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An Analytic Model for Surface Ground Motion With Spall Induced by Underground Nuclear Tests

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Abstract

This report provides a detailed presentation and critique of a model used to characterize the *surface* ground motion following a contained, spalling underground nuclear explosion intended for calculation of the resulting atmospheric acoustic pulse. Some examples of its use are included. Some discussion of the general approach of ground motion model parameter extraction, not dependent on the specific model, is also presented.

Introduction

The surface ground motion model discussed here was developed by S. I. Warshaw of Lawrence Livermore National Laboratory (LLNL) for use in calculations of the atmospheric disturbance following an underground nuclear test. The procedure described in this report for applying the model was developed by the author. The primary focus of the discussion is the surface ground motion. However, a few examples of the atmospheric disturbance calculations are included. The model is specifically limited to events in which spall occurs, but it could be adapted for non-spalled events.

Section 1: The Ground Motion Model

During an underground nuclear test (UGT) an array of surface accelerometers is placed at various distances and directions from surface ground zero (SGZ). The recorded acceleration time histories, together with the velocity histories (usually derived by integration but sometimes measured directly), form a typical ground motion data set. Ground motion stations further away from SGZ generally experience accelerations of lesser magnitude, and ground motion stations at different azimuths often record substantially different acceleration histories, even when the stations are at the same distance from SGZ. Therefore, a fully comprehensive ground motion model would have to take into account the complexities of geology and terrain variations. The model described here is relatively simple and assumes that (a) the medium through which the shock wave passes is homogeneous, (b) the ground surface is flat and horizontal, and (c) the ground motion is cylindrically symmetrical. In addition, our model is specifically intended to treat spall, since it occurs quite frequently in contained UGTs.

Our basic model is that the surface acceleration experienced at a given distance from SGZ after a device is exploded is a piecewise linear function of time. The integral of this acceleration function is a velocity function; both of these functions are shown in Fig. 1. Each of the 11 vertices, which occur at times t_1 through t_{11} on the acceleration curve, and each of the four velocity extrema labeled v_a , v_d , v_v , and v_b is provided with a mathematical model of some form. In addition, the time intervals $t_5 - t_4$ and $t_7 - t_6$ have models. (Hereafter, $t_5 - t_4$ is denoted Δt .) The interval from t_5 to t_6 is the period of ballistic free-fall characteristic of UGT spallation. It will normally have an acceleration of $-g$. The acceleration spikes at t_2 and t_7 are due to the initial blast wave and the subsequent slapdown of the spalled layer, respectively.

The specific mathematical models for the various points on the acceleration and velocity curves fall into three classes:

1. A function of distance from SGZ.
2. A constant proportion of the acceleration of gravity.
3. A constant ratio between values at two points.

Figure 1. The 10-segment piecewise linear acceleration time history model and its associated piecewise quadratic velocity time history.

The acceleration and velocity extrema a_2 , a_7 , v_{α} , and v_{β} and the time interval Δt are in the first class. We have used two different radial functions for these points. The first, developed by Warsaw, is of the form

$$y = \frac{A}{1 + \left(B \frac{RC}{ZB}\right)^C},$$

where RC is the surface distance from SGZ, ZB is the explosive placement depth, and A, B, and C are parameters to be estimated. Each of the above five quantities using this radial function (which we will refer to as the Warsaw model) is allowed to have separate parameter values.

The second function is the generally accepted¹ power law for a_2 and v_{α} . It is of the form

$$y = K \left(\frac{Y RC^2 + ZB^3}{Y^{1/3}} \right)^{-E},$$

where Y is proportional to the energy released in the explosion, and K and E are the parameters to be estimated. We refer to this model as the Bernreuter model because we first learned of it through Bernreuter, Jackson, and Miller,² who used it for determining how to protect delicate diagnostic instrumentation against violent UGT induced surface motions.

Table 1 gives the variable names for each of the radial parameters we have been using in our codes. Some examples of the range of shapes these functions can take are given in Figs. 2 and 3. For the purposes of comparison all are scaled to take on a maximum value of 1.0.

The Bernreuter model applies to the initial velocity and acceleration peaks, but it does not appear to have been used for any subsequent extrema. Given that one has decided to use this model for v_{α} or a_2 , there are two possibilities for v_{β} and a_7 , both of which we have used. The first is to use the Warsaw model for v_{β} and a_7 , while the second is to assume a linear relationship between a_2 and a_7 and between v_{α} and v_{β} . That is, $a_7 = Fa_2$ or $v_{\beta} = Fv_{\alpha}$, where the parameter F is to be estimated by linear regression. In the latter case the radial dependence of a_7 has the same shape as that of a_2 , differing only by a scale factor, and similarly for v_{β} and v_{α} . In the first case a_7 and a_2 will generally drop off at different rates at different

Table 1. Radial parameters of the first class. These parameters are used to determine the values of the points of the model that have a radial dependence. (See the appendix for details.)

Point	Model					
	Warsaw			Bernreuter		
	A	B	C	K	E	F
a_2	APOS	FA	XPA			
$a_2(Y^{1/3})$				ACON	AEXP	
a_7	ANEG	FB	XPB			
$a_7(Y^{1/3})$				ACON	AEXP	ARAT
v_{11}	VPOS	FV	XPV	VCON	VEXP	
v_{11}	VNEG	FW	XPW	VCON	VEXP	VRAT
$t_5 - t_4$	DTSH	FT	XPT			

distances from SGZ. The parameters K, E, and F of the Bernreuter model will have different values for a_2 and a_7 than for v_{11} and v_{11} .

Given any set of parameters for the Bernreuter model it is easy to find a set of parameters for the Warsaw model which will match this fit to the data. In practice this is what we do when we decide to use the Bernreuter model for a particular set of data.

The point v_3 has a model which combines a radial dependence with a constant ratio. Explicitly, this is

$$v_3 = v_{11} \left[1.0 - (1.0 - F8) \left[1 + \left(F7 \frac{RC}{ZB} \right)^2 \right]^{-1} \right].$$

Note that when $RC = 0$, the model reduces to $v_3 = (F8)v_{11}$.

Except for a_{10} , the remaining points on the acceleration curve are all members of the second class of models: their magnitudes are a proportion of the acceleration of gravity (g) that does not depend on the distance from SGZ. In some cases the constant of proportion is a parameter to be estimated, and in other cases it is assumed by the model to be 0. Table 2 gives the details. For example, the model says that $a_1 = (F1)g$ and $a_5 = (-F7)g$.

The remaining velocity extrema v_3 and v_{11} , a_{10} , and the time interval $t_7 - t_6$ have the third type of model, shown in Table 3. The model says, for example, that $v_3 = (F4)v_{11}$. All remaining time and velocity points are determined using simple constant acceleration calculations relating times, accelerations, and velocities. Formulas for these are given in the appendix.

Finally, the model must specify a surface arrival time for the seismic shock wave. For this, the speed of sound in the earth (VB) is estimated from the data using the following relationship:

$$t_2 = \frac{\sqrt{RC^2 + ZB^2}}{VB}.$$

This estimate corresponds to the time at which a uniformly expanding spherical blast wave first reaches the flat surface of the ground.

A complete set of ratio parameters F1 through F8 and an appropriate set of radial parameters (Warsaw model for Δt and Warsaw or Bernreuter parameters for the extrema), together with ZB and VB, are sufficient to specify the entire array of points t_1 to t_{11} , a_1 to a_{11} , v_1 to v_{11} , v_{11} , v_{11} , v_3 , and v_3 for any RC. The model includes an additional parameter that is essential for calculating the atmospheric acoustic pulse. This is RCMAX, the radius beyond which no significant surface ground motion contribution to the acoustic pulse is assumed to occur. For the purpose of determining RCMAX, we assume that this is also close to the end of the spill zone.

Figure 2. The Warsaw radial function. Four sets of parameters show the range of possible shapes the function can have. $ZB = 100$ and $RC =$ the distance (in m) from SGZ.

Figure 3. The Bernreuter radial function. One set of parameters shows the typical shape. $RC =$ the distance (in m) from SGZ. $Y = 1.0$ and $ZB = 100.0$.

Table 2. Ratio parameters of the second class. These parameters are used to determine the value of the points of the model that are not radially dependent.

Data point	Parameter	Constant
a_1		0.0
a_3	F1	
a_4	F1	
a_5	- F7	
a_6	- F7	
a_8	F2	
a_9	- F2	
a_{11}		0.0

Table 3. Ratio parameters of the third class. These parameters specify a constant ratio between certain points on the model.

Ratio of data points	Parameter
a_{10}/a_8	F3
v_7/v_{11}	F4
v_8/v_7	- F5
$(t_7 - t_6)/(t_2 - t_1)$	F6
v_3/v_{11}	F8 ^a

^aThis is only true at RC = 0.

Before presenting some applications of this model, it will be useful to make some general remarks about the fitting procedure.

The Warsaw radial model is nonlinear in the parameters and is therefore fit using a nonlinear least squares routine.³ The Bernreuter model is linear in the logarithms of the variables and can be fit using ordinary linear regression. The primary distinction between these procedures is that the Bernreuter model minimizes relative errors, whereas the Warsaw model minimizes absolute errors. This is most important at large radii, where the data values are small. In such a case an error that would be sufficiently small in absolute value if it were near SGZ may be excessively large in relative value. For this reason the Bernreuter model tends to provide better fits at large radii.

We specify RCMAX to ensure that the ground motion beyond RCMAX will be considered negligible. This in turn indicates that the radial fits to the data should have a small value at RCMAX. This does not always happen so in some cases a constraint must be imposed during the fitting calculations.

In order to implement the constraint, a value for RCMAX and an appropriately small value for the fit at RCMAX must be chosen. The latter is provided by Bernreuter et al., who state that the spall zone will normally extend to about where the peak particle velocity (v_{11}) has fallen to about 2 ft/s, or 0.6 m/s. In some cases data have been collected at locations beyond the extent of spall (indicated by lack of free-fall on the acceleration record). Then RCMAX can be estimated by interpolation between a data point within the spall zone and another outside it. In the not uncommon absence of such data there can be considerable difficulty; extrapolating data within the spall zone to a distance that will decrease the velocity to 0.6 m/s often results in excessively large values of RCMAX.

An alternative, given by Eisler and Chilton,⁴ is to look at the time difference $t_6 - t_1$. They find that this value decreases with radius to a certain point, becoming constant thereafter. They suggest that this point is the limit of spallation and so is a suitable approximation to RCMAX. This method requires more extensive accelerometer coverage than is usually available.

One important statistical aspect of the radial data should be mentioned. With most methods of data fitting, the analyst commonly assumes that the data have equal variances. This is most definitely not the case in this situation. Close to SGZ one would expect to find little variation in the medium through which the explosive shock wave passes, and so we expect the data to have small variances. In the middle ranges, the shock wave can pass on a slant through radically different ground materials to reach accelerometers at different azimuths. Hence, a larger variance is expected due to variations in geology and multiple reflections in different strata. At large radii the magnitudes are small, so the variances must necessarily be small. In terms of fitting, this means we should expect good fits close to SGZ and at large radii but be tolerant of relatively large errors at the middle distances.

The ratio parameters are usually obtained in two steps. The first takes the geometric mean of the available data ratios. This may produce good results, but often does not, especially for the parameters for points after slapdown. Second, the parameters are adjusted as necessary to improve the overall velocity record fits, and sometimes to make the model acceleration and velocity records look reasonable. By looking reasonable, we mean that the overall magnitudes and durations of the third and fourth velocity peaks are somewhat smaller than the first and second. This will be discussed in more detail in Section 4.

Section 2. Model Fits to Event A

Here we present the fit of our model to the first of two sets of data. The first, called Event A, is an example of a relatively poor fit. The second, Event B, shows an excellent fit and will be discussed in Section 3.

2.1. Radial Model Fits

Figures 4–8 present the Warsaw and Bernreuter fits for the radial parameters for Event A. The data at a radius of 15 m will be referred to as being at SGZ. The Bernreuter fits were obtained using the value $Y = 1.0$.

The radial fits for the initial (a_2) and slardown (a_7) acceleration peaks (Figs. 4–5) illustrate some of the problems that we often encountered.

The scatter of the data for a_2 is such that no real decrease in amplitude in the first 1000 m from SGZ is apparent. The fit reflects this fact and suggests a ground motion more like a piston than a flexible diaphragm. There is some possibility of rejecting the two extreme values (85 m/s^2 at 836 m and 22 m/s^2 at 707 m), but the raw data records do not justify this, nor does doing so significantly affect the fit. The value for APOS, 47.40, is very close to the mean of the 12 points having a radius less than 1500 m. This is appropriate, because the mean is the least squares point estimate of a set of data when other variables (in this case RC) are irrelevant.

The Bernreuter fit for a_2 (Fig. 4) has a nice shape and goes through the data fairly nicely if the extreme value at 836 m is ignored. However, it overestimates the acceleration at SGZ rather badly, and its sum of squared residuals is almost twice that of the Warsaw fit.

The measured data for a_7 (Fig. 5) is disturbing in several respects. Many of the data records show clipping of the slardown acceleration peak. One such is the record at a radius of 1131 m; like several

Figure 4. Comparison of the (a) Warsaw and (b) Bernreuter radial model fits to the initial acceleration peak a_2 for Event A. $ZB = 640.0$ and $Y = 1.0$. The sum of squared residuals for curve (a) is 2572.9 and for curve (b) 5109.1.

Figure 5. Comparison of the (a) Warshaw and (b) Bernreuter radial model fits to the slapdown acceleration peak a_7 for Event A. ZB = 640.0 and Y = 1.0. The sum of squared residuals for curve (a) is 37543.0 and for curve (b) 52894.0.

Figure 6. Comparison of the (a) Warshaw and (b) Bernreuter radial model fits to the minimal velocity peak v_a for Event A. ZB = 640.0 and Y = 1.0. The sum of squared residuals for curve (a) is 3.3561 and for curve (b) 3.8858.

Figure 7. Comparison of the (a) Warshaw and (b) Bernreuter radial model fits to the first negative velocity peak v_{β} for Event A. $ZB = 640.0$ and $Y = 1.0$. The sum of squared residuals for curve (a) is 14.992 and for curve (b) 19.710.

Figure 8. The Warshaw radial fit to the shoulder drop-off time $\Delta t = t_5 - t_4$ for Event A. The Bernreuter model is not used for this variable. $ZB = 640.0$ and the sum of squared residuals is 0.12850.

others it is clipped at 23 g. The correct value is presumably some unknown amount >23 g. However, there is an apparently non-clipped data record at SGZ with a value of 24 g. We can conclude that the slapdown peak at 1131 m is at least as large as at SGZ and possibly larger. In addition, there are several data records with smaller radii, smaller magnitudes, and no indication of clipping (283 m, 329 m). This is contrary to our expectation that acceleration magnitudes generally decrease with distance from SGZ.

Since the ground motion as a whole has a finite extent, a_7 necessarily has a radial dependence. However, the clipping of the data prevents us from determining if the form of our radial function is appropriate, and if not, what form would be.

To obtain Warshaw parameters for a data set of this type, we must choose *some* data values. It is tempting to replace the clipped data with larger values, but there is no justifiable way to determine them. Since we believe that variability is generally small at SGZ, the fit should be close to these values. There is already some unknown degree of error if we fit the clipped data, and this error will increase if we try to get closer to the intermediate range data with the smaller values. This fit was therefore achieved by fixing ANEG at 235.2, which is the measured value of the data at SGZ, and by adjusting the other two parameters for a least squares fit to the data as they are, subject to the constraint that the value be small at RC MAX. Because the choices of RC MAX and the acceleration value at RC MAX are somewhat arbitrary, these parameters are not unique.

The Bernreuter fit in this case (Fig. 5) is taken to be the same shape as the Bernreuter fit for a_2 but multiplied by the factor 3.75 representing the ratio between the data values for a_7 and a_2 . We do not consider this fit acceptable because it totally ignores the plateau aspect of the data and is probably too large at very large radii such as 3000 m. The excellent match at SGZ is accidental. There are no data points shown at the radii of 2560 m and 3929 m because no spall is evident there.

In our experience v_{11} almost always has a very nice radial dependence and is therefore easy to fit. This data set is no exception. The Warshaw and Bernreuter fits for v_{11} (Fig. 6) are similar. The Warshaw model has a slightly better fit at SGZ and the extreme radii. The Bernreuter model fits slightly better in the intermediate range (283 m to 1250 m).

For this data set RC MAX can be determined in conjunction with fitting v_{11} . Since the acceleration records at 2560 m and 3929 m do not show spall and the record at 1250 m does, we assume that the limit of the extent of spall is between these radii. To estimate its location we took a linear interpolation between the values of v_{11} at 1250 m and 2560 m to find the radius where the peak velocity is approximately 0.6 m/s. The result, 2300 m, was then used as a constraint in the fitting code: only those Warshaw parameter sets which satisfy $v_{11} < 0.6$ m/s at $RC = 2300$ m are considered acceptable. For future work, we suggest that a more appropriate interpolation would be log-log-linear in the slant distance $\sqrt{RC^2 + ZB^2}$ which in this case would have given an estimate of 1890 m.

In practice, there are many local minima with approximately the same sum of squared errors when the non-linear least squares routine applies a constraint of this sort. It is thus possible to choose certain desirable features for the fit, such as an exact fit at SGZ. Such a choice is appropriate because the data have more variability in the middle ranges than at SGZ. In other words, we chose to have a better fit at SGZ at the expense of a poorer fit in the intermediate ranges.

The data for v_{11} (Fig. 7) shows a large decrease in the first 200 m. The data between 200 m and 1250 m show very little drop-off, so the fit in this region tends to overestimate the data inside 500 m and underestimate the data beyond 500 m. When there is a sharp fall-off in the data close to SGZ, the least squares fit tends to be very steep as well. This in turn tends to produce overestimates at SGZ. For this reason such data usually must be fit by fixing VNEG at a value which produces a good fit at SGZ, and then making sure that the fit is good at large radii and has a relatively small sum of squared errors.

The Bernreuter fit to v_{11} (Fig. 7) would be acceptable if the error at SGZ were not so great. However, since this fit is actually the fit to v_{11} multiplied by a scale factor representing the ratio v_{11}/v_{12} , there is no room to adjust the fit. Another way of looking at this is that the correlation between v_{11} and v_{12} is quite good except at SGZ.

The time interval Δt (Fig. 8) is generally very difficult to obtain from the data records. Δt turns out to be most useful for determining the width of the velocity peak, so we often obtain values from the velocity records rather than the acceleration records. When the data are obtained from the velocity records there is no precise way to specify where t_1 and t_2 occur on the record. We usually take them to be quite close to the

peak v_{cr} , which occurs between them. This often produces poor fits to the overall velocity records. In these cases we re-evaluate our initial estimates of Δt and adjust the data and parameters at will. This degree of freedom is not appropriate for the other radial parameters.

In virtually every data set we have seen, the value of Δt at SGZ is near 0.4 s, while beyond some critical radius all the values are less than 0.1 s. In some cases the values remain high out to the critical radius and then suddenly drop. This can lead to rather extreme parameter values, especially for the exponent.

The values of the ratio parameters were obtained by making first guesses and then adjusting for good fit. A discussion of the considerations involved in this process is found in Section 4.

2.2. Velocity Record Fits

Table 4 provides the complete set of parameters used to calculate these model ground motions. Table 5 lists the azimuth and distance from SGZ for each data record. Figures 9a-m are plots showing the model velocities superimposed on the velocity data records for Event A.

Table 4. Parameter values for Event A used to calculate the model fits shown in Figs. 9a-m and 18c-d.

Warshaw radial		Ratio		Bernreuter radial		Geological	
APOS	47.40	F1	0.85	ACON	2.61 E+05	ZB	640.0
FA	0.32	F2	0.75	AEXP	1.29	VB	1735.0
XPA	9.24	F3	0.65	ARAT	3.75	RCMAX	2300.0
ANEG	235.20	F4	0.41	VCON	8.17 E+04		
FB	0.42	F5	0.67	VEXP	1.53		
XPB	6.16	F6	0.16	VRAT	1.31		
VPOS	4.60	F7	0.85	Y	1.0		
FV	1.00	F8	0.66				
XPV	1.49						
VNEG	-7.39						
FW	1.45						
XPW	1.66						
DTSH	0.50						
FT	5.48						
XPT	1.57						

Table 5. Accelerometer locations for Event A.

Radius RC (m)	Azimuth	Figure shown	Spall
15	SW	9a	yes
15	SE	9b	yes
283	SW	9c	yes
329	NW	9d	yes
385	S	9e	yes
466	SE	9f	yes
707	SW	9g	yes
792	SE	9h	yes
836	NE	9i	yes
1131	SW	9j	yes
1250	NE	9k	yes
2560	SE	9l	no
3929	SE	9m	no

Perhaps the most disconcerting feature of this data is that, in 8 of the 11 data records with observed spall, the velocity does not return to zero. (See Figs. 9b, c, and e.) This clearly indicates that there may be serious problems in instrument calibration or in data collection, reduction, and presentation. Since these symptoms are present in every data set we have seen, it is equally clear that a full understanding of ground surface behavior during a UGT will not be possible until better data are available. These problems greatly hinder our use of the model. However, since the data prior to slapsdown are usually very good, we

Figure 9. Velocity time history fits for Event A at various distances from SGZ. Each figure (a) through (m) shows the fit of the model velocity record to an accelerometer's data velocity record at a different radius.

suspect that the extreme forces occurring during slapdown may have been enough to alter the accelerometer calibration in these cases.

With regard to both v_α and v_β the choice of radial fit can easily be seen in the superimposed plots. For instance, the fits are quite good at SGZ (Figs. 9a-b), whereas the intermediate ranges show deviations of varying magnitudes. A fit such as the one at 1250 m (Fig. 9k) may look rather bad when viewed individually, but by referring to the radial fits for v_α and v_β , we can see that these deviations were necessary in order to satisfy other considerations.

In cases where the radial fit is a global minimum, we can indeed claim that any improvement at a particular radius will cause an even larger deterioration of the fit of at least one other radius. In cases where the radial fit is a local minimum due to some constraint, there may be several competing fits. It may

Fig. 9 continued.

then be possible to improve the fit at a particular location without decreasing the overall sum of squared errors. Whenever this is the case, we choose to give priority to SGZ.

The evaluation of the fit after slapdown, which depends primarily on the ratio parameters, presents problems. It is usually impossible to do at those radii for which the velocity data does not return to zero. If the velocity records that do return to zero have shapes too different from our model, evaluation is difficult or impossible. For this data set only the data record at 15 m SE (Fig. 9b) is useable, so we judge the ratio parameters acceptable because the fit to this record is acceptable.

Fig. 9 continued.

Fig. 9 continued.

Section 3. Model Fits to Event B

This section shows how our model fits the data from Event B. Section 3.1 contains the radial parameter fits in Figs. 10–14, and Section 3.2 contains the superimposed velocity plots. The Bernreuter fits were calculated using the value $Y = 1.0$.

3.1. Radial Model Fits

Figure 10 shows the Bernreuter fit to the data for a_2 . The unconstrained Warsaw fit, not shown here, is essentially the same. Clearly, the lack of data beyond 701 m causes the least squares fit to predict an absurdly large value for a_2 at large radii. Fits of this sort can be improved by introducing estimates of data with small values at large radii (for the Bernreuter model) or applying a magnitude constraint (for the Warsaw model), but we must be able to decide where and how small. This is difficult, because all of the data records appear to be well within the spall zone. Thus, we have no direct way of estimating the extent of spall and no way of estimating RCMAX.

In some of the first data we looked at for other events, the extent of spall appeared to be approximately five times the burial depth. Although we have since come to believe that using this rule of thumb generally produces an overestimate, we used it here and obtained $RCMAX = 3500$ m. For the magnitude constraint at RCMAX we used acceleration and velocity values comparable to those observed in other data sets, such as Event A, which had data at comparably large radii. Since this procedure leaves considerable room for many choices of RCMAX and magnitude at RCMAX, these parameters are not unique in any sense. Other parameter sets could easily produce equally good fits. It is worth noting here that we only use the 0.6 m/s criterion for determining RCMAX when we have enough data to interpolate.

The Warsaw and Bernreuter fits to a_2 (Fig. 11) were obtained using the constraint and estimated data, respectively. Clearly, the Warsaw fit is better.

The data for a_1 (Fig. 12) suggest rather strongly that something is wrong, but there are no indications on the data records as to whether the problem is with the data at SGZ or at 700 m. The Warsaw fit was obtained only by applying the constraint. Based on our experience with other data sets, we are inclined to be suspicious of the SGZ data; this is why we have settled for a fit that badly overestimates at SGZ. The Bernreuter fit is not significantly different from the Warsaw fit. The data record at 350 m was not included in the fitting procedure, since it showed double (and perhaps triple) spall. When this happens it is not clear which slapdown peak to use.

For Event B, as for Event A, the v_a data shows a nice radial dependence (Fig. 13). The Warsaw fit is slightly better, due mainly to its fit at SGZ. The Warsaw fit predicts 0.6 m/s at $RC = 2000$ m, which suggests that the fit is satisfactory even if we overestimated the extent of spall by as much as 1500 m.

Both of the fits to v_a are satisfactory (Fig. 14). Since they are so alike, the choice of which to use is almost arbitrary. We chose the Warsaw fit because it does a better job with the three data records at 701 m. The data record at 350 m was not used for v_a , since it was not clear where v_a occurred on the velocity record.

The data shown on Fig. 15 are our initial guesses for Δt . They produced very bad fits of the first velocity peak widths on the superimposed velocity plots. The fit is to the adjusted data, which is not shown. The adjustment procedure is as follows. Taking each data record separately, we determine by trial and error the value of Δt that produces a good fit of the velocity peak width. These new values are then fit as if they had been obtained directly from the data records. This approach was very successful with this data set.

Figure 10. The Bernreuter unconstrained radial fit to the initial acceleration peak a_2 for Event B. The data set contained no points at large radii. Without an appropriate constraint, the fit predicts excessively large values of a_2 at large distances from SGZ. $ZB = 701.0$ and $Y = 1.0$. The sum of squared residuals is 25.717.

Figure 11. Comparison of the constrained (a) Warsaw and (b) Bernreuter radial model fits to the initial acceleration peak a_2 for Event B. $ZB = 701.0$ and $Y = 1.0$. The sum of squared residuals for curve (a) is 1388.0, and for curve (b) 3470.0.

Parameters	
(a)	
ANEG	= 193.66
FB	= 1.15
XPB	= 2.19
(b)	
ACON	= 7.33 E+08
AEXP	= 2.42
ARAT	= 2.13

Figure 12. Comparison of the constrained (a) Warsaw and (b) Bernreuter radial model fits to the initial acceleration peak a_7 for Event B. ZB = 701.0 and Y = 1.0. The sum of squared residuals for curve (a) is 52830.0, and for curve (b) 58914.0.

Parameters	
(a)	
VPOS	= 4.22
FV	= 0.91
XPV	= 1.91
(b)	
VCON	= 1.76 E+07
VEXP	= 2.31
VRAT	= 1.38

Figure 13. Comparison of the constrained (a) Warsaw and (b) Bernreuter radial model fits to the initial acceleration peak v_α for Event B. ZB = 701.0 and Y = 1.0. The sum of squared residuals for curve (a) is 0.79862, and for curve (b) 1.7369.

Figure 14. Comparison of the constrained (a) Warshaw and (b) Bernreuter radial model fits to the initial acceleration peak v_β for Event B. ZB = 701.0 and Y = 1.0. The sum of squared residuals for curve (a) is 0.47155, and for curve (b) 0.85540.

Figure 15. Our initial guesses for Δt using the Warshaw radial fit. We used adjusted data, which we produced by taking each data record separately and determining by trial and error the value of Δt that produces a good fit of the velocity peak width. ZB = 701.0 and Y = 1.0. The sum of squared residuals is 7.735 E-02.

3.2. Velocity Record Fits

Table 6 lists the parameters used to calculate the superimposed velocity plots found in Figs. 16a-g. Table 7 is a list of the station radii and azimuths. All are within the spill zone.

We were able to obtain good to excellent fits to the first positive and negative peaks (v_{α} and v_{β}), both in magnitude and width, at all locations except 350 m/s, where it is not clear what a good fit would be. The second two velocity peaks (v_{γ} and v_{δ}) are not quite so good. We should mention that when there are few data records, and several of them at the same radii, fitting all of them is relatively easy.

Table 6. Parameter values for Event B used to calculate the model fits shown in Figs. 16a-g and 20.

Warshaw radial		Ratio		Bernreuter radial		Geological	
APOS	62.16	F1	1.13	ACON	7.33 E+08	ZB	701.0
FA	0.53	F2	0.75	AEXP	2.42	VB	1843.0
XPA	4.09	F3	0.58	ARAT	2.13	RCMAX	3500.0
ANEG	193.66	F4	0.34	VCON	1.76 E+07		
FB	1.15	F5	0.65	VEXP	2.31		
XPB	2.19	F6	0.29	VRAT	1.38		
VPOS	4.22	F7	0.92	Y	1.0		
FV	0.91	F8	0.44				
XPV	1.91						
VNEG	-6.40						
FW	1.01						
XPW	2.18						
DTSH	0.41						
FT	4.99						
XPT	1.32						

Table 7. Accelerometer locations for Event B.

Radius RC (m)	Azimuth	Figure shown
15	S	16a
15	SW	16b
350	S	16c
701	NW	16d
701	NE	16e
701	SW	16f
769	S	16g

Figure 16. Velocity time history fits for Event B at various distances (in m) from SGZ. Each figure (a) through (g) shows the fit of the model velocity record to an accelerometer's data velocity record at a different radius.

Fig. 16 continued.

Section 4. Ratio Parameters

The first ratio parameter, F1, specifies the magnitude of the shoulder following the first acceleration peak. Figure 17 shows schematic examples of the first portion of four typical acceleration records. From these it can be seen that the Warsaw model is evidently too simplistic for these cases.

Figure 17a is typical of accelerations very close to SGZ. Beyond some radius (500 m for Event A) the shoulder has usually disappeared, as seen in Fig. 17d. The remaining examples are typical of medium ranges. The arrows indicate approximately where the data values for a_3 and a_4 should be taken. In the case of Event A only five of the acceleration records yielded values for a_3 . Four of the seven data records for Event B were readily interpretable. The value of F1 is the geometric mean of the useable data. F1 has a small effect on the width of the first velocity peak.

The parameter F2 specifies the magnitudes of the accelerations a_8 and a_9 . Acceleration records in this region often look more like noise than any easily described function, so the model here is definitely an over-simplification. Obtaining values is very difficult, but if there are any data records from which a value for a_8 or a_9 can be plausibly determined, we use their geometric mean for an initial guess for F2. The main function of F2 is to determine, along with F4, the time interval $t_9 - t_8$.

F4 determines the value of v_7 , the second velocity peak, as a proportion of v_{11} . Thus v_7 has a radial dependence. The geometric mean of the ratio v_7/v_{11} is used for F4. Generally, the magnitude of the ground motion will decrease with time, so we expect F4 to be less than 1.0.

The time interval $t_9 - t_8$ is calculated with the equation

$$t_9 - t_8 = 4 \frac{F4}{F2} \frac{v_{11}}{g} .$$

There are usually at least one or two records with good velocity data after slapdown, so F2 and F4 are adjusted as necessary to obtain a good fit for $t_9 - t_8$ on these records. Only one station for Event A qualified (15 m SE, Fig. 9b), so the adjustment procedure was easy. Adjustment was also necessary for Event B.

The parameters F3 and F5 relate in much the same way as F2 and F4. F3 should be small because it represents an acceleration with no radial dependence (in fact, our model requires $F3 < 1.0$ for computational purposes), and F5 should be less than 1.0 for the same reason as is F4. Fits for F3, the final acceleration peak, are usually hard to obtain because the model is excessively simple. The initial guess for F5 will be the geometric mean of the ratios v_8/v_7 from those data records which are useable. F3 and F5 together control $t_{11} - t_9$, the width of the final velocity peak, relative to $t_9 - t_8$, via the following relationship:

$$t_{11} - t_9 = \frac{F5(1 + F3)}{2F3} (t_9 - t_8) .$$

Thus, in practice it is possible to specify the magnitude of the final velocity peak using F5 and then adjust its width with F3 until it matches as well as possible whatever velocity records are clean enough to use.

For Event B six of the seven velocity records were of sufficient quality so that the fit for the final two velocity peaks could be adequately assessed and therefore improved.

The parameters F2, F3, F4, and F5 are collectively adjusted so that good data records are matched by the model, or so that the velocity model looks reasonable if there is no good data.

The parameter F6 specifies $t_7 - t_6$, the slapdown peak rise time, relative to $t_2 - t_1$, the initial peak rise time. The time scale on most of our data records is such that $t_7 - t_6$ cannot be measured with much accuracy. However, it tends to be in the 0.01 to 0.03 s range. Typical values for $t_2 - t_1$ are 0.05 to 0.09 s. We have found values of F6 ranging from 0.1 to 0.5. The observable effects of this parameter on the model velocity are small and limited to the width of the first negative velocity peak.

The parameter F7 specifies the slope of the velocity between times t_5 and t_6 . Under conditions of single spall the slope will be $-g$ because the spalled layer is in free-fall, and so F7 is 1.0. However, it is

Figure 17. Typical first acceleration peak profiles showing the possible range of shapes. Arrows indicate plausible values for a_3 or a_4 .

fairly common for there to be some sort of disturbance during this interval (often a double spill) that changes the average acceleration. We adjust F7 to deal with this. Many of the Event A accelerometers recorded an effect of this sort. It is quite pronounced at stations 329 m and 385 m (Figs. 9d-e) as a bump between v_α and v_β and rather subtle at the two 15 m stations (Figs. 9a-b). F7 was adjusted to compensate at the 15 m stations for this disturbance. The fit for Event B was improved by a small adjustment in F7, although in only one case (350 m, Fig. 16c) was this type of disturbance large.

We mentioned earlier that the shoulder between t_3 and t_4 disappears as the radius increases. This means that the difference between v_3 and v_α diminishes with radius. Since the disappearance of the shoulder also affects the time interval $t_5 - t_4$, the radial dependence of the relationship between v_α and v_3 should also involve the radial dependence of $t_5 - t_4$. Thus, the parameterization of v_3 is

$$v_3 = v_\alpha \left\{ 1 - (1 - F8) \left[1 + \left(FT \frac{RC}{ZB} \right)^{XPT} \right]^{-1} \right\}$$

In essence, F8 specifies the ratio v_3/v_α at SGZ, and FT and XPT specify the rate at which the ratio v_3/v_α approaches 1.0 as the radius increases. Since the parameters FT and XPT are already determined by the $t_5 - t_4$ fit, F8 must be determined from the data at SGZ. Strictly speaking, the stations at SGZ do not have $RC = 0$, but we consider them close enough. F8 is then the geometric mean of the ratios v_3/v_α from the SGZ stations. If, however, no records are close enough to SGZ, then obtaining F8 (and most of the other parameters as well) will be quite difficult. Finding v_3 on the velocity record can be difficult; we usually look for a change in slope somewhere before v_α . In Fig. 9a (Event A) v_3 is slightly less than 3 m/s.

The geometric mean is used for the parameters F1 through F7 because the quantities they estimate are ratios, i.e., $F1 = a_3/g$. The geometric mean is the only measure of central tendency that is invariant under inversion of the data, i.e., we could have used $F1 = g/a_3$. Our experience suggests that a reasonable range of values for the ratio parameters is 0.2 to 1.5; one should be wary of values larger than this.

Section 5. Atmospheric Acoustic Pulses

The principal purpose of an analytic model of the surface ground motion from a UGT is to estimate the acoustic signal that is launched into the atmosphere as a result. These signals are calculated by the approximation of Huyghens-Kirchhoff integration over the moving ground surface. This surface is considered as a distributed source of mutually interfering acoustic wavelets. The theory behind this calculation is well-known.⁵⁶ The response (i.e., the acoustic fluid velocity signal) at a point in the atmosphere directly above a UGT can be calculated from the following retarded integral over the ground surface:

$$v(t) = \frac{1}{4\pi} \int_0^{2\pi} \int_0^{R_0} \left[\frac{1 + \cos \theta}{a_0 R} a_n \left(t - \frac{R}{a_0} \right) + \frac{\cos \theta}{R^2} v_n \left(t - \frac{R}{a_0} \right) \right] r dr d\theta$$

where R is the slant distance from the observation point to a point on the moving ground surface, a_0 is the atmospheric speed of sound (V_A in our modelling parameter set), a_n and v_n are the vertical acceleration and velocity functions of our modelling, and $r dr d\theta$ is an element of the ground surface area. The limit of integration over the moving ground is the radius R_0 , corresponding to our $RCMAX$, and θ is the angle the slant vector R makes with the vertical. In this calculation we assume the atmosphere is uniform, i.e., a_0 is constant.

In this section we show several examples of the acoustic fluid velocity pulse resulting from our analytic parameterization of the surface ground motions. We also show how changes in some of the parameters affect the shape of this acoustic pulse profile. In all these examples, except as noted, the observation point is located 10 km above the ground and 1 km away from the vertical passing through SGZ, and a_0 is taken to be a mean value of 324 m/s over this altitude range.

Figures 18a-c show acoustic pulses resulting from different fits to the surface ground motions for Event A, using the Warshaw model; Fig. 18d shows a pulse using the Bernreuter model.

Only F3, F4, and F5 were changed in going from Fig. 18a to 18b. This had a substantial effect on the latter part of the pressure pulse. In going from Fig. 18b to 18c $RCMAX$, $VPOS$, FV , XPV , $VNEG$, FW , XPW , $F2$, and $F7$ were changed. Again there was a substantial effect on the pulse. The parameter values for Fig. 18c are the final values. Figures 13c-d allow us to compare the effect on the pressure pulse of the two different sets of radial functions for the velocity and acceleration extrema discussed in Section 2. As is evident, the acoustic differences are not very great.

Figures 19a-b show that the effects of differences between the radial models are not necessarily so small. The pulse in Fig. 19a was calculated using Warshaw's, whereas the pulse in Fig. 19b was figured using Bernreuter's. All the other parameters are the same. These pulses are 8.75 km above the ground surface.

Figure 20 is the fluid velocity pulse for the parameters from Event B. Its shape is quite similar to that of Event A (Fig. 18c); from this we can conclude that, from an acoustic point of view, the differences in the ground motion between Events A and B are not that great.

(a) Parameters			
RCMAX	=	3200.0	FT = 5.48
VA	=	324.0	XPT = 1.57
VB	=	1735.0	F1 = 0.85
ZB	=	640.0	F2 = 0.50
APOS	=	47.40	F3 = 0.99
FA	=	0.32	F4 = 0.13
XPA	=	9.24	F5 = 1.44
ANEG	=	235.20	F6 = 0.16
FB	=	0.42	F7 = 1.00
XPB	=	6.16	F8 = 0.66
VPOS	=	4.44	Y = 1.0
FV	=	0.85	ACON = 2.61E+05
XPV	=	1.26	AEXP = 1.29
VNEG	=	-7.35	ARAT = 3.75
FW	=	1.96	VCON = 8.17E+04
XPW	=	0.79	VEXP = 1.53
DTSH	=	0.500	VRAT = 1.31

(b) Parameters			
RCMAX	=	3200.0	FT = 5.48
VA	=	324.0	XPT = 1.57
VB	=	1735.0	F1 = 0.85
ZB	=	640.0	F2 = 0.50
APOS	=	47.40	F3 = 0.65
FA	=	0.32	F4 = 0.41
XPA	=	9.24	F5 = 0.67
ANEG	=	235.20	F6 = 0.16
FB	=	0.42	F7 = 1.00
XPB	=	6.16	F8 = 0.66
VPOS	=	4.44	Y = 1.0
FV	=	0.85	ACON = 2.61E+05
XPV	=	1.26	AEXP = 1.29
VNEG	=	-7.35	ARAT = 3.75
FW	=	1.96	VCON = 8.17E+04
XPW	=	0.79	VEXP = 1.53
DTSH	=	0.500	VRAT = 1.31

Figure 18. Acoustic pulses for Event A, calculated at a point 10 km above and 1 km away from SGZ. (a) and (b) show pulses using the Warshaw radial model and preliminary parameter values, for comparison with (c) the final values using the Warshaw radial model, and (d) the final pulse using the Bernreuter radial model.

Parameters					
RCMAX	=	2300.0	FV	=	1.00
VA	=	324.0	XPV	=	1.46
VB	=	1735.0	VNEG	=	-7.39
ZB	=	640.0	FW	=	1.45
APOS	=	47.40	XPW	=	1.66
FA	=	0.32	DTSH	=	0.500
XPA	=	9.24	FT	=	5.48
ANEG	=	235.20	XPT	=	1.57
FB	=	0.42	F1	=	0.85
XPB	=	6.16	F2	=	0.75
VPOS	=	4.60	F3	=	0.65
			F4	=	0.14
			F5	=	0.67
			F6	=	0.16
			F7	=	0.85
			F8	=	0.66
			Y	=	1.0
			ACON	=	2.61E+05
			AEXP	=	1.29
			ARAT	=	3.75
			VCON	=	8.17E+04
			VEXP	=	1.53
			VRAT	=	1.31

Fig. 18 continued.

Parameters

RCMAX	=	2800.0	FT	=	6.05
VA	=	329.0	XPT	=	1.11
VB	=	1565.0	F1	=	2.36
ZB	=	565.0	F2	=	0.89
APOS	=	146.90	F3	=	0.35
FA	=	1.70	F4	=	0.44
XPA	=	1.48	F5	=	0.75
ANEG	=	205.80	F6	=	0.25
F8	=	4.76	F7	=	0.90
XPB	=	0.91	F8	=	0.55
VPOS	=	6.27	Y	=	1.0
FV	=	1.66	ACON	=	2.84E+08
XPV	=	1.09	AEXP	=	2.33
VNEG	=	-7.21	ARAT	=	1.68
FW	=	3.90	VCON	=	1.05E+07
XPW	=	1.56	VEXP	=	2.28
DTSH	=	0.300	VRAT	=	0.79

Figure 19. Comparison of acoustic pulses created with the Warshaw and Bernreuter radial models. These pulses were calculated at a point 8.75 km above and 1 km away from the vertical through SGZ. (a) uses the Warshaw radial model, while (b) uses the Bernreuter radial model.

Parameters

RCMAX= 3500.0	DTSN= 0.410
VA= 324.0	FT= 4.99
VB= 1843.0	XPT= 1.32
ZB= 701.0	F1= 1.13
APDS= 62.16	F2= 0.75
FA= 0.53	F3= 0.58
XPA= 4.09	F4= 0.34
ANEG= 193.66	F5= 0.65
FB= 1.15	F6= 0.29
XPB= 2.19	F7= 0.92
VPOS= 4.22	F8= 0.44
FV= 0.91	
XPV= 1.91	
VNEG= -6.40	
Fw= 1.01	
XPW= 2.18	

Figure 20. Acoustic pulse for Event B using the Warshaw radial model and the final set of parameters shown in Table 6. This pulse was calculated at a point 10 km above and 1 km away from the vertical through SGZ.

Section 6. Data Used for the Model

We treated all the data as received, i.e., without modification.

Figures 21a-b are two examples of actual accelerometer records. They illustrate some of the aspects of the data (compare with Fig. 17). Most of the data was taken by Los Alamos National Laboratory. In general, the format and presentation of this data is quite useful for our purposes. The time scales of the plots are particularly appropriate for our needs.

We have experienced three sources of difficulty with this data. The first is that the slapdown acceleration peaks are frequently clipped. The problems this causes were discussed in Section 2.1.

Second, noise in the acceleration records often distorts the velocity records. This happens because the velocity records we have been using are not measurements in the true sense of the word but are obtained by integrating the acceleration records. As we mentioned in Section 2.2, this primarily affects the data during and after slapdown. However, the data sometimes appears to be distorted prior to slapdown.

Third, the accelerometer coverage on various explosive events ranges from very sparse to barely adequate for our needs. The largest data set we have for one event contains 14 surface acceleration records, some of which are only partially useful. Many events have data sets with no records beyond the presumed extent of spall, which is a troublesome omission if one wishes to calculate atmospheric pressure pulses. Figure 10 shows an example of a problem this limited coverage can cause.

In order to make a crude estimate of how many accelerometers might be necessary to thoroughly cover the ground surface, we would like to determine how much surface area a single accelerometer can adequately represent. We have found that accelerometer records taken less than 200 m apart can be significantly different, but this distance probably represents an extreme; a more typical distance might be 400 m. Let us therefore assume that a single accelerometer adequately covers the area within a radius of 200 m about itself. It follows that, within a radius of 1000 m about SGZ, at least 25 accelerometers are needed to cover the entire surface:

$$n = \pi r_1^2 / \pi r_2^2 = (r_1 / r_2)^2 = (1000 / 200)^2 = 25$$

Figure 21. Accelerometer records taken at SGZ. These records show some of the problems we often encounter, such as (a) noise and (b) clipping, as well as details that our model does not fully account for.

If we suppose that accelerometers more than 1000 m from SGZ are good for coverage within a radius of 500 m about themselves, then another 32 accelerometers are needed to cover the surface between 1000 and 3000 m. We believe that any study of the ground motion attempting to take into account the fact the ground motion is azimuthally variable will require at least this degree of coverage.

Section 7. Critique of the Present Model

This model is an initial guess at a time profile and radial dependence of what are hoped to be the most relevant gross features of UGT induced ground surface motions.

The major problems in the time profile are in the shoulder a_3 and a_4 and the reverberations a_8 , a_9 , and a_{10} . As indicated in Section 4 and Fig. 17, the model for the shoulder cannot encompass the wide variety of acceleration profiles seen here. However, it does a reasonable job of approximating them.

The reverberations after slapdown are more of a problem. Figure 21b is a good example of an acceleration record that does not look remotely like our model after the slapdown peak. For future work, it might be appropriate to have some sort of parameterization that allows $a_8 < 0$, $a_9 > 0$ and so on, or that allows fewer or more acceleration peaks after slapdown.

In terms of the radial dependence, the variation in the data for a_2 (in many cases) and for a_7 (in most cases) is so great that no monotone function will fit them; very well. The variation is probably due to the fact that the ground motion is *not* cylindrically symmetric; if this is so, then any cylindrically symmetric model will have this problem. An asymmetric model might use an azimuthal modulation of the existing radial modelling.

It has turned out to be more effective for parameterizing this model to estimate F2 through F5 using the widths and magnitudes of v_6 and v_7 , even though F2 and F3 actually specify accelerations. If we had been able to anticipate this it might have been better to have the parameters specify the widths of v_6 and v_7 directly.

The parameterization of v_3 is somewhat unsatisfactory because there are parameter sets for which $v_3 > v_4$, and thus $t_3 > t_4$ if RC is larger than some critical value. This critical value depends on several of the other parameters in a complex fashion. An ideal model would have simple constraints on each parameter, independent of the others, such that whenever the constraints are met, basic properties of the model (such as $t_4 > t_3$) are not violated. This parameterization of v_3 does not have this property. We have solved the problem by adjusting v_3 to be slightly less than v_4 whenever the critical radius is exceeded.

Section 8. Overview of the Modelling Process

This section provides general comments that do not depend on the specific model and parameterization used.

First, it is clear that any complete model for UGT induced ground motion must include a time profile for the ground motion and a radial dependence of some sort. Our approach has been to estimate the radial dependencies of specific points on the profile independently of their effect on the profile's shape and then evaluate the fit of the overall profile. The azimuthal dependence has been ignored or averaged out.

The estimation of the radial dependencies is a problem best handled by conventional statistical methods. The methods are straightforward: depending on the radial function, some sort of least squares

method is appropriate. Subtleties arise when a constraint of the sort discussed in Sections 1 and 2.1 causes local minima, or when we try to take into account the fact that the data do not have homogeneous variances.

Difficulties arise when there is insufficient data (see Fig. 20); then the investigator's arbitrary choice is required. This in turn leads to radial fits that are not best in any statistical sense and opens up the possibility that different investigators could obtain substantially different predictions based on the same data.

We have not attempted to evaluate the fit of the overall time profiles by statistical methods because evaluating the error of the fit would require extremely clean data. Miscellaneous noise and calibration errors in the data would inflate the estimate of the error of the fit. Also, some parts of the time profile are obviously more important than others, and we don't know enough yet to determine appropriate statistical weightings.

As a consequence, our evaluation of the overall profile fits is basically visual. We look at the entire set of superimposed plots, eliminate as many gross errors as possible, and attempt to balance out the more subtle errors so that some are in one direction and some are in the other. The discussion in Section 3.2 gave some indication of this process.

If it were practical to evaluate the fit of the profile by statistical methods, it might be possible to take a general approach in which the radial dependencies were evaluated simultaneously with the profile fits. Then the radial dependencies might apply to the entire profile rather than to selected points on it.

Section 9. Summary

We have presented some details of a specific ground motion model (Section 1); two examples of its application, including some discussion of the choices involved in obtaining the radial parameters (Sections 2 and 3); and a separate discussion of our methods for choosing the ratio parameters (Section 4). Section 5 presented examples of the model's use in calculating the acoustic air pressure pulses resulting from the ground motion; Section 6 discussed the quality of the data we worked with; Section 7 critiqued our model; and Section 8 discussed general aspects of our approach to fitting the model.

The specific model is only a beginning attempt at modelling UGT induced ground motion in which spall is present. It is capable of dealing with a wide variety of radial dependencies and can lead to a substantial variety of air pressure pulses. Despite its sometimes severe limitations and gross oversimplifications, we believe it has done a surprisingly good job of reproducing several sets of ground motion data.

Acknowledgments

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Appendix

This appendix contains formulas for all of the points t_1 through t_{11} , t_a , t_3 , t_7 , t_8 , v_1 through v_{11} , v_a , v_3 , v_7 , v_8 , and a_1 through a_{11} , which are used in our work.

The following symbol definitions are used:

g is the acceleration of gravity (9.8 m/s^2),

ZB is the depth of burial,

VB is the average speed of propagation of the shockwave from the point of explosion to the surface,

RC is the surface distance from SGZ to a ground motion recording location. The symbols $F1$ through $F8$, $APOS$, FA , XPA , $ANEG$, FB , XPB , $VPOS$, FV , XPV , $VNEG$, FW , XPW , $DTSH$, FT , and XPT are parameters to be estimated.

Except for a_{10} , v_7 , and v_8 , all of the following points are expressed solely in terms of the above parameters:

$$t_2 = \frac{1}{VB} \sqrt{RC^2 + ZB^2}$$

$$a_1 = a_{11} = 0$$

$$a_2 = \frac{APOS}{1 + \left(FA \frac{RC}{ZB}\right)^{XPA}}$$

$$a_3 = a_4 = (F1)g$$

$$a_5 = a_6 = (-F7)g$$

$$a_7 = \frac{ANEG}{1 + \left(FB \frac{RC}{ZB}\right)^{XPB}}$$

$$a_8 = (F2)g$$

$$a_9 = (-F2)g$$

$$v_a = \frac{VPOS}{1 + \left(FV \frac{RC}{ZB}\right)^{XPV}}$$

$$a_{10} = (F3)a_8$$

$$v_1 = v_8 = v_9 = v_{11} = 0$$

$$v_3 = \frac{VNEG}{1 + \left(FW \frac{RC}{ZB}\right)^{XPW}}$$

$$v_7 = (F4)v_a$$

$$v_8 = (-F5)v_7$$

$$\Delta t = t_5 - t_4 = \frac{DTSH}{1 + \left(FT \frac{RC}{ZB}\right)^{XPT}}$$

$$v_3 = v_a \left\{ 1 - (1 - F8) \left[1 + \left(FT \frac{RC}{ZB}\right)^{XPT} \right]^{-1} \right\}$$

The remaining points are expressed in terms of the parameters, the above points, and each other.

$$t_1 = t_2 - \frac{2v_3}{a_3 + 2a_2}$$

$$t_5 = t_4 + \Delta t$$

$$t_3 = t_2 + \frac{2v_3}{a_3 + 2a_2}$$

$$t_6 = t_5 + \frac{v_6 - v_5}{a_5}$$

$$t_4 = t_3 + \frac{v_4 - v_3}{a_3}$$

$$t_8 = \frac{a_7 t_6 - a_6 t_7}{a_7 - a_6}$$

$$t_a = \frac{a_4 t_5 - a_5 t_4}{a_4 - a_5}$$

$$t_7 = t_6 + F6(t_2 - t_1)$$

$$t_8 = t_7 - \frac{2v_7}{a_8 + a_7}$$

$$t_7 = \frac{t_8 + t_9}{2}$$

$$t_6 = t_8 + 4 \frac{v_3}{a_8}$$

$$t_5 = t_9 + 2 \frac{v_6}{a_9}$$

$$t_{10} = t_9 + F3 \frac{2v_8}{a_9}$$

$$t_{11} = t_{10} - 2 \frac{v_{10}}{a_{10}}$$

$$v_2 = \frac{a_2 v_3}{a_3 + 2a_2}$$

$$v_4 = v_1 - \frac{1}{2} a_1^2 \frac{\Delta t}{a_4 - a_5}$$

$$v_5 = v_4 + \frac{1}{2} (a_4 + a_5) \Delta t$$

$$v_6 = v_3 - \frac{1}{2} a_6^2 \frac{t_7 - t_6}{a_6 - a_7}$$

$$v_7 = v_6 + \frac{1}{2} (a_6 + a_7) (t_7 - t_6)$$

$$v_{10} = v_5 + \frac{1}{2} a_{10} (t_{10} - t_5)$$

The generic relationship is that for any pair of adjacent points,

$$v_{i+1} - v_i = \frac{1}{2} (a_{i+1} + a_i) (t_{i+1} - t_i) \quad .$$

It should be noted that if $v_4 < v_3$ then $t_4 < t_3$, and the model should not allow this. Since it is possible to produce sets of parameters that cause this but are otherwise acceptable, there should be some sort of check for this condition in any code that calculates these values. In our computer codes, we set $v_3 = v_4 - 10^{-4}$ whenever $(v_4 - v_3) < 10^{-4}$. In addition, the above definitions require $F3 < 1$, and $F8 < 1$. Except for VNEG, all parameters are positive. Note that the parameter RCMAX, discussed in the text, does not appear here. RCMAX is not necessary for the *calculation* of the ground motion from a set of the above parameters, but it is very important for the *estimation* of the parameters. It is also essential for the calculation of the atmospheric acoustic pulse.