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Media in Two Dimensions with Applications to
Nuclear Repository Design

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Numerical Calculation of Thermal Convection in Porous
Media in Two Dimensions with Applications to
Nuclear Repository Design

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Abstract

Thermal convection induced by nuclear waste heating could be an important mechanism in the transport of radioactive nuclides to the biosphere. A computer code HP2D was developed to calculate this effect. The code calculates transient and steady state heat and fluid flow in porous media. It allows for anisotropy of hydraulic conductivities, temperature dependent fluid density and viscosity. A heat source and a variety of temperature and flow boundary conditions can be easily employed. The code solves two coupled governing equations. The time dependent heat transport is solved by an alternating direction implicit technique; and the steady state flow of water is solved by a Gauss-Seidel Poisson solver. A sample calculation of thermal convection about a repository is given.

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INTRODUCTION

Thermal convection induced by nuclear-waste heating could be an important mechanism in the transport of radioactive nuclides to the biosphere. Thermal convection may redirect a portion of the prevalent horizontal flow through a repository, and up to a bed of higher permeability (an aquifer), and subsequently to the surface water system. (See Fig. 1 and 2) The strength of the thermal convection pattern is related to the permeability of the region and the magnitude of the thermal loading. The quantities of interest are the rate of flow through the repository (gallons per minute, gpm) and the time it takes for radioactive nuclides to reach the surface.

At this time, it appears that the most extensive calculations involving thermal convection have been reported in (OWI/TM-36, V-21).¹ In that report, migration times for nuclides to reach the top of the surface of the basin from repositories in shale, basalt, and granite were estimated from the velocity field to be on the order of 500 years. The equations used in these calculations assumed anisotropic porous media and horizontal fluid flow through the repository. The equations for heat flow and fluid flow were solved independently.

In this report we consider the coupled equations. The theory and the numerical method are briefly described. Simple examples of flow around a waste repository in bedded salt are provided. These examples were motivated by a requested contribution to the Bedded Salt Report⁵ for the Nuclear Regulatory Commission.

THEORY

The basic equations describing thermal convection in porous media are the equations of continuity, Darcy's Law, and energy, in conjunction with an equation of state. They are simplified by the Boussinesq approximation and may be written as follows²

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

$$u = \frac{-k_x}{\mu} \frac{\partial p}{\partial x} \quad (2)$$

$$v = \frac{-k_y}{\mu} \left[\frac{\partial p}{\partial y} + \rho g \right] \quad (3)$$

$$\rho_s C_s \frac{\partial T}{\partial t} + \rho C \left[u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right] = \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right] + q \quad (4)$$

$$\rho = \rho_0 [1 - \beta(T - T_0)] \quad (5)$$

where

- u velocity in x direction
- v velocity in y direction
- K_x permeability in x direction
- K_y permeability in y direction

μ	viscosity
P	pressure
ρ	density of the fluid
C	specific heat of the fluid
ρ_s	density of the solid
C_s	specific heat of the solid
T	temperature
λ	coefficient of heat conductivity
q	heat source
ρ_0	initial density
β	coefficient of thermal expansion

Nondimensionalization of the above equations show that the solution depends on the Rayleigh number based on a heat source given by

$$Ra = \left(\frac{\beta}{\alpha \lambda} \right) q K_v H^3 \quad (6)$$

The material constants for bedded salt are

β	coefficient of thermal expansion of water .0002 (1/K)
α	$\lambda/\rho C_p$ thermal diffusivity $2.55 \times 10^{-6} (m^2/sec)$
λ	thermal conductivity 2.8 BTU/(hr ft F) = 4.84 (w/(m k))
ρ	density of salt 135 (lb/ft ³)
C_p	specific heat of salt .21 (BTU/(lb F))
q	heat source 35(KW/arce) = .865(w/m ³)

$K_v = \frac{k_v g}{v}$ hydraulic conductivity (cm/sec)

k_v permeability (cm^2)

g gravity $9.8 \text{ (m/sec}^2\text{)}$

v kinematic viscosity of water $1.007 \times 10^{-6} \text{ (m}^2\text{/sec)}$ at 20°C
(an improved calculation would be to use a temperature dependent viscosity)

H depth of salt 200 (m)

NUMERICAL METHOD

HP2D in its present form is based on a two dimensional model. The method of solution is similar to that employed by Lau,³ FAUST-MERCER,⁴ and GWTHERM.^{1,4} The velocity is written in terms of a stream function and then Eqs. 1 to 3 are combined to form a Poisson equation with a temperature dependent right hand side. The energy equation is written in terms of the stream function and temperature.

The numerical procedure consists of solving the energy equation for the temperature by alternating direction implicit method and solving the flow field with a Gauss-Seidel Poisson solver using the temperature obtained from the energy equation.

RESULTS

It remains questionable whether thermal convection would play a role in bedded salt depositories since some finite level of permeability is required to sustain the convective flow and bedded salt has very low permeability. Also, it would be instructive to consider a salt layer that has been fractured by some cause and that could sustain porous flow.

Simply substituting some of the higher values for the permeability of salt found in the Geoscience Handbook⁶ and extrapolating from the shale, basalt, and granite calculation of (OWI/TM-36, V-21)¹ resulted in time of migration to the surface of the order of 1,000 years. In general, migration times scale inversely with the permeability of the medium and the depth of the repository.

In the calculations reported here the entire multilayer basin geology was not modeled, preliminary calculations for a repository in a 200-m thick salt bed have been made. Horizontal fluid flow was assumed to be zero so that convection cells were confined to the bedded salt area.

Preliminary calculations for a repository in bedded salt have been made for a thermal loading of 35 kW/acre and 180 kW/acre at a depth of 100 meters below an upper aquifer. The temperature contours, streamlines, and vector plots for 100 years after decommissioning are shown in Figs. 3 and 4. The value of the temperature are in agreement with independent heat conduction calculations.⁷ The results show that the convection patterns are localized for the 35-kW/acre case and are much larger for the 180-kW case. This further supports the need for limiting the thermal loading of underground repositories. The results, however, are still in a preliminary state and an

exhaustive set of cases has not yet been completed to allow general qualitative and quantitative conclusions to be drawn. Further, a three-dimensional calculation may be necessary for conclusive results.

FINAL REMARKS

The HP2D code is more convenient to use and will be able to confirm or improve upon the results of GWTERM or other decoupled treatments. Furthermore, the addition of marker particles or specie equations may improve the accuracy of the calculation of the time of migration of radioactive nuclides to the surface.

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Figure 1 Repository configuration for preliminary calculations has non-conducting boundaries, no flow top and bottom, and prescribed flow on the left and right boundaries.

Streamlines

Fig. 2: Sample calculation showing how convection patterns redirect a portion of the prevalent horizontal flow through a repository.

FIG. 3: Thermal convection results for the $q = 180$ kW/acre.
 ($T = 100$ y, $K_V = 1 \times 10^{-10}$ cm/s and $K_H = 1 \times 10^{-10}$ cm/s)

FIG. 4: Thermal convection results for $q = 35 \text{ kW/acre}$.
 $(T = 100 \text{ y}, K_V = 1 \times 10^{-10} \text{ cm/s and } K_H = 1 \times 10^{-8} \text{ cm/s})$