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Estimating the Cerenkov Neutron

Sensitivity of Suprasil (SiO₂)

and Lucite (C₅H₈O₂)

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Fast neutrons* hitting a sample of suprasil will produce light. This light is Cerenkov light produced by "high energy" (>.2 MeV) electrons traversing the transparent medium. Neutrons produce these electrons in a two step process. First, the neutrons inelastically scatter with either the silicon (Si) or the oxygen (O) atoms producing inelastic gamma rays. Some of these gammas, in turn, will also react producing, primarily, Compton electrons with some pair production electrons possible. The majority of these Compton electrons will have enough energy to induce Cerenkov light production. For intermediate energy neutrons (2-20 MeV), this process is efficient enough to produce a relatively simple neutron detector with some desirable properties.

Suprasil is radiation resistant synthetic quartz which "typically" shows no appreciable damage (browning) below 10⁸ rad/g absorbed dose. This dose is approximately 30 times the equivalent dose required to darken Lucite. The geometry considered here is a cylinder

* Slow (thermal) neutrons can also produce Cerenkov light by producing neutron capture gamma rays. These neutrons are not considered in this report.

7.62cm high times 4.44cm dia. A FW-114A PD is mounted on the back end of the cylinder -- in the beam. All faces of the cylinder, except the one next to the PD, are painted black. The appropriate physical constants are: density, $\rho = 2.2\text{g/cm}^3$ and index of refraction, $n = 1.47$ @ 400nm.

To estimate the Cerenkov/neutron sensitivity we need to know a number of Cerenkov properties. The basic Cerenkov light criteria is that the electron's velocity must exceed the velocity of light -- in the medium, i.e.

$$V_e \geq C/n(v)$$

This formula is not quite as simple as it first appears since the index of refraction, n , is also a function of the emitted light frequency, v . Since n typically gets larger for shorter wavelengths, the threshold velocity is lower in the U.V. than the "visible". If we ignore this dispersion ($n(v) = n_0 = \text{const.}$) the number of photons emitted per centimeter of electron pathlength can be written

$$dN_C/dX = 370(E_2 - E_1) \sin^2\theta_C *$$

where E = energy of photons observed in eV and

$$\sin^2\theta_C = (1 - 1/\beta^2 n^2)$$

$$\beta = V/c$$

For a S-20 photocathode, $E_2 = 3.6\text{eV}$ (340nm) and $E_1 = 1.3\text{eV}$ (950nm) and the total number of photons emitted in this "observable" range is:

$$dN_C/dX = 850 \sin^2\theta_C.$$

* V. Ashford, et.al. in "Proceedings of the 1978 ISABELLE Summer Workshop", P.99, BNL 50885, 1978.

For constant n , $\sin\theta_c$ is a slowly varying function of β only. For electron energies $E > 1$ MeV ($\beta = .814$) β rapidly approaches its asymptotic value of 1. Assuming, for the moment, $\beta \sim 1$ we can write

$$\sin^2\theta_c \approx (1 - 1/n_o^2) \approx .54$$

and

$$dN_C/dX \approx 450 \text{ photons/cm } (E \geq 1\text{MeV}).$$

The total number of Cerenkov photons produced by an electron of energy E can be written

$$N_C = 450 \int dX \approx 450 R(E)$$

where $R(E)$ is the electron range in cm. Now $R(E)$ for electrons has a very simple relationship ($E > 1\text{MeV}$) i.e.

$$R(E) = (.52E - .09)/\rho \text{ cm}$$

where ρ is the material density (g/cm^3). Substituting we get

$$N_C \approx 450 (.52E - .09)/\rho$$

For suprasil with $\rho = 2.2 \text{ g/cm}^3$

$$N_C \approx 105(E - .17)$$

A more accurate fit of the actual dN/dX vs. E curve (See Figure 1.) taking the variation of β into account gives a similar but slightly different equation for SiO_2 i.e.

$$N_C \approx 107 (\bar{E} - .48) \quad E > 1\text{MeV}$$

Since N_C is a linear, first order function of $\bar{E}$, the average $\bar{N}_C$ is simply related to the average electron energy $\bar{E}$, i.e.

$$\bar{N}_C \approx 107(\bar{E} - .48)$$

Now $\bar{E}$ is the average electron energy deposited per gamma interaction.

Figure 2 shows $\bar{E}$ vs E_γ obtained for SiO_2 from a tabulation of Plechaty,

et. al.* Combining the results shown in Figure 1 and Figure 2, we can obtain the number, N_I , of Cerenkov photons produced per gamma interaction (See Figure 3). The total number of Cerenkov photons produced per gamma of energy E_γ is:

$$N_C(E_\gamma) = A_n \sigma(E_\gamma) N_I(E_\gamma) R(\rho, z)$$

Where A_n = number of atoms/cm³

σ_γ = total gamma interaction cross section

$R(\rho, z)$ = gamma pathlength in the suprasil

Figure 4 shows the total gamma cross section and Figure 5 shows the number of Cerenkov photons produced per gamma interaction.

In the case of neutron produced inelastic gamma rays, a spectrum of gamma rays is typically produced. Table 1 shows a tabulation of the gammas produced in oxygen at 14.8 MeV. Figures 5 & 6 show gamma spectrum measured in silicon and Figure 7 shows the oxygen gamma spectrum produced by neutrons between 12 and 14 MeV. The total number of gammas, $N_\gamma(E_n)$, produced in a Cerenkov radiator of length L for a parallel beam of neutrons is:

$$N_\gamma(E_n) = A_n \int_0^L e^{-A_n \sigma_{\text{non}}(E_n) z} \int \frac{d\sigma(E_\gamma)}{dE} dE dz$$

Where σ_{non} = neutron nonelastic cross section at E_n

$d\sigma/dE$ = differential gamma production cross section.

* E.F. Plechaty, D.E. Cullen, R.J. Howerton, "Tables and Graphs of Photon Interaction Cross Sections from 1.0 KeV to 100 MeV Derived from LLL Evaluated Nuclear Data Library", UCRL - 50400, V.6, Rev. 1, 1975.

TABLE - 1
160 Gamma Production Cross Sections at 14.8 MeV*

E (MeV)	Reaction	Level Decay	Transition Probability	(n,Xγ)
.5 - 1.5	--	--	--	37.7mb
1.78	$^{160}\text{O}(n,n'\gamma)^{160}\text{O}$	8.88 - 7.12	14% }	3.0
1.96	$^{160}\text{O}(n,n'\gamma)^{160}\text{O}$	8.88 - 6.92	5 }	
2.74	$^{160}\text{O}(n,n'\gamma)^{160}\text{O}$	8.88 - 6.14	74	43.7
3.09	$^{160}\text{O}(n,\alpha)^{13}\text{C}$	3.09 - 0.00	100	20.6
3.68	$^{160}\text{O}(n,\alpha)^{13}\text{C}$	3.68 - 0.00	100	63.6
3.85	$^{160}\text{O}(n,\alpha)^{13}\text{C}$	3.86 - 0.00	76	49.8
4.43	$^{160}\text{O}(n,n'\alpha)^{12}\text{C}$	4.43 - 0.00	100	21.8
6.14	$^{160}\text{O}(n,n'\gamma)^{160}\text{O}$	6.14 - 0.00	100	172.1
6.92	$^{160}\text{O}(n,n'\gamma)^{160}\text{O}$	6.92 - 0.00	100	44.8
7.12	$^{160}\text{O}(n,n'\gamma)^{160}\text{O}$	7.12 - 0.00	100	38.6
8.88	$^{160}\text{O}(n,n'\gamma)^{160}\text{O}$	8.88 - 0.00	7	<u>4.9</u>

Total 499.3mb

Each one of these gammas have their own cross section vs energy and energy threshold. Hence, a detailed evaluation can (and does) get messy.

* F. Hjenberg-SeLove and T. Lauritsen, Nuclear Physics II, No. 1, 1959.

The non-elastic neutron cross section is used instead of the total cross section since elastic collisions usually are peaked very much forward and typically do not remove any "significant" energy from the neutron.

The total number of neutron induced, $N_C(E_n)$, Cerenkov photons incident on the PD photocathode is the product of a neutron and gamma interaction plus light collection efficiency, i.e.

$$N_C(E_n) = A_n^2 \int_0^E \frac{d\sigma(E_\gamma)}{dE} \sigma_\gamma(E_\gamma) N_I(E_\gamma) dE \int_0^L \int_0^W e^{-AN^\sigma} \text{non}^Z R(\rho, z) G(\rho, z) d\rho dz$$

Where $G(\rho, z)$ = fraction of the light produced that intercepts the PD.

Assuming the gammas/electrons are produced isotropically $G(\rho, z)$ can be specified for a given point inside the radiator as the fractional solid angle subtended by the PD., i.e.

$$G(\rho, z) = .5 * (1 - \cos \phi) \cos \alpha$$

where $\phi = \phi(\rho, z)$ = angle subtended by PD

$$\alpha = \alpha(\rho, z) = \text{off axis angle}$$

The first "energy integral", I_1 , involves the gamma energy only, at a given neutron energy, and can be performed if $d\sigma(E_\gamma)/dE$ has been measured. The $d\sigma(E_\gamma)/dE$ cross section term for silicon and oxygen has been measured for a number of neutron energies. (See Figure 5-7). * These reports give more detailed spectra, typically resolution of Figure 7.

* J.L. Dickens, T.A. Love, and G.L. Morgan, "Gamma-Ray Production from Neutron Interactions with Silicon for Incident Neutron Energies between 1.0 and 20.0 MeV: Tabulated Differential Cross Section", ORNL TM-4389, 1973.

G.L. Morgan and G.T. Chapman, "The $0(N, X\gamma)$ Reaction Cross Section for Incident Neutron Energies between 6.5 and 20.0 MeV", ORNL-5575 1979.

The $\sigma_Y(E_Y)$ cross section is obtainable (See Figure 4) from Plechaty's et. al. tabulation op. cit. It is operationally easier to split the calculation of the first integral into its silicon and oxygen components, since these two cross sections are tabulated separately.

In an effort to reduce the number of data points required as inputs to the calculation the components of the first integral were investigated further. Figure 5 shows $\sigma_Y(E_Y)N_I(E)$ plotted against E_Y . As can be seen, this function is nearly linear in E . A fit of this function gives

$$\sigma_Y(E)N_I(E) = 180(E_Y - .316)$$

for $1.5 < E_Y < 10$ this equation is accurate to $\pm 5\%$. Since most of the inelastic gammas are above 1.5 MeV, this implies that the first integral (over energy) can be reduced to a simpler form with acceptable accuracy, i.e.

$$I_1(E_n) = A_n^2 180(\bar{E}_Y - .316) \sigma(E_n)$$

In this form, much work can be saved since $\sigma(E_n)$ and $\bar{E}_Y(E_n)$ are tabulated for oxygen and the differential cross sections are not needed. Figure 8 and Figure 9 show the average gamma energy and gamma production cross sections for oxygen. Unfortunately, these quantities were not tabulated for silicon and the differential gamma production cross section had to be used at each neutron energy. Figures 10 and 11 show a summary of these calculations for Si and O respectively. Figure 12 shows the sum of one Si and two O (SiO_2) "energy" integrals.

The two "geometry integrals" over ω and z are a bit messy but solvable. The equivalent of these two integrals written explicitly out for a Monte Carlo integration is:

$$I_2 = \left[\sum_{i=1}^N \text{EXP}(-A_n \sigma_{\text{non}} z) * R * \Omega \right] / N$$

where $z \equiv$ distance into Cerenkov converter neutron travels to point P .

$R \equiv$ distance from point, P , inside converter to outside of converter.

$\Omega^* \equiv$ fraction of light that can escape from point P to photodiode.

$N \equiv$ number of points selected.

In a Monte Carlo calculation of this integral, points are selected uniformly at random throughout the converter volume for a fixed cross section (σ_{non}) and the average, I_2 , found. Figure 13 shows the calculated value of this integral as a function of the nonelastic cross section, σ_{non} . Also shown is a least square fit to the somewhat noisy Monte-Carlo results.

The value of this integral depends only on the geometry of the Cerenkov converter and the neutron inelastic cross section (See Figure 4). By using the coefficients of the second order fit the "geometry integral" vs. neutron energy (nonelastic cross section) can be obtained as seen in Figure 15. The product of the energy

* The calculation of Ω is not, strictly speaking, correct since Ω should be calculated along L and not just at point P . This was not deemed worth the extra trouble since the Cerenkov light produced is very nearly uniform in the Cerenkov converter.

integral (Figure 12) and the geometry integral (Figure 15) gives the number of Cerenkov photons produced versus neutron energy that intercept the photocathode per incident neutron. (See Figure 16).

To calculate the neutron sensitivity of this detector, the photocathode's light efficiency is needed. Figure 17 shows a measured S-20 photocathode response. This photocathode is probably more efficient than typical as the peak efficiency at 2.6 eV (477nm) is .3 electrons/photon and a more typical value might be .2 electron/photon.

The average photocathode sensitivity is given by

$$\bar{\epsilon} = \frac{\int_{E_1}^{E_2} \epsilon(E) N(E) dE}{\int_{E_1}^{E_2} N(E) dE}$$

Ignoring dispersion $N(E) = N_0 \equiv \text{constant}$ and the integrals above reduce to:

$$\bar{\epsilon} = \frac{\int_{E_1}^{E_2} \epsilon(E) dE}{(E_2 - E_1)}$$

For the photocathode sensitivity shown in Figure 14:

$$\bar{\epsilon} = .187.$$

The implied neutron sensitivity is shown in Figure 18. The absolute accuracy of this calculation should be as good as the accuracy of the approximations and cross sections used. This would imply $\pm 20\%$. But, the uncertainty could be greater due to light collection efficiency -- critical angle scattering, etc.; effects that were not considered. Also possible are incorrect normalizations done in the

calculations. All of these effects should not significantly affect the shape of the sensitivity curve -- only its magnitude.

The neutron/Cerenkov sensitivity of lucite* ($C_5H_8O_2$) can also be determined by the process just outlined for SiO_2 . Figure 19 shows the inelastic gamma production cross sections for oxygen and carbon. The inelastic carbon cross section being obtained by multiplying the measured** differential cross section at 125 degrees by 4π . Carbon is nearly unique in that only one inelastic gamma ray is produced, 4.43 MeV, for neutrons in the energy range of interest.

Figure 20 shows the geometry and energy integrals for $C_5H_8O_2$. The geometry integral is calculated by including the elastic hydrogen cross section in q_{non} since neutrons colliding with hydrogen suffer appreciable energy loss and direction change. Figure 21 shows the number of Cerenkov photons incident on the PD per incident neutron. This is obtained by multiplying the energy and geometry integrals together. Figure 22 shows the implied neutron sensitivity -- under the same assumptions used to derive the SiO_2 sensitivity. Figure 23 shows a comparison of the two sensitivities.

Conclusions

The mechanisms and energy dependence of neutron induced Cerenkov light production in lucite and quartz (suprasil) can be reasonably

* Lucite is the Dupont trade name for methylmethacrylate. The calculation here assumes UVT (ultra-violet transmitting) lucite. Other trade names for UVT methylmethacrylate are plexiglas II (UVT), and some acrylic plastics.

** J.K. Dickens, G.L. Morgan, G.T. Chapman, T.H. Love, E. Newman and F.G. Perey; "Cross Sections for Gamma Ray Production by Neutrons for 22 Elements Between $Z = 3$ and $Z = 82$ ", NSE 62, P515, 1977.

well understood. Both materials are dependent upon the inelastic neutron cross section and inelastic gammas produced. This gives both materials useful high "effective" neutron sensitivity thresholds corresponding to the neutron inelastic scattering thresholds. These thresholds would be lower in thick detectors due to neutron moderation and subsequent capture gamma production.

Despite the apparent superior performance of suprasil vs. lucite, it cannot be recommended as a neutron detector without more investigation. As reported by Doyle and Dickinson*, quartz may have as much as 70% of the light produced present in slow (non-Cerenkov) luminescence components. Table 2 shows a summary of their data. Whether suprasil has these same light components or not should be investigated before suprasil is used as a Cerenkov detector.

Table 2

Material	Decay component	Light fraction	Material	Decay component	Light fraction
Quartz	prompt	0.30	LiF	prompt	0.13
	21 μ sec	.12		137 μ sec	0.36
	110 μ sec	.43		710 μ sec	0.43
	390 μ sec	.08		3.3 msec	0.08
	2.37 msec	.07			
Sapphire	prompt	.23	Lucite	prompt	1
	4.3 μ sec	.006		15 μ sec	1.0×10^{-4}
	34 μ sec	.005		42 μ sec	1.8×10^{-4}
	470 μ sec	.03		650 μ sec	6.8×10^{-4}
	1.58 msec	.16		2.06 msec	2.5×10^{-3}
	4.29 msec	0.57			
			H ₂ O	prompt	1
				60 μ sec	$\sim 7 \times 10^{-5}$

* J.E. Doyle and W.C. Dickinson, "The Cerenkov Response of Lucite and Quartz to Gamma and Fast Neutron Radiation", UCRL-7032, 1962.

CERENKOV LIGHT PRODUCED BETWEEN 1.3 - 3.6 EV VS. ELECTRON ENERGY.
 FILES .. CL/EL S.. CL/EL L.. CL/EL/L

ELECTRON ENERGY DEPOSITED PER GAMMA INTERACTION IN SiO_2 .
FILES .. ENE-

PRODUCT OF GAMMA CROSS SECTION AND Cerenkov LIGHT PRODUCTION.
FILES .. CL*GX5

TOTAL CROSS SECTION (BARNS)

SiO₂ GAMMA TOTAL CROSS SECTION VS GAMMA ENERGY.
FILES .. GSXECT

Figure 4

NUMBER OF CERENKOV PHOTONS PRODUCED FOR GAMMA INTERACTION.
FILES .. CG/GE

SILICON (N,XG) DIFFERENTIAL GAMMA PRODUCTION CROSS SECTIONS.

FILES .. ES110 0.. ES110 1.. ES111 2.. ES112 4.. ES114 7.. ES117

SILICON (N,XG) DIFFERENTIAL GAMMA PRODUCTION CROSS SECTIONS.

FILES .. ES12 2.. ES12 3.. ES13 4.. ES14 5.. ES15 6.. ES16
7.. ES17 8.. ES18 9.. ES19

OXYGEN (N,XG) DIFFERENTIAL GAMMA PRODUCTION CROSS SECTIONS.
FILES .. OE13E

AVERAGE GAMMA ENERGY PRODUCED BY INELASTIC NEUTRON COLLISIONS IN OXYGEN.
FILES .. OXEG

Figure 9

OXYGEN INTEGRATED GAMMA PRODUCTION CROSS SECTION VS. NEUTRON ENERGY.
FILES .. O-XS

INTEGRAL OF LIGHT PRODUCED/NEUTRON VS. NEUTRON ENERGY.
FILES .. SILIGHT +.. SILIGHT

CERENKOV LIGHT PRODUCTION PROBABILITY DUE TO OXYGEN.
FILES .. OXLL

SUM OF SILICON AND OXYGEN LIGHT PRODUCTION IN SiO_2 .
 FILES .. SiO2LIGHT S.. SiLIGHT O.. OXLL

GEOMETRY FACTOR INTEGRAL FOR 7.62CM X4.44CM DIA. S102.
FILES .. GE04FIT +.. GE03.5

NEUTRON NON-ELASTIC CROSS SECTION IN SiO_2 .
FILES .. XSNON

GEOMETRY FACTOR INTEGRAL VS. NEUTRON ENERGY.
FILES .. GE04/EN

CERENKOV LIGHT/NEUTRON FROM 7.62CM X 4.44CM DIA. S10**2.
FILES .. S102L*GEO

MEASURED FW-114A (S-20) PHOTOCATHODE SENSITIVITY.
FILES .. S20E

Figure 18

CERENKOV NEUTRON SENSITIVITY FOR PD .18 EFFICIENT.
FILES .. SENS/S102

OXYGEN AND CARBON INELASTIC GAMMA PRODUCTION CROSS SECTIONS.
FILES .. OGX5 C.. CGX5

COMPONENTS OF THE NEUTRON INDUCED Cerenkov LIGHT IN LUCITE (C5H8O2)
 FILES .. CL5+OL2 O.. OL2 C.. CL5 G.. GEO/LUC

CERENKOV PHOTONS/NEUTRON --GEOMETRY FACTOR

LUCITE-NEUTRON INDUCED Cerenkov LIGHT PRODUCTION.
FILES .. CHOL*GEO

CALCULATED NEUTRON SENSITIVITY OF LUCITE CERENKOV DETECTOR (7.62CM X 4.44CM)
FILES .. SENS/LUC

COMPARISON OF SUPRASIL (S102) AND LUCITE (CSH802) NEUTRON SENSITIVITY.
 FILES .. SENS/S102 L.. SENS/LUC