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Nonperturbative Electromagnetic Muon-Pair Production with Capture in Peripheral Relativistic Heavy-Ion Collisions

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ABSTRACT

We discuss preliminary calculations of impact-parameter-dependent probabilities and cross sections for muon-pair production with capture of the negative muon into the K -shell of the target caused by the time-dependent electromagnetic fields generated in peripheral relativistic heavy-ion collisions. Our approach is nonperturbative in that we calculate probabilities by solving the time-dependent Dirac equation on a three-dimensional Cartesian lattice using the basis-spline collocation method. Use of the axial gauge for the electromagnetic potentials produces an interaction easier to implement on the lattice than the Lorentz gauge.

I. INTRODUCTION

Lepton-pair production has been widely discussed as a possible tool to help probe the formation and the decay of the quark-gluon plasma phase of matter in ultrarelativistic heavy-ion collisions [1]. Suggestions by several authors indicate that other sources of lepton pairs might possibly mask the leptonic signals from the plasma phase [2–4]. Electromagnetic lepton-pair production caused by highly stripped heavy ions in relativistic motion is estimated to be a major contribution to the physical background for the plasma pairs as the fields involved contain large Fourier components.

The electromagnetic production of lepton pairs with heavy ions is fundamentally different from the production mechanism with protons or electrons because the coupling constant is strongly enhanced; e.g., for very heavy systems ($Au + Au, U + U$) $Z\alpha \approx 0.5$. Applying low-order perturbation theory to this process in the high-energy regime results in probabilities which violate unitarity and cross sections which violate the Froissart bound [5, 6]. This evidence, along with nonperturbative model studies, clearly suggests that higher-order QED effects will be important at the Relativistic Heavy-Ion Collider (RHIC) [7].

In addition to these fundamental questions regarding lepton-pair production and nonperturbative QED, a valid description of electromagnetic lepton-pair production is important for both detector design and collider performance at experimental facilities such as the Relativistic Heavy-Ion Collider. Pair production with capture of the negative lepton into atomic bound states changes the charge state of the heavy ion leading to a decrease in the luminosity lifetime of the collider [2].

There is a long history of the use of perturbative methods in studying the electromagnetic production of lepton-pairs and a review article exists which discusses recent developments in this field [8]. However, as mentioned above, we desire to study nonperturbative effects in lepton-pair production. One nonperturbative approach discussed for use at mildly relativistic energies is a coupled-channels calculation which shows a five-fold increase in the cross section over perturbative predictions for electron capture into the K-shell of a participant in a fixed-target collision of $Pb + Pb$ at $1.2 \text{ GeV}/A$ [9].

In this paper, we discuss our nonperturbative approach which is applicable at ultrarelativistic energies. Beginning with the QED Lagrange density, we make reasonable assumptions about the nature of the lepton and radiation fields in peripheral relativistic heavy-ion collisions which simplify the equations of motion to the time-dependent Dirac equation and the classical Maxwell equations. In doing this, we maintain the field theoretical tools for calculating particle production. We implement the solution of the Dirac equation using the lattice basis-spline collocation method [10, 11]. In these preliminary calculations we limit ourselves to muon-pair creation with capture into the K -shell, neglecting, for now, the more probable and more nonperturbative electron capture process. We do this because the necessary $1s$ bound state of the target is easier to compute for the muonic atom than for the electronic atom with our lattice methods. We neglect free-pair production for now, as this process requires the evolution in time of many states, whereas the capture process requires time-evolution of only a few states.

In developing our approach to nonperturbative lepton-pair production, we have performed $1 + 1$ and $3 + 1$ dimensional model calculations [3, 10, 12]. In the $3 + 1$ dimensional calculation for the system $^{197}\text{Au} + ^{197}\text{Au}$ at collider energies of 0.2, 1.0 and 2.0 GeV/A , we estimated the probabilities for muon-pair production with capture into the K -shell to be between 1.0×10^{-3} and 7.0×10^{-2} at the grazing impact parameter of $b = 8.8\lambda_c$ [12]. This work gave the first clear computational indication for the nonperturbative nature of lepton-pair production in relativistic heavy-ion collisions, though it required small lattices and a screened projectile interaction to make the size of the calculation manageable. Here we improve upon these three-dimensional calculations by using the physical interaction, i.e. no screening. This is made possible by taking advantage of our freedom in choosing the gauge for the electromagnetic interaction.

II. THEORETICAL FRAMEWORK

Our formalism of nonperturbative lepton-pair production in relativistic heavy-ion collisions follows closely the derivation given in Ref. [12]. We begin with the standard QED Lagrange density operator ($\hbar = c = m_0 = 1$)

$$\mathcal{L}_{QED} = \hat{\psi}^\dagger \gamma^0 [\gamma^\mu i \partial_\mu - 1] \hat{\psi} - \frac{1}{4} \hat{F}_{\mu\nu} \hat{F}^{\mu\nu} - \hat{j}^\mu \hat{A}_\mu, \quad (1)$$

where

$$\hat{j}^\mu = \hat{j}_{lept}^\mu + \hat{j}_{ext}^\mu = e \hat{\psi}^\dagger \gamma^0 \gamma^\mu \hat{\psi} + \hat{j}_{ext}^\mu \quad (2)$$

denotes the total four-current density operator which consists of the lepton current and the conserved external current generated by the colliding heavy nuclei. By

varying the action integral with respect to the field operators $\hat{\psi}$ and $\hat{A}_\mu$, we obtain the Euler-Lagrange equations of motion for the quantum fields

$$[\gamma''(i\partial_\mu + e\hat{A}_\mu) - 1]\hat{\psi}(x) = 0, \quad (3)$$

$$\partial_\mu \hat{F}^{\mu\nu}(x) = j^\nu(x). \quad (4)$$

These equations of motion are difficult to solve so we will now make the following simplifying assumptions. First, we neglect the leptonic current because the external heavy-ion current is much larger. Second, we shall assume that the radiation field may be treated classically. Hence, Eqs. (3) and (4) decouple. We now have the problem of solving the time-dependent Dirac equation for the lepton field $\hat{\psi}$ interacting only with an external, classical four-vector potential A_μ^{ext} which is determined independently by the Maxwell equations.

We study the electromagnetic production of lepton pairs in a reference frame in which one of the nuclei, henceforth referred to as the target, is at rest. The target nucleus and the muon interact via the static Coulomb field, A_T^0 . The only time-dependent interaction, $A_P^\mu(t)$, arises from the classical motion of the projectile. Thus, it is natural to recast the Dirac equation in Schrödinger form

$$[H_F + H_P]\hat{\psi}(\vec{r}, t) = i\frac{\partial}{\partial t}\hat{\psi}(\vec{r}, t), \quad (5)$$

where the static Furry Hamiltonian, which describes the lepton field in the presence of the strong external Coulomb field of the target nucleus, is given by

$$H_F = -i\vec{\alpha} \cdot \nabla + \beta - eA_T^0, \quad (6)$$

and the time-dependent interaction between the lepton field and the projectile is

$$H_P(t) = e\vec{\alpha} \cdot \vec{A}_P(t) - eA_P^0(t). \quad (7)$$

Following the usual practice, we expand the field operator $\hat{\psi}(\vec{r}, t)$, defined in Eq. (5), into complete orthonormal sets of single-particle basis states. First we consider the Furry basis $\{\chi_i(\vec{r})\}$, i.e. the stationary eigenstates of the Furry Hamiltonian H_F defined in Eq. (6)

$$H_F\chi_i(\vec{r}) = E_i\chi_i(\vec{r}), \quad (8)$$

which are also proper in- and out-states for asymptotic times $|t| \rightarrow \infty$, where the interaction $H_P(t)$ is zero

$$\hat{\psi}(\vec{r}, t) = \sum_i \hat{a}_i(t)\chi_i(\vec{r})\exp(-iE_it). \quad (9)$$

From the anticommutation relations for the fermion field operators $\hat{\psi}$ and $\hat{\psi}^\dagger$, one readily shows that the $\hat{a}$'s are operators which describe the creation and annihilation of leptons in the static Coulomb field of the target nucleus. They define what one may call the mathematical vacuum $|0\rangle$, i.e., $\hat{a}_i|0\rangle = 0$. We also expand the fermion field operator in terms of the time-dependent basis $\{\phi_j(\vec{r}, t)\}$ of solutions to the full Dirac Hamiltonian $H_F + H_P(t)$

$$[H_F + H_P(t)]\phi_j(\vec{r}, t) = i \frac{\partial}{\partial t} \phi_j(\vec{r}, t). \quad (10)$$

The operator-valued expansion coefficients, denoted by $\hat{\alpha}_j$, are quasi-particle destruction operators

$$\hat{\psi}(\vec{r}, t) = \sum_j \hat{\alpha}_j \phi_j(\vec{r}, t). \quad (11)$$

The QED ground state, in the presence of the undercritical external field of the target nucleus, $|\Phi_0\rangle$, is a many-lepton state of time-independent one-particle solutions of the Furry Hamiltonian where all states with energies less than $-m_0c^2$ are occupied, i.e. a single Slater determinate of the form

$$|\Phi_0(-\infty)\rangle \equiv |\Phi_0\rangle = \prod_{i < F} (\hat{a}_i^\dagger) |0\rangle, \quad (12)$$

where $i < F$ denotes states below the Fermi energy $E_f = -mc^2$. The time-evolved QED ground state is

$$|\Phi_0(t)\rangle = \hat{U}(t, t_0 \rightarrow -\infty) |\Phi_0\rangle, \quad \hat{U}^\dagger(t, t_0) \hat{U}(t, t_0) = 1, \quad (13)$$

where $\hat{U}(t, t_0)$ is the time-evolution operator defined in the Schrödinger picture by

$$\hat{U}(t, t_0) = e^{-i(H_F + H_P(t))(t - t_0)}. \quad (14)$$

Equations (9), (11), and (12) are consistent with the particle-hole (Dirac sea) picture with both positive- and negative-energy states. However, we wish to work in the space-time picture with physical particles and antiparticles. Therefore, we now define antilepton creation and annihilation operators with respect to the QED ground state $|\Phi_0\rangle$ via

$$\hat{b}_i \equiv \hat{a}_i^\dagger, \quad \hat{b}_i^\dagger \equiv \hat{a}_i, \quad i < F. \quad (15)$$

Note that these new operators describe creation and annihilation of antiparticles in the static Coulomb field of the target nucleus. Also, the QED ground state behaves as the Furry-vacuum state for the creation of physical particles

$$\hat{a}_p^\dagger |\Phi_0\rangle = |\chi_p^{(+)}\rangle, \quad \hat{a}_p |\Phi_0\rangle = 0, \quad p > F, \quad (16)$$

where $|\chi_p^{(+)}\rangle$ is a Furry state with positive energy. We also define a new set of quasiparticle and quasiantiparticle creation operators with respect to the time-evolved Furry-vacuum $|\Phi_0(t)\rangle$

$$\hat{\beta}_i \equiv \hat{\alpha}_i^\dagger, \quad \hat{\beta}_i^\dagger \equiv \hat{\alpha}_i, \quad i < F. \quad (17)$$

We now equate the two representations of the field operator, Eqs. (9) and (11), using the representations defined in Eqs. (15) and (17) to establish the following connection between these two sets of Fock space operators

$$\hat{a}_p(t) = \sum_{s>F} \hat{\alpha}_s \langle \chi_p^{(+)} | \phi_s^{(+)}(t) \rangle e^{iE_p t} + \sum_{r<F} \hat{\beta}_r^\dagger \langle \chi_p^{(+)} | \phi_r^{(-)}(t) \rangle e^{iE_p t}, \quad p > F. \quad (18)$$

Having derived all the necessary field theoretical tools, we now evaluate the probability for lepton-pair production with capture in relativistic heavy-ion collisions. First, using Eq. (18) and the anticommutation relations for the $\hat{\alpha}$'s, we calculate the expected number of leptons produced in state $p > F$ to be

$$\langle \hat{n}_p(t) \rangle = \langle \Phi_0(t) | \hat{a}_p^\dagger(t) \hat{a}_p(t) | \Phi_0(t) \rangle = \sum_{r<F} |\langle \chi_p^{(+)} | \phi_r^{(-)}(t) \rangle|^2, \quad p > F. \quad (19)$$

Since the expectation value $\langle \hat{n}_p(+\infty) \rangle$ in Eq. (19) is positive-definite and normalized to unity as a consequence of the Pauli principle and the unitarity of the time-evolution operator, $\langle \hat{n}_p(+\infty) \rangle$ is interpreted as the inclusive probability P_p that a lepton will be created in state $p > F$.

In many experimental situations, one is interested only in the probability for producing a lepton of a specific energy regardless of its angular momentum. Calculating this requires that one averages the probabilities for all leptons produced with a given energy. However, if one is interested in pair production with capture, there will be no need for such averaging over the magnitude of the total angular momentum because of the discrete nature of the spectrum. Only averaging over the momentum projections is required; e.g. the probability of creating a lepton with energy E_p is

$$P_{E_p} = \frac{1}{2\mu_p + 1} \sum_{\mu_p = -j_p}^{+j_p} P_p = \frac{1}{2\mu_p + 1} \sum_{\mu_p = -j_p}^{+j_p} \langle \hat{n}_p(+\infty) \rangle. \quad (20)$$

From Eqs. (19) and (20) we see that to compute probabilities for lepton-pair production, we project time-evolved single-particle states onto static Furry states, i.e. compute single-particle transition amplitudes. Measurable probabilities are the asymptotic ($t \rightarrow \infty$) limit of the squares of these amplitudes. With this in mind, we identify these asymptotic transition amplitudes with matrix elements of the scattering operator, $\hat{S} \equiv \hat{U}(+\infty, -\infty)$, defined in the Furry basis as

$$\langle \chi_j | \hat{S} | \chi_i \rangle = S_{i \rightarrow j} = \langle \chi_j | \phi_i(\infty) \rangle. \quad (21)$$

We use the time-reversal symmetry of the Dirac equation to reduce the effort needed to compute capture probabilities by applying to Eqn. (20) the principle of semi-detailed balance [13]

$$\sum_{\mu_a = -j_a}^{j_a} \sum_{\mu_b = -j_b}^{j_b} |S_{a \rightarrow b}| = \sum_{\mu_a = -j_a}^{j_a} \sum_{\mu_b = -j_b}^{j_b} |S_{b \rightarrow a}|. \quad (22)$$

For example, to compute Eq. (20), we must solve for the entire time-evolved negative energy continuum. Rewriting Eq. (20), using Eqs. (19), (21), and (22), we obtain an expression where only $2j_p + 1$ time-dependent solutions of the Dirac equation are required

$$P_{E_p} = \frac{1}{2\mu_p + 1} \sum_{\mu_p = -j_p}^{+j_p} \sum_{r < F} |S_{p \rightarrow r}|^2, p > F. \quad (23)$$

In the case of capture into the K -shell of the target, one needs to compute only two time-dependent Dirac states $|\phi_{1s}^{\pm 1/2}(t)\rangle$. Currently, we are forced to assume that both magnetic substates contribute equally to P_{E_1} , as our method chosen to compute static bound states is unable to solve for degenerate eigenvectors.

III. NUMERICAL IMPLEMENTATION

In the course of Section II, we reduced the problem of electromagnetic pair production in relativistic heavy-ion collisions to that of computing single-particle excitation amplitudes in the Furry basis. We implement the solution of the Dirac equation (Eq. (8)), necessary for computing these excitation amplitudes, using the lattice basis-spline collocation method (BSCM) which is discussed in detail in Refs. [10, 11]. In this method, quantum-state vectors and coordinate-space operators are given by expansions in terms of basis-spline functions and represented on a spatial lattice. We write the BSCM lattice representation of the Dirac equation for the Furry Hamiltonian in matrix notation as

$$\mathbf{H_F} \chi_i = E_i \chi_i, \quad (24)$$

where χ_i gives the value of the spinor only at the N^3 lattice points. In effect, the BSCM reduces the partial differential equation to a series of linear algebraic equations which may be solved (at least partially) using iterative techniques. This method is very efficient and allows us to avoid notorious pathologies, e.g. fermion doubling, and to maintain basic conservation laws on the lattice [10]. We use three-dimensional Cartesian coordinates which avoid the pathologies of rotating frames and the complicated metrics of spherical coordinate systems. The time-dependent electromagnetic fields exhibit no useful symmetry in any case.

The complete eigensolution of $\mathbf{H_F}$, providing its full spectrum of stationary states, currently exceeds the state-of-the-art in computational capabilities due to the rank of $\mathbf{H_F}$ being $N^3 \times 4 \times 2$ where the lattice size N can be as large as 100. For this reason, we compute the $1s$ state needed for K -shell capture cross sections by partial eigensolution of $\mathbf{H_F}$ using an efficient iterative Lanczos algorithm [14, 15]. The Lanczos algorithm has features which are attractive for our purposes; the memory requirements are small and the method approximates extremal eigenvalues in the spectrum very well. However, the algorithm has the limitation that it cannot determine the multiplicity of any eigenvalue, i.e. the solutions are not eigenstates of the projection of the total angular momentum. A description of this algorithm as we apply it can be found elsewhere [16].

We approximate the continuum states of $\mathbf{H_F}$ by requiring the eigenstates of the lattice representation of the free Dirac Hamiltonian, $H_0 = -i\vec{\alpha} \cdot \nabla + \beta$, to be orthogonal to the bound states of $\mathbf{H_F}$. Thus, we impose onto the free Dirac continuum the correct initial condition that transition S -matrix elements between the bound and continuum states are zero before switching on the interaction. Even though the free Dirac continuum states are known analytically, we cannot use these states in our lattice calculation by simply evaluating these functions at the lattice points, as such states would not be eigenstates of our lattice Hamiltonian. We must construct the eigenstates of H_0 which we denote as $\xi_{n,\lambda,s}$.

$$H_0 \xi_{a,\lambda,s} = \lambda E_a \xi_{a,\lambda,s}; \lambda = \pm 1, \quad (25)$$

where s is the helicity [17]. Therefore, we define our Furry continuum states by modifying the states $\xi_{a,\lambda,s}$

$$\tilde{\chi}_{a,\lambda,s} \equiv \xi_{a,\lambda,s} - \sum_{b=1}^{n_b} \chi_{E_b,\kappa_b}^{\mu_b} \left(\chi_{E_b,\kappa_b}^{\dagger\mu_b} \xi_{a,\lambda,s} \right). \quad (26)$$

Currently, the sum above includes only the $1s$ bound state.

We approximate the scattering operator $\hat{S}$ by a Taylor series expansion of the lattice representation of the time-evolution operator U . We begin and end time-evolution at finite times $\pm T$, where we may neglect the projectile's interaction with the lepton. Time is discretized in the sense that the interactions are taken as constant in each of a series of successive small intervals $(t, t + \tau)$. In each interval, the solution of the lattice representation of the time-dependent single-particle Dirac equation, Eq. (10), is obtained by

$$\phi(t + \tau) = U(t + \tau, t) \phi(t) = \left[1 + \sum_{n=1}^N \frac{[-i\tau(H_F + H_P(t))]^n}{n!} \right] \phi(t). \quad (27)$$

All numerical procedures discussed for implementing our lattice methods reduce to a series of *matrix* $\times$ *vector* operations which can be efficiently executed on vector or parallel supercomputers without explicitly storing the matrix in memory.

IV. ELECTROMAGNETIC INTERACTION

We saw in Eqs. (6) and (7) that the physics of lepton pair production is defined by the electromagnetic fields of two particles in relative motion, and that these fields enter the Hamiltonian via the dimensionless interaction energy between the lepton and the colliding nuclei $\bar{A}^\mu = -eA^\mu$. We assume a spherical and homogeneous charge density for both nuclei, as finite nuclear size effects are important in the $1s$ state for heavy leptons. We also neglect recoil effects. Therefore, in the fixed-target frame of reference, the projectile moves with constant velocity β_f along a straight-line trajectory in z -direction with impact parameter b .

Since the Dirac equation is covariant under a gauge transformation of the EM potentials, the gauge may be chosen for convenience in any problem. The most familiar gauge used in problems with electric sources is the Lorentz gauge, defined by the condition, $\partial_\mu A^\mu = 0$. Since we assume the projectile to move with constant velocity, the time-dependent EM interaction between the projectile and muon in the Lorentz gauge can be generated by a Lorentz-boost of the static Coulomb field. The Lorentz factors for the fixed target and collider frames are related by $\gamma_f = 2\gamma_c^2 - 1$. Figure 1 shows the lattice representation of the temporal component of the lepton's interaction energy with the target and projectile, $\bar{A}^0(\vec{r}, t) = \bar{A}_p^0(\vec{r}) + \bar{A}_t^0(\vec{r}, t)$, in a $100 \text{ GeV}/A$ collision of $^{197}\text{Au} + ^{197}\text{Au}$ in the collider frame. Features of the interaction in the Lorentz gauge cause difficulties for our lattice methods. First, the interaction is very large as it is proportional to γ_f , and sharply peaked in the boost direction $\hat{e}_3$. Its width is inversely proportional to γ_f . Also, the long-range nature of the Coulomb potential remains after the boost as the Lorentz

gauge interaction behaves asymptotically as $1/z$. The large magnitude of the interaction requires that the time step be kept small so as to ensure a proper expansion of the time-evolution operator. Also, the long-range z dependence of the interaction requires that we start numerical calculations at large distances between projectile and target. Another major difficulty lies in representing spiked functions on a finite lattice. These features combine to make realistic, three-dimensional calculations too demanding computationally [12].

Figure 1. Depicted is a slice taken at $y = 0$ of the temporal component of the lepton's interaction energy with the target and projectile for the Lorentz gauge in a collision of $^{197}\text{Au} + ^{197}\text{Au}$ at a collider energy of $100 \text{ GeV}/A$. The center of the projectile nucleus is located at the point $(0, 8.732 \lambda_c, -10 \lambda_c)$; the target is located at the origin. A square mesh with 51×51 points is used to represent the interactions. The narrow spike of the time-dependent interaction is cut off. Actually, the extrema of this interaction is $-141,210 m_0 c^2$.

Another gauge choice exists which, for relativistic energies, produces an interaction that is better represented on the lattice [18]. The new gauge we use is the axial gauge in which the component of the vector potential in the direction of the projectile's motion is required to be zero, i.e.

$$\vec{A}_P^3[r'(t)] \rightarrow \tilde{A}_P^3[r'(t)] = \vec{A}_P^3[r'(t)] + \partial_z \Lambda[r'(t)] \equiv 0. \quad (28)$$

The new interaction is found by using the function Λ in a gauge transformation

$$\vec{A}_P^\mu[r'(t)] \rightarrow \tilde{A}_P^\mu[r'(t)] = \vec{A}_P^\mu[r'(t)] - \partial^\mu \Lambda[r'(t)]. \quad (29)$$

Figure 2 shows the x component of the axial gauge interaction of the muon with the projectile, $\tilde{A}_p^1[r'(t)]$, for the 100 GeV/A collision discussed in figure 1. The following features of the axial gauge interaction enable its easy representation on the lattice. First, the temporal component, $\tilde{A}_p^0[r'(t)]$, is very small as it is inversely proportional to γ_f . The dominate components in the interaction, $\tilde{A}_p^1[r'(t)]$ and $\tilde{A}_p^2[r'(t)]$, have a step function behavior in the large γ_f limit. Step functions require fewer lattice points than spiked functions to ensure a faithful representation on a mesh. Also, almost all of the energy dependence for the interaction is contained in the slope of the step in the x and y components which is proportional to γ_f ; the extrema are independent of γ_f for large γ_f . As a result, the axial gauge interaction is largely independent of γ_f in the large γ_f limit. Therefore, all three of the numerical difficulties associated with the Lorentz gauge interactions are removed in the axial gauge.

Figure 2. Depicted is a slice taken at $y = 0$ of the x component of the lepton's interaction energy with the projectile for the axial gauge in a 100 GeV/A collision of $^{197}\text{Au} + ^{197}\text{Au}$ in the collider frame. The center of the projectile nucleus is located at the point $(0, 8.732 \lambda_c, -10 \lambda_c)$. A square mesh with 51×51 points is used to represent the interaction.

V. RESULTS AND CONCLUSIONS

The first step in calculating the probability for muon-pair creation with the capture of the negative muon into the K -shell is the calculation of the $1s$ state of the Furry Hamiltonian. The static Coulomb interaction between the target

nucleus and muon is evaluated for a homogeneous, spherical nuclear charge distribution with a radius $R = 1.40 \times A^{1/3} \text{ fm}$ where A is the atomic number. Using the basis-spline collocation method and the iterative Lanczos algorithm referred to in Section III, we compute the muonic $1s$ state of ^{197}Au on a uniform cubic lattice within a box extending from $-20\lambda_c$ to $+20\lambda_c$ in the three Cartesian coordinates where λ_c is the muon Compton wavelength. We perform the calculation on uniform lattices varying in size from $N = 16$ to 39. We require for the convergence of each solution that the energy fluctuation $\eta = [\langle H^2 \rangle - \langle H \rangle^2]^{1/2}$ be less than 10^{-6} . For the $N = 16$ lattice, the Lanczos algorithm required about 120 recursions to obtain a fluctuation of 3×10^{-7} . The Cray-2 CPU time required to perform these static calculations varies from 1 minute for $N = 16$ to 40 minutes for $N = 39$. The energy eigenvalues obtained for different lattices are shown in figure 3. The accuracy of our static solution is checked by performing an integration of the radial Dirac equations which gives an energy of $0.9127m_\mu c^2$ for the muonic $1s$ state in ^{197}Au . We observe a N^{-2} convergence of the static lattice solutions.

Figure 3. Energies for the $1s$ muonic state of ^{197}Au computed with the basis-spline collocation method as a function of the lattice size N . The energies approach a limiting value as the lattice size is increased.

We perform the time-development of the muonic $1s$ state of ^{197}Au , i.e. solve Eq. (10), on the 16^3 lattice under the influence of the time-dependent external fields produced in the collision of $^{197}\text{Au} + ^{197}\text{Au}$ at a collider energy of 100 GeV/A . A maximum number of 15 terms in the Taylor series expansion of the time-evolution operator is sufficient to preserve conservation of the norm of the wavefunction at the 1 part in 10^6 level. This is an important indicator for the numerical accuracy of the time-development of the Dirac spinor. We use the axial gauge for the interaction of the muon with the projectile. Initially, the projectile nucleus is

positioned at $(0, b, -200 \lambda_c)$ giving rise to a negligible interaction with the muon at the position of the target nucleus which is fixed at the center of the cubic lattice. We evolve the wavefunction in time for 2000 time steps with $\Delta t = 0.2/\beta_f$ in units of λ_c/c , stopping the evolution when the projectile is positioned at $(0, b, 200 \lambda_c)$. One such time-dependent run for a given impact parameter requires 33 minutes of Cray-2 CPU time.

The dependence of the muon capture probability on the beam energy is shown in figure 4 by varying the energy between 0.1 and 100 GeV/A in the collider frame of reference for the grazing impact parameter $b = 8.732 \lambda_c$. Notice that the capture probability is independent of the beam energy for $E_c \geq 16$ GeV/A. The energy independence of the interaction of the muon with the projectile in the high-energy limit was discussed in Section IV in the context of the axial gauge.

Figure 4. Muon capture probability into the K-shell in collisions of $^{197}\text{Au} + ^{197}\text{Au}$ as a function of beam energy in the collider frame at the grazing impact parameter $b = 8.732 \lambda_c$. The calculations are performed on a uniform cubic lattice with 16^3 collocation points. These results have yet to converge because of the use of small lattice sizes. We give a subjective estimate of the error in the probabilities from this limitation by the indicated error bars.

The dependence of the muon capture probability on impact parameter at a collider energy of 100 GeV/A is shown in figure 5. For peripheral impact parameters (i.e. $b \geq 8.732 \lambda_c$), the capture probability decreases proportionally to b^{-2} as shown by the fit to the results in figure 5. We calculate the total cross section for muon capture into the K-shell of ^{197}Au by integrating the capture probabilities over b with the formula

$$\sigma = 2\pi \int_0^\infty b P(b) db. \quad (30)$$

We cut off the integral at $b = 700 \lambda_c$ as the b^{-2} dependence of the probability results in a logarithmic divergence. Therefore, our estimate of the total cross section for muon-pair production with K-shell capture in collisions of $^{197}\text{Au} + ^{197}\text{Au}$ at 100 GeV/A is 0.57 b with 0.43 b coming from peripheral collisions.

Figure 5. Probability for muon capture into the K-shell as a function of the impact parameter in units of λ_c for $^{197}\text{Au} + ^{197}\text{Au}$ collisions at a collider energy of 100 GeV/A. We show a b^{-2} fit to the probabilities for impact parameters greater than grazing. Error bars from the subjective error estimates are smaller than the data indicators because of the large logarithmic scale.

We stress that our nonperturbative calculations are preliminary and have not converged essentially because of the size of the collocation lattice dictated by the computational expense and, therefore, the results should be viewed only as an order-of-magnitude estimate. We estimate that realistic calculations for μ^- capture require 60 – 100 lattice points in each direction. We plan to meet the computational demands of such large lattices by implementing the calculation on the Intel iPSC/860 hypercube parallel supercomputer at the Oak Ridge National Laboratory.

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