[redacted]
DOE/ER/01569-150

OBSERVATION OF QUANTIZED CIRCULATION IN ROTATING SUPERFLUID ^{4}HE

A THESIS

SUBMITTED TO THE FACULTY OF THE GRADUATE SCHOOL

OF THE UNIVERSITY OF MINNESOTA

BY

PATRICK WILLIAM KARN

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

MASTER OF SCIENCE

APRIL 1979

DISCLAIMER

\- - -

OBSERVATION OF QUANTIZED CIRCULATION IN ROTATING SUPERFLUID ^{4}HE

\- - -

A THESIS

SUBMITTED TO THE FACULTY OF THE GRADUATE SCHOOL

OF THE UNIVERSITY OF MINNESOTA

BY

PATRICK WILLIAM KARN

IN PARTIAL FULFILLMENT OF THE REQUIREMENTS

FOR THE DEGREE OF

MASTER OF SCIENCE

\- - -

APRIL 1979

\- - -

CONTENTS

I. INTRODUCTION

The observation of quantized circulation states in superfluid ^4He has had a long and tantalizing history. The first predictions of quantized circulation in superfluid helium were made by Onsager[1] and Feynman.[2] Since the velocity of flow of the helium atoms was assumed to change very little over atomic distances, it was proposed that the helium could be described by a wave function with a coherent phase ϕ such that the momentum of flow per particle is given by

$$m\bar{v}_s = \hbar \text{ grad } \phi, \tag{1}$$

where $\bar{v}_s$ is the superfluid velocity, $\hbar$ is Plancks' constant divided by 2π, and m is the mass of the helium atom. This condition implies that the flow is irrotational, that $\bar{\nabla} \times \bar{v}_s = 0$. The line integral around any closed curve is defined as the circulation κ around that curve:

$$\kappa = \oint \bar{v}_s \cdot \overline{dl}. \tag{2}$$

In a simply-connected region it follows from Stokes' theorem that since $\bar{\nabla} \times \bar{v}_s = 0$ the circulation κ will be zero. However, in a multiply-connected region, for example where there is a solid obstacle, or if there is a singularity in the liquid such as a vortex, κ need not be zero. In order for the helium wave function to be single-valued, the phase ϕ must increase by a multiple of 2π as the integral is taken around the obstacle. Therefore,

$$\kappa = 2\pi\, n \frac{\hbar}{m} = n \frac{h}{m} \tag{3}$$

where n = 0, ±1, ±2,.. A vortex line is a singularity in the fluid around which there is a non-zero circulation; vortex lines are thought to exist with single quantum units of circulation, i.e., n = ±1, for a vortex.

The first experiment to detect the quantization of circulation in superfluid helium was reported by Vinen in 1961.[3] He used a thin wire immersed in helium inside a cylindrical container. The wire, which lay along the cylinder axis, was put into transverse vibration by passing a current pulse through it in the presence of a steady transverse magnetic field. The circulation of the helium, which was produced by rotating the entire container, would then alter the time rate of precession of the plane of vibration of the wire.

For a cylindrically symmetric wire, the two lowest modes of vibration are two circularly polarized degenerate modes. When circulation is present around the wire, the degeneracy is lifted and there is a splitting of the angular frequencies of the two modes given by:

$$\Delta\omega_\kappa = \frac{\rho_s \kappa}{\mu} , \tag{4}$$

where ρ_s is the superfluid density and μ is the wire mass per unit length plus the mass of the liquid which is displaced by the wire. When the wire is pulsed, both modes are excited equally, and the result is vibration in a plane which precesses with angular frequency $\frac{\Delta\omega_\kappa}{2}$ in the same direction as the fluid is rotating.

As the wire vibrations decay and as the plane of the wire's vibration precesses, an emf is induced in the wire by its motion in the magnetic field. This signal can be detected as a slowly damped beat pattern with period $\frac{2\pi}{\Delta\omega_\kappa}$.

There is always some asymmetry in an actual wire, and the normal modes turn out to be non-degenerate plane-polarized modes in the absence of circulation. When circulation is present, the modes are elliptically polarized. There is then an inherent frequency splitting denoted by $\Delta\omega_o$, called the zero-circulation splitting. If the cell and fluid rotate during the measurement process, a further adjustment must be made for a correction due to the Coriolis force denoted by $\Delta\omega_r$. Then the total observed splitting is given by

$$(\Delta\omega)^2 = (\Delta\omega_o)^2 + (\Delta\omega_\kappa - \Delta\omega_r)^2 \tag{5}$$

where $\Delta\omega_r = 2\omega_r$, where ω_r is the angular-frequency of rotation of the apparatus. The quantity $\frac{2\pi}{\Delta\omega}$ is called the node time and is the observed period between the nodes or zeros in the wire signal which appear when the wire motion is excited and detected in a properly-oriented magnetic field. The quantity $\frac{2\pi}{\omega_r}$ is the rotation period of the cryostat and can be easily measured. When $\Delta\omega_\kappa = \Delta\omega_r$, then $\Delta\omega = \Delta\omega_o$, and the node time is the zero-circulation node time. Thus $\Delta\omega_o$ can be assumed to equal the largest value of $\Delta\omega$ observed during the run when it can be assumed that $\Delta\omega_\kappa$ takes on the value $\Delta\omega_r$ at at least one point during the run. Once the constant $\Delta\omega_o$ is known, $\Delta\omega_\kappa$ can be calculated from the values of the observed node times and rotation periods, and the circulation in quantum units can be found from the expression

$$n = \frac{\mu}{\rho_s} \frac{m}{h} (\Delta\omega_\kappa)$$

$$n = \frac{\mu}{\rho_s} \frac{m}{h} \left\{ \Delta\omega_r \pm [(\Delta\omega)^2 - (\Delta\omega_o)^2]^{1/2} \right\} \qquad (6)$$

This discussion of the theory is meant only to indicate what the observable quantities are. For a detailed description of the wire motion, refer to the M.S. thesis of Starks.[4]

Vinen found that circulations of one quantum unit were particularly stable, but he also observed a continuum of circulation values.

In an attempt to extend Vinen's work, Whitmore and Zimmermann repeated his experiment using the same basic technique.[5] One major difference was that observations of the state of the helium could not be done while the cryostat was in rotation but were carried out for periods of several hours after rotation stopped. The circulation of the helium was found to occur more favorably at integral quantum unit values. However, during the period of observation, the circulation would change spontaneously in a continuous manner and even change direction. Whitmore and Zimmermann also found that by going to larger wire sizes, larger values of circulation could be detected, up to $n = \pm 3$.

Kral and Zimmermann modified the experiment so that observations could be done while the entire apparatus was in rotation.[6] Their procedure was to accelerate the experimental cell slowly from rest to a maximum velocity over a period of several hours. Then the apparatus was decelerated to rest at the same rate. Acceleration commenced in the opposite direction and was again followed by deceleration. Their results were quite different from the previous experiment. Stable circulation was found at both integral and non-integral multiples of the quantum unit. There was also a lack of reproducibility in the results from run to run.

In taking readings on the circulation while the liquid was in rotation, they hoped to observe the effect of angular velocity on the circulation. They found that the circulation was history dependent. During acceleration, the helium tended to remain at lower levels, while during deceleration, the circulation was seen at higher levels, so that a hysteresis curve was traced out in a graph of circulation level vs. rotational velocity. The observed non-integral quantum-unit circulations could be explained, it was thought, by a vortex being attached to the wire at some point along its length. The circulation changes by one unit at the point where the vortex is pinned to the wire. As a result, since the observed circulation is an average of the actual circulation all along the length of the wire, vortex pinning will yield non-integral quantum values for that observed circulation.

In an attempt to reduce or eliminate the metastability in the observed circulation, Starks made two changes in the experiment. He used a smaller wire than had previously been tried by Kral. This change was made because Whitmore and Zimmermann had observed a smaller range of circulation levels when they used a smaller wire. It was assumed that this reduction in metastability could also be achieved when the apparatus was rotating. Kral and Zimmermann had used wires from 60 to 90 μm in diameter, while Starks used a 25 μm diameter wire. The other difference was that the experiment was done at lower temperatures. While the previous work was performed at temperatures above 1 K, the observations made by Starks were at a temperature less than 0.5 K. Whitmore and Zimmermann had observed above 1 K that transitions between levels took place more quickly at lower temperatures, presumably because of the smaller amount of normal fluid present in the sample at lower temperatures.

The metastability was in fact reduced in Stark's experiment, and clear evidence was obtained for quantized circulation. The observed levels were stable within a run and reproducible from run to run. The transitions between levels were rapid changes. However, for a series of four runs with a single wire the stable levels did not seem to occur at integral quantum units. The observed levels appeared at $n = 1.82$ and $n = 2.73$ times h/m. It was noticed, however, that if the levels were divided by a scale factor of 0.91, the values $n = 2$ and $n = 3$ were obtained. No reason was found, at that time, to explain the need for this scale factor, although the previous theory about vortex pinning and the effect of a vortex line near the wire were offered. More will be said about the scale factor later.

This present work was begun as a direct extension of the work of Starks. The apparatus is virtually the same.

Two main intentions motivated this work. The first was simply to obtain more data so that a wider base of observations could be examined. This end was facilitated by an automated data-taking system assembled by Starks after his M.S. thesis work was completed. The second intention was to go to an even smaller wire size. Two wire sizes were used in the experiment, a 34 μm diameter wire and a 15 μm diameter wire which we shall call Wires #1 and #2, respectively. The larger wire enabled observations of circulations out to the n = ±4 levels. The smaller wire was chosen to try to reduce the metastability. Finally, a set of runs was made with 0.1% ^{3}He in the ^{4}He sample in order to observe the effect of a small amount of ^{3}He normal fluid on the circulation.

II. APPARATUS AND PROCEDURE

A. Overall Apparatus.

The experimental apparatus and the electronics are almost identical to those described in the theses of Kral[7] and Starks. The reader is referred to these writings for details. One item that differs is the automated data-taking system used in the experiment. This system recorded for each minute the clock time at the beginning of the minute, the time period displayed on the rotation period counter at the beginning of the minute, the clock time at which the rotation period counter last completed a reading, and the six node time readings taken during the minute, the wire being pulsed every ten seconds. The node time readings were the time intervals between the 1st and 2nd nodes of the wire signal.

The system consisted of a set of logic and level-conversion boards to modify the outputs of the counters and clock to TTL compatible voltages. These signals were then fed through SERDEX[8] modules which converted the parallel bits to serial and transmitted them to a teletype machine by a twisted pair lead. The data were then typed out by the teletype and also were put on a paper tape for later use.

B. Preparation of the Wire Sample.

The wire was actually a quartz fiber which was coated with thin layers of chromium and gold to make it electrically conducting. The fiber was pulled from a 4.0 mm (5/32 inch) diameter quartz rod which had been heated to the melting point in an oxygen-gas flame. Sections of the fiber were then checked for the desired diameter and uniformity with a micrometer caliper accurate to 3 μm (0.1 mil). These were cut to 18 cm lengths and placed in a spit for deposition of the metal films in a bell jar. The gas flame served to give fairly clean fibers, but further cleaning was effected in the bell jar. After the jar was evacuated to approximately 10^{-6} Torr, pure oxygen gas was introduced to bring the pressure up to the $10\text{-}20 \times 10^{-3}$ Torr range. Then two aluminum electrodes, one on each side of the spit, were charged to a potential difference of 2000 VAC using an ungrounded transformer. This voltage ionized the oxygen, and the ions in the glowing discharge bombarded the surfaces of the fibers, removing any contaminants. After about 30 minutes of ion bombardment, the current was turned off and the bell jar was pumped back down to 10^{-6} Torr.

A layer of chromium approximately 180 Å in thickness was then evaporated onto the fibers while the spit rotated. The chromium bonds well to the quartz and forms a good surface for the gold to adhere to. Gold does not stick to quartz very well. The gold was deposited directly after the chromium to a thickness of 1000 Å. The thickness of the metal films was measured to ±10 Å by a film thickness monitor using a vibrating quartz crystal.

In previous work, gold had been evaporated directly onto the quartz fiber. This procedure led to many failures of the wires at various stages of preparation for a run, due to the gold flaking off and causing the wire to have an infinite or unstable resistance. The advent of the use of an intermediate layer of chromium resulted in stable and useful wires which could go for periods as long as a year with no change in their resistances.

The wires were examined and measured under a microscope for proper diameter, uniform cross section and gold layer, and absence of dust. A candidate for the sample wire was selected, cut to a length of about 10 cm, checked for electrical continuity, and then weighed on a microbalance to determine the mass per unit length.

The wire was then cut to a length of 6.9 cm and prepared for mounting in the experimental cell. The metal coating was scraped off about 3 mm of each end to allow epoxy cement[9] to make a good bond and hold the wire firmly in the mounting posts. After this cement was set, a fillet of conducting silver epoxy cement[10] was applied to the gold coating and the posts at each end to provide good electrical contact.

When the conducting epoxy had set, the wire was again tested for conductivity. If it was conducting, it was placed in the cell. The wire tension was given a preliminary adjustment by letting one end of the wire and post hang free inside the cell before the set screws were tightened.

The resistance was checked with a resistance bridge, and the vibration frequency was observed on an oscilloscope. If the signal looked strong, noise-free, and consistent, the wire was given a test run in a vacuum can immersed in liquid nitrogen. The frequency would generally change as the wire was cooled from room temperature to 77 K in an unpredictable but reproducible manner.

Since the wire and the cell were both made of quartz, the thermal contraction of both should have been about the same. However, the tension of some wires was observed to increase on cooling while for others it decreased. This presumably has to do with the relative lengths of the wire and the cell. Consequently, the wire frequency had to be closely monitored during the initial cool-down to 77 K. There was little further change in the frequency on cooling to liquid helium temperature.

The frequency could be adjusted in a test jig in which one of the mounting posts was attached to a flexible diaphragm. The desired frequency was somewhere in the range of 400 to 700 Hz at low temperatures for a good sinusoidal response to a current pulse. At lower frequencies, the tension was too small and resulted in the wire being too loose, so that it tended to flop around. At higher frequencies, the tension was too large and resulted in too much damping of the signal.

C. Preparation for Cool-Down.

When the wire tension had been properly adjusted and the signal seemed satisfactory, the quartz cell containing the wire was attached to the cryostat by set screws, to a flange which was mechanically and thermally linked to the ^{3}He pot. The cell can was screwed on over the cell; an indium gasket made the vacuum seal. The cell can was evacuated through a special pump-out capillary which was separated from the cell fill line. The capillary was a 0.25 mm (10 mil) i.d. cupronickel tube which was wrapped around the ^{3}He pot and passed through the cell flange. Since it was only about 10 cm long, it provided a quick means of leak checking the indium seal. In the past, leak checking had to be done through the cell fill line which extended to the top of the cryostat and included a long section of 0.10 mm (4 mil) i.d. capillary. This resulted in a 10 to 15 minute lapse time between helium entering the cell through a leak and being detected with the leak checker at the top of the cell fill line.

The other advantage of this arrangement was that it afforded an easy means to back-fill the cell with pure helium gas once the leak checking was complete. This practice reduced the possibility of there being water vapor, air, or other contaminants in the cell or cell fill line which could freeze onto the wire or plug the line when the cryostat was cooled to 77 K and lower. After the cell was filled to slightly more than one atmosphere of helium, the pump-out line was sealed with a bead of solder.

The next step was to check the electrical leads before putting in place the vacuum can which isolated the ^{4}He pot, ^{3}He pot, and the cell from the helium bath. These included the ^{4}He pot, ^{3}He pot, and cell thermometers and the ^{4}He pot and cell heaters. The thermometers were 500 Ω Spear carbon resistors, and the heaters were 500 Ω loops of Evanohm wire wrapped around the ^{4}He pot and cell.

If the connections were satisfactory, the vacuum can was put in place, with an indium gasket making the seal between the top of the can and the flange. The vacuum space and seal were checked for leaks from the outside and from the inside at the ^{4}He pot.

The cryostat was then lowered into the inner dewar for cool-down to 77 K, using liquid nitrogen in the outer dewar and nitrogen gas in the inner. The cool-down took about six hours and was generally done over night. After the cryostat had been brought to 77 K and the wire had been checked for proper vibration frequency, preparations were made for liquid helium transfer.

The nitrogen gas was pumped out of the inner dewar, and the dewar was flushed with dry helium gas. The vacuum can was given a small quantity of dry hydrogen gas. The hydrogen provided a good transfer medium to remove heat from the contents of the vacuum can to the helium vapor outside the can. Hydrogen freezes out at 14 K however, so the vacuum is acceptable when the helium starts to condense.

The initial helium transfer took approximately 30 minutes and used about five liters of liquid.

After transfer was completed, the ^{4}He pot was filled from the bath, and pumping was started to cool the ^{4}He pot, ^{3}He pot, and cell further. After the ^{3}He pot reached 3 K, which took about three hours, ^{3}He gas could be condensed in through the pump line. When most of the ^{3}He was condensed in the pot, it was pumped on with a mechanical pump to lower the temperature to about 0.45 K. The vapor which had been pumped away was returned as liquid by a recirculating line running from the back of the pump down to the pot. The removal and return of the ^{3}He gas would come into steady state after about ten hours, with almost all of the ^{3}He in the pot.

The cell was filled with ^{4}He after the ^{3}He pot had been pumped on for a couple of hours. A ballast can on the dewar stand was pressurized to 250 psig with ^{4}He gas, and this gas was allowed to condense into the cell. This amount of gas was enough to fill the cell and part of the cell line with liquid so that the pressure at the top of the cell line was one atmosphere.

The period of vibration of the wire was observed before the cell filling and after. The period after filling was always longer by about 2 to 3%. This increase occurred because when the cell has liquid in it, the wire must push liquid out of the way in order for it to vibrate. This effectively increases the mass of the wire by an amount equal to the mass of the helium which would be contained in the volume of the wire. This effect is called the hydrodynamic correction to the mass of the wire.

The change in wire frequency can, in fact, be used to check the size of the hydrodynamic correction. The mass of the wire and the measured frequency before cell filling will yield a force constant for the wire's vibration. Assuming that the tension in the wire remains the same during cell filling, the frequency of vibration of the wire when it is immersed in the liquid should yield the bare wire mass plus the hydrodynamic correction.

The frequency change has been observed for five cell fillings with Wire #1. The inferred change in the effective wire mass was within ±0.1 μg/cm of the hydrodynamic correction of 1.32 μg/cm calculated from the measured wire diameter. Two cell fillings were observed for Wire #2 and again, the change in the effective wire mass agreed with the hydrodynamic correction to within ±0.02 μg/cm out of 0.25 μg/cm.

D. Running of the Experiment.

When the experiment was ready to proceed, the cell line and ^{3}He lines were closed off and disconnected. Helium was transferred to make the bath level as high as possible. Then a basket approximately 10 cm in length and about 3 cm in diameter filled with molecular seive[11] was lowered from the top of the cryostat down to just above the ^{4}He pot inside the ^{3}He pump line. The molecular sieve would pump ^{3}He by adsorption onto its surface and cool the cell to as low as 0.37 K. The damping of the liquid in the cell would drop, allowing observation of the node pattern of the wire.

In this experiment, the time from the first node to the second node was recorded rather than the time from the pulse to the first node as described in Starks's thesis and earlier work. This practice eliminated the concern about the effects of any possible amplifier delay as described in those works. Presumably, the time of both nodes was delayed by the same amount so that amplifier delay canceled out.

The orientation of the magnet was adjusted so that the signal possessed true nodes of zero amplitude. The VCO and tuned amplifier were adjusted to the wire's vibration frequency. If everything was working properly at this point, the drive belts were attached to the turntable and rotation was begun. Also started was a second motor whose purpose was to increase the rate at which the rotation drive ran by continually turning up the drive rate dial. This resulted in a slow acceleration of the cryostat. The apparatus was accelerated from rest to 3.7 radians per second over a period of about 2.25 hours. When the maximum velocity was reached, deceleration to rest was begun, then acceleration and deceleration in the opposite direction. It took about nine hours to carry out these four stages of rotation.

III. RESULTS

A. Presentation of Data.

A total of 22 runs was made with two different wires. The properties of the wires are given in Table 1. These data include diameters, masses per unit length, normal operating resistances and frequencies of vibration at 0.4 K, and peak displacement and velocity amplitudes of the wire and peak emfs generated by the wires after a 0.5 V pulse applied to the bridge. The run numbers for each wire are also given.

The results of the runs are displayed in Table 2. These data include pressures in the cell line at the top of the cell during the run, pulse amplitudes to the bridge during the run, the zero circulation node times observed, and the observed quantum units of circulation derived from the stable levels of each run. Stable levels were taken to be those for which there were periods of at least 10 minutes duration during which the circulation did not change by more than 5%.

Examples of the data are shown in Figures 1 and 2 representing Wires #1 and #2, respectively. The plots are of

TABLE 1

Wire #	Diameter	Mass per Unit Length	Resistance	Frequency	Run #'s
1	34.0±0.5 μm	23.1±0.5 μg/cm	825 Ω	670 Hz	1-15
2	14.9±0.5 μm	4.4±0.5 μg/cm	520 Ω	430 Hz	16-22

Responses to 0.5 V Pulse to Bridge

Wire #	Initial Displacement Amplitude	Initial Velocity Amplitude	Initial Amplitude of Induced emf
1	2.4 μm	1.0 cm/s	0.06 mV
2	28.0 μm	7.8 cm/s	0.46 mV

TABLE 2

Run #	Pressure (Torr)	Pulse Amplitude (V)	Zero Circulation Node Time (ms)	Observed Quantum Unit
1	≥50	0.6	650	≤1.016±.014
2	≥50	0.6	650	≤1.015±.009
3	≥760	0.6	650	≤1.126±.051
4	≥100	0.6	650	≤1.033±.027
5	70	0.6	650	.955±.003
6	760	0.6	650	1.016±.011
7	300	0.6	636	.998±.032
8	60	0.6	636	1.005±.011
9	140	0.6	679	1.014±.010
10	130	0.6	652	1.006±.012
11	140	0.6	666	1.013±.003
12	760	0.6	657	1.010±.003
13	170	0.6	668	1.012±.032
14	760	0.6	660	1.011±.015
15	140	0.6	670	1.013±.017
16	130	0.3	750	.988±.001
17	760	0.3	750	.986±.003
18	130	0.2	750	.987±.001
19	760	0.2	750	.985±.002
20	380	0.3	750	.986±.014
21	380	0.4	750	.990±.012
22	380	0.5	750	.996±.010

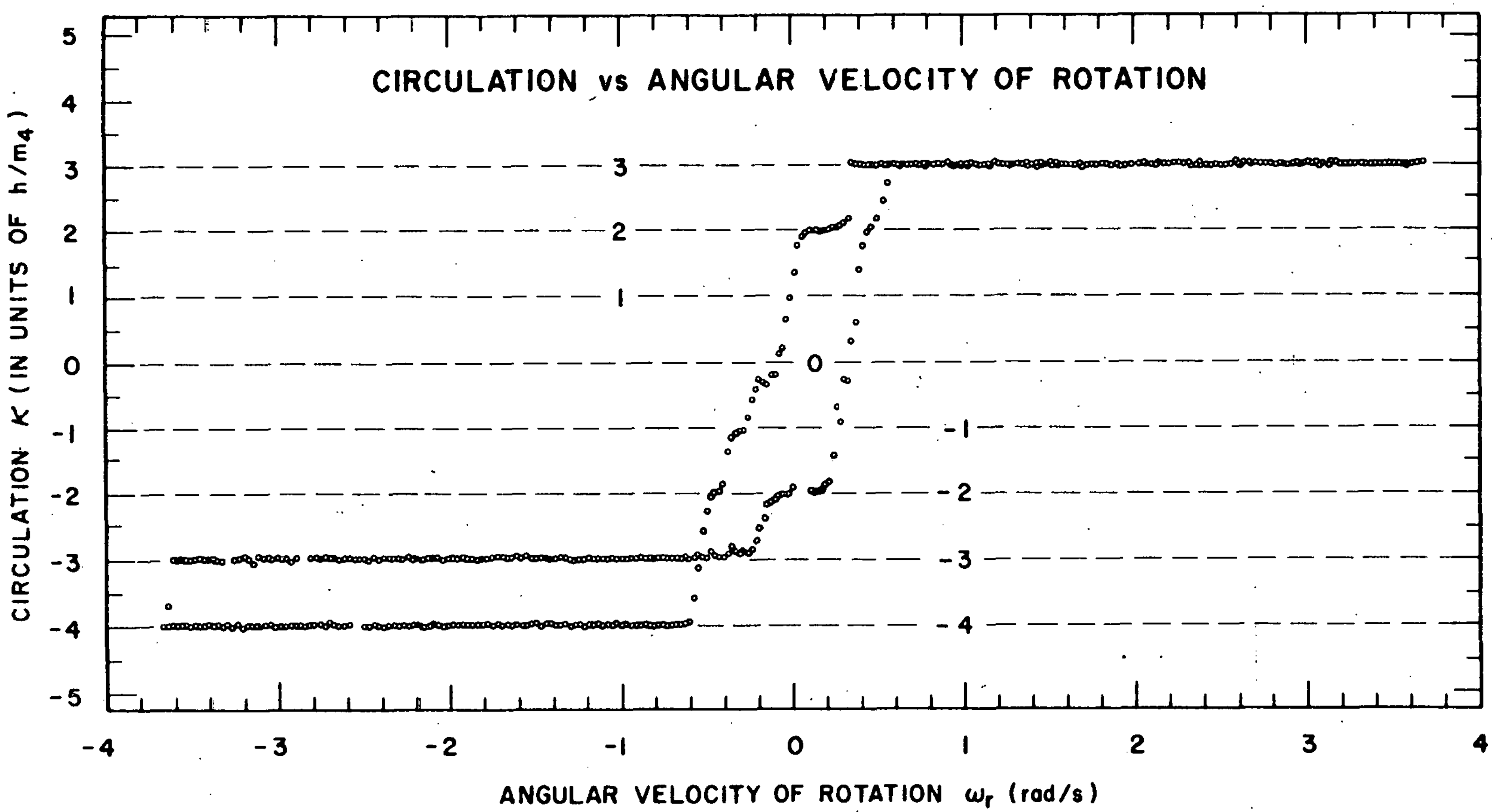

Figure 1: Wire #1, Run #2

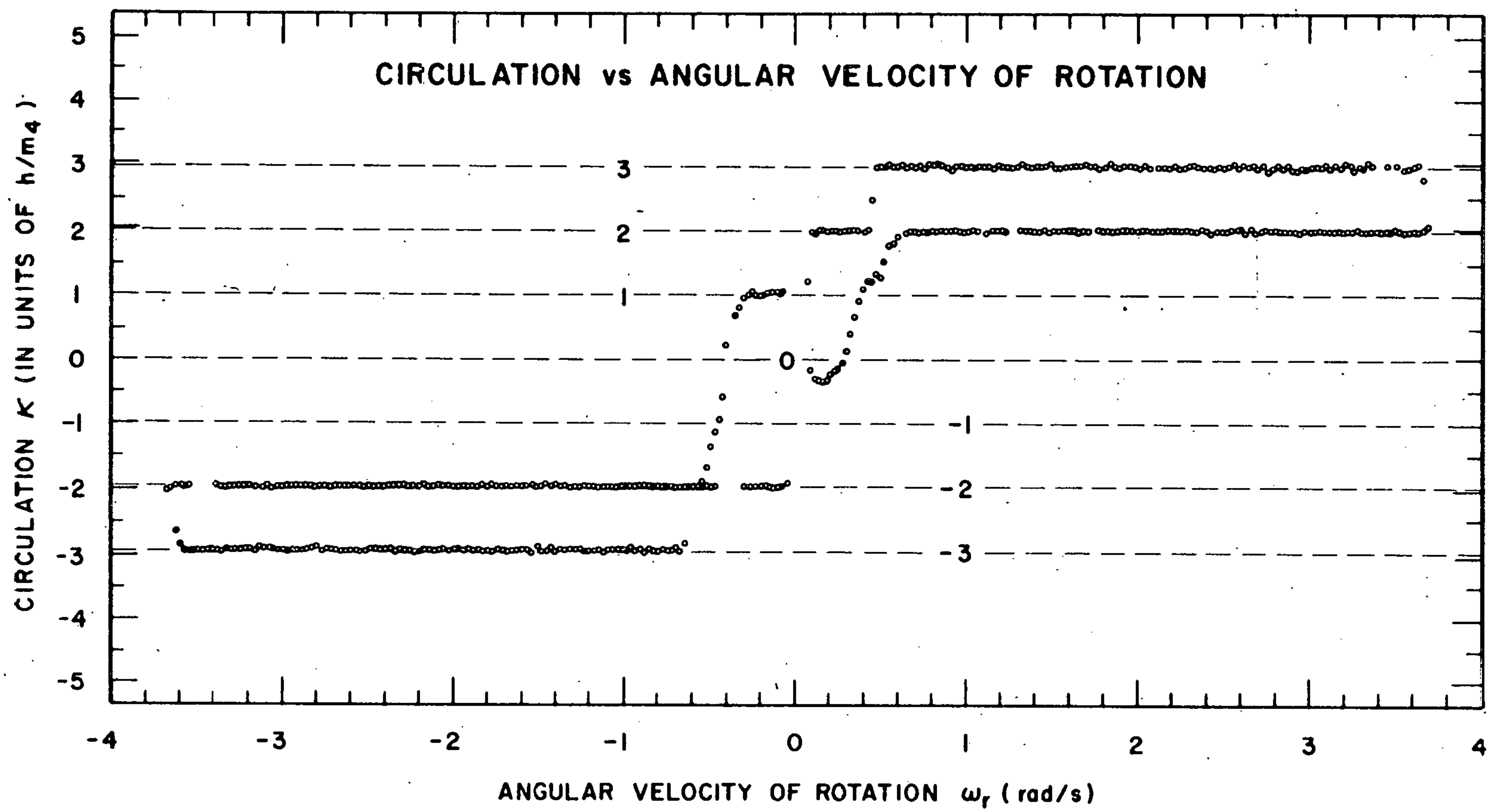

Figure 2: Wire #2, Run #16

observed circulation in units of h/m vs. rotation speed in radians per second. Throughout the experiment, positive circulation was defined, arbitrarily, to be motion in the counterclockwise sense and negative circulation to be motion in the clockwise direction, as seen from above.

The runs were generally done in groups. That is, a set of three or more runs were done on successive days after the cell had been filled once, with the temperature of the cell never going above the λ point. Runs 1 to 4, 5 to 8, 10 to 15, 16 to 19, and 20 to 22 were all done in such groups. The results of these runs tend to agree more within a group than with results from other groups of runs for a given wire, although the discrepancies are never very large. The figures, which are representative of the best data taken, show evidence for quantization of circulation at the expected levels during the long periods of stable observed values of κ. The observed levels repeat themselves within each run and are reproducible from run to run for each wire.

B. Discussion.

The observed quantum units given in Table 2 show a spread about unity for Wire #1 and are slightly under unity for Wire #2. The uncertainties given for these values are statistical uncertainties derived from the data points which represented periods of stable circulation during each run and do not include the relatively large uncertainties in the masses per unit length of the wires. The range of these values is meant only to indicate the reliability of determination of the observed quantum unit. All of the results from runs for which the cell line pressure is known accurately are within the uncertainties inherent in the effective mass per unit length μ of the wire, which is the least well known quantity in Equation (6). The reliability of μ is limited by the accuracy with which we could weigh masses in the 20 to 50 μgram range. The wire masses in Table 1 represent the best results based on repeated weighings of the wires on two separate microbalances.

What about the other quantities in Equation 6? The quantity $\Delta\omega_r$ is the Coriolis force correction equal to twice the rotational angular frequency ω_r of the cryostat. The quantity $\Delta\omega_r$ ranges in value from 0 to approximately ±7.4 rad/sec. The quantity ω_r is determined throughout the run by recording the rotation period of the cryostat and fitting these periods to a curve, so that the rotational frequency can be given for any required clock time. This fitting was done by using a least squares fitting routine, and the errors between observed and predicted rotation periods was less than 0.5% in all runs.

The quantity $\Delta\omega$ is the angular frequency associated with the observed node time. It ranged in value from

8 rad/sec to 48 rad/sec for Wire #1 and to 95 rad/sec for Wire #2. The quantity $\Delta\omega_o$ is the angular frequency associated with the node time when no circulation is present. It was always approximately 8 rad/sec. During the stable portions of each run, where the circulation was constant and the values given in Table 2 were determined, $(\Delta\omega)^2$ was much greater than $(\Delta\omega_o)^2$ so that the calculated circulation was insensitive to errors in $\Delta\omega_o$.

This leaves the question of how accurately $\Delta\omega$ was measured. From an electronics points of view, the accuracy of $\Delta\omega$ is not in question. The counter used to measure $\Delta\omega$ has been checked against other counters and found in agreement. The node time counter was triggered by a lock-in amplifier tuned to the wire frequency and locked onto the wire signal. The counter would turn on when the lock-in signal crossed zero at the first node and would turn off when the lock-in signal crossed zero at the second node. The time at which the nodes occurred was made uncertain by the vibrational periods of the wires, corresponding to the frequencies which are given in Table 1. Visual inspection of the wire signal, lock-in signal, and counter gating showed that the counter starts and stops occurred at the times of zero crossing of the lock-in signal to within ±0.1 msec and that the lock-in signal crossed zero at the time of the nodes to within the time resolution of of the node, limited by the wires' vibrational period. This was estimated to be ±1 msec. This uncertainty represents an error of 1.5% to 0.1% in the measured node time.

C. Other Results.

In an attempt to examine the effect of a small amount of normal fluid on the circulation, Runs 20 to 22 were made with 0.1% by mole fraction of ^3He added to the ^4He sample. The effect of the viscous ^3He was readily apparent in the damping of the wire. Measurements of the damping were made during Run #18 with the pure ^4He sample at a temperature of 0.42 K. The characteristic damping time was found to be approximately 720 msec. With ^3He added to the sample the damping time during Run #20 was measured to be 80 msec at the same temperature. This made measuring node times greater than 250 msec impossible. The value of the zero-circulation node time of 750 msec was taken from Runs 16 to 19 which used the same wire. Run #22 is shown in Figure 3.

This diagram has a couple of noteworthy features. The stable level in Run #22 occurs at the same value of κ as the 2nd levels in Runs 16 to 19. This level is also seen in Runs 20 and 21. There is no indication that the presence of the ^3He has altered the equilibrium value of κ from that

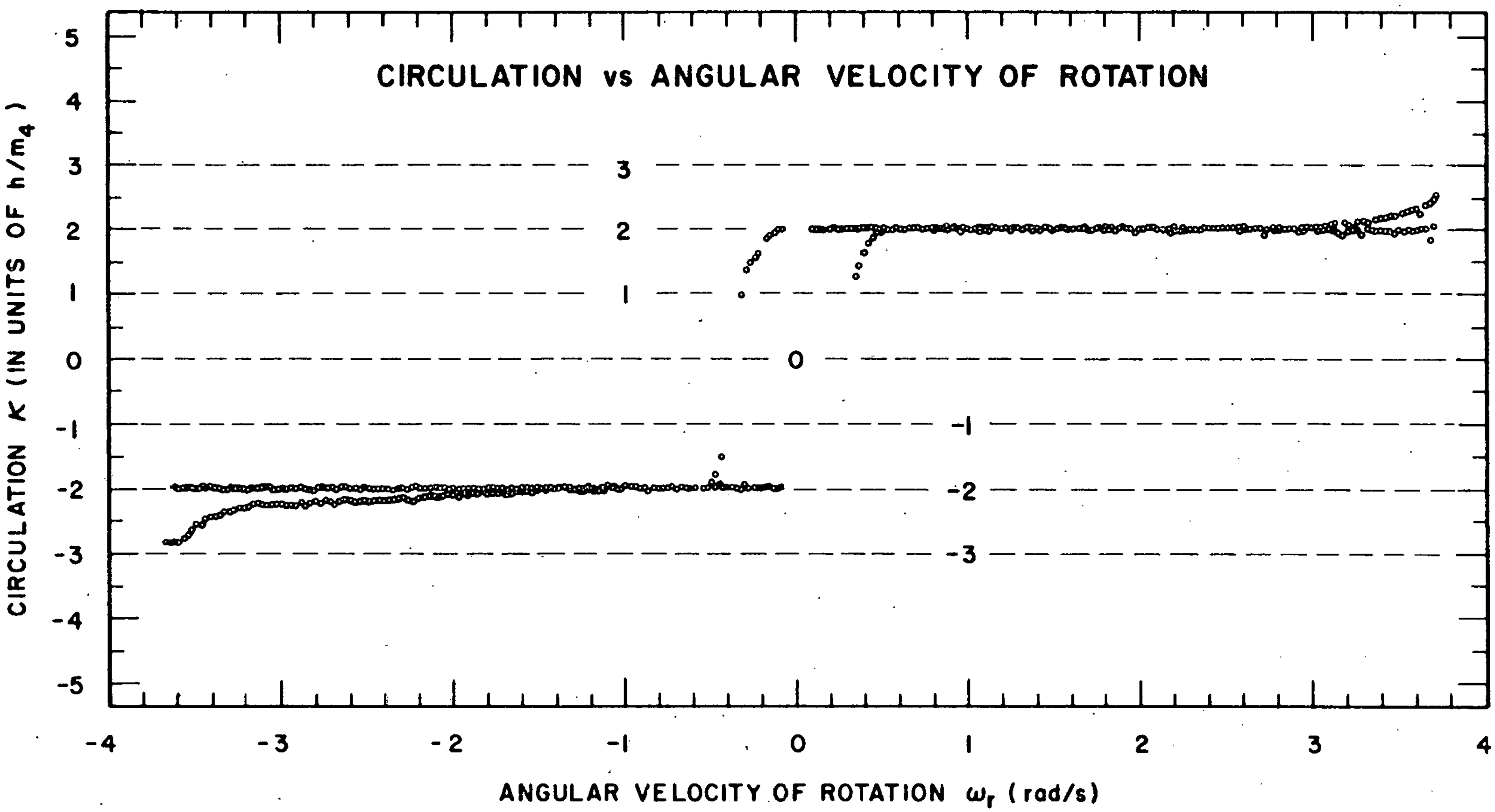

Figure 3: Wire #2, Run #22

observed for the pure ^{4}He sample. The other feature of note is that Level 3 of the circulation is not seen as in Run #16 (Figure 2). The wire was excited above the 2nd level at the maximum angular velocity points in each direction, but then slowly decayed back to the 2nd level. This decay was also seen in Runs 20 and 21. It was not seen in any of the runs with a pure ^{4}He sample with either of the wires. This is apparently a result of the influence of the small amount of ^{3}He present.

During most of the runs, the bridge was given a 10 V pulse at the points of highest rotational velocity of the cryostat. This pulse usually excited the circulation to a higher level. This behavior raised a suspicion that the circulation was not always at its equilibrium value. To try to test this idea and to attempt to get better data, Runs 9 through 15 were performed with 10 V pulses to the bridge every 10 or 15 minutes. It was thought that this might allow the circulation to reach its equilibrium state at each value of angular velocity. For some of these runs, the large pulses were given throughout the run, and for others the large pulses were administered during only half the run. In between the large pulses, the pulse size was the normal 0.6 V.

The results of these runs were disappointing. Rather than assuring that the circulation was in equilibrium, the periodic large pulses seemed to introduce noise and metastability into the observed circulations, especially at higher rotation rates. The circulation values obtained from sections of the runs where stable levels were observed are the same as for the non-pulsed runs with Wire #1, but there were many fewer useful data.

One other effect that was explored was that of varying the pressure of the liquid in the cell. This was done by putting more or less liquid helium in the cell fill line above the cell. The cell was always full of liquid. This fact is known because liquid helium at 0.4 K has a very low vapor pressure, and the measured pressure in the cell was always maintained at at least 50 Torr. However, for some runs, the pressure was brought up to an atmosphere, which implies that the level of the liquid in the cell fill line was somewhere in the bath at 4.2 K.

The change in pressure from one run to the next was expected to change the observed values of κ slightly because ρ_s, the superfluid density, changes by slightly more than 1% when the pressure goes from 0 to 760 Torr. The observed values of κ were in fact slightly higher during the runs made at higher pressure. This effect has been corrected for in the data in Table 2.

D. Comment on Previous Work.

It has become apparent during this work that the scale factor observed by Starks was due to inaccurate weighing of his wire. The problem of weighing a mass less than 100 μg to a fraction of 1 μg is not a trivial matter.

The source of the error is the use of the optical scale of the microbalance in the Physics Department. This scale has a range of 100 μg, so that it is very convenient for weighing the wires. However, checking the scale against a moving rider which effectively adds 100 μg to one side of the balance reveals that the optical scale is indicating a mass too small by 7.5 ± 1.6%. This error is verified by weighing the wires on a microbalance in Chemistry. The two balances agree at the 1 mgram level. While the results in this work have been corrected for this error, those of Starks have not, hence at least a major part of the 9% discrepancy between his observed values and the quantum unit circulation. Although the error in his work falls within the uncertainty in the mass values, there may be a residual discrepancy on the order of 1 to 2%.

One additional set of data for this experiment exists which has not yet been discussed. This is a set of four runs made with a 25 μm diameter wire published by Karn, Starks, and Zimmermann.[12] The results of these runs are very mysterious. Stable levels of circulation were observed but with a scale factor of 0.76. Even correcting for the maximum error in wire mass still leaves a scale factor of 0.85. This data was taken between that of Starks' M.S. thesis and the present data and the analysis was carried out in the same manner as in the present work. No major changes took place in the procedure or apparatus since Starks' thesis work was done.

IV. CONCLUSIONS

This work has extended that of Starks to both larger and smaller wire diameters in an effort to further reduce metastability in the observed circulation. It has also produced a much larger data base for analysis. It also provided an opportunity to examine the effects of varied operating conditions on the circulation such as the addition of ^{3}He normal fluid to the superfluid sample, the effect of recurrent large pulses, and changes of pressure.

Although the accuracy of the results is limited by uncertainties in the wire mass determination, the fact that 1% changes in κ due to changes in pressure in the cell line could be detected gives an indication of the sensitivity of the experiment.

More consideration will have to be given to wire mass measurement in any future work. However, the consistency of the present results gives the best evidence of quantization of circulation in rotating superfluid helium.

V. ACKNOWLEDGEMENTS

I would like to thank my advisor, Professor William Zimmermann, Jr. for his guidance and help throughout this work. I must also express my appreciation to Mr. Daniel Starks, who spent much of his own personal time in getting me started in this work.

I am grateful to my laboratory partners, Mr. Paul Schubert and Dr. Fred Wirth for their suggestions, help and friendship. I also thank Mr. Lawrence Bot for his help with the data analysis.

This work was funded by the Department of Energy (DOE) under contract number EY-76-S-02-1569 for which I am grateful.

Finally, I wish to thank my wife, Karen, for being a good listener and for her patience over the last 20 months.

VI. REFERENCES

1. L. Onsager, Nuovo Cimento 6, Supp. 2, 249 (1949).
2. R. P. Feynman, in Progress in Low Temperature Physics, edited by C. J. Gorter (North Holland Pub. Co., Amsterdam, 1955).
3. W. F. Vinen, Proc. Roy. Soc. (London) A260, 218 (1961).
4. D. R. Starks, M.S. thesis, University of Minnesota, 1976 (unpublished).
5. S. C. Whitmore and W. Zimmermann, Jr., Phys. Rev. 166, 181 (1968).
6. S. F. Kral and W. Zimmermann, Jr., Phys. Rev. B 11, 3334 (1975).
7. S. F. Kral, Ph.D. thesis, University of Minnesota, 1973 (unpublished).
8. Analog Devices, Inc., Norwood, Massachusetts 02062.
9. Stycast 1266, Emerson & Cuming, Inc., Northbrook, Illinois.
10. ACME type E Solder No. 3021, ACME Chemicals & Insulation Co., New Haven, Connecticut 06505.
11. Linde type 13X. Linde Corporation, 120 S. Riverside Plaza, Chicago, Illinois 60606.
12. P. W. Karn, D. R. Starks, and W. Zimmermann, Jr., Journal de Physique 39, Colloque C6, 170 (1978).