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Abstract

A detailed description of the studies performed under contract DE-AC02-79-ER10386.A000 "Particle Deposition in Granular Media" during the period June 1, 1979 to date is presented. These studies include the design and construction of apparatus for filtration experiments and a complete trajectory analysis for the calculation of the initial collection efficiency of granular media. The results of the trajectory analysis have been used to develop a generalized correlation of the collection efficiency.

I. Introduction

The progress report summarizes the studies carried out under the contract entitled "Particle Deposition in Granular Media" awarded to Syracuse University from the Department of Energy, Office of Basic Energy Sciences during the period June 1, 1979 to date. The report is prepared and submitted as part of submission renewal proposal application.

The long term objective of the study is to obtain quantitative understanding of the deposition phenomenon in granular media arising from the flow of fluid-particle suspensions through the media and to obtain the predictive capability of filter performance, which can be used as a basis of rational design. The main feature of the deposition phenomenon is the continuous change of the media structure. As a result, both the collection efficiency and pressure drop across filter bed vary with time. An understanding of this time-dependent behavior represents the central issue in the study of granular bed filtration.

Two major problems have been considered since the inception of this study. The first problem concerns with the gathering of filter performance data under controlled conditions, which can be used to assess the validity of the comparison theoretical studies. For this purpose, the design and construction of an experimental apparatus for granular filtration experiment is undertaken. It is expected that the apparatus will be ready for experimental work by the end of the current contract year.

The second problem studied concerns with the calculation of the initial collection efficiency of granular media using the classical trajectory analysis and the constricted tube model for characterizing the granular media. The results obtained is then used to establish a correlation for the prediction of the initial collection efficiency.

A detailed description of the results of these two studies are given in the following sections.

II. Design and Construction of Apparatus for Filtration Experiment

A schematic diagram of the experimental apparatus is shown in Fig. II-1. This experimental apparatus was designed and constructed to give a uniform flow of dried and neutralized aerosol particles of known size (or size distribution) at velocities from a few up to 50 cm/sec to be filtered in an experimental packed bed filter. A photograph of the apparatus is shown in Fig. II-2.

The apparatus can be broadly divided into the following sections: (1) the aerosol generation section, (2) the neutralizing section, (3) the solution section, (4) the test (filter) section and (5) the sampling section.

The aerosol generation section employs the use of two types of devices. A collision nebulizer (BGI, CN-1, Waltham, Massachusetts 02154), is used to generate aerosols (monodispersed) from suspensions of either polystyrene latex spheres (1.091 μm diameter) or polyvinyltoluene latex spheres (2.02 μm diameter) (The Dow Chemical Company, Indianapolis, Indiana, 46206). Air at 35 psi is forced through the nozzles of the atomizer to produce large droplets. The spray is then passed through a heated glass tube to remove the water adhered to the particles and another glass tube half filled with silica gel to remove water vapor present in the air stream.

The other type of device for aerosol generator is a fluidized bed aerosol generator (Thermo-Systems, Inc., Model 3410, St. Paul, Minnesota) which disperses solid particles directly into the air. The fluidized bed generator is capable of generating aerosols of much higher concentration than the collision atomizer. However the particle size range will be much greater since monodispersed particles are available only in aqueous suspensions. In addition, a Wright Dust Feeder is also available for the same purpose.

A neutralizer (Thermo-Systems, Inc., Model 3012, St. Paul, Minnesota) is used to reduce the electrostatic charge of the aerosols generated by either

Fig. II-1 Schematic Diagram of Experimental Apparatus

Schematic Representation of Filtration Apparatus

- (1) Inlet Air Supply Line Valve
- (2) Inlet Air Drier (Silica Gel)
- (3) Inlet Air Fibrous Filter
- (4) Inlet Air Pressure Regulator
- (5) Flow Control Valve (Capillary type)
- (6) Rotameter to measure Inlet Air Rate
- (7) Collision Nebulizer for Aerosol Generation
- (8) Electrical Heater
- (9) Temperature Control for Electrical Heater
- (10) A.C. Power Supply
- (11) Diffusion Drier (Silica Gel)
- (12) Aerosol Neutralizer
- (13) Aerosol Tunnel
- (14) Isokinetic Inlet Aerosol Measurement Probe
- (15) Contraction Duct to provide Plug Flow
- (16) Manometer to measure Pressure Drop across Filter Bed
- (17) Granular Filter Bed
- (18) Isokinetic Outlet Aerosol Measurement Probe
- (19) Absolute Filter

Fig. II-2 Photograph of Experimental Apparatus

types of devices. After leaving the neutralizer, the aerosol enters the top of the 6" diameter aerosol tunnel, where it will be diluted by air jets directed upwards from holes on to 1/4" diameter circular copper tubes concentric with the aerosol tunnel.

The contraction tunnel (lower part of the aerosol tunnel) is designed to provide a uniform aerosol flow field. Its design follows that originally proposed by Tsien (1943). The contraction tunnel is reduced from 6" and the upper end to 1.5" at the lower end. The contraction tunnel was made by applying layers of epoxy resin and fiberglass on a smooth waxed wooden pattern. The experimental filter was made of plexiglass cylinder with 1 9/16" I.D. and a height of 4". Glass beads of uniform size (200 ~ 400 μ m) will be used as filter grains.

The determination of aerosol concentration will be made in two ways: (a) with the use of air electrostatic aerosol sample (Thermo-Systems, Inc., Model 3100, St. Paul, Minnesota) or (b) a particle counter (Climet Instrument Co., Model 208, Redlands, California 92373). A sampling probe is provided and placed 15" downstream from the experimental filter since the Stokes number in all the experiments is expected to be low, the condition of other kinetic sampling will not be required, thus simplifying the experimental effort.

Trial runs are being conducted now. The apparatus will be ready for experiment by the end of the current contract year.

III. Calculation of Initial Collection Efficiency of Granular Media - Trajectory Analysis

The initial collection efficiency of granular filter bed is an important quantity in the description of the dynamic behavior of deep bed filtration. Although a number of investigators have calculated the initial collection efficiency of the trajectory analysis for granular bed (Parestky et al., 1971; Pendse et al., 1978) these results were found to be an order of magnitude lower than corresponding experimental values. A more accurate prediction is obviously needed.

In the present study, a complete trajectory analysis is used to calculate the initial collection efficiency of granular media using the constricted tube model for the media representation. Payatakes et al. (1974 a, b) pioneered the use of the constricted tube model in the study of deep bed filtration. Their work however was concerned primarily with hydrosol filtration and the effort of particle inertia was not considered. The earlier work of Pendse et al. (1978) used one type of the constricted tube configuration in this calculation. As shown in the present study, the particular configuration considered by them was not suitable for aerosol deposition.

The results presented below are divided into three parts. The first part gives a general description of the constricted tube model and the various possible tube wall configurations. In the second part, the calculation of the collection efficiency based on the use of the different types of tubes is made. The results are compared with available experimental data. From the comparison, a generalized correlation is proposed and tested.

III-1. Geometric Representation of Granular Bed

III-1.A. Modelling of Porous Media

For the study of particle deposition, granular deep filter may be considered to be composed of a series of unit bed elements (UBE) of

thickness, ℓ , each of which in turn consists of a number of geometrically similar collectors (unit cell) [Fig. III-1]. For homogeneous filter bed in its clean state all the unit bed elements are identical.

The geometry and size distribution of these collectors depends upon the porous media model used to characterize the bed and there is a multitude of choice in this respect. Generally speaking, porous media models can be classified into two categories, namely external flow models and internal flow models. (Figure III-2).

The external flow models are based upon a geometric description of packing grains and the flow field resulting from such a formulation is of the external flow type. Consideration of the pore space therefore can be made only in a secondary manner. Among the external flow models are the isolated sphere model (Maxwell, 1892), Brinkman's model (Brinkman, 1947), Happel's model, (Happel, 1958) and the modified Happel's model (Spielman and FitzPatrick, 1973) which have been used in the past in the study of deep bed filtration.

In contrast to the external flow models, one can focus the attention on the pores of the packed bed. Models formulated on such a basis are of the internal flow type. The pores in their connected form, serve as flow channels and the surface of the pore acts as the collecting surface. The advantage of the internal flow models is their potential capability of including the clogging effect due to deposition [Tien et al., 1979]. The classical Kozeny model represents the pore space by a bundle of straight capillaries. Scheidegger (1957) and more recently Dullien (1975) have provided a review of this approach. Such an oversimplified approach, however, cannot provide predictions with sufficient accuracy without the introduction of certain extraneous parameters (for example,

GRANULAR FILTER BED

SERIES OF UNIT BED ELEMENTS

CONSTRICED TUBE TYPE UNIT CELLS

FIG. III-1 SCHEMATIC REPRESENTATION OF A GRANULAR FILTER BED

ISOLATED SPHERE MODEL

SPHERE IN CELL MODEL

(A) EXTERNAL FLOW MODELS

CAPILLARY MODEL

PARABOLIC WALL

SINUSOIDAL WALL
CONSTRICTED TUBE MODELS

HYPERBOLIC WALL

(B) INTERNAL FLOW MODELS

FIG. III-2 VARIOUS REPRESENTATIONS OF UNIT CELLS

the tortuosity factor). Furthermore, the rectilinear streamlines of fluid provided by capillary model are unrealistic and at variance with the converging-diverging flow characteristic inherent in granular bed.

A higher order representation of granular porous media may result if one replaces capillaries in the Kozeny model with entities of different geometrical features. This forms the basis of the so-called constricted tube model. In the context of particle deposition, the collectors are assumed to be the unit cells in the shape of constricted tubes. The size of a constricted tube is described by the following quantities: d_m , the maximum diameter corresponding to entrance and exit sections, d_c , the constriction (or throat) diameter, and h , its height.

III-1.B. Constricted Tube Model of Granular Bed

Although the possible use of the constricted tube geometry to represent granular media was first mentioned more than two decades ago (Petersen, 1958); a complete model formulation was obtained only recently (Payatakes et al., 1973a). Payatakes et al. defined the necessary model parameters, proposed procedures for their determination, and obtained model validation through comparison with experimental data. The model parameters include: N_c , the number of constricted tubes per unit bed cross-sectional area; ℓ , the length of the unit bed element; h , the height of the unit cell; d_m , the maximum diameter of the unit cell; and d_c , the constriction diameter of the unit cell. The size distribution of pore constrictions can be obtained from the capillary pressure versus saturation data. A summary of the relationships between the filter bed properties and the model parameters is given in Table (III-1). Although the size of a unit cell is specified in terms of h , d_m and d_c , the actual shape of the constricted tube wall is left arbitrary. In the original formulation of Payatakes et

Table (III-1)

Determination of Porous Media Model Parameters

(Case of Uniform Unit Cell Size)

- (1) Number of constricted tubes per unit area, N_c

$$N_c^* = N_c \cdot d_g^2 = \frac{6 \cdot \epsilon^{1/3} \cdot (1 - S_w)^{1/3} (1 - \epsilon)^{2/3}}{\pi} \quad (\text{III-1})$$

- (2) Length of unit bed element, ℓ

$$\ell^* = \frac{\ell}{d_g} = \left[\frac{\pi}{6} \frac{1}{(1 - \epsilon)} \right]^{1/3} \quad (\text{III-2})$$

- (3) Constriction diameter, $d_c^{(\dagger)}$

$$d_c^* = \frac{d_c}{d_g} = 0.35 = 2 r_1^* \text{ (approximately)} \quad (\text{III-3})$$

- (4) Height of the unit cell, h

$$h = d_g \quad (\text{III-4})$$

- (5) Maximum diameter of the unit cell, d_m

$$d_m^* = \frac{d_m}{d_g} = \left[\frac{\epsilon (1 - S_{wi})}{(1 - \epsilon)} \right]^{1/3} = 2 r_2^* \quad (\text{III-5})$$

[†] d_c and S_{wi} can be determined from capillary pressure-saturation data.

al. (1973) the wall geometry is assumed to be parabolic. Other geometries (sinusoidal, hyperbolic) have also been suggested (Fedkiw and Newman, 1977; Venkatesan and Rajagopalan, 1979). (See Figure III-2b). The different flow fields corresponding to these different wall geometries will be examined since fluid velocity is inherently important in the calculation of particle deposition by trajectory analysis.

III-1.C. Flow Field Inside the Constricted Tube

The flow field within a constricted tube with a specific wall geometry, in principle, can be obtained by the solution of the appropriate Navier-Stokes equations. Payatakes et al. (1973b) have outlined a numerical procedure to solve the complete Navier-Stokes equations (that is, including the fluid inertia terms) for the flow of Newtonian fluid through a periodically constricted tube extending to infinity in both directions, the period of variation being equal to the height of a unit cell h . For creeping flow (no fluid inertia) approximate solutions corresponding to different tube geometries have been obtained with the collocation method (Fedkiw and Newman, 1977; Neira and Payatakes, 1978, 1979). An iterative method for the solution of the flow problem for the case of sinusoidal wall geometry has also been established (Deiber and Schowalter, 1979).

In contrast to the periodicity condition used by Payatakes et al., one may argue that the flow field needs to be defined only within the unit cell itself, thus eliminating the pre- and post-extensions of the unit cell to form an infinite periodically constricted tube. The boundary conditions imposed on the governing equations are the no-slip condition on the tube wall, axisymmetry and the constant volumetric flow rate at any given cross-section. No extraneous boundary condition is

imposed at the entrance or exit sections. Such an approach was employed by Chow and Soda (1972) in their approximate solution for the flow through a constricted tube with arbitrary wall geometry. The solution was obtained by perturbation in terms of the ratio of the mean wall radius to the characteristic length along the axis (over which significant change in wall-radius takes place). The stream function is expressed explicitly in terms of the Reynolds number, N_{Re} , the ratio of the amplitude of variation of wall radius to the mean radius, ϵ' , and the perturbation parameter, δ' . Similarly, Venkatesan and Rajagopalan (1979) have applied the exact analytical solution for creeping flow through a constricted tube with hyperbolic wall. The method of solution proposed by Chow and Soda (1972), though approximate, yields the most general solution available for the flow field within a constricted tube. In the limit that the perturbation parameter, δ' , goes to zero (i.e. constricted tube with negligible constriction), the stream function reduces to the simple Hagen-Poiseuille flow form

$$\psi'_0 = \frac{\psi}{q} = \frac{1}{2\pi} \left[\left(\frac{r}{r_w} \right)^4 - \left(\frac{r}{r_w} \right)^2 \right] \quad (\text{III-6})$$

where q is the volumetric flow rate through a constricted tube. Equation (III-6) therefore represents the zeroth order solution for the flow through a constricted tube with arbitrary shape.

A summary of available flow field solutions for various types of constricted tubes is given in Table (III-2).

III-1.D. Effect of Wall Geometry

Having a number of flow field expressions corresponding to different tube geometry at hand, a comparison among them can be readily made. The

Table (III-2)

List of Available Flow Field Solutions for Constricted Tube Type Unit Cells

(I) Unit Cell (no extraneous periodicity conditions)

Flow Type		Creeping Flow ($N_{Re,s} = 0$)			Inertial Flow ($N_{Re,s} > 0$)		
Wall Geometry		parabolic	sinusoidal	hyperbolic	parabolic	sinusoidal	hyperbolic
I(a)	Simplified solution	yes	yes	yes	yes	yes	yes
I(b)	Perturbation solution	yes ⁽¹⁾	yes ⁽¹⁾	yes ⁽¹⁾	yes ⁽¹⁾	yes ⁽¹⁾	yes ⁽¹⁾
I(c)	Exact analytical solution	no	no	yes ⁽²⁾	no	no	no

(II) Extended Unit Cell (infinite periodic constricted tube)

Flow Type		Creeping Flow ($N_{Re,s} = 0$)			Inertial Flow ($N_{Re,s} > 0$)		
Wall Geometry		parabolic	sinusoidal	hyperbolic	parabolic	sinusoidal	hyperbolic
I(a)	Finite difference solution	yes ⁽³⁾			yes ⁽³⁾		
I(b)	Collocation solution	yes ⁽⁴⁾	yes ⁽⁵⁾				
I(c)	Geometric solution		yes ⁽⁷⁾				

Note:

- (1) Chow and Soda (1972)
- (2) Venkateshan and Rajagopalan (1979)
- (3) Payatakes, Tien and Turian (1973b)
- (4) Neira and Payatakes (1978)
- (5) Fedkiw and Newman (1977)
- (6) Neira and Payatakes (1979)
- (7) Deiber and Schowalter (1979)

comparison includes both the prediction of overall pressure drop and the fluid velocity profile.

(a) Pressure Drop

The pressure drop-flow relationship can be studied in terms of the relationship between the friction factor, f_s , and the Reynolds Number, $N_{Re,s}$, defined as

$$f_s = \left(-\frac{dp}{dz}\right) \frac{d_g}{2 \rho V_s^2} \quad (\text{III-7a})$$

and

$$N_{Re,s} = \frac{d_g V_s \rho}{\mu} \quad (\text{III-7b})$$

From Equation (III-7a) and (III-7b) one has

$$(f_s \cdot N_{Re,s}) = \left(-\frac{dp}{dz}\right) \frac{d_g^2}{2 \mu V_s} \quad (\text{III-7c})$$

Based on the Kozeny theory of the flow through packed bed, one obtains the following expression under laminar flow regime:

$$(f_s \cdot N_{Re,s}) = 72 \left[\frac{(1 - \epsilon)^2}{2\epsilon^3} \right] \quad (\text{III-8})$$

From a comparison with experimental data it was found that the constant in the above equation should be 150 instead of 72. (See Blake-Kozeny equation in Bird et al., 1960). The difference between the theoretical and empirical constant is commonly attributed to the so-called tortuosity effect.

Similar expressions can be obtained with the use of the constricted tube model. For example, if the tube wall geometry is of the parabolic type, the following equation was obtained (Payatakes et al., 1973b; Neira and Payatakes, 1978)

$$(f_s \cdot N_{Re,s}) = \frac{2 (-\Delta p_l^*) \langle d \rangle_c \langle d \rangle_q^2}{\pi l^* N_c^* \langle d \rangle_c^3} \quad (\text{III-9})$$

where $(-\Delta p_1^*)$ is the normalized pressure drop corresponding to $N_{Re,c}$
 $(= \frac{h v_o \rho}{\mu})$ equal to one.

For the case of hyperbolic geometry, from the results of Venkateshan and Rajagopalan (1979) one has:

$$(f_s \cdot N_{Re,s}) = \frac{24 g(\xi_o, \zeta_o)}{\pi N_c^* d_c^{*4}} \quad (III-10)$$

where

$$\xi_o = \left[\left(\frac{d_m^*}{d_c^*} \right)^2 - 1 \right]^{1/2} \quad (III-11a)$$

$$\zeta_o = [(\xi_o d_c^*)^2 + 1]^{-1/2} \quad (III-11b)$$

and

$$g(\xi_o, \zeta_o) = \frac{1}{\xi_o \zeta_o} \frac{(1 + \zeta_o)^2}{(1 + 2 \zeta_o)} \left[\frac{\xi_o}{1 + \xi_o^2} + \tan^{-1} \xi_o \right] \quad (III-11c)$$

Fedkiw and Newman (1977) have obtained the bed friction factor as a function of the unit cell dimensions for the constricted tube with sinusoidal wall. The friction factor, f_B , defined by Fedkiw and Newman is based on the specific interfacial area, a , of the filter grains and has an implicit porosity dependence as follows:

$$f_B = \frac{36 \epsilon^3}{a} \left(- \frac{dp}{dz} \right) \frac{1}{\rho v_s^2} \quad (III-12)$$

Rewriting this equation, one has

$$(f_s \cdot N_{Re,s}) = (f_B \cdot N_{Re,B}) \cdot \left[\frac{(1 - \epsilon)^2}{2 \epsilon^3} \right] \quad (III-13)$$

where $N_{Re,B} = \left(\frac{\rho U_s}{a \mu} \right)$. Figure III-7 of Fedkiw and Newman's work presents $(f_B \cdot N_{Re,B})$ as a function of r_A and A where $r_A = \left(\frac{d_c^* + d_m^*}{4} \right)$ and $A = \left(\frac{d_m^* + d_c^*}{4} \right)$.

To assess the effect of the tube wall geometry on the prediction of the overall pressure drop, four kinds of granular bed with different

porosity were considered. The corresponding model parameters were evaluated in accordance with the procedures established before [Payatakes et al. 1973a, see Table (III-I) for summary] and the values of $(f_s \cdot N_{Re,s})$ were calculated from Equations (III-9), (III-10) and (III-13) corresponding to a wall geometry of parabolic, hyperbolic and sinusoidal types respectively. These results are given in Table (III-3). Also included in the tables are the values of $f_s \cdot N_{Re,s}$ predicted from the Blake-Kozeny equation [i.e. Equation (III-8) with a constant of 150 instead of 72] and those predicted with the use of Happel's model. The comparison is also shown in Fig. III-3 in which the predicted values of $(f_s \cdot N_{Re,s})$ are plotted against bed porosity. Also included in this figure are the values of $(f_s \cdot N_{Re,s})$ as a function of bed voidage, ϵ , based on Happel's model and the classical Blake-Kozeny equation. The dependence of $(f_s \cdot N_{Re,s})$ on ϵ is qualitatively similar in all these cases. Both the constricted tube model and Happel's model overestimate the pressure drop as compared with the Blake-Kozeny equation. The effect of the wall geometry is found to be insignificant.

(b) Velocity Profiles

For a more direct and detailed assessment of the effect of the wall geometry, the axial and radial velocity profiles within a constricted tube with parabolic and sinusoidal wall geometry and corresponding to the conditions of $d_c^* = 0.352$ and $d_m^* = 0.795$ are shown in Figures III-4, III-5, III-6, and III-7. Similar comparison of the velocity profiles for the hyperbolic wall and parabolic wall type unit cells has been given by Venkatesan and Rajagopalan (1979). The velocity profiles for the tube with hyperbolic wall are similar to the perturbation solution for the parabolic wall geometry. Both v_z and v_r are scaled with respect to v_0 ,

Table (III-3)

Effect of Constricted Tube Shape on Pressure Drop Prediction

Granular Bed Medium ϵ	I glass beads 0.38	II glass beads 0.40	III sand 0.41	IV sand 0.47
Unit Cell Dimensions:				
$d_c^{*†}$	0.352	0.377	0.337	0.364
$d_m^{*†}$	0.795	0.807	0.804	0.860
Predictions based on con- stricted tube model				
$(f_s \cdot N_{Re,s})$ parabolic sand	637	454	628	460
$(f_s \cdot N_{Re,s})$ sinusoidal sand	603	436	492	311
$(f_s \cdot N_{Re,s})$ hyperbolic wall	567	460	644	499
$(f_s \cdot N_{Re,s})$ Blake-Kozeny Equation	523	422	379	203
$(f_s \cdot N_{Re,s})$ Happel's Model	569	460	414	227

Note:

[†]See Payatakes (1973), pp. 71, 180, 189.

FIG. III-3 COMPARISON OF PRESSURE DROP PREDICTIONS BASED ON CONSTRICTED TUBE MODELS WITH DIFFERENT WALL GEOMETRIES

FIG. III-4 AXIAL VELOCITY PROFILES FOR PARABOLIC TUBE GEOMETRY

I: $z^*=0.5$ (THROAT)

II: $z^*=0.25$

III: $z^*=0.0$ or 1.0
(ENTRANCE OR EXIT)

$$d_m^* = 0.795$$

$$d_c^* = 0.352$$

(A) SIMPLIFIED SOLUTION

(B) PERTURBATION SOLUTION

(C) COLLOCATION SOLUTION

FIG. III-5 RADIAL VELOCITY PROFILES FOR PARABOLIC
TUBE GEOMETRY

FIG. III-6 AXIAL VELOCITY PROFILES FOR SINUSOIDAL TUBE GEOMETRY

FIG. III-7 RADIAL VELOCITY PROFILES FOR SINUSOIDAL TUBE GEOMETRY

FIG. III-8 SCHEMATIC REPRESENTATION OF LIMITING TRAJECTORY

the average velocity at the constriction (i.e. throat). The differences in the axial velocity profiles for different geometries are rather insignificant, but the radial velocity profiles near the entrance and exit sections of the unit cell exhibit strong dependence on the wall shape.

The results shown in Figures III-4 and III-7 indicate that all these solutions predict zero radial velocity component at the throat section of the unit cell.

Although the rates of certain heat and mass transfer processes like convective diffusion in granular media are found to be insensitive to minor changes in velocity components (Venkatesan and Rajagopalan, 1978). Point values of fluid velocity in the calculations of particle trajectories. In the following sections, the effect of different tube geometries on the prediction of the collection efficiencies of the bed will be examined in detail.

III.2 Calculation of Collection Efficiency - Simple Case

The principle of the trajectory analysis is illustrated in Fig. (III-8). If the granular bed is represented by the constricted tube model, the retention of particle in the bed can be viewed as the deposition of particles on the wall of the constricted tube as the particle suspension flows through the tube. Generally speaking, when a particle enters the tube, it may escape (i.e. flows out the tube) or be collected. In either case, the outcome can be determined from the trajectory of the particle. A particle may be considered as collected if its trajectory

leads to a contact between the particle and the tube wall (assuming that the particle will not bounce off).

If the Brownian diffusion effect is unimportant, particle trajectories are deterministic. For particles entering the tube, their trajectories are uniquely determined by their initial positions at the inlet and different trajectories do not intersect one another. If one assumes that a particle which enters the tube at a radial position of $r^* = (r_g^*)$, (Point L), makes contact with the tube wall at its exit[†], it follows that all particles which are collected by the tube wall must have their initial positions between points L and K while those which escape must have initial positions between points Z and L'. The trajectory beginning at point L is known as the limiting trajectory (See Fig. III-8). Accordingly, the collection efficiency of the unit cell, $(\eta_o)_c$, defined as the fraction of particle entering the unit cell which becomes collected, is given as

$$(\eta_o)_c = \frac{\psi [0, r_g] - \psi [0, r_o]}{\psi [0, 0] - \psi [0, r_o]} \quad (\text{III-14})$$

where ψ is the stream function of the flow within the tube, and r_o is the farthestmost position that an entering particle may be, and it is dependent upon the particle size and the shape of the tube wall. r_o can be calculated readily from geometrical consideration for a given tube geometry and particle size.

[†]As shown later, almost all the particles are deposited on the upper half of the tube. Accordingly, for practical purposes, the limiting trajectory can be considered to be the one which makes contact with the tube wall at its throat.

For the simple case where all the unit cells are the same size, the collection efficiency of the unit bed element, η_0 , becomes the same as that of $(\eta_0)_c$. Furthermore it is interesting to note that the collection efficiency, η , and the filter coefficient, λ , are related by the expression

$$\lambda = \frac{1}{2} \ln \frac{1}{1 - \eta} \quad (\text{III-15})$$

III-2.A. Collection Efficiency Due to Interception Alone

The interception collection efficiency can be obtained readily from the knowledge of the stream function within the tube. In this case, particle trajectories coincide with streamlines. The collection efficiency under creeping flow conditions is given as

$$(\eta_0)_I = \frac{\psi [0.5, (r_1^* - 0.5 N_R)] - \psi [0, r_0^*]}{\psi [0, 0] - \psi [0, r_0^*]} \quad (\text{III-16})$$

where the subscript I denotes interception. The limiting trajectory is the particle trajectory passing through $[0.5, (r_1^* - 0.5 N_R)]$. (r_0^*) is the farthestmost radial position at the entrance (i.e. $z^* = 0$), which can be occupied by an entering particle. r_1^* is the throat radius, $(d_c^*/2)$. N_R is the relative size parameter defined as d_p/d_g . As stated before, (r_0^*) depends on the size of the particle and the size and shape of the tube wall. For the sinusoidal geometry (r_0^*) is equal to $(r_2^* - 0.5 N_R)$, where $r_2^* = (d_m^*/2)$. For other cases, it can be easily calculated from geometrical consideration.

A comparison of the calculated values of $(\eta_0)_I$ based on the various flow fields corresponding to the three different tube geometries and with $d_c^* = 0.352$ and $d_m^* = 0.795$ is shown in Table (III-4). The calculated results are found to be of the same order of magnitude. The simplified flow field expressions with parabolic and sinusoidal geometries [columns

Table (III-4)

The Collection Efficiency Due to Interception Alone, $(\eta_0)_I$, (Under Creeping Flow Conditions)

Based on Different Tube Geometries and Flow Field Expressions

TUBE GEOMETRY							Sphere-in-cell
	Parabolic Wall			Sinusoidal Wall		Hyperbolic Wall	Happel's Model
N_R (d_p/d_g)	Simplified Solution	Perturbation Solution	Collocation Solution	Simplified Solution	Perturbation Solution	Analytical Solution	Analytical Solution
0.001	0.000021	0.000036	0.000035	0.000029	0.000038	0.000036	0.000044
0.005	0.00052	0.00089	0.00087	0.00065	0.00093	0.00009	0.00017
0.01	0.0020	0.0035	0.0034	0.0025	0.0036	0.00036	0.0011
0.02	0.008	0.013	0.013	0.01	0.014	0.014	0.017
0.05	0.045	0.073	0.074	0.056	0.076	0.075	0.109
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)

(2) and (5) in Table (III-4)] yield lower values of $(\eta_0)_I$ as compared to the other four cases. The agreement among the four cases; the analytical solution with hyperbolic geometry, the perturbation solutions with sinusoidal and parabolic geometries and the collection solution with parabolic geometry, is remarkably good in spite of their significant geometrical differences.

Also included in the table are the calculated values of $(\eta_0)_I$ based on Happel's model with $\epsilon = 0.38$ [a value consistent with the dimensions of $d_m^* = 0.795$ and $d_c^* = 0.352$, see Table (III-3)]. In general, the values of $(\eta_0)_I$ based on Happel's model are higher than those based on the constricted tube models. It is interesting to note that the relationship between the collection efficiency η and the filter coefficient λ , for Happel's model (Rajagopalan and Tien, 1976) is

$$\lambda = \frac{3(1-\epsilon)}{2d_g} \ln \left(\frac{1}{1-\eta_{\text{Happel}}} \right) \quad (\text{III-17})$$

By comparing the two $\lambda - \eta$ relationships [Equations (III-15) and (III-17)], one observes that for a given value of λ , η_{Happel} is always greater than η . The ratio $(\eta_{\text{Happel}}/\eta)$ depends on the bed porosity and varies from 1.06 to 1.004 for $0.38 < \epsilon < 0.47$.

In summary, for all the three tube geometries, the constricted tube model is found to yield consistent predictions of $(\eta_0)_I$ for $N_R < 0.05$. Moreover, $(\eta_0)_I$ is found to be insensitive to minor variations in the velocity field resulting from the use of different methods of solutions. Accordingly, an analytical expression for $(\eta_0)_I$ can be readily obtained from the simplified expression of the stream function. With the assumption that $\psi [0, r_0^*]$ is equal to $\psi [0, r_2^*]^\dagger$, expression for $(\eta_0)_I$ is found to be

[†]This is true for small values of N_R .

$$(\eta_o)_I \cong N_R^2 \left\{ \frac{4}{d_c^2} - \frac{4 N_R}{d_c^3} - \frac{N_R^2}{d_c^4} \right\} \quad (III-18)$$

III-2.B. Effect of Fluid Inertia on Particle Collection Due to Interception

The effect of fluid inertia on the interception collection efficiency can be assessed from the perturbation solution of Chow and Soda (1972). In Table (III-5), $(\eta_I)_o$ corresponding to two different values of N_R is given as a function of $N_{Re,s}$ assuming that the constricted tube is of the sinusoidal geometry. It should be noted that the perturbation solution is valid on the condition that the product of the Reynolds number, $N_{Re,s}$ and the perturbation parameter, δ' , is small. In terms of the dimensions of unit cells representative of a typical granular filter bed, Chow and Soda's solution is only applicable for $N_{Re,s} < 20$. This limitation is however ignored and the use of the perturbation solution is extended to $N_{Re,s} = 100$ since no other solution is available. The high Reynolds number results given in Table (III-5) therefore should only be regarded as approximation.

The effect of fluid inertia is shown by plotting the ratio, $(\eta_o)_I / (\eta_o)_{I, N_{Re,s}}$ against $N_{Re,s}$ (Fig. III-9). Also included in the figure is the combined collection efficiency due to interception and inertial impaction. For low Reynolds number, the fluid inertia effect can be represented as

$$\frac{(\eta_o)_I}{(\eta_o)_{I, N_{Re,s} = 0}} = 1 + 0.04 N_{Re,s} \quad (III-19)$$

The validity of this expression will be tested against experimental data as given in later sections.

III-3. Calculation of Collection Efficiency with Trajectory Analysis

The deposition of aerosol particles in granular bed, under most circumstances, is controlled by the effect of particle inertia.

Table (III-5)

Variation of $(\eta_0)_I$ With $N_{Re,s}$ Based
on Sinusoidal Geometry and Perturbation Solution

$N_{Re,s}$	N_R	
	0.003	0.0124
0.0	0.0003	0.0055
1.0	0.0003	0.006
10.0	0.0005	0.0083
25.0	0.0008	0.0129
50.0	0.0011	0.0165
75.0	0.0012	0.0173
100.0	0.0013	0.0178

CONSTRICTED TUBE WITH SINUSOIDAL WALL

FIG. III-9 EFFECT OF FLUID INERTIA ON INITIAL COLLECTION EFFICIENCY

The particle trajectories, generally speaking, differ from fluid streamlines. The determination of the limiting trajectory (for the calculation of the collection efficiency) therefore has to be made from the integration of the equations of particle motion. This will be explained in the following sections.

III-3.A. Trajectory Equations

The equations of particle motion whose solution gives particle trajectories are simply a consequence of the dynamic equilibrium of various forces. The forces considered in the present study are gravity, particle inertia and fluid drag. The force due to the Brownian diffusion is excluded since it is relatively unimportant for particle of diameter greater than 2 μm . Another notable exclusion is the electrical forces. This is not because they are unimportant, but because the study of the role of electrical forces in particle capture in itself can form a separate task.

The formulation of the equations of particle motion is based on the assumption that the particle concentration is sufficiently low so that the motion of one particle does not interfere with those of others. Furthermore, the particle motion is assumed to be axisymmetric as a consequence of the axisymmetry of the fluid flow through the constricted tube unit cell.

The force balance on a suspended particle along the z- and r-directions gives:

$$\frac{4}{3} \pi a_p^3 \rho_p \frac{d^2 z}{dt^2} = \frac{6 \pi a_p \mu}{c_s} \left(v_z - \frac{dz}{dt} \right) + \frac{4}{3} \pi a_p^3 (\rho_p - \rho_f) g \quad (\text{III-20a})$$

$$\frac{4}{3} \pi a_p^3 \rho_p \frac{d^2 r}{dt^2} = \frac{6 \pi a_p \mu}{c_s} \left(v_r - \frac{dr}{dt} \right) \quad (\text{III-20b})$$

Equations (III-20a) and (III-20b) can be written into dimensionless form. First the following dimensionless variables are introduced.

$$\begin{aligned} z^* &= z/h \\ r^* &= r/h \\ t^* &= \frac{tv_0}{h} \end{aligned} \quad (\text{III-21})$$

where h is the height of the unit cell and v_0 is the average velocity of fluid at the throat of the unit cell.

With the use of these variables, Equation (III-20a) and (III-20b) can be rewritten as follows:

$$\frac{d^2 z^*}{dt^{*2}} = \frac{2}{N_{St}} \left[\frac{v_z^*}{z^*} - \frac{dz^*}{dt^*} + N_G' \right] \quad (\text{III-22})$$

$$\frac{d^2 r^*}{dt^{*2}} = \frac{2}{N_{St}} \left[v_r^* - \frac{dr^*}{dt^*} \right] \quad (\text{III-23})$$

where

$$N_{St}' = \frac{4 a_p^2 \rho_p v_0 c_s}{9 \mu h} \quad (\text{III-24})$$

$$N_G' = \frac{2 (\rho_p - \rho_f) g a_p^2 c_s}{9 \mu v_0} \quad (\text{III-25})$$

v_z^* and v_r^* are the fluid velocity components normalized by v_0 , and c_s is the Cunningham slip factor.

It is customary to express the inertia and gravity parameters, N_{St}' and N_G' , in terms of the superficial velocity, V_s , and grain diameter, d_g . Using the constricted tube model with the assumption that all the constricted tubes are of the same size, one gets

$$h = d_g \quad (\text{III-26})$$

$$v_0 = \frac{V_s}{N_c \left(\frac{\pi d_g^2}{4} \right)} \quad (\text{III-27})$$

where N_c is the number of constricted tubes per unit cross-section of the bed, d_c is the constriction diameter and V_s is the superficial velocity.

Defining N_{St} and N_G as

$$N_{St} = \frac{4 a_p^2 V_s \rho_p c_s}{9 \mu d_g} \quad (III-28)$$

and

$$N_G = \frac{2 a_p^2 (\rho_p - \rho_f) g c_s}{9 \mu V_s} \quad (III-29)$$

one gets

$$N_{St}' = N_{St} \cdot \left(\frac{v_o}{V_s} \right) \quad (III-30)$$

$$N_G' = N_G / \left(\frac{v_o}{V_s} \right) \quad (III-31)$$

III-3.B. Initial and Terminal Conditions of the Particle Trajectory

The solution of Equation (III-22) and (III-23) requires initial conditions, namely, the starting position of the particle and its velocity. The starting position of the particle is taken to be at the entrance of the tube, (z_o^*, r_o^*) ; and the initial velocity components are assumed to be

$$\left(\frac{dz^*}{dt^*} \right)_o = v_{z,o}^* + \left(\frac{U_t}{v_o} \right) = v_{z,o}^* + N_G' \quad (III-32)$$

$$\left(\frac{dr^*}{dt^*} \right)_c = v_{r,o}^* \quad (III-33)$$

where U_t is the terminal settling velocity of the suspended particle in unbounded fluid.

The purpose of the solution of trajectory equations is to determine the outcome of approaching particles as they move through the constricted tube. There are two possible outcomes. A particle may be collected by the tube wall or it may escape. The criteria used for this determination are:

1. A particle is assumed to be collected whenever at least one point of its trajectory satisfies the conditions

$$y = a_p \quad (\text{III-34})$$

where y is the distance of the particle center from the tube wall.

Notice that Equation (III-34) has incorporated collection due to interception by accounting for the finite size of particles [i.e. the particle is assumed to be collected when the particle surface touches the tube wall].

2. If the particle trajectory exhibits a positive value of $\frac{dy}{dt}$ before the condition of Equation (III-34) is met, the particle is assumed to have escaped the unit cell. $(\frac{dy}{dt}) > 0$ indicates that the particle has gone past the point of its trajectory which corresponds to $y = y_{\text{minimum}}$ and moves away from the wall.

III-3.C. Method of Calculation

A trial and error procedure is used to determine the location of the limiting trajectory from which the collection efficiency can be calculated. The procedure involved is as follows. Assume that the initial position of the limiting trajectory at the inlet of the constricted tube is $(0, r_\ell)$. Let the estimated value of r_ℓ be r_i in the i -th approximation and the corresponding interval of uncertainty be $2\Delta r_i$. In other words, r_ℓ belongs to the interval $(r_i - \Delta r_i, r_i + \Delta r_i)$. The particle trajectory corresponding to an initial position of $(0, r_i)$ can be obtained from the integration of the equations of particle motion [i.e. Equations (III-22) and (III-23)]. If the trajectory suggests the escape of the particle, r_ℓ must be between r_i and $(r_i + \frac{\Delta r_i}{2})$ and a new estimate of r_ℓ can be taken to be $r_{i+1} = (r_i + \frac{\Delta r_i}{2})$ and $\Delta r_{i+1} = \frac{\Delta r_i}{2}$. On the other hand, if

the trajectory calculation indicates that the particle is collected, the next estimate of r_ℓ becomes $r_{i+1} = (r_i - \frac{\Delta r_i}{2})$, and $\Delta r_{i+1} = \frac{\Delta r_i}{2}$. As a beginning, one may take $r_1 = \frac{r_0}{2}$ and $\Delta r_1 = \frac{r_0}{2}$. In the present work, it was found that an estimate of r_ℓ which gives an accuracy of $\pm 1\%$ (for the collection efficiency) can be found with five to eight trials.

The numerical integrations were performed in double precision (accuracy up to 16 significant digits) using the fourth-order Runge-Kutta method. The time increment, (Δt^*) used was 0.01. A variable step algorithm was used to limit the relative change in either z - or r -coordinates in a single step within 1%.

III-3.D. Results

Using the method presented before, computer runs were made to calculate the collection efficiency, η_0 , for specified values of the dimensionless parameters, N_{St} , N_R , $N_{Re,s}$ and N_G , corresponding to different tube geometries and flow field solutions. The major part of the calculations were made with $N_G = 0$ and $N_{Re,s} = 0$. The effect of gravitational force as stated before was found to be insignificant for small values of N_G , (see later discussions), while the effect of the fluid inertia can be approximately accounted for with the use of Equation (III-19). The conditions used in the calculation are summarized in Table (III-6).

The results of the calculation is shown in Fig. III-10a and III-10b. It is obvious that the calculated results are very sensitive to the tube geometry and the flow field expression used. This is particularly true for the predicted dependence of η_0 on the Stokes number, N_{St} . When the simplified flow expressions are used, the results indicate that η_0 decreases with the increase of N_{St} (curves BB and DD). Similar trends also exist if the tube geometry is assumed to be parabolic and the flow

Table (III-6)

Conditions Used in Trajectory Calculations

Shown in Figures (III-10a) and (III-10b)

Tube dimensions	$d_m^* = 0.795, d_c^* = 0.352^\dagger$
Tube geometry	Sinusoidal wall, Parabolic wall
Flow field expressions	Perturbation solution (both geometrics) Collocation solution (Parabolic wall) Simplified Expression (both geometrics)
N_R	0.003, 0.0124
$N_{Re,s}$	0
N_G	0
N_{St}	0.001 to 1.0

† Corresponding to a granular bed with $\epsilon = 0.38$

FIG. III-10 (a) VARIATION OF (η_0) WITH N_{St} , $N_R = 0.0124$

FIG. III-10(b) VARIATION OF (η_0) WITH N_{st} , $N_R = 0.003$

Note: Fig. III-10a and III-10b

Curve

- A-A: Perturbation solution, sinusoidal wall
- B-B: Simplified solution, sinusoidal wall
- C-C: Perturbation solution, parabolic wall
- D-D: Simplified solution, parabolic wall
- E-E: Collocation solution, parabolic wall
- F-F: Pure interception efficiency
- G-G: Geometric mean of η -values based on (A-A) and (E-E)
- H-H: $\{N_{St} + (\eta)_{G-G}\}$, Equation (III-36)
- θ : Experimental data (Doganoglu, 1975)

field is given by the perturbation solution (curve CC), although the decrease is much more gradual and the magnitude of the calculated collection efficiency predicted in this case is much greater than that based on the simplified flow field expression. For the same tube geometry (parabolic) but with the flow field expression given by the collocation solution, η_0 is found, at first, to decrease slowly the N_{St} until $N_{St} \approx 0.06$ and then increase abruptly (curve EE). By comparing curves EE with CC, it is also obvious that the flow field given by the perturbation solution predicts a significant inertial effect at relatively low Stokes number ($N_{St} = 0.001$) while the expression given by the collocation solution predicts no effect. At low N_{St} , curve EE approaches curve FF which represents the collection efficiency due to interception alone. On the other hand, an entirely different trend on the dependence of η_0 on N_{St} is observed if the tube geometry is assumed to be sinusoidal and the flow field expression is given by the perturbation solution (curve AA). In this case, η_0 is found to be a monotonically increasing function of N_{St} . At low value of N_{St} , curve AA is close to curve CC (parabolic geometry, perturbation solution) but at high values of N_{St} , it approaches curve EE (parabolic geometry, collocation solution). A comparison with experimental data suggests that curves AA and EE can be considered as the upper and lower bounds in the prediction of η_0 . This point will be elaborated later.

The reason of this strong dependence on the tube geometry and various flow field solutions is not difficult to discern. The collection efficiency is strongly determined by the initial conditions used in the integration of the equations of particle motion and the local velocity of fluid near the entrance of the unit cell. Since the initial particle

velocity is assumed to be that of the fluid, the orientation and placement of streamlines near the inlet of the unit cell becomes a critical factor. The simplified flow field expression [i.e. Equation (III-6)] predicts the streamlines near the tube wall at its entrance move along a direction away from the wall. A natural consequence of this is low efficiency. Furthermore, with such an orientation for the streamline, the collection efficiency will decrease as the particle inertia increases. This explains the behavior of curve BB and DD as shown in Figs. III-10a and III-10b.

The use of a sinusoidal geometry and a more complete flow field expression implies that the streamlines at the entrance of the unit cell are parallel to the axial direction, and persist along that direction over a certain distance before they move toward the center. This is a much more favorable situation for particle collection, compared with the situation provided by the simplified flow field expression. As a result, the predicted value of η_0 is much higher. Greater particle inertia, which implies that a particle will more likely retain its initial direction along its flight, means greater likelihood for capture since the initial particle direction is oriented toward the wall. This explains that η_0 is found to be a monotonically increasing function of N_{St} .

The use of the parabolic geometry means that, at the entrance, the streamlines near the tube wall are necessarily oriented toward the center of the tube if no extraneous conditions (i.e. principle of periodicity) are imposed. Thus the use of the flow field expression given by the perturbation solution gives results which show a decrease of η_0 with the increase of N_{St} (curve CC). The flow field expression of the same geometry given by the collocation solution, on the other hand, was obtained with the use of the principle of periodicity. As a result, the streamlines

at the entrance are parallel to the axis, but change their direction very quickly as they move into the cell. Accordingly, for lower N_{St} , the collection efficiency decreases with N_{St} . However, when the particle possesses sufficiently large initial velocity, the inertial effect becomes dominant. This is shown by the large increase of η_0 with N_{St} and the fact that curve EE approaches curve AA at high N_{St} .

The fact that different tube geometries and flow field expressions yield vastly different predictions gives rise to the question as to which of the geometry and flow field expressions should be used for the prediction of the collection efficiency. To provide some insight to this question, a comparison between the experimental data of Dagonoglu (1975) and these predictions is made. Dagonoglu's data, corrected for the gravitational and fluid inertia effects, are compared with the predictions (see Figs. III-10a and III-10b). These comparisons indicate that the data disagree with any single set of predictions. Instead the experimental data are found to be within the range of the predicted values. This leads to the natural and perhaps not too surprising conclusion that not a single geometry and flow field expression provides sufficient accurate description for the study of particle deposition where the inertial effect is important. A combination of the various cases, namely by considering the unit cells to be of different geometries, may provide practical solution to this problem. For example, if the geometrical mean of the values given by curves AA and EE is considered, the resulting curve, GG, gives a much better agreement with the data.

Improvement of such an approximation can be made to account for the expected dependence of η_0 and N_{St} . Since the results given by the two AA curves (Figs. III-10a and III-10b) can be approximated within the range of the Stokes' number, $10^{-3} < N_{St} < 0.2$, by the expression

$$\eta_0 = \text{constant} + N_{St} \quad (III-35)$$

one may conclude that η_0 can be given as

$$\eta_0 = N_{St} + (\eta_0)_{G-G} = N_{St} + \sqrt{(\eta_0)_{AA} \cdot (\eta_0)_{EE}} \quad (III-36)$$

which is shown by curve HH. A more complete discussion on the development of a correlation based on η_0 is given in the next section.

III-4 Development of Correlation for the Prediction of η_0

The last section of this report is devoted to a discussion on the establishment of a generalized correlation which can be used for the prediction of the initial collection efficiency of granular filter beds when the inertial impaction is the dominant mechanism of collection. Such a correlation is not available at present time even though granular sand filters have been widely used for gas cleaning. Several empirical correlations have been proposed in the past. These expressions are usually based on a rather small amount of data (usually the investigator's own results) covering a limited range of relevant parameters. As a result, most of these expressions do not include all the relevant dimensionless groups which are known to have strong influence on collection efficiency.

There are a number of reasons which have retarded the development of a generalized correlation of η_0 . The necessary experimental data are insufficient at the present time. Furthermore, the accuracy of this data is not high. This, plus the fact that the collection efficiency depends upon a large number of variables (and therefore large number of dimensionless parameters), make it unlikely that a generalized correlation can be established from experimental data alone.

The trajectory calculation presented in this report indicates that with the use of the constricted tube model, the predicted η_0 is of the

same order of magnitude as the experimental value. This represents a significant improvement over the earlier predictions (Paretsky et al., 1971; Pendse et al., 1978). Through a combination of the predictions based on different tube geometries, the predicted values can be made fairly close to those of experimental data. The trajectory calculation also permits a systematic examination of the effect of a large number of dimensionless groups over considerable ranges, which is often difficult to implement experimentally.

The correlation developed in this work is based upon the results of the trajectory calculation and their comparison with available data. The trajectory calculation reveals the functional relationship between n_0 and the various dimensionless groups, while the experimental data is used to obtain the proper combination of the unit cell geometries.

III-4.A. Available Experimental Data

The experimental data used are those reported by Knetting and Beeckmans (1974a) Doganoglu (1975) and Melcher et. al. (1978). All these data were obtained under conditions where the dominant mechanism of collection was inertial impaction. Data of Doganoglu and Melcher et al. were obtained using liquid aerosols; thus the possible effect due to deposited particles was negligible. Knettig and Beeckmans' data were obtained over a relatively short period of filtration. Thus the collection efficiency they obtained can be considered to be that of the initial period. The ranges of relevant dimensionless parameters covered by these experiments are

$$0.0004 < N_R < 0.02$$

$$0.0 < N_{Re,s} < 140$$

$$0.001 < N_{St} < 0.1$$

$$10^4 < N_{Re} < 10^9$$

$$0.00005 < N_G < 0.01$$

The conditions used in these experiments are tabulated in Table (III-7). Table (III-8) summarizes these experimental data. Calculations of $(\eta_o)_{\text{expt}}$ are based on Equation (III-17), which relates η with filter coefficient, λ .

A number of other investigators have also reported data on collection efficiency of granular filter bed when the inertial effect is dominant (Paretsky, 1971; Meisen and Mathur, 1974; Thambimuthu et al., (1978). Unfortunately, these data were reported with incomplete information or only graphically and therefore cannot be used in this work. The correlations developed on the basis of these data, however, are included in the subsequent comparisons.

III-4.B. Existing Correlations of η_o

A number of correlations for the estimation of the collection efficiency of granular bed have been proposed in the past. The correlations are either for the total collection efficiency or collection efficiency due to one (inertial impaction) or two (inertial impaction and gravitation) mechanisms.

(i) Correlation by Paretsky (1972) [quoted in Dagonoglu (1975)]

$$(\eta_o)_i = 2.0 N_{St}^{1.13} \quad \text{(III-37)}$$

(ii) Correlation by Meisen and Mathur (1974)

$$(\eta_o)_i = 0.00075 + 2.6 N_{St} \quad \text{(III-38)}$$

Table (III-7)

Summary of Experimental Conditions

Investigator	Doganoglu (1975)	Knettig and Beeckmans (1974)	Melcher et. al. (1978)
Granular medium	glass beads	glass beads	glass beads
Aerosol particles	DOP (dioctyl- phthalate)	Uranine and Methylene blue (1:2 wt ratio)	DOP (dioctyl- phthalate)
Particle diameter	1.35 μm , 1.75 μm	0.8 μm - 2.9 μm	1.5 μm - 4 μm
Grain diameter	110 μm , 600 μm	425 μm	500 μm - 400 μm
Flow direction	downflow	upflow	downflow

Serial Numbers of
Data Listed in

Tables (III-8) to (III-10f)	19 through 54	1 through 18	55 through 60
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Symbols used in

Figures (III-11) to (III-16)	0	X	Δ
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Table III-8

Summary of Experimental Data

SR. NO.	d_p μm	d_g μm	V_s cm/s	ϵ_0	$(\eta_0)_{\text{expt}}$
1	1.50	500.	20.00	0.39	0.0063
2	1.50	500.	51.00	0.39	0.0337
3	2.53	500.	20.00	0.39	0.0139
4	2.53	500.	51.00	0.39	0.0763
5	4.00	500.	20.00	0.39	0.0375
6	4.00	500.	51.00	0.39	0.0914
7	1.50	1000.	20.00	0.38	0.0336
8	1.50	1000.	100.00	0.38	0.0767
9	2.53	1000.	20.00	0.38	0.0730
10	2.53	1000.	100.00	0.38	0.1559
11	4.00	1000.	20.00	0.38	0.1404
12	4.00	1000.	100.00	0.38	0.2097
13	1.50	4000.	40.00	0.36	0.0026
14	1.50	4000.	240.00	0.36	0.2456
15	2.53	4000.	40.00	0.36	0.0433
16	2.53	4000.	240.00	0.36	0.5817
17	4.00	4000.	40.00	0.36	0.2290
18	4.00	4000.	240.00	0.36	0.8125
19	1.35	109.	0.98	0.40	0.0328
20	1.35	109.	2.02	0.40	0.0355
21	1.35	109.	2.69	0.40	0.0343
22	1.35	109.	3.83	0.40	0.0371
23	1.35	109.	3.83	0.40	0.0369
24	1.35	109.	4.92	0.40	0.0253
25	1.35	109.	6.04	0.40	0.0259
26	1.35	109.	8.69	0.40	0.0302
27	1.35	109.	10.53	0.40	0.0278
28	1.35	109.	12.37	0.40	0.0367
29	1.35	109.	13.19	0.40	0.0572
30	1.35	109.	19.53	0.40	0.0838

Table III-8 (continued)

31	1.75	109.	0.98	0.40	0.0489
32	1.75	109.	2.02	0.40	0.0577
33	1.75	109.	2.69	0.40	0.0501
34	1.75	109.	3.83	0.40	0.0354
35	1.75	109.	3.83	0.40	0.0385
36	1.75	109.	4.92	0.40	0.0412
37	1.75	109.	8.69	0.40	0.0415
38	1.75	109.	10.53	0.40	0.0441
39	1.75	109.	12.37	0.40	0.0442
40	1.75	109.	13.19	0.40	0.0737
41	1.75	109.	19.53	0.40	0.0927
42	1.35	596.	2.86	0.40	0.0012
43	1.35	596.	3.83	0.40	0.0007
44	1.35	596.	12.37	0.40	0.0001
45	1.35	596.	19.51	0.40	0.0001
46	1.35	596.	31.46	0.40	0.0029
47	1.35	596.	43.80	0.40	0.0081
48	1.75	596.	2.86	0.40	0.0051
49	1.75	596.	3.83	0.40	0.0042
50	1.75	596.	6.04	0.40	0.0030
51	1.75	596.	12.37	0.40	0.0033
52	1.75	596.	19.51	0.40	0.0024
53	1.75	596.	31.46	0.40	0.0063
54	1.75	596.	43.80	0.40	0.0151
55	0.80	425.	8.19	0.38	0.0023
56	0.80	425.	11.20	0.38	0.0025
57	1.60	425.	8.19	0.38	0.0035
58	1.60	425.	11.20	0.38	0.0037
59	2.90	425.	8.19	0.38	0.0125
60	2.90	425.	11.20	0.38	0.0142

(iii) Correlation by Knetting and Beeckmans (1974b)

$$(\eta_o) = 3.76 \cdot 10^{-3} - 0.454 N_{St} + 9.68 N_{St}^2 - 16.2 N_{St}^3 \quad \text{for } N_{St} > 0.0416 \quad (\text{III-39})$$

(iv) Correlation by Meltcher et. al. (1978)

$$(\eta_o)_i = \beta N_{St} \quad (\text{III-40a})$$

where the dependence of β on $N_{Re,s}$ for $N_{Re,s} < 100$ may be expressed as:

$$\beta = \begin{cases} 1.0 & \dots N_{Re,s} < 100 \\ 0.01 N_{Re,s} & \dots 100 > N_{Re,s} < 1000 \\ 10 & \dots N_{Re,s} > 1000 \end{cases} \quad (\text{III-40b})$$

(v) Correlation by Dagonoglu (1975)

$$(\eta_o)_{G,i} = 2.89 N_{St} + 6.89 N_G \quad \text{for small grains } d_g = 100 \mu m \quad (\text{III-41a})$$

and

$$(\eta_o)_{G,i} = 0.0583 N_{Re,s} N_{St} + 1.42 N_G \quad \text{for large grains } d_g = 600 \mu m \quad (\text{III-41b})$$

(vi) Correlation by Thambimuthu et. al. (1978)

$$(\eta_o)_i = 10^5 \cdot N_{St}^3 \quad \text{for } 10^{-3} < N_{St} < 10^{-2} \quad (\text{III-42})$$

Most of the correlations were developed with the investigator's own data and their differences are quite significant. Some of these differences undoubtedly are due to the widely different conditions used to obtain the data. For example, the correlation of Meltcher et. al. (1978) is based on data of high Reynolds number [$10 < N_{Re,s} < 640$] while most of the other correlations are based on results corresponding to small $N_{Re,s}$ [$N_{Re,s} \ll 10$]. The expression of Meisen and Mathur (1974) was based on spouted bed data with the assumption that within a spouted bed, the annular region is in the incipient stage (i.e. about to be fluidized).

Since these expressions are based on limited data, the incorporation of all the relevant dimensionless parameters is not possible. Their use as generalized correlations for the estimation of η_0 is therefore questionable.

III-4.C. Procedure for Developing Correlation

The proposed correlation is established with the following premises:

(i) The dominant mechanism of collection is by inertial impaction and interception. The contribution by other mechanisms such as gravitational and Brownian diffusion are secondary. The total collection can be approximated by the addition of the individual contributions.*

(ii) The effect of the fluid inertia on the collection efficiency due to inertial impaction and interception revealed by the trajectory calculation (see Fig. III-9) is assumed to be valid. With the use of Equation (III-19), one has

$$\frac{(\eta_0)_{I,i}}{[(\eta_0)_{I,i}]_{N_{Re,s}=0}} = 1 + 0.04 N_{Re,s} \quad (III-43)$$

where the subscripts, I, i, denote contributions due to interception and inertial impaction.

Equation (III-36) is used to estimate $[(\eta_0)_{I,i}]_{N_{Re,s}=0}$. This is tantamount to assuming that the findings of the trajectory calculation, as shown in Figs. III-10a and III-10b through a combination of unit cell geometries, provide a good description for granular media.

$$[(\eta_0)_{I,i}]_{N_{Re,s}=0} = N_{St} + [\{(\eta_0)_{I,i}\}_p \cdot \{(\eta_0)_{I,i}\}_s]^{1/2} \quad (III-44)$$

* This assumption of additivity is incorrect in the strict sense as pointed out by Löffler (1979). However when the inertial impaction is the dominant mechanism, the use of this assumption should not introduce any significant error.

where the subscripts p and s refer to the parabolic geometry (collocation solution for flow field) and sinusoidal geometry (perturbation solution for the flow field) and the respective collection efficiencies are at $N_{Re,s} = 0$ and small values of N_{St} such that the collection efficiency is independent of N_{St} .

The results of Figs. III-10a and III-10b suggest that, with the use of the parabolic tube geometry and with the flow field given by the collocation solution (i.e. curve EE) and at small values of N_{St} , $[(\eta_0)_{I,i}]_p$ is a function of N_R only and is close to $[(\eta_0)_I]_p$. Furthermore, from the simplified flow expression, $[(\eta_0)_I]_p$ is given by Equation (III-18), or

$$[(\eta_0)_{I,i}]_p = N_R^2 \cdot \left\{ \frac{4}{d_c^{*2}} - \frac{4 N_R}{d_c^{*3}} - \frac{N_R^2}{d_c^{*4}} \right\} \quad (III-18)$$

On the other hand, the collection efficiency due to inertial impaction and interception at $N_{Re,s} = 0$ and low N_{St} obtained with the use of the sinusoidal geometry and the flow field given by the perturbation solution, is found to be [Equation (A-1), Appendix].

$$[(\eta_0)_{I,i}]_s = 0.2302 N_R^{0.08243} \quad (III-45)$$

The derivations of the above expression from the results of the trajectory calculations are given in Appendix.

By combining Equations (III-34), (III-44), (III-45) and (III-18), the final correlation is established as follows

$$(\eta_0)_{I,i} = (1 + 0.04 N_{Re,s}) \cdot [N_{St} + \{0.2302 (4 - \frac{4N_R}{d_c^{*2}} - \frac{N_R^2}{d_c^{*2}})^{1/2} \cdot (\frac{N_R^{1.041215}}{d_c^{*2}})\}] \quad (III-46)$$

III-4.D. Comparisons

The validity of the generalized correlation proposed in the preceding section can be assessed by comparing its agreement with experimental data with those of other correlations. For this purpose, the data listed in Table (III-8) will be used. First, the experimental value of the collection efficiency has to be corrected for contributions due to mechanisms other than interception and inertial impaction. The collection

efficiencies due to interception. Brownian diffusion and gravity are known to be (Rajagopalan and Tien, 1976)

$$(\eta_o)_I = 1.5 A_s (1 - \epsilon)^{2/3} N_R^2 \quad (\text{III-47})$$

$$(\eta_o)_D = 4 A_s^{1/3} N_{Pe}^{-2/3} \quad (\text{III-48})$$

$$(\eta_o)_G = (1 - \epsilon)^{2/3} N_G \quad (\text{III-49})$$

where the subscripts, I, D, and G denote interception, Brownian diffusion and gravity, respectively. The corrected experimental values used for comparison are

$$(\eta_o)_{\text{expt},i} = (\eta_o)_{\text{expt}} - [(\eta_o)_I + (\eta_o)_D + (\eta_o)_G] \quad (\text{III-50})$$

$$(\eta_o)_{\text{expt},I,i} = (\eta_o)_{\text{expt}} - [(\eta_o)_D + (\eta_o)_G] \quad (\text{III-51})$$

$$(\eta_o)_{\text{expt},I,i,G} = (\eta_o)_{\text{expt}} - (\eta_o)_D \quad (\text{III-52})$$

Equation (III-46) as well as the various earlier correlations are compared with the appropriate experimentally determined values of η . The relative contribution due to the individual mechanisms (i.e. interception, Brownian diffusion and gravitation) can be estimated by using Equations (III-47) - (III-49). As shown in Table (III-9), these contributions are relatively modest in most cases (see values under column with headings f_R , f_G and f_D) and the effect of the inertial impaction is dominant. However, for a number of runs reported by Dagonoglu (1975), the total collection was found to be considerably less than the individual contributions (see cases 42 - 45, Table (III-9). These data therefore were not used in the comparison.

Table III-9

Dimensionless Parameters and Collection Efficiency Contributions

SR. NO.	$N_{Re,s}$	N_R	N_G	N_{Pe}	N_{St}	f_R	f_G	f_D
1	6.67	0.0030	0.0004	6.3E6	0.0061	6.25	4.27	6.42
2	17.00	0.0030	0.0001	1.6E7	0.0155	1.16	0.31	0.64
3	6.67	0.0051	0.0010	1.1E7	0.0166	8.04	5.27	2.05
4	17.00	0.0051	0.0004	2.7E7	0.0423	1.46	0.33	0.20
5	6.67	0.0080	0.0025	1.7E7	0.0405	7.44	4.76	0.56
6	17.00	0.0080	0.0010	4.3E7	0.1033	3.05	0.77	0.12
7	13.33	0.0015	0.0004	1.3E7	0.0030	0.32	0.81	0.77
8	66.67	0.0015	0.0001	6.3E7	0.0152	0.14	0.07	0.12
9	13.33	0.0025	0.0010	2.1E7	0.0083	0.41	1.01	0.25
10	66.67	0.0025	0.0002	1.1E8	0.0415	0.19	0.09	0.04
11	13.33	0.0040	0.0025	3.4E7	0.0202	0.54	1.29	0.10
12	66.67	0.0040	0.0005	1.7E8	0.1012	0.36	0.17	0.02
13	106.67	0.0004	0.0002	1.0E8	0.0015	0.30	5.36	2.62
14	640.00	0.0004	0.0000	6.0E8	0.0091	0.00	0.01	0.01
15	106.67	0.0006	0.0005	1.7E8	0.0041	0.05	0.87	0.11
16	640.00	0.0006	0.0001	1.0E9	0.0249	0.00	0.01	0.00
17	106.67	0.0010	0.0012	2.7E8	0.0101	0.02	0.40	0.02
18	640.00	0.0010	0.0002	1.6E9	0.0607	0.01	0.02	0.00
19	0.07	0.0124	0.0062	6.0E4	0.0011	20.58	13.66	27.25
20	0.15	0.0124	0.0030	1.2E5	0.0023	19.02	6.12	15.55
21	0.19	0.0124	0.0023	1.7E5	0.0031	19.68	4.76	13.29
22	0.28	0.0124	0.0016	2.4E5	0.0044	18.20	3.09	9.71
23	0.28	0.0124	0.0016	2.4E5	0.0044	18.30	3.11	9.76
24	0.36	0.0124	0.0012	3.0E5	0.0056	26.69	3.53	12.05
25	0.44	0.0124	0.0010	3.7E5	0.0069	26.07	2.81	10.27
26	0.63	0.0124	0.0007	5.3E5	0.0100	22.36	1.67	6.91
27	0.76	0.0124	0.0006	6.5E5	0.0121	24.29	1.50	6.60
28	0.89	0.0124	0.0005	7.6E5	0.0142	18.40	0.97	4.49
29	0.95	0.0124	0.0005	8.1E5	0.0151	11.80	0.58	2.76
30	1.41	0.0124	0.0003	1.2E6	0.0224	8.06	0.27	1.45

Table III-9 (continued)

31	0.07	0.0161	0.0102	7.8E4	0.0018	23.20	15.01	15.38
32	0.15	0.0161	0.0049	1.6E5	0.0038	19.66	6.17	8.05
33	0.19	0.0161	0.0037	2.1E5	0.0051	22.64	5.34	7.66
34	0.28	0.0161	0.0026	3.0E5	0.0072	32.05	5.30	8.54
35	0.28	0.0161	0.0026	3.0E5	0.0072	29.47	4.88	7.87
36	0.36	0.0161	0.0020	3.9E5	0.0092	27.54	3.55	6.22
37	0.63	0.0161	0.0012	6.9E5	0.0163	27.34	1.99	4.23
38	0.76	0.0161	0.0009	8.4E5	0.0198	25.73	1.55	3.50
39	0.89	0.0161	0.0008	9.8E5	0.0232	25.67	1.32	3.14
40	0.95	0.0161	0.0008	1.0E6	0.0248	15.39	0.74	1.80
41	1.41	0.0161	0.0005	1.6E6	0.0367	12.24	0.40	1.10
42	1.14	0.0023	0.0021	9.6E5	0.0006	18.04	123.82	113.39
43	1.52	0.0023	0.0016	1.3E6	0.0008	31.08	159.23	160.73
44	4.92	0.0023	0.0005	4.2E6	0.0026	223.75	354.98	529.64
45	7.75	0.0023	0.0003	6.6E6	0.0041	372.92	375.11	651.49
46	12.50	0.0023	0.0002	1.1E7	0.0066	7.85	4.90	9.97
47	17.40	0.0023	0.0001	1.5E7	0.0091	2.76	1.24	2.81
48	1.14	0.0029	0.0035	1.3E6	0.0010	7.30	48.83	22.96
49	1.52	0.0029	0.0026	1.7E6	0.0013	9.02	45.03	23.34
50	2.40	0.0029	0.0017	2.6E6	0.0021	12.66	40.09	24.19
51	4.92	0.0029	0.0008	5.4E6	0.0042	11.32	17.51	13.42
52	7.75	0.0029	0.0005	8.5E6	0.0067	15.67	15.36	13.70
53	12.50	0.0029	0.0003	1.4E7	0.0108	5.95	3.62	3.78
54	17.40	0.0029	0.0002	1.9E7	0.0150	2.49	1.09	1.27
55	2.32	0.0019	-0.0003	1.2E6	0.0009	7.17	8.83	54.51
56	3.17	0.0019	-0.0002	1.6E6	0.0012	6.63	5.97	40.90
57	2.32	0.0038	-0.0010	2.3E6	0.0033	19.86	21.19	22.57
58	3.17	0.0038	-0.0008	3.2E6	0.0045	17.90	14.71	17.38
59	2.32	0.0068	-0.0032	4.2E6	0.0104	17.44	18.76	4.27
60	3.17	0.0068	-0.0024	5.8E6	0.0142	15.39	12.11	3.06

The results of comparisons between the experimentally determined η and the correlation proposed in this work [i.e. Equations (III-46)] and the earlier ones are summarized in Table (III-10 a-f) as well as shown in Figs. III-11 to III-16 in which the ratio of the predicted collection efficiency to the experimental value, $\frac{(\eta_o)_{\text{predicted}}}{(\eta_o)_{\text{expt}}}$ is plotted as a function of $(\eta_o)_{\text{expt}}$. In making the comparisons, the correlation proposed by Knetting and Beeckman was not used since all the data listed in Table (III-9) were obtained with $N_{St} < 0.0416$.

The degree of agreement between correlations and experiment can be seen from the scattering of data points away from the horizontal line representing $(\eta_o)_{\text{predicted}}/(\eta_o)_{\text{expt}} = 1$. As seen from Figs. III-11 to III-16, the correlation established in this work [i.e. Equation (III-46)] gives much better prediction than all the previous correlations, with the exception of Dagonoglu's. Even though the degree of agreement of Dagonoglu's correlation with experimental data, on the average, is similar to that of Equation (III-46), it agrees much better with Dagonoglu's own data (indicated by the symbol η in Fig. III-14) than those obtained by other investigators. Equation (III-45) also represents a complete correlation since it incorporates all the theoretically significant dimensionless parameters.

Although the present correlation is more general and represents considerable improvement over previously established correlations, it by no means should be considered as definitive. The correlation requires further and more extensive testing against experiments. For this purpose, more accurate data on aerosol filtration in granular beds are urgently needed. It should also be mentioned that in the trajectory calculation and in the development of the correlation of this study, it is assumed that particle deposition occurs once a particle makes contact with the

Comparison of Experimental Data with Predicted Values
Based on the Correlation of Paretsky (1972) [Eq. (III-37)]

SR. NO.	$(\eta_o)_{\text{expt}}$	$(\eta_o)_{\text{predicted}}$	$\frac{(\eta_o)_{\text{predicted}}}{(\eta_o)_{\text{expt}}}$
1	0.0052	0.0063	1.2004
2	0.0330	0.0180	0.5467
3	0.0117	0.0195	1.6567
4	0.0747	0.0561	0.7501
5	0.0327	0.0534	1.6323
6	0.0878	0.1537	1.7511
7	0.0329	0.0029	0.0869
8	0.0765	0.0176	0.2306
9	0.0718	0.0089	0.1239
10	0.1554	0.0548	0.3527
11	0.1377	0.0244	0.1771
12	0.2085	0.1503	0.7210
13	0.0024	0.0013	0.5525
14	0.2455	0.0099	0.0403
15	0.0429	0.0041	0.0948
16	0.5816	0.0308	0.0529
17	0.2280	0.0111	0.0489
18	0.8123	0.0844	0.1039
19	0.0127	0.0009	0.0734
20	0.0211	0.0021	0.0999
21	0.0214	0.0029	0.1362
22	0.0256	0.0043	0.1694
23	0.0254	0.0043	0.1708
24	0.0146	0.0058	0.3940
25	0.0158	0.0073	0.4604
26	0.0209	0.0110	0.5250
27	0.0188	0.0136	0.7238
28	0.0279	0.0163	0.5840
29	0.0485	0.0176	0.3616
30	0.0756	0.0273	0.3617
31	0.0228	0.0016	0.0713
32	0.0382	0.0037	0.0963
33	0.0323	0.0051	0.1575
34	0.0192	0.0076	0.3953
35	0.0223	0.0076	0.3403
36	0.0258	0.0101	0.3891
37	0.0276	0.0191	0.6934
38	0.0305	0.0238	0.7731
39	0.0309	0.0285	0.9226
40	0.0605	0.0306	0.5067
41	0.0800	0.0478	0.5972
48	0.0011	0.0008	0.7196
49	0.0010	0.0011	1.1473
50	0.0007	0.0018	2.6501
51	0.0019	0.0042	2.1617
52	0.0013	0.0070	5.2294
53	0.0055	0.0119	2.1792
54	0.0144	0.0174	1.2068
55	0.0011	0.0007	0.6668
56	0.0015	0.0010	0.7082
57	0.0028	0.0032	1.1194
58	0.0030	0.0045	1.5193
59	0.0122	0.0115	0.9453
60	0.0133	0.0164	1.2312

* [See Note (1)]

44%*

31%*

100%*

Comparison of Experimental Data with Predicted Values

Based on the Correlation of Meisen and Mathur (1974) [Eq. (III-38)]

SR. NO.	$(\eta_o)_{\text{expt}}$	$(\eta_o)_{\text{predicted}}$	$\frac{(\eta_o)_{\text{predicted}}}{(\eta_o)_{\text{expt}}}$	
1	0.0052	0.0166	3.1733	
2	0.0330	0.0410	1.2444	
3	0.0117	0.0439	3.7335	
4	0.0747	0.1107	1.4812	
5	0.0327	0.1060	3.2424	
6	0.0878	0.2692	3.0666	
7	0.0329	0.0087	0.2629	
8	0.0765	0.0403	0.5265	
9	0.0718	0.0223	0.3108	
10	0.1554	0.1085	0.6984	
11	0.1377	0.0534	0.3877	
12	0.2085	0.2640	1.2659	
13	0.0024	0.0047	1.9867	*[See Note (1)]
14	0.2455	0.0245	0.0996	
15	0.0429	0.0115	0.2690	
16	0.5816	0.0654	0.1125	
17	0.2280	0.0271	0.1187	
18	0.8123	0.1587	0.1954	33%*
19	0.0127	0.0037	0.2897	
20	0.0211	0.0068	0.3214	
21	0.0214	0.0088	0.4105	
22	0.0256	0.0122	0.4753	
23	0.0254	0.0122	0.4791	
24	0.0146	0.0154	1.0553	
25	0.0158	0.0188	1.1899	
26	0.0209	0.0267	1.2784	
27	0.0188	0.0322	1.7105	
28	0.0279	0.0376	1.3470	
29	0.0485	0.0401	0.8260	
30	0.0756	0.0590	0.7803	
31	0.0228	0.0055	0.2431	
32	0.0382	0.0106	0.2780	
33	0.0323	0.0139	0.4304	
34	0.0192	0.0195	1.0152	
35	0.0223	0.0195	0.8739	
36	0.0258	0.0248	0.9589	
37	0.0276	0.0432	1.5663	
38	0.0305	0.0522	1.7091	
39	0.0309	0.0612	1.9803	
40	0.0605	0.0652	1.0776	
41	0.0800	0.0962	1.2024	
48	0.0011	0.0033	2.9819	
49	0.0010	0.0042	4.3133	
50	0.0007	0.0061	8.7702	
51	0.0019	0.0118	6.1088	
52	0.0013	0.0181	13.6022	
53	0.0055	0.0287	5.2432	
54	0.0144	0.0397	2.7608	42%*
55	0.0011	0.0031	2.8409	
56	0.0015	0.0040	2.7089	
57	0.0028	0.0094	3.3236	
58	0.0030	0.0125	4.2380	
59	0.0122	0.0278	2.2861	*
60	0.0133	0.0378	2.8381	0%

Comparison of Experimental Data with Predicted Values

Based on the Correlation of Meltcher et. al. (1978) [Eq. (III-40a,b)]

SR. NO.	$(\eta_o)_{\text{expt}}$	$(\eta_o)_{\text{predicted}}$	$\frac{(\eta_o)_{\text{predicted}}}{(\eta_o)_{\text{expt}}}$
1	0.0052	0.0061	1.1652
2	0.0330	0.0155	0.4699
3	0.0117	0.0166	1.4114
4	0.0747	0.0423	0.5658
5	0.0327	0.0405	1.2383
6	0.0878	0.1033	1.1762
7	0.0329	0.0030	0.0923
8	0.0765	0.0152	0.1987
9	0.0718	0.0083	0.1155
10	0.1554	0.0415	0.2668
11	0.1377	0.0202	0.1470
12	0.2085	0.1012	0.4855
13	0.0024	0.0016	0.6850
14	0.2455	0.0584	0.2376
15	0.0429	0.0044	0.1032
16	0.5816	0.1592	0.2737
17	0.2280	0.0108	0.0474
18	0.8123	0.3888	0.4786
19	0.0127	0.0011	0.0887
20	0.0211	0.0023	0.1099
21	0.0214	0.0031	0.1444
22	0.0256	0.0044	0.1715
23	0.0254	0.0044	0.1729
24	0.0146	0.0056	0.3861
25	0.0158	0.0069	0.4394
26	0.0209	0.0100	0.4778
27	0.0188	0.0121	0.6425
28	0.0279	0.0142	0.5077
29	0.0485	0.0151	0.3117
30	0.0756	0.0224	0.2963
31	0.0228	0.0018	0.0808
32	0.0382	0.0038	0.0994
33	0.0323	0.0051	0.1566
34	0.0192	0.0072	0.3754
35	0.0223	0.0072	0.3232
36	0.0258	0.0092	0.3577
37	0.0276	0.0163	0.5919
38	0.0305	0.0198	0.6479
39	0.0309	0.0232	0.7523
40	0.0605	0.0248	0.4097
41	0.0800	0.0367	0.4588
48	0.0011	0.0010	0.8857
49	0.0010	0.0013	1.3596
50	0.0007	0.0021	2.9599
51	0.0019	0.0042	2.1996
52	0.0013	0.0067	5.0148
53	0.0055	0.0108	1.9640
54	0.0144	0.0150	1.0418
55	0.0011	0.0009	0.8289
56	0.0015	0.0012	0.8452
57	0.0028	0.0033	1.1759
58	0.0030	0.0045	1.5324
59	0.0122	0.0104	0.8556
60	0.0133	0.0142	1.0699

*[See Note (1)]

33%*

25%*

100%*

Comparison of Experimental Data with Predicted Values

Based on the Correlation of Doganoglu (1975) [Eq. III-41a,b)]

SR. NO.	$(\eta_o)_{\text{expt}}$	$(\eta_o)_{\text{predicted}}$	$\frac{(\eta_o)_{\text{predicted}}}{(\eta_o)_{\text{expt}}}$	
1	0.0055	0.0018	0.3343	
2	0.0331	0.0152	0.4580	
3	0.0125	0.0050	0.4007	
4	0.0750	0.0413	0.5511	
5	0.0345	0.0122	0.3541	
6	0.0885	0.1010	1.1409	
7	0.0332	0.0018	0.0552	
8	0.0765	0.0590	0.7704	
9	0.0725	0.0050	0.0690	
10	0.1555	0.1608	1.0340	
11	0.1395	0.0122	0.0875	
12	0.2089	0.3928	1.8804	*[See Note (1)]
13	0.0025	0.0092	3.6674	
14	0.2456	0.3402	1.3851	
15	0.0432	0.0251	0.5796	
16	0.5817	0.9279	1.5952	
17	0.2289	0.0612	0.2673	
18	0.8124	2.2662	2.7894	44%*
19	0.0171	0.0462	2.6985	
20	0.0232	0.0275	1.1847	
21	0.0230	0.0246	1.0681	
22	0.0267	0.0237	0.8854	
23	0.0265	0.0237	0.8921	
24	0.0155	0.0249	1.6040	
25	0.0165	0.0270	1.6367	
26	0.0214	0.0337	1.5753	
27	0.0192	0.0389	2.0250	
28	0.0283	0.0444	1.5693	
29	0.0489	0.0469	0.9601	
30	0.0758	0.0669	0.8822	
31	0.0300	0.0756	2.5175	
32	0.0417	0.0451	1.0805	
33	0.0349	0.0402	1.1517	
34	0.0210	0.0388	1.8446	
35	0.0241	0.0388	1.6076	
36	0.0273	0.0407	1.4920	
37	0.0284	0.0551	1.9406	
38	0.0312	0.0637	2.0415	
39	0.0315	0.0727	2.3114	
40	0.0610	0.0768	1.2592	
41	0.0803	0.1096	1.3640	
48	0.0036	0.0050	1.4003	
49	0.0028	0.0038	1.3555	
50	0.0019	0.0026	1.4074	
51	0.0025	0.0024	0.9446	
52	0.0017	0.0037	2.2083	
53	0.0057	0.0083	1.4537	
54	0.0145	0.0155	1.0675	67%*
55	0.0009	0.0005	0.5881	
56	0.0013	0.0005	0.3963	
57	0.0021	0.0019	0.9233	
58	0.0024	0.0019	0.7917	
59	0.0098	0.0060	0.6115	
60	0.0116	0.0060	0.5173	83%*

Comparison of Experimental Data with Predicted Values

Based on the Correlation of Thambimuthu et. al. (1978) [Eq. (II-42)]

SR. NO.	$(\eta_o)_{\text{expt}}$	$(\eta_o)_{\text{predicted}}$	$\frac{(\eta_o)_{\text{predicted}}}{(\eta_o)_{\text{expt}}}$
1	0.0052	0.0225	4.3051
2	0.0330	0.3724	11.2889X
3	0.0117	0.4559	38.8039X
4	0.0747	7.5588	101.1563X
5	0.0327	6.6408	203.0594X
6	0.0878	110.1133	1254.1896X
7	0.0329	0.0028	0.0853
8	0.0765	0.3509	4.5887X
9	0.0718	0.0570	0.7939
10	0.1554	7.1229	45.8389X
11	0.1377	0.8301	6.0279X
12	0.2085	103.7622	497.6099X
13	0.0024	0.0004	0.1483X
14	0.2455	0.0758	0.3087
15	0.0429	0.0071	0.1662
16	0.5816	1.5385	2.6454X
17	0.2280	0.1038	0.4551
18	0.8123	22.4126	27.5922X 5.5%*
19	0.0127	0.0001	0.0112
20	0.0211	0.0012	0.0590
21	0.0214	0.0029	0.1375
22	0.0256	0.0085	0.3312
23	0.0254	0.0085	0.3338
24	0.0146	0.0180	1.2301
25	0.0158	0.0333	2.1094
26	0.0209	0.0991	4.7489
27	0.0188	0.1763	9.3759X
28	0.0279	0.2858	10.2246X
29	0.0485	0.3464	7.1373X
30	0.0756	1.1246	14.8738X
31	0.0228	0.0006	0.0274
32	0.0382	0.0055	0.1431
33	0.0323	0.0129	0.4000
34	0.0192	0.0373	1.9439
35	0.0223	0.0373	1.6733
36	0.0258	0.0790	3.0561
37	0.0276	0.4352	15.7792X
38	0.0305	0.7743	25.3586X
39	0.0309	1.2553	40.6342X
40	0.0605	1.5219	25.1603X
41	0.0800	4.9403	61.7780X
48	0.0011	0.0001	0.0848
49	0.0010	0.0002	0.2333
50	0.0007	0.0009	1.2632
51	0.0019	0.0076	3.9374
52	0.0013	0.0297	22.3305
53	0.0055	0.1246	22.7395X
54	0.0144	0.3362	23.3809X 11%*
55	0.0011	0.0001	0.0681
56	0.0015	0.0002	0.1299
57	0.0028	0.0036	1.2889
58	0.0030	0.0093	3.1412
59	0.0122	0.1129	9.2749
60	0.0133	0.2887	21.6903X 16.67%*

Comparison of Experimental Data with Predicted Values

Based on the Correlation of Present Work [Eq. (III-46)]

SR. NO.	$(\eta_o)_{\text{expt}}$	$(\eta_o)_{\text{predicted}}$	$\frac{(\eta_o)_{\text{predicted}}}{(\eta_o)_{\text{expt}}}$
1	0.0056	0.0116	2.0671
2	0.0334	0.0312	0.9349
3	0.0129	0.0277	2.1529
4	0.0758	0.0799	1.0537
5	0.0355	0.0620	1.7476
6	0.0906	0.1877	2.0723
7	0.0330	0.0070	0.2107
8	0.0766	0.0612	0.7993
9	0.0721	0.0167	0.2312
10	0.1557	0.1614	1.0370
11	0.1385	0.0374	0.2701
12	0.2093	0.3864	1.8464
13	0.0024	0.0099	4.1570
14	0.2456	0.2519	1.0260
15	0.0429	0.0250	0.5841
16	0.5816	0.6778	1.1654
17	0.2280	0.0585	0.2565
18	0.8123	1.6419	2.0212
19	0.0194	0.0145	0.7465
20	0.0278	0.0157	0.5658
21	0.0281	0.0166	0.5885
22	0.0324	0.0179	0.5540
23	0.0322	0.0179	0.5574
24	0.0214	0.0193	0.9010
25	0.0225	0.0206	0.9156
26	0.0276	0.0239	0.8653
27	0.0256	0.0262	1.0251
28	0.0347	0.0285	0.8217
29	0.0553	0.0296	0.5346
30	0.0824	0.0378	0.4585
31	0.0341	0.0193	0.5649
32	0.0495	0.0213	0.4299
33	0.0436	0.0226	0.5183
34	0.0305	0.0248	0.8143
35	0.0336	0.0248	0.7392
36	0.0372	0.0270	0.7260
37	0.0389	0.0346	0.8877
38	0.0419	0.0383	0.9144
39	0.0422	0.0421	0.9961
40	0.0718	0.0438	0.6093
41	0.0913	0.0571	0.6256
48	0.0015	0.0042	2.8146
49	0.0013	0.0046	3.4194
50	0.0011	0.0056	5.1775
51	0.0023	0.0087	3.7666
52	0.0017	0.0127	7.4311
53	0.0059	0.0207	3.5273
54	0.0148	0.0305	2.0677
55	0.0013	0.0031	2.4302
56	0.0016	0.0035	2.1635
57	0.0035	0.0079	2.2606
58	0.0036	0.0095	2.6205
59	0.0144	0.0192	1.3402
60	0.0155	0.0242	1.5588

*[See Note (1)]

50%*

58%*

33%*

Note (For Tables III-10a to III-10f)

1. The values of fraction of data points for which $(\eta_o)_{\text{predicted}}$ is within factor of 0.5 and 2.0 of $(\eta_o)_{\text{expt}}$, for the three sets of experimental data; are indicated by superscript * in each of these tables.
2. For case of the correlation proposed by Thambimuth et. al. (1978), the Stokes' number corresponding to number of data points is found to be outside the range of N_{St} of the experimental data from which the correlation was obtained, namely $0.001 < N_{St} < 0.01$. These data points are indicated by X in Table III-10e.

FIG. III - II COMPARISON OF EXPERIMENTAL DATA WITH CORRELATION BY PARETSKY (1972)

FIG. III-12 COMPARISON OF EXPERIMENTAL DATA WITH CORRELATION BY MEISEN AND MATHUR (1974)

FIG. III-13 COMPARISON OF EXPERIMENTAL DATA WITH CORRELATION BY MELTCHER ET.AL.(1978)

FIG. III-14 COMPARISON OF EXPERIMENTAL DATA WITH CORRELATION BY DOGANOGLU (1975)

FIG. III-15 COMPARISON OF EXPERIMENTAL DATA WITH CORRELATION BY THAMBIMUTHU ET.AL.(1978)

EQU. III-46

EXPERIMENTAL

○ DOGANOGLU, 1975

x KNETTIG AND BEECKMANS, 1974a

△ MELTCHER, et.al., 1978

$\frac{(\eta_0)_{\text{PREDICTED}}}{(\eta_0)_{\text{expt.}}}$

$(\eta_0)_{\text{expt.}}$

FIG. III-16 COMPARISON OF EXPERIMENTAL DATA WITH PRESENT CORRELATION

collector and the possibility of particles bouncing upon contact is ignored. Particle bouncing is known to be important in fibrous filtration (Loffler, 1979). Since the packing density of granular beds is much greater than that of fibrous beds, the likelihood of particles bouncing off is expected to be less in granular bed. It is however not clear whether this bouncing-off effect can be completely ignored. This is another area of study which merits the attention of future investigators.

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NOTATIONS

a	specific surface area of filter grains
a_p	particle radius
C_s	Cunningham's Slip Factor
d_c	constriction diameter of constricted tube
d_c^*	d_c/d_g
$\langle d_c \rangle$	average constriction diameter
d_g	grain diameter
d_m	maximum diameter (entrance and exit) of constricted tube type unit cell
d_m^*	d_m/d_g
f_B	friction factor (Eq. III-12)
f_s	friction factor (Eq. III-7a)
h	height of unit cell
ℓ	length of unit bed element, or center to center distance between two spheres
N_c	number of constriction per unit bed cross sectional area
N_G	gravity parameter, $2 a_p^2 (\rho_p - \rho_f) g c_s / 9 \mu V_s$
N_G'	$2 a_p^2 (\rho_p - \rho_f) g c_s / 9 \mu V_0$
N_R	relative size parameter, d_p/d_g
$N_{Re,o}$	Reynolds number, $(2 a V_o \rho) / \mu$
$N_{Re,s}$	Reynolds number, $(d_g V_s \rho) / \mu$
N_{St}	Stokes' Number, $\frac{4 a_p^2 V_s \rho_p c_s}{9 \mu d_g}$
N_{St}'	$(4 a_p^2 \rho_p V_o c_s) / (9 \mu h)$
q	volumetric flow rate through a given unit cell
r_w	wall radius
r_w	r_w/r_m
S_{wi}	irreducible saturation
t	time

v_0	mean velocity at throat section of constricted tube, $q/\pi r_0^2$
v_r	radial component of velocity
v_z	axial velocity component
v_z^*	v_z/v_0
v_s	superficial velocity
z	axial distance, bed depth

Greek Letters

δ	perturbation parameter, r_m/h
ϵ	bed porosity
ϵ'	amplitude of variation of tube radius, $(\frac{r_2^* - r_1^*}{2})$
ψ	stream function
ψ^*	normalized stream function
$\hat{\psi}$	$\psi/\hat{V}_m r_m^2$
η_0	initial collector efficiency of unit bed element
$(\eta)_c$	collection efficiency of unit cell
$(\eta_0)_I$	initial collection efficiency contribution due to interception
$(\eta_0)_i$	initial collection efficiency contribution due to particle
λ	filter coefficient
λ_0	initial filter coefficient
μ	fluid viscosity
ρ_f	fluid density
ρ_p	particle density

Subscripts

G	gravity
i	particle inertia
I	interception