector and the gamma-ray source are accurately movable. A used gamma-ray scanner is refitted to hold the detector-collimator assembly, of which the position can be changed manually along three dimensions. A Hiplot* digital plotter, which is controlled by the computer with a minimum increment of 0.005 in., is put under the collimator to move the source and map its trace on a grid paper simultaneously. For various heights and depths of the collimator, position response was measured with the source moved step by step along a straight line. One hundred and sixty steps were set with a 0.05 in. increment for each step. The total travel of the source is thus 8 in.

Appendix I is the acquisition program written in the Interactive Basic Language of the NOVA-2.

COLLIMATOR GEOMETRY

The response of a collimator is actually a problem of geometry, and may be obtained by calculating the solid angle subtended to a given point in space, as shown in Fig. 2.

* Manufacturer: Houston Instrument, Austin, Texas.

Let P_1 , P_2 , and P_3 be planes parallel to each other, with the source S on P_1 and the two ends of the collimator on P_2 and P_3 , respectively. The upper and lower bounds of the collimator are circles O'' and O' , respectively. Their orthogonal projection on P_1 is circle O . For any small area $dxdy$ at point Q on P_3 , the solid angle to S is:

$$d\Omega = 1/\rho^2 \cos\theta dxdy \quad (1)$$

where ρ is the distance between S and Q ; θ is the angle between the plane perpendicular to line SQ and plane P_3 .

If point O is the origin and point S is on the X -axis, expression (1) can be written as:

$$d\Omega = \frac{w}{((x-l)^2 + y^2 + w^2)^{3/2}} dxdy \quad (1')$$

where l is the distance between O and S , and w the distance between P_1 and P_3 .

Thus, the solid angle of the upper bound of the collimator to point S is:

$$\begin{aligned} \Omega &= \iint_D \frac{w}{((x-l)^2 + y^2 + w^2)^{3/2}} dxdy \\ &= \int_{y_1}^{y_2} \left[\frac{x_2 - l}{\sqrt{(x_2 - l)^2 + y^2 + w^2}} - \frac{x_1 - l}{\sqrt{(x_1 - l)^2 + y^2 + w^2}} \right] / (y^2 + w^2) dy \end{aligned} \quad (2)$$

where D is the overlap of circle O'' and the image of O' projected from point S to plane P_3 , which may be shown to be a circle with center O''' and radius

$$r' = r(1 + d/h) \quad (3)$$

where r is the inner radius of the collimator, h is the distance between P_1 and P_2 , and d between P_2 and P_3 .

Four geometrical transitions will occur as point S moves outward from the origin. These are illustrated in Fig. 3, where the integral areas

are shaded and their boundaries are as follows:

1. when $0 \leq \ell < r$ (Fig. 3a)

$$x_1 = -\sqrt{r^2 - y^2}$$

$$x_2 = \sqrt{r^2 - y^2}$$

$$y_1 = -r$$

$$y_2 = r$$

2. when $r \leq \ell < r \sqrt{1+2h/d}$ (Fig. 3b)

$$x_1 = -\sqrt{r^2 - y^2}$$

$$x_2 = \begin{cases} \sqrt{r^2 - y^2} & \text{when } |y| \geq \sqrt{r^2 - \left(\frac{r^2 d + 2r^2 h - \ell^2 d}{2\ell h} \right)^2} \\ \sqrt{\left(\frac{rw}{h} \right)^2 - y^2} - \frac{\ell d}{h} & \text{when } |y| < \sqrt{r^2 - \left(\frac{r^2 d + 2r^2 h - \ell^2 d}{2\ell h} \right)^2} \end{cases}$$

$$y_1 = -r$$

$$y_2 = r$$

3. when $r \sqrt{1+2h/d} \leq \ell < r(1+2h/d)$

(Fig. 3c)

$$x_1 = -\sqrt{r^2 - y^2}$$

$$x_2 = \sqrt{\left(\frac{rw}{h} \right)^2 - y^2} - \frac{\ell d}{h}$$

$$y_1 = -\sqrt{r^2 - \left(\frac{r^2 d + 2r^2 h - \ell^2 d}{2\ell h} \right)^2}$$

$$y_2 = \sqrt{r^2 - \left(\frac{r^2 d + 2r^2 h - \ell^2 d}{2\ell h} \right)^2}$$

4. when $r(1+2h/d) \leq \ell$

(Fig. 3d)

is the null case.

Expression (2) has been numerically integrated and the program used appears in Appendix II.

COMPARISON BETWEEN EXPERIMENT AND CALCULATION

For simplification, only the relative geometrical dependence, i.e., the shape of the response curve, is compared while the absolute efficiency of the detector and the radioactivity of the source are represented by a constant, k .

Suppose the following expression is true:

$$C = kf(\ell) + b \quad (4)$$

where ℓ is the distance from the source to the axis of the collimator; c , the count acquired in a given time interval; $\Omega = f(\ell)$, the calculated solid angle; and b , the background. The total variance between experimental and calculated data S then will be:

$$S = \sum_{i=1}^n (C_i - kf(\ell_i) - b)^2 \quad (5)$$

Following the principle of least squares, we obtain

$$k = \frac{\sum_{i=1}^n C_i \sum_{i=1}^n f(\ell_i) - n \sum_{i=1}^n (C_i f(\ell_i))}{\left(\sum_{i=1}^n f(\ell_i) \right)^2 - n \sum_{i=1}^n f^2(\ell_i)} \quad (6)$$

$$b = \frac{\sum_{i=1}^n f(\ell_i) \sum_{i=1}^n (C_i f(\ell_i)) - \sum_{i=1}^n C_i \sum_{i=1}^n f^2(\ell_i)}{\left(\sum_{i=1}^n f(\ell_i) \right)^2 - n \sum_{i=1}^n f^2(\ell_i)} \quad (7)$$

The original position of the source may not be exactly on the axis of the collimator, causing an additional offset error. To reduce this error, a center-shift program was written, which located the real symmetrical axis of the response curve and then shifted the entire curve left or right to fit the calculation.

After the processing mentioned above, a good correspondence has been achieved for a collimator of 0.5 in. diameter under various conditions, shown in Fig. 4 through Fig. 6. The solid curve is calculated; the dotted, experimental. For the purpose of drawing a smooth curve, Lagrangian interpolation using 0.005 in. steps was performed. Eighty-one points in 0.05 in. steps were calculated by numerical integration to obtain the leading half of each curve. The remaining 80 points constituting the trailing half were obtained by symmetry.

It should be noted that the parameters of the calculated curves are the same as those of the corresponding experimental ones, except the depth d of the collimator, which was made 0.07 in. deeper for the calculations. An X-ray examination of the detector showed the NaI(Tl) crystal to be 0.07 in. behind the window surface of the detector (Fig. 7). Accounting for this distance filled the gap between calculation and experiment, as illustrated by comparison of Fig. 4 with Fig. 8. The former is calculated with 0.07 in. added to the depth, the latter, without.

It is noted also that the tops of the experimental curves are rather rough and sloping and do not fit the calculated curves properly. This is because the crystal and the collimator, which have the same diameter, are not exactly aligned, axially, with each other. Close examination of the

X-rays of Fig. 7 show the crystal to be slightly off center in one, and tilted off axis in the other which was obtained after revolving the detector under the X-ray tube. The edge effects caused by such a misalignment with the collimator axis are responsible for the sloping tops of the curve. The experimental points of Fig. 6 were obtained after rotating the detector-collimator assembly approximately with respect to the source axis, thus lessening the effect of the misalignment. This phenomenon occurs only when the incident angle is small, and thus does not influence the sloped parts of the curve seriously.

CORRECTION FOR PENETRATION

Correction has been made for the penetrability of both the collimator and the detector.

In the calculation discussed previously, we supposed that any part of the collimator, no matter how thin, can stop all photons which travel across it, as shown by ray a in Fig. 9. Actually, that is not true, because for photoelectric interactions there is a probability of $e^{-\mu t}$ that the photons will pass through the collimator, where μ is the linear absorption coefficient and t is the thickness of the collimator they pass through. This will make the effective diameter of the collimator larger than its real size, necessitating a correction. The higher the energy of the incident photons, the larger the collimator appears.

As for the detector, correction depends on its diameter. When the diameter is equal to that of the collimator, some photons, as shown by ray b in Fig. 9, may exit the edge of the crystal before any interaction occurs. In this case, the photons will not be detected even though they have entered the front surface of the crystal. Thus, the collimator will appear

longer for such photons, and a second correction is needed. When the diameter of the crystal is large compared to the diameter of collimator such that all photons passing through the collimator have a constant probability of being detected, regardless of the interaction depth, no such correction is necessary. The correction made here is for detectors of which the diameter is equal to that of the collimator and the thickness is so large that few photons can pass through perpendicularly. The depth of the collimator is also assumed to be large enough that few photons penetrate through to the edge of the crystal.

Suppose there are three parallel planes P_1 , P_2 , and P_3 (Fig. 10), and a cylinder OO'' is perpendicular to the planes as in Fig. 2. The section of the cylinder between P_2 and P_3 stands for the collimator and the section above P_3 stands for the detector. C is a point on the surface of the detector and source S is moving on P_1 along the x axis. Joining C and S with a straight line, it will cross the wall of the cylinder at D and B , and plane P_2 at A . If B is above P_2 , the photons that go along SC are attenuated by the collimator. Line segment $\overline{AB}$ is the absorption thickness of the collimator and $\overline{DC}$ is the distance the photon must travel in the detector unless interaction occurs.

Let the coordinates of point C be (x, y, z) . Because $z=w$, we have:

$$u_1 = \overline{CD} = \left(\frac{\frac{r^2}{l(l-x) + \sqrt{r^2(l-x)^2 + l^2y^2 - l^2y^2}}}{(l-x)^2 + y^2} - 1 \right) \sqrt{(l-x)^2 + y^2 + w^2}$$

$$u_2 = \overline{AB} = \left(\frac{\frac{l(l-x) - \sqrt{r^2(l-x)^2 + r^2y^2 - l^2y^2}}{(l-x)^2 + y^2} - \frac{h}{w}}{1} \right) \sqrt{(l-x)^2 + y^2 + w^2}$$

Expression (1) and (2) can be corrected as follows:

$$d\Omega = \frac{w}{((x-l)^2 + y^2 + w^2)^{3/2}} \left(1 - e^{-\mu_d u_1}\right) e^{-\mu_c u_2} dx dy \quad (8)$$

and

$$\begin{aligned} \Omega = & \iint_{D_1} \frac{w}{((x-l)^2 + y^2 + w^2)^{3/2}} \left(1 - e^{-\mu_d u_1}\right) dx dy + \\ & + \iint_{D_2} \frac{w}{((x-l)^2 + y^2 + w^2)^{3/2}} \left(1 - e^{-\mu_d u_1}\right) e^{-\mu_c u_2} dx dy \end{aligned} \quad (9)$$

where μ_d is the linear absorption coefficient of the detector, and μ_c of the collimator.

The integration (9) is performed on the whole front surface of the detector, which is divided into two parts (Fig. 11)--one is the exposed area D_1 which is directly exposed by gamma-rays from the source, and the other is the shaded area D_2 , shielded by the collimator. We have found that D_1 is exactly the same as D listed above in Eq. (2) and D_2 is as follows:

1. When $0 < l < r$, D_2 is null. All points on the front surface of the detector belong to D_1 (Fig. 11a).

2. When $r \leq l < r\sqrt{1+2h/d}$

(Fig. 11b)

$$x_1 = \sqrt{\left(\frac{rw}{h}\right)^2 - y^2} - \frac{\ell d}{h}$$

$$x_2 = \sqrt{x^2 - y^2}$$

$$y_1 = -\sqrt{r^2 - \zeta^2}$$

$$y_2 = \sqrt{r^2 - \zeta^2}$$

where

$$\zeta = \frac{r^2 d + 2r^2 h - \ell^2 d}{2\ell h}$$

3. When $r\sqrt{1 + \frac{2h}{d}} \leq \ell < r\left(1 + 2\frac{h}{d}\right)$ (Fig. 11c)

$$x_1 = \begin{cases} -\sqrt{r^2 - y^2} & \text{while } |y| \geq \sqrt{r^2 - \zeta^2} \\ \sqrt{\left(\frac{rw}{h}\right)^2 - y^2} - \frac{\ell d}{h} & \text{while } |y| < \sqrt{r^2 - \zeta^2} \end{cases}$$

$$x_2 = \sqrt{r^2 - y^2}$$

$$y_1 = -r$$

$$y_2 = r$$

4. When $\ell \geq r\left(1 + 2\frac{h}{d}\right)$ (Fig. 11d)

$$x_1 = -\sqrt{r^2 - y^2}$$

$$x_2 = \sqrt{r^2 - y^2}$$

$$y_1 = -r$$

$$y_2 = r$$

Expression (9) is calculated by means of numerical integration.

The results have been compared with expression (2), calculated without correction. Since the experiments were done with a low energy (60 keV) source, ^{241}Am , for which the half-value layer of lead is only about 0.005 in. [2], only a slight difference is observed (Fig. 12), but as shown by the square dots, with increasing energy, i.e., decreasing μ , the correction will become more and more significant (see Appendix III).

REFERENCES

- [1] Laurer, G.R. and M. Eisenbud. A multicrystal detection system for localization and measurement of photon emitters without external collimation. Nucl. Instrum. Methods 140: 481, 1977.
- [2] Radiological Health Handbook, p. 137. Public Health Service Publication No. 2016, 1970.

FIGURE 1
Data Acquisition System

Data Acquisition System

FIGURE 2

**The Geometry of a Cylindrical
Collimator to a Point Source**

The Geometry of a Cylindrical Collimator to a Point Source.

FIGURE 3

**Boundaries of Variables when Source S
Goes Outwards from the Origin**

Boundaries of Variables when S goes Outwards from the Origin

FIGURE 4

Point Source Response

H = 4 in.

D = 0.63 in.

..... = experimental

—— = calculated

FIGURE 5

Point Source Response

H = 4 in.

D = 1.07 in.

..... = experimental

—— = calculated

FIGURE 6

Point Source Response

H = 4 in.

D = 1.32 in.

.... = experimental

— = calculated

FIGURE 7

Radiograph of NaI(Tl) Detector

FIGURE 8

Point Source Response Showing

H = 4 in.

D = 0.56 in.

..... = experimental

—— = calculated

FIGURE 9
Possible Photon Paths Through
Crystal and Collimator

Possible Photon Paths through Crystal and Collimator.

- a) interaction in the collimator.
- b) no interaction in the crystal.

FIGURE 10

The Absorber Thickness Along the Path
from Source S to a Given Point C
on the Detector

The Absorber Thickness along the Path from Source S to a given Point C on the Detector.

FIGURE 11

Boundaries of Shaded Area D_2 When
Source Travels Outward from the Origin

Shaded Area D_2 when Source Travels Outward

FIGURE 12

**Comparison of Experimental 60 keV, Calculated
60 keV and Calculated 200 keV Point Source Responses**

APPENDIX I
Data Acquisition Program

APPENDIX I/IV

THE ACQUISITION PROGRAM WRITTEN IN THE INTERACTIVE BASIC LANGUAGE
OF THE NOVA-2

```

10 PRINT "CRYSTAL LOCATION: X, Y=";
15 INPUT X,Y
16 PRINT
20 PRINT "COLLIMATOR HEIGHT & DEPTH: H,Z=";
25 INPUT H,Z
30 PRINT
40 PRINT "FIELD SIZE (X1 TO X1+P, Y1 TO Y1+Q), X1,Y1,P,Q=";
50 INPUT X1,Y1,P,Q
60 PRINT
70 PRINT "SOURCE INCREMENT (IN.) D=";
80 INPUT D
90 PRINT
100 PRINT "LIVE TIME T=";
110 INPUT T
120 PRINT
130 PRINT "DO YOU WANT A MARK AFTER EVERY SOURCE INCREMENT"
131 PRINT "(IF YOU WANT, TYPE 1, OTHERWISE TYPE 0)";
132 INPUT N
133 PRINT
135 IF N=1 GOTO 150
140 GOSUB 600
150 CALL 1,106,T
160 CALL 10,30,0
170 LET V= INT (D/.005+.001)
180 LET P= INT (P/D/5+.001)
185 LET Q= INT (Q/D+.001)
190 LET D= INT (P/8)
200 LET W= INT (P-8*D+.001)
201 LET S= LCN (645)- LCN (644)+ LCN (643)- LCN (642)+2
202 LET R= LCN (641)- LCN (640)+1
205 DIM G(7,33)
210 GOTO 800
230 FOR J=1 TO D
240   FOR I=1 TO 8
250     GOSUB 400
255     GOSUB 470
260   NEXT I
265   IF N=1 GOTO 271
270   GOSUB 600
271   PRINT 1000+K*100+J;"DATA";
273   FOR I=1 TO 7
275     PRINT M(I);", ";
277   NEXT I
278   PRINT M(8)
80 NEXT J
82 LET J=D+1
285 RETURN

```

```

290 FOR I=1 TO W
300   GOSUB 400
305   GOSUB 470
310 NEXT I
311 LET I=W+1
312 GOSUB 400
313 PRINT 1000+K*100+D+1;"DATA";
314 FOR I=1 TO W
315   PRINT M[I];", ";
318 NEXT I
319 PRINT M[W+1]
320 RETURN
321 FOR C=1 TO 20
330   LET A= LOG 3: LOG 5:6
340 NEXT C
350 CALL 9,2, 0,8,119
370 CALL 10,1,121
375 CALL 10,P*V,118
380 CALL 10,V,116
384 IF N=1 GOTO 390
385 CALL 10,5,122
390 LET A= LOG 3: LOG 5:6
392 CALL 9,2, 0,8, 0
394 RETURN
395 CALL 10,30, 0
398 END
400 CALL 4,4
410 CALL 5
420 CALL 9, 0,2,49, 0
430 CALL 1,96,15& LCN (97)
440 CALL 5
450 IF 1& DIB (49)=1 GOTO 440
460 CALL 5
462 LET M[I]= INT ( R0I ( 0)-((( R0I (1)+ R0I (2))/S)*R)
463 LET G[K,8*(J-1)+I-1]=M[I]
464 RETURN
470 CALL 9,2, 0,8,119
472 IF N= 0 GOTO 480
474 CALL 10,5,122
480 CALL 10,1,121
490 CALL 10,V,114
494 CALL 10,10,127
495 PRINT
500 CALL 10,1, 0
505 LET A= LOG 3: LOG 5:8
510 CALL 9,2, 0,8, 0
530 RETURN
600 FOR C=1 TO 10
610   LET A= LOG 3: LOG 4:5
620 NEXT C
630 CALL 9,2, 0,8,119
640 CALL 10,5,122
645 CALL 10,1,121
647 PRINT
650 LET A= LOG 3: LOG 4:5
660 CALL 9,2, 0,8, 0
670 RETURN
690 LET
700 FOR C=1 TO 20
701   LET A= LOG 3: LOG 5:7
702 NEXT C

```

```

705 CALL 9,2, 0,8,119
710 CALL 10,V,112
720 PRINT
730 LET A= LOG 3: LOG 5:6
740 CALL 9,2, 0,8, 0
750 RETURN
760 FOR C=1 TO 20
765   LET A= LOG 3:5
770 NEXT C
775 CALL 9,2, 0,8,119
780 CALL 10,5,122
785 CALL 10,1,121
787 PRINT
788 CALL 10,1, 0
790 LET A= LOG 3: LOG 5:7
792 CALL 9,2, 0,8, 0
795 RETURN
800 FOR K= 0 TO 6
810   GOSUB 230
820   IF K=1 GOTO 1000
830   IF K=2 GOTO 1020
840   IF K=3 GOTO 1050
850   IF K=4 GOTO 1070
860   IF K=6 GOTO 1100
870 NEXT K
1000 GOSUB 700
1010 GOTO 870
1020 GOSUB 290
1025 GOSUB 760
1030 GOSUB 321
1040 GOTO 870
1050 GOSUB 321
1060 GOTO 870
1070 GOSUB 290
1075 GOSUB 760
1080 GOSUB 700
1090 GOTO 870
1100 GOSUB 290
1105 GOSUB 760
1110 GOTO 395
1200 CALL 10,30, 0
1205 FOR K= 0 TO 6
1210   FOR J= 0 TO D-1
1220     PRINT 1000+K*100+J+1;"DATA";
1230     FOR I= 0 TO 6
1240       PRINT G[K,8*J+I];", ";
1250     NEXT I
1260     PRINT G[K,8*J+7]
1270   NEXT J
1272   IF K= 0 GOTO 1330
1273   IF K=1 GOTO 1330
1274   IF K=3 GOTO 1330
1275   IF K=5 GOTO 1330
1280   PRINT 1000+K*100+D+1;"DATA";
1290   FOR I=1 TO W
1300     PRINT G[K,8*D+I-1];", ";
1310   NEXT I
1320   PRINT G[K,8*D+W]
1330 NEXT K
1340 CALL 10,30, 0
1350 END

```

APPENDIX II

Program Used for Numerical
Integration of Expression (2)

APPENDIX II/IV

THE PROGRAM USED FOR NUMERICALLY INTEGRATING EXPRESSION (2)

```

*
1 PRINT "R,D,H=";
2 INPUT R,D,H
3 PRINT
4 LET E=.01
5 LET Q=.01
6 DIM M[80]
7 DIM P[160]
8 DIM U[30]
9 LET U[0]=70
15 LET W=H+D
20 FOR J= 0 TO 80
25 LET L=J/20
30 IF L>R*(1+2*H/D) GOTO 500
35 IF L>R* SQR (1+2*H/D) GOTO 550
40 IF L>R GOTO 600
45 LET A=-R
50 LET B=R
65 GOSUB 110
70 LET M[J]=T
75 NEXT J
76 GOTO 905
78 PRINT 1700;"REM DATA CALCULATED FOR R,D,H=";R;D;H
79 FOR J= 0 TO 15
80 PRINT 1701+J;"DATA";
81 FOR I= 0 TO 3
82 PRINT M[J*5+I];", ";
83 NEXT I
84 PRINT M[J*5+4]
85 NEXT J
87 PRINT 1717;"DATA";M[80]
99 LET N=1
100 GOTO 3070
110 REM SIMPSON'S NUMERICAL INTEGRATION
111 LET G=B-A
120 LET X=A
130 GOSUB 700
140 LET V=F
150 LET X=B
160 GOSUB 700
170 LET V=V+F
180 LET N=N+1
190 LET X=X+.5*G
200 LET S= 0
210 FOR I=1 TO N
220 LET X=X+G
230 GOSUB 700
240 LET S=S+F
260 NEXT I
270 LET T=(4*S+V)*G/6

```

```

80  IF N=1 GOTO 350
290 IF ABS (G)>Q GOTO 350
300 LET X=T-Q
310 IF ABS (T)<1E-10 GOTO 330
320 LET X=X/T
330 IF ABS (X)<E GOTO 400
350 LET Q=T
360 LET V=2*S+V
370 LET N=N+N
380 LET G=.5*G
390 GOTO 190
400 RETURN
500 LET T= 0
510 GOTO 70
550 LET A=R*R*D+2*R*R*H-L*L*D
555 LET A=- SQR (R*R-A*A/4/L/L/H/H)
560 LET B=-A
570 GOTO 65
600 LET K=R*R*D+2*R*R*H-L*L*D
605 LET K= SQR (R*R-K*K/4/L/L/H/H)
610 LET A=-R
620 LET B=R
630 GOTO 65
700 IF L>R* SQR (1+2*H/D) GOTO 800
710 IF L>R GOTO 850
715 LET Y=- SQR (R*R-X*X)
716 LET Z=-Y
720 LET C= 0
721 LET U[1]=Y-L
722 LET U[2]=X*X+W*W
723 LET U[3]=U[1]*U[1]+U[2]
725 GOTO 745
726 IF C=1 GOTO 732
727 LET U[5]=F
728 LET C=1
729 LET U[1]=Z-L
730 GOTO 722
732 LET F=F-U[5]
735 RETURN
740 LET F=U[1]/(U[2]* SQR (U[3]))*W
741 GOTO 726
745 LET F=U[1]/(U[2]* SQR (U[3]))*(1+W*U[ 0]-W*W*(1/U(3)+2/U(2)))/U(0)
746 GOTO 726
800 LET Y=- SQR (R*R-X*X)
810 LET Z= SQR (R*R*W*W/H/H-X*X)-L*D/H
820 GOTO 720
850 LET Y=- SQR (R*R-X*X)
860 IF ABS (X)>K GOTO 890
870 LET Z= SQR (R*R*W*W/H/H-X*X)-L*D/H
880 GOTO 720
890 LET Z=-Y
900 GOTO 720

```


APPENDIX III

Program Used for Numerical
Integration of Expression (9)

APPENDIX III/IV

THE PROGRAM USED FOR NUMERICALLY INTEGRATING EXPRESSION (9)

```

1 * PRINT "R,D,H=";
2 INPUT R,D,H
3 PRINT
4 LET E=.01
5 LET Q=.01
6 DIM M[80]
7 DIM P[160]
8 DIM U[30]
9 LET U[29]=60.96
10 LET U[30]=140.4
15 LET W=H+D
20 FOR T= 0 TO 16
25   LET L=T/4
26   LET C= 0
30   IF L>R*(1+2*H/D) GOTO 500
35   IF L>R* SQR (1+2*H/D) GOTO 550
40   IF L>R GOTO 600
45   LET A=-R
50   LET B=R
65   GOSUB 2500
66   LET M[T]=U[26]
67   PRINT M[T],
68   LET C=1
70   IF L>R*(1+2*H/D) GOTO 920
71   IF L>R* SQR (1+2*H/D) GOTO 930
72   IF L>R GOTO 940
73   GOTO 76
74   GOSUB 2500
75   LET M[T]=M[T]+U[26]
76   PRINT M[T]
77 NEXT T
78 CALL 10,30, 0
79 FOR J= 0 TO 4
80   PRINT 1701+J;"DATA";
81   FOR I= 0 TO 3
82     PRINT M[J*5+I];", ";
83   NEXT I
84   PRINT M[J*5+4]
85 NEXT J
87 PRINT "CAL. DATA FOR R,D,H,U1,U2=";R;D;H;U[29];U[30]
88 CALL 10,30, 0
99 LET N=1
100 END
110 REM      SIMPSON'S NUMERICAL INTEGRATION
111 LET G=B-A
120 LET X=A
130 GOSUB 700
140 LET V=F
150 LET X=B
160 GOSUB 700
170 LET V=V+F
180 LET N=N+1
190 LET X=X+.5*G
200 LET S= 0
210 FOR I=1 TO N

```

```

220 LET X=X+G
230 GOSUB 700
240 LET S=S+F
250 NEXT I
70 LET T=(4*S+V)*G/6
280 IF N=1 GOTO 350
290 IF ABS (G)>0 GOTO 350
300 LET X=T-0
310 IF ABS (T)<1E-10 GOTO 330
320 LET X=X/T
330 IF ABS (X)<E GOTO 400
350 LET 0=T
360 LET V=2*S+V
370 LET N=N+N
380 LET G=.5*G
390 GOTO 190
400 RETURN
500 LET U[26]= 0
510 GOTO 66
550 LET A=R*R*D+2*R*R*H-L*L*D
555 LET A=- SQR (R*R-A*A/4/L/L/H/H)
560 LET B=-A
570 GOTO 65
600 LET U[8]=(R*R*D+2*R*R*H-L*L*D)/2/L/H
605 LET U[8]= SQR (R*R-U[8]*U[8])
610 LET A=-R
620 LET B=R
630 GOTO 65
700 IF L>R* SQR (1+2*H/D) GOTO 800
710 IF L>R GOTO 850
715 LET U[13]=- SQR (R*R-X*X)
716 LET U[14]=-U[13]
720 GOTO 2820
800 LET U[13]=- SQR (R*R-X*X)
810 LET U[14]= SQR (R*R*W*W/H/H-X*X)-L*D/H
820 GOTO 2820
850 LET U[13]=- SQR (R*R-X*X)
860 IF ABS (X)>U[8] GOTO 890
870 LET U[14]= SQR (R*R*W*W/H/H-X*X)-L*D/H
880 GOTO 2820
890 LET U[14]=-U[13]
900 GOTO 2820
920 LET A=-R
922 LET B=R
924 GOTO 74
930 LET A=-R
931 LET U[8]=(R*R*D+2*R*R*H-L*L*D)/2/L/H
932 LET B=R
933 LET U[8]= SQR (R*R-U[8]*U[8])
934 GOTO 74
940 LET A=(R*R*D+2*R*R*H-L*L*D)/2/L/H
941 LET A=- SQR (R*R-A*A)
942 LET B=-A
945 GOTO 74
950 IF L>R*(1+2*H/D) GOTO 960
951 IF L>R* SQR (1+2*H/D) GOTO 970
952 LET U[13]= SQR (R*R*W*W/H/H-X*X)-L*D/H
4 LET U[14]= SQR (R*R-X*X)
9 RETURN

```

```

960 LET U[13] = - SQR (R*R-X*X)
962 LET U[14] = -U[13]
965 RETURN
970 IF ABS (X) < U[8] GOTO 975
973 LET U[13] = - SQR (R*R-X*X)
974 GOTO 976
975 LET U[13] = SQR (R*R*W*W/H/H-X*X) - L*D/H
976 LET U[14] = SQR (R*R-X*X)
979 RETURN
1001 DATA 0, 0, 9.232E-13, 1.249E-4, .001247, 3.2003E-3
2500 REM NUMERICAL INTEGRATION
2510 LET U[1] = 1
2520 LET U[11] = 1
2530 LET U[2] = (B-A)/U[1]
2540 LET U[12] = 1/U[11]
2550 LET N = 1
2560 GOTO 2610
2570 LET U[25] = U[23]*U[2]/6
2575 LET U[27] = U[24]
2580 LET N = 0
2585 LET U[1] = U[1] + U[1]
2590 LET U[11] = U[11] + U[11]
2595 LET U[2] = U[2]*.5
2600 LET U[12] = U[12]*.5
2610 LET U[23] = 0
2615 LET U[24] = 0
2620 LET Q = 0
2625 LET K = 1
2630 LET X = B
2640 FOR I = 1 TO 2
2645 GOTO 2805
2650 LET X = A
2655 NEXT I
2660 LET U[23] = .5*U[23]
2662 LET U[24] = .5*U[24]
2665 IF U[1] = 1 GOTO 2680
2667 LET K = 2
2670 FOR I = 2 TO U[1]
2672 GOTO 2800
2675 NEXT I
2680 LET U[23] = .5*U[23]
2682 LET Q = 1
2685 LET K = 3
2687 LET X = A - .5*U[2]
2690 FOR I = 1 TO U[1]
2692 GOTO 2800
2695 NEXT I
2700 LET U[23] = 4*U[23]
2705 LET U[24] = 2*U[24]
2710 IF N = 1 GOTO 2570
2720 LET U[26] = (U[27]*.5 + U[23])*U[2]/6
2725 LET X = U[26] - U[25]
2730 IF ABS (U[26]) < .000001 GOTO 2740
2735 LET X = X/U[26]
2740 IF ABS (X) - E >= 0 GOTO 2760
2750 RETURN
2760 LET U[25] = U[26]
2765 LET U[27] = U[27]*.5 + U[24]
2770 GOTO 2585
2800 LET X = X + U[2]

```

```

2805 IF C= 0 GOTO 700
2810 GOSUB 950
2820 LET U[15]=(U[14]-U[13])*U[12]
2825 LET U[21]= 0
2830 LET Y=U[13]-.5*U[15]
2835 FOR J=1 TO U[11]
2840 LET Y=Y+U[15]
2842 GOSUB 2885
2845 LET U[21]=F+U[21]
2850 NEXT J
2855 LET U[22]=U[21]
2860 IF N+Q= 0 GOTO 2880
2861 LET Y=U[13]
2862 GOSUB 2885
2863 LET U[22]=.5*F+U[22]
2864 LET Y=U[14]
2865 GOSUB 2885
2866 LET U[22]=.5*F+U[22]
2869 IF U[11]=1 GOTO 2880
2870 LET Y=U[13]
2875 FOR J=2 TO U[11]
2876 LET Y=Y+U[15]
2877 GOSUB 2885
2878 LET U[22]=F+U[22]
2879 NEXT J
2880 LET U[23]=U[23]+U[15]*(U[21]+U[22])/3
2881 LET U[24]=U[24]+U[15]*U[22]/3
2882 IF K=1 GOTO 2650
2883 IF K=2 GOTO 2675
2884 IF K=3 GOTO 2695
2885 LET U[3]=(L-Y)*(L-Y)+X*X
2886 IF U[3]>1E-15 GOTO 2888
2887 LET U[3]=1E-15
2888 LET U[4]= SQR (U[3]+W*W)
2889 LET U[5]= SQR (R*R*U[3]-L*L*X*X)
2890 LET U[6]= ABS (((L*(L-Y)+U[5])/U[3]-1)*U[4])
2891 LET F=W/U[4]+3*(1-2.71828+(-U[29]*U[6]))
2892 IF C= 0 GOTO 2899
2893 LET U[7]= ABS (((L*(L-Y)-U[5])/U[3]-H/W)*U[4])
2894 LET F=F*2.71828+(-U[30]*U[7])
2899 RETURN

```

*

APPENDIX V

Computer Program to Develop
Synthesized Response Using
a 9x9, 81-Crystal Array

```

1 REM    CALC RESP & PROD OF SQUARE 2-DIM CRYST ARRAY
2 REM    DX(34) IS THE X COOR      IN INCHES OF DET#34
3 REM    X(22) IS X COOR OF
NCHES
4 REM    PROGRAM NAME:SQARRAY
5 DIM C(250)
6 DIM R(120)
7 DIM X(30),Y(30),PROD(100)
8 DIM DX(100),DY(100)
10 OPEN #1:"DSK1,HALFINCH",INTERNAL
20 FOR I=80 TO 0 STEP -1
30 INPUT #1:C(I)
35 C(I)=C(I)-200
36 IF C(I)>=0 THEN 40
37 C(I)=0
40 NEXT I
50 CLOSE #1
90 MAXP=0
91 MAXR=0
100 PRINT "    ENTER LOCATIONS OF
105 PRINT "    ENTER 100 TO STOP "
110 PRINT "COORDINATES (IN INCHES)
120 FOR I=1 TO 30
130 PRINT I
140 INPUT "X? ":X(I)
142 IF X(I)=100 THEN 170
145 X(I)=X(I)/.05
150 INPUT "Y? ":Y(I)
155 Y(I)=(8-Y(I))/.05
160 NEXT I
170 PRINT " ";I-1;"SOURCES WERE ENTERED"
200 GOSUB 500
210 FOR B=0 TO 99
215 R(B)=0
220 N=DX(B)/.05
230 M=DY(B)/.05
240 FOR A=1 TO I-1
250 Z=SQR((N-X(A))*(N-X(A))+(M-Y(A))*(M-Y(A)))
260 R(B)=R(B)+C(Z)
270 NEXT A
271 IF R(B)<MAXR THEN 275
272 MAXR=R(B)
275 NEXT B
276 GOSUB 1000
279 PRINT "":
280 INPUT "RESP,PRODH,PRODR,PRODV,PRODL,OR PRODAVG ? ":F$
285 IF F$="RESP" THEN 287 ELSE 300
287 GOSUB 1110
288 INPUT "PRINT ? ":K$
289 IF K$="Y" THEN 290 ELSE 294
290 GOSUB 1500
294 INPUT "CONTINUE TO PROD ? ":M$
295 IF M$="Y" THEN 279 ELSE 90
300 FOR B=0 TO 88
310 IF B-10<0 THEN 315 ELSE 350
315 PRODH=0
    GOTO 355
    PRODH=R(B)*R(B)*R(B-10)*R(B-10)*R(B+10)*R(B+10)
355 IF B-11<0 THEN 356 ELSE 360
356 PRODR=0
357 GOTO 370
360 PRODR=R(B)*R(B)*R(B-11)*R(B-11)*R(B+11)*R(B+11)

```

DX(34)=APPROX 3
SOURCE #22 IN I

SOURCES AROUND COORD. 4,4

OF SOURCE #:

```

370 IF B-1<0 THEN 372 ELSE 375
372 PRODV=0
373 GOTO 380
375 PRODV=R(B)*R(B)*R(B-1)*R(B-1)*R(B+1)*R(B+1)
380 IF B-9<0 THEN 382 ELSE 385
382 PRODL=0
385 GOTO 410
PRODL=R(B)*R(B)*R(B-9)*R(B-9)*R(B+9)*R(B+9)
410 PRODAVG=(PRODH+PRODR+PRODV+PRODL)/4
412 IF F$='PRODH' THEN 414 ELSE 418
414 PROD(B)=PRODH
416 GOTO 440
418 IF F$='PRODR' THEN 420 ELSE 424
420 PROD(B)=PRODR
422 GOTO 440
424 IF F$='PRODV' THEN 426 ELSE 430
426 PROD(B)=PRODV
428 GOTO 440
430 IF F$='PRODL' THEN 432 ELSE 436
432 PROD(B)=PRODL
434 GOTO 440
436 PROD(B)=PRODAVG
440 IF PROD(B)<MAXP THEN 446
445 MAXP=PROD(B)
446 NEXT B
450 GOSUB 1110
451 INPUT 'PRINT ? ':K$
452 IF K$='Y' THEN 485 ELSE 486
485 GOSUB 1500
486 MAXP=0
487 INPUT 'CONTINUE WITH SAME SOURCES ? ':G$
488 IF G$='N' THEN 90 ELSE 279
500 FOR K=0 TO 9
510 FOR L=0 TO 9
520 DX(K*10+L)=K
530 DY(K*10+L)=L
540 NEXT L
550 NEXT K
560 RETURN
1000 OPEN #2:'DSK1.HEXDATA'
1010 FOR U=1 TO 12
1020 INPUT #2:A$
1030 CALL CHAR(U+99,A$)
1040 NEXT U
1050 CLOSE #2
1060 RETURN
1110 PRINT ::::::::::::::::::::::::::::
1150 FOR ROW=4 TO 20 STEP 2
1151 CALL HCHAR(ROW+1,6,58-ROW/2,1)
1160 FOR COL=8 TO 24 STEP 2
1165 B=10*(COL-8)/2+(ROW-4)/2
1167 IF F$='RESP' THEN 1170
1168 CHR=PROD(B)*11/MAXP+100
1169 GOTO 1180
1170 CHR=R(B)*11/MAXR+100
1180 CALL COLOR(CHR/11,2,12)
1190 CALL HCHAR(ROW,COL,CHR,2)
1200 CALL HCHAR(ROW+1,COL,CHR,2)
1210 NEXT COL
1220 NEXT ROW
1225 PRINT TAB(7);'0 ':'1 ':'2 ':'3 ':'4 ':'5 ':'6 ':'7 ':'8 '
1230 RETURN
1500 OPEN #3:'RS232/1.BA=9600'
1501 PRINT #3:'
SQARRAY: 'F$::
'I-1:'SOURCES-
RE LOCATED AT:'

```



```

1502 FOR J=1 TO I-1
1503 PRINT #3: "      ";X(J)*.05;";";8-Y(J)*.05;
1504 NEXT J
1505 PRINT #3: ":::::"
1510 FOR ROW=24 TO -2 STEP -1
1512 IF (ROW+1)/3=INT((ROW+1)/3) THEN 1516
    4 PRINT #3: "      ";
1515 GOTO 1520
1516 PRINT #3: (ROW+1)/3; "      ";
1520 FOR COL=0 TO 8
1530 B=10*COL+INT(ROW/3+.9)-(2*INT(ROW/3+.9)-8)
1535 IF F$='RESP' THEN 1540
1537 CHR=PROD(B)*11/MAXP
1538 GOTO 1550
1540 CHR=R(B)*11/MAXR
1550 IF CHR<10.5 THEN 1560
1555 PRINT #3: "#####";
1557 GOTO 1700
1560 IF CHR<9.5 THEN 1570
1565 PRINT #3: "*****";
1566 GOTO 1700
1570 IF CHR<8.5 THEN 1580
1575 PRINT #3: "XXXXX";
1576 GOTO 1700
1580 IF CHR<7.5 THEN 1590
1585 PRINT #3: "!!!!!!";
1586 GOTO 1700
1590 IF CHR<6.5 THEN 1600
1595 PRINT #3: "=====";
1597 GOTO 1700
1600 IF CHR<5.5 THEN 1610
1605 PRINT #3: "////////";
1607 GOTO 1700
1610 IF CHR<4.5 THEN 1620
1615 PRINT #3: "::::::::::";
1617 GOTO 1700
1620 IF CHR<3.5 THEN 1630
1625 PRINT #3: " / / / ";
1627 GOTO 1700
1630 IF CHR<2.5 THEN 1640
1635 PRINT #3: " : : : ";
1637 GOTO 1700
1640 IF CHR<1.5 THEN 1650
1645 PRINT #3: ".....";
1647 GOTO 1700
1650 IF CHR<0.5 THEN 1660
1655 PRINT #3: " . . . ";
1657 GOTO 1700
1660 PRINT #3: "      ";
1667 GOTO 1700
1700 NEXT COL
1705 PRINT #3: "
1710 NEXT ROW
1720 PRINT #3: "      0      1      2      3      4      5      6      7      8*:::::::::::::
:::::::::::::::::::::
1730 CLOSE #3
1740 RETURN

```

APPENDIX VI

Computer Program to Develop Three-
Dimensional Plot of Response to Point Source

```

LIST
001 DIM M[1,180]
1002 DIM P[200]
1003 DIM U[10]
1004 LET F=.3708
1005 LET R=15
1006 LET S=-20
1007 LET T=10
1008 LET K=1
1009 DIM V[10]
1010 FOR I= 0 TO 80
1012   READ P[I]
1013 NEXT I
1015 FOR I=100 TO 105 STEP 2
1016   PRINT "X";(I-100)/2;"", Y";(I-100)/2;"=";
1017   INPUT P[I],P[I+1]
1018   PRINT
1019 NEXT I
1030 FOR I= 0 TO 1
1031   FOR J= 0 TO 180
1032     LET M[I,J]=-700
1037   NEXT J
1038 NEXT I
1040 LET B=-900
1041 LET C=-650
1042 LET L=-900
1043 LET V[0]= 0
1044 LET V[1]= 0
1045 LET A=10+ INT (3* LOG (P[0])/ LOG 10)
1046 LET A=( INT (P[0]+3/A)+1)*A
1047 LET H=-650
1050 FOR Y=-3.25 TO 3.25 STEP .1
1060   FOR N= 0 TO 160
1061     LET X=N/20-4
1065     LET G=1
1066     FOR J=100 TO 105 STEP 2
1070       LET I= SQR ((Y-P[J+1])*(Y-P[J+1])+(X-P[J])*(X-P[J]))*20*K
1075       IF I<=80 GOTO 1080
1076       LET Z= 0
1077       GOTO 1091
1080       LET Z= INT (I)
1090       LET Z=P[Z]+(I-Z)*(P[Z+1]-P[Z])
1091       LET G=G*Z
1095     NEXT J
1100     LET Z=G/A*200
1110     LET E=(Y-S)* COS F-(Z-T)* SIN F
1120     LET D=R*X/E
1130     LET E=R*((Y-S)* SIN F+(Z-T)* COS F)/E
1140     LET D= INT (D*200)
1141     LET E= INT (E*200)
1146     LET I= INT (D/10+90)
1151     IF V[0]=1 GOTO 1185
1152     IF M[0,I]+(D/10+90-I)*(M[0,I+1]-M[0,I])>=E GOTO 1299
1153     IF I<V[1] GOTO 1160
1154     LET M[I,V[1]]=C+(10*V[1]-B-900)*(E-C)/(D-B)
1155     LET V[1]=V[1]+1
1156     GOTO 1153

```

Three Dimension
Plotting

(multi-products)

```

1160 LET B=D
1170 IF (E-H)<5 GOTO 1190
1175 LET V[0]=1
1180 GOTO 1190
1185 LET V[0]=0
1190 IF D>=L GOTO 1210
1200 LET U[8]=L-D
1205 GOTO 1220
1210 LET U[4]=D-L
1220 LET L=D
1240 LET U[1]=1
1245 LET U[10]=1
1250 IF E>=C GOTO 1270
1260 LET U[6]=C-E
1265 GOTO 1280
1270 LET U[2]=E-C
1280 LET C=E
1290 GOSUB 3500
1295 IF V[0]=0 GOTO 1302
1296 LET X=X-.025
1298 GOTO 1065
1299 LET M[1,I]=M[0,I]
1300 LET V[1]=V[1]+1
1302 LET H=E
1303 NEXT N
1304 FOR J=0 TO 180
1306 LET M[0,J]=M[1,J]
1307 NEXT J
1308 LET V[1]=0
1310 NEXT Y
1320 IF B>-900 GOTO 1340
1330 LET U[4]=-900-B
1335 GOTO 1350
1340 LET U[8]=B+900
1350 IF C>-650 GOTO 1380
1360 LET U[2]=-650-C
1370 GOTO 1390
1380 LET U[6]=650+C
1390 GOSUB 3500
1400 END
1401 FOR I=0 TO 80

```

*

**New York University Medical Center
Institute of Environmental Medicine**

550 FIRST AVENUE • NEW YORK, N.Y. 10016 • (212) 679-3200