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Theoretical Overview
--Hot and Dense QCD in Equilibrium

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THEORETICAL OVERVIEW[†]

— HOT AND DENSE QCD IN EQUILIBRIUM —

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Static and dynamical properties of QCD at finite temperature and density are reviewed. Non-perturbative aspects of the QCD plasma and the modification of the hadron properties associated with the chiral transition are discussed on the basis of lattice data, effective theories and QCD sum rules. Special emphasis is laid on the importance of the finite baryon density to see the effects of the restoration of chiral symmetry in experiment.

1. Introduction

The QCD coupling constant $g(\mu)$ decreases logarithmically as one increases the typical scale μ of the process because of the asymptotic freedom. In the deep inelastic lepton-hadron scattering, μ is chosen to be the 4-momentum transfer $\sqrt{Q^2}$. In QCD medium at high T (temperature) or ρ (density), μ will be proportional to temperature or chemical potential, which is the reason why we expect a weakly interacting system of quarks and gluons called quark-gluon plasma (QGP).¹

The experimental aim of the QGP physics is to create and detect this new phase in the laboratory, while in the theoretical side an unified description of the equilibrium system in the entire $T - \rho$ plane is a big challenge. A number of attempts have been proposed on the latter issue so far: (i) perturbative QCD and its improved version in high $T - \rho$ region, (ii) the lattice QCD for T around T_c ($\rho=0$), and (iii) effective theories and QCD sum rules in the low $T - \rho$ region. A tentative goal of these studies is to cover the phase diagram in $T - \rho$ plane by extending the applicable range of (i), (ii) and (iii). In this review, some of the recent theoretical progress are discussed along the path shown in Fig.1: (I) $\rightarrow$ (II) $\rightarrow$ (III).

Here it will be instructive to give a measure of the critical parameters where hadron-quark transition starts to take place: The critical density ρ_c (the critical temperature T_c) can be estimated by the close packing density (temperature) of nucleons (pions) as

$$\rho_c = (4\pi R_N^3/3)^{-1}, \quad n_\pi(T_c) = (4\pi R_\pi^3/3)^{-1}, \quad (1)$$

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with R_N (R_π) being the nucleon (pion) radius and n_π being the pion number density at finite T . Eq.(1) gives $\rho_c \sim 3\rho_0$ and $T_c \sim 200\text{MeV}$. The critical field for the transition can be also estimated by $gE_c = \sigma \simeq (420\text{MeV})^2$ with g being the strong coupling constant (electric charge) for color electric (usual electric) field. These numbers are not hopelessly large to reach in the relativistic heavy ion collisions. Also, it is remarkable that more sophisticated calculations confirm the above simple estimates.

Fig.1: A schematic phase diagram in $T - \rho$ plane. (I) is a region with $T > T_c$ and $\rho=0$ (section 2), (II) is a region with $T \leq T_c$ and $\rho=0$ (section 3), and (III) is a region with $T = 0$ and $\rho \neq 0$ (section 4).

2. QCD at high temperature ($T > T_c$, $\rho=0$)

2.1. Multi scales at $T > T_c$

It is now well known that there exists non-perturbative physics in the infrared scale even if T is considerably larger than T_c ,² which is summarized as the hierarchic scale structure:

$$\dots < g^2T < gT < T, \quad (2)$$

with g being the running coupling constant at finite T . gT is a scale of the electric screening mass of gluons ($m_{el} = gT\sqrt{N_c/3 + N_f/6}$) and g^2T is related to the magnetic screening mass. As far as one stays in low orders of g , we have a consistent perturbation theory for static and dynamical properties of the system. Perturbative calculation of the energy density, pressure etc³ and the resummation of the hot thermal loops⁴ are the typical examples. On the other hand, if one goes to higher orders or probes the system with an energy scale $O(g^2T)$, perturbation theory starts to break down,² e.g. at $O(g^6)$ for the free energy and at $O(g^4)$ for the magnetic mass. In this case, one has to sum up infinite numbers of diagrams to get physical results, which is certainly a non-perturbative task. g^2T is also related to the coupling constant of 3D QCD as a high T limit of 4D QCD; confinement of 3D QCD suggests again the non-perturbative nature of g^2T .⁵

To understand the coexistence of non-perturbative (NP) and perturbative (P) physics at high T , let us introduce a scale K in momentum space which separates the NP and P domains (Fig.2). Since the NP region is confined in a finite volume in momentum space, it does not affect the bulk properties of the system in high enough T where the typical frequency of quarks and gluons is (a few) $\times T \gg K$. However, if one probes the low frequency region or decreases T toward T_c , effect of the NP region emerges.

This feature is actually seen in the lattice QCD simulations of the energy density $\mathcal{E}$, pressure $\mathcal{P}$ and the hadronic correlations in the scalar-pseudoscalar channel, which we will see in the following subsections.

Fig.2: Separation of scales at high T .

2.2. Energy density and pressure on the lattice

The solid lines in Fig.3 are $\mathcal{E}$ and $\mathcal{P}$ for pure gauge system ($SU_c(3)$) on the $16^3 \times 4$ lattice,⁶ where the integral definition of $\mathcal{P}$ ($= (T/V) \int \partial \ln Z / \partial \beta \cdot d\beta$ with $\beta = 2N_c/g^2$) and a non-perturbative β -function⁷ are used. Due to these prescriptions, the unphysical behavior of the data in ref.8 (boxes in Fig.3) are removed and simultaneously the interaction measure $\Delta \equiv (\mathcal{E} - 3\mathcal{P})/T^4$ is reduced by factor 2.

Fig.3: $\mathcal{E}/T^4$ and $\mathcal{P}/T^4$ (solid lines) from ref.6 along with the data (open boxes) in Ref.8. The point $2T_c$ is deduced by using a non-perturbative β -function.

Now one can extract both **P** and **NP** physics from Fig.3:

Stefan-Boltzmann limit (perturbative physics): At high enough T , it seems that both $\mathcal{E}$ and $\mathcal{P}$ approach the Stefan-Boltzmann (SB) limit (with g^2 corrections), which means weakly interacting massless gluons dominate the bulk properties of the system.

Cutoff scale K (non-perturbative physics): As T approaches T_c , a sizable discrepancy between $\mathcal{E}$ and $3\mathcal{P}$ shows up; in particular $\mathcal{P}$ is much smaller than its SB value. This discrepancy can be seen even up to $\sim 2T_c$ (note that the difference of $\mathcal{E}$ and $3\mathcal{P}$ starts from g^4 in perturbation theory),³ which suggests that the system is not at all a free gluon gas up to $\sim 2T_c$.

Now what is the physics behind this **NP** aspect? To get a rough idea, let us take a "cutoff model" to simulate the **NP** physics,^{9,10}

$$\ln Z_{QCD}(T, V) = \ln Z_{NP}(|\vec{p}| < K) + \ln Z_P(|\vec{p}| > K) \quad , \quad (3)$$

where a cutoff $K(T)$ is introduced as a scale separating the gluon momentum in the

partition function Z_{QCD} . Perturbation theory is supposed to work above $K(T)$, while only massive (probably color singlet) modes exist below $K(T)$. To determine the absolute value and T dependence of K , lattice data of $\mathcal{E}$ and $\mathcal{P}$ (after the extrapolation to the continuum limit) in $SU_c(2)$ gauge theory are used in Fig.4, which gives⁹ $K=2.9T_c[T_c/(T-T_c)]^{0.3}$. K is found to be as large as $3T_c$ at $T \simeq 2T_c$. Furthermore, by using the same K , one can simultaneously fit the color averaged potential between heavy quarks at finite T .¹⁰ The dashed line in Fig.5 is the result of the simple Debye screening which fails to fit the lattice data at large separation, while the cutoff model fits the data very well without introducing extra free parameters. Physical reason of this excellent agreement is rather simple: because of the low momentum cutoff of the gluon propagator, the Debye mass gets reduced near T_c and hence the potential becomes long range.

Fig.4: Fitting of the $SU_c(2)$ data of $\mathcal{E}$ and $\mathcal{P}$ (circles and boxes) by the cutoff model (solid lines).⁹

Fig.5: $SU_c(2)$ lattice data on $-V(r,T)/T$ vs rT (circles) and the prediction of the cutoff model (solid lines).¹⁰

Although the above cutoff model seems to work well phenomenologically, it is not an unique way to reproduce the discrepancy between $\mathcal{E}$ and $3\mathcal{P}$. In fact, the trace anomaly in QCD tells us that $\mathcal{E}-3\mathcal{P}$ is nothing but the gluon condensate

$$\langle \Theta_\mu^\mu \rangle_T = \mathcal{E} - 3\mathcal{P} = \frac{\beta}{4\alpha_s} \langle G^2 \rangle_T . \quad (4)$$

Therefore, one can interpret the non-zero value of $\mathcal{E} - 3\mathcal{P}$ above T_c by $\langle G^2 \rangle_T$. The direct calculation of $\langle G^2 \rangle_T$ on the lattice has been done by Di Giacomo et al.¹¹ and Su H. Lee.¹² The latter work shows

$$\langle E^2 \rangle_{T \simeq 1.5T_c} \simeq -\langle B^2 \rangle_{T \simeq 1.5T_c} \simeq (0.6 - 0.7) \langle E^2 \rangle_0 , \quad (5)$$

which is consistent with the same quantities extracted from the Monte Carlo data of $\mathcal{E}$ and $\mathcal{P}$ by Adami, Hatsuda and Zahed.¹³ Whether the non-vanishing gluon condensate can reproduce $V(r,T)$ or not is an interesting open question.

There is also a recent interesting observation on the non-trivial nature of the high T phase by Mardor and Svetitsky.¹⁴ They have analysed the free energy of the hadron bubble inside QGP using the MIT bag model (Fig.6):

$$F(R) = \Delta P \left(\frac{4\pi}{3} R^3 \right) + \sigma(4\pi R^2) + \alpha(8\pi R) + \dots \quad (6)$$

Fig.6: Free energy of the hadron bubble as a function of its radius R near $T_c=150\text{MeV}$.¹⁴ Top to bottom: $T=155\text{MeV}$, $T=T_c$ and $T=147\text{MeV}$.

Because of the smallness of the surface tension σ and negativeness of the curvature term α , they found an instability of the plasma against the creation of the hadron bubbles even for T much larger than T_c . They call this phenomena as “swiss cheese instability” of QGP.¹⁵ This kind of structure will certainly affect the $\mathcal{E}$ - $\mathcal{P}$ relation and can be a physical reason for the non-vanishing gluon condensate at high T . The hadron bubble could also be a seed of fluctuations creating heavy elements in the early universe: note that this can occur even if the actual QCD phase transition is not the 1st order. Model independent confirmation of this swiss cheese instability on the lattice will be interesting.¹⁶

2.3. Hadronic correlations above T_c

Whether there are hadronic modes above T_c is an interesting problem related to the NP nature of the system. This question was first addressed by Hatsuda and Kunihiro¹⁷ and by DeTar¹⁸ independently in a different context. The former focused on the fluctuation of the order parameter above T_c and has predicted low-mass parity-degenerate excitations ($\pi - \sigma$) above T_c .^{17,19} The latter raised an issue of the confinement above T_c and argued hadronic modes in all channels.^{18,20} Lattice simulations of the hadronic screening mass have been performed later by DeTar and Kogut²¹ (Fig.7(a)) and was subsequently studied by other groups.²²

The hadronic screening mass $\mu(T)$ is defined by

$$\int dxdydz \langle \mathcal{O}_H(\vec{x}, \tau) \mathcal{O}_H(\vec{0}, 0) \rangle_T \longrightarrow \exp[-\mu(T) |z|] \quad , \quad (7)$$

with $\mathcal{O}_H$ being the operators having quantum numbers of hadrons.

Fig.7(a): Monte Carlo data for the screening masses of baryons ($N_+ - N_-$), vector mesons ($\rho - a_1$) and scalar mesons ($\pi - \sigma$) on $10^3 \times 6$ lattice.²¹ Extrapolation to the chiral limit is made.

Fig.7(b): Recent compilation of the screening masses on the lattice by Gocksch.²³

From Fig.7, we can learn the following things.

Eletsky-Ioffe limit (perturbative physics): Above the critical temperature of chiral transition, one finds (i) there is a parity doubling ($\pi - \sigma$, $\rho - a_1$ and $N_+ - N_-$), which is a clear signal of the restoration of chiral symmetry, and (ii) screening masses for mesons approach $2\pi T$ and those for baryons approach $3\pi T$. This is an evidence for the *massless free quarks* above T_c . In fact, as was pointed out by Eletsky and Ioffe (EL),²⁴ the hadronic screening masses at high T are essentially controlled by the lowest Matsubara frequency of quarks (πT);

$$\mu(T) \longrightarrow 2\pi T \quad (\text{mesons}), \quad 3\pi T \quad (\text{baryons}) . \quad (8)$$

This is just a kinematical and channel independent statement, and the lattice data show that this is actually satisfied even near T_c (except for the $\pi - \sigma$ channel).

We have neglected $O(g^2)$ corrections to the EL limit in the above argument. However, g^2 on the lattice is not negligibly small ($g^2 \sim 1$) and it is important to calculate the perturbative correction to the hadronic screening masses both in the continuum and lattice QCD. Such perturbative correction certainly introduces channel dependence of $\mu(T)$, which might explain the different behavior of $\mu(T)$ between $\rho - a_1$ and $\pi - \sigma$. (Note that the spin-spin force due to the one gluon exchange is attractive (repulsive) for the pion (rho-meson).)

Soft modes (non-perturbative physics): As one sees in Fig.7, the screening mass in the $\pi - \sigma$ channel is considerably smaller than the EL limit ($2\pi T$). This is confirmed also by other groups; one of them is shown in Fig.8 where the dashed line corresponds to the EL limit in the continuum. The small $\mu_{\sigma-\pi}$ suggests that there remains sizable residual interaction only in the scalar-pseudoscalar channel above T_c . The physics behind this

phenomena has been already discussed by Hatsuda and Kunihiro¹⁷ before the lattice data. They showed that there is a sizable fluctuation of the order parameter around T_c as far as the chiral transition is not strong 1st order. This is because the free energy at and just above T_c has quite a shallow minimum around $\langle \bar{q}q \rangle_T = 0$ and a long range correlation of the order parameter is induced both in the spacial and time correlations, i.e.

$$\langle \bar{q}q(\vec{x}, t) \bar{q}q(\vec{0}, 0) \rangle_T \longrightarrow \text{softening around } T_c. \quad (9)$$

The large spacial correlation corresponds to the small screening mass observed on the lattice (see also the recent analysis by Gocksch with the use of the quark σ -model²³), while the large time correlation corresponds to a low mass *hadronic* excitation above T_c in $\pi - \sigma$ channel. Since this mode above T_c has good properties of a quasi-particle (low mass and narrow width),^{17,26} we called it as “soft mode” which is a complete analogue of the fluctuation of Cooper pairs in the low dimensional superconductor (Maki-Aslamazov-Larkin effect).²⁷ Effect of such a light color-singlet mode on the static and dynamical properties of the quark-gluon plasma is an interesting future problem.

Fig.8: Screening mass of $\pi - \sigma$ on the $8^2 \times 24 \times 4$ lattice ($n_f a = 0.05$).²⁵

Fig.9: Prediction for the real mass of $\pi - \sigma$ in an effective model.²⁶

Hadronic wave function: Recently, DeGrand et al.²⁸ reported a lattice measurement of “wave functions” for $\rho - a_1$ and $\pi - \sigma$ above T_c in the Coulomb gauge, which gives more detailed information on the nature of the hadronic modes. The data show a sizable correlation of $q - \bar{q}$ pairs near the origin of the relative coordinate in both $\pi - \sigma$ and $\rho - a_1$ channels, while the correlation gets weak for $\rho - a_1$ at large separation. A simplest interpretation of this data is the following: in any channels, there still remains a sizable perturbative attraction due to color Coulomb interaction since the effective mass of quarks is heavy (πT).²⁹ This effect will induce a short range correlation in the relative “wave function”. As for $\sigma - \pi$, we already know that there is a considerable (probably non-perturbative) residual interaction,¹⁷ therefore there is no surprise about its larger correlation over $\rho - a_1$. (Of course, there is also a chance to get a larger correlation in $\pi - \sigma$ just from the spin dependent part of the screened one gluon exchange.)

The lattice data for the screening masses and the wave functions seem to be quite

consistent with the above simple picture i.e.

$$\begin{aligned} & \text{(massless quarks with short - range correlations)} \\ & + \text{(large fluctuation of the order parameter)} . \end{aligned} \quad (10)$$

However, at the present stage, one cannot exclude the possibility of a more sophisticated theory suggesting the existing of hadronic poles above T_c .^{18,30}

Quark susceptibility: The flavor singlet and non-singlet quark-number susceptibilities at finite T have been measured on the lattice by Gottlieb et al.³¹;

$$\frac{1}{VT} \int d^3x d^3y \langle j_0^{S,NS}(\vec{x}, 0) j_0^{S,NS}(\vec{y}, 0) \rangle_T \equiv \chi^{S,NS} , \quad (11)$$

where $j_0^S = u^\dagger u + d^\dagger d$ and $j_0^{NS} = u^\dagger u - d^\dagger d$. This quantity has been often confused with the baryon number susceptibility. Recently Kunihiro clarified the explicit relation between $\chi^{S,NS}$ and the vector-meson (ω and ρ) correlation functions at finite T .³² The lattice data shows an enhancement of $\chi^{S,NS}$ across T_c , which simply indicates the liberation of quarks in the vector channel. This is consistent with the result of the screening masses in the vector channel and supports the simple picture in the above (10).

3. Physics at $T \leq T_c$

3.1. Order of the phase transition

The determination of the order of the QCD phase transition at finite T is one of the central issues of the recent lattice QCD study.³³ As for the pure gauge system without dynamical fermion ($m_q = \infty$), the center symmetry ($Z(3)$ in $SU_c(3)$ case) controls the confinement-deconfinement phase transition. The effective $Z(3)$ spin model predicts the 1st order transition and the lattice studies with finite scaling analyses support this feature.³⁴ Although it is of 1st order, the transition is much weaker than that seen before on the smaller lattice. Once one introduces dynamical fermions, $Z(3)$ symmetry is explicitly broken. However, as far as m_q is large enough, one can still study the phase transition based on this approximate symmetry.

On the other hand, in the opposite limit where m_q is zero, chiral symmetry instead of $Z(3)$ symmetry takes place to control the phase transition with $\langle \bar{q}q \rangle_T$ as an order parameter. For finite quark masses ($m_{u,d} = O(10\text{MeV})$ and $m_s = O(200\text{MeV})$), chiral symmetry is explicitly broken again, but one can still study the phase transition based on the approximate chiral symmetry. It should be remarked here that the chiral restoration at finite T might not simply mean the transition to QGP or the dissociation of the gluon condensate, which is discussed by Brown, Bethe and Pizzochero³⁵ and by Patkós.³⁶

Fig. 10 is an illustration of the m_q dependence of the phase transition based on the lattice data.³⁷ Although the chiral transition near $m_q = 0$ seems to be 1st order, the phase transition is not observed for the realistic values of $m_{u,d,s}$ (the dashed circle).

There are several predictions for the order of the chiral transition on the basis of the σ -models,³⁸ NJL models³⁹ and instanton liquid model.⁴⁰ While the prediction of the order and m_q dependence of the transition is a difficult task, the global behavior of $\langle \bar{q}q \rangle_T$ is rather easy to access and is actually more important in relation to the experimental observables.

Fig.10: A schematic phase diagram in $m_{u,d}-m_s$ plane based on the lattice data.³⁷

3.2. Global behavior of $\langle \bar{q}q \rangle_T$

In Fig.11(a), (b) and (c), typical behaviors of the quark condensate at finite T are shown. Fig.11(b) is a result of the Nambu-Jona-Lasinio (NJL) model⁴¹ as an effective theory of QCD,⁴² where thermal quarks give rise to the restoration of chiral symmetry at high T . The solid line (dashed line) corresponds to the light (heavy) quark condensate with the current mass $m_u = 5.5$ MeV ($m_s = 138$ MeV). The light quark condensate changes rapidly around $T_c \simeq 200$ MeV, while the heavy one has only smooth change across this temperature. The reason is simple: it is harder to excite heavy quark than light quarks.

The lattice measurement of the same quantities confirms this simple picture,⁴³ which is given in Fig.11(a). Fig.11(c) is a result of the chiral Lagrangian approach where the thermal pions and heavy mesons are responsible for the restoration of chiral symmetry.⁴⁴

Fig.11(a): $\langle \bar{q}q \rangle_T$ on the lattice around T_c taken from ref.43. (compiled by W. Weise)

Fig.11(b): $\langle \bar{q}q \rangle_T$ in the NJL model.⁴¹

Fig.11(c): $\langle \bar{q}q \rangle_T / \langle \bar{q}q \rangle_0$ in the chiral model.⁴⁴

As was pointed out by Cleymans et al.,⁴⁵ T_c in any of these approaches is predicted as

$$T_c \simeq 2f_\pi \sqrt{3/N_c} \sim 2f_\pi \sqrt{3N_f/(N_f^2 - 1)} \sim 200 \text{ MeV}, \quad (12)$$

where the 1st estimate is from the dynamical models such as the ladder QCD and the NJL model, while the 2nd estimate is from the pion gas model without interactions. The two different approaches (quark loop vs meson loop) giving the similar result suggest a quark - hadron duality in the estimate of T_c . The mismatching of N_c and N_f dependence in (12) would be resolved if one takes into account the higher loops in the quark side (to get the N_f dependence) and the summation of higher-meson contributions in the hadron side.

A message from the above calculations is that $\langle \bar{q}q \rangle_T$ decreases slowly (proportional to T^2) at low T and changes rapidly near $T_c \simeq 2f_\pi$. Associated with this change of the ground state characterized by $\langle \bar{q}q \rangle_T$, one can expect the changes of the elementary modes of excitations (mesons and baryons), which is actually well-known phenomena in condensed matter and nuclear physics. Furthermore, the decays of such hadrons in medium can be used as a thermometer of the hot and/or dense matter.

3.3. Hadrons at finite T

Serious attempts to calculate the hadron properties at finite T have been initiated by the works of Pisarski,⁴⁶ Bochkarev-Shaposhnikov,⁴⁷ Hatsuda-Kunihiro,^{17,26} Hashimoto-Miyamura-Hirose-Kanki⁴⁸ and Matsui-Satz⁴⁹ and it is now extensively examined by a number of authors with various approaches (chiral Lagrangians, chiral perturbation theory, the Skyrme model, the virial expansion, the Nambu-Jona-Lasinio model, ladder QCD, instanton liquid model, QCD sum rules and so on). In relation to the restoration of chiral symmetry, the low lying hadrons made of u and d quarks (such as π , σ , ρ , A_1 , ω , N , $\Delta \dots$) are particularly interesting since their properties are mostly determined by the quark condensate in the medium $\langle \bar{q}q \rangle_T$ as QCD sum rules tells us. The following is a short summary of our current understanding of such light hadrons in medium.

(1) Low T theorems:

At enough low temperatures, chiral symmetry gives a constraint on the T dependence of several physical quantities.⁵⁰ For example, in the chiral limit,

$$\frac{\langle \bar{q}q \rangle_T}{\langle \bar{q}q \rangle_0} \simeq 1 - \Theta \frac{N_f^2 - 1}{3N_f}, \quad \frac{f_\pi(T)}{f_\pi} \simeq 1 - \Theta \frac{N_f}{6}, \quad \frac{\mu_\pi(T)}{m_\pi} \simeq 1 + \Theta \frac{1}{6N_f}, \quad (13)$$

where $\Theta \equiv T^2/4f_\pi^2$ and μ_π being the screening pion mass. If one wants to go beyond low T limit, predictions start to become model dependent.

(2) Hadron masses below $0.8 T_c$:

As we have already seen in the previous section, $\langle \bar{q}q \rangle_T$ does not have considerable change below $\sim 0.8T_c$. Since the low lying hadron masses (M_{hadron}) are strongly correlated with the quark condensate, M_{hadron} is expected not to change so much in this region. This is actually confirmed by various approaches: pion and nucleon masses in the virial expansion approach,⁵¹ ρ -meson in the QCD sum rules¹³ and in the chiral model,⁵² pion and kaon in the Nambu-Jona-Lasinio model⁵³ and in the collective potential approach.⁵⁴ In Fig. 12, the ρ -meson mass at finite T is shown as an example, where the QCD sum rule with Borel method is used.¹³ (As for much improved treatment, see ref.55.)

Fig.12: The ρ meson mass at finite T in the QCD sum rules.¹³

The change of the pion mass is particularly weak, since the Gell-Mann-Oakes-Renner relation still holds for the real part of the masses below T_c .^{26,42}

$$M_{NG}^* \sim \sqrt{-m_q \langle \bar{q}q \rangle_T / f_\pi^{*2}}. \quad (14)$$

This shows that, because of the extra mass scale m_q , T dependence of M_{NG}^* is weaker than that of other hadrons. (Note that $f_\pi^*(T)$ decreases as T increases,⁵⁶ which balances the decrease of $\langle \bar{q}q \rangle_T$ and makes M_{NG}^* almost constant.) In Fig.13, a calculation of $M_{\pi,K}^*$ in the $SU_f(3)$ NJL model is shown.

Fig.13: Masses of pion and kaon, and also the kaon decay constant (f_K) at finite T in an effective theory.⁵³

(3) Hadron masses beyond $0.8 T_c$:

Many of the approximation scheme break down near T_c , so it is rather difficult to predict the hadron properties near T_c although the physics there is quite interesting. In spite of such difficulties, there are several theories which predict the scaling of the hadron masses associated with the decrease of $\langle \bar{q}q \rangle_T$. For example, a naive use of the QCD sum rule tells^{12,13,57}

$$M_\rho^* \sim \langle \bar{q}q \rangle_T^{1/3} \sim [1 - T^2/T_c^2]^{1/6}, \quad (15)$$

which suggests a decrease of the ρ -meson mass near T_c (see Fig.12). Brown and Rho have derived a scaling relation for the low lying hadron masses based on the low energy realization of QCD trace anomaly⁵⁸

$$\frac{M_N^*}{M_N} \sim \frac{M_{\rho,\omega}^*}{M_{\rho,\omega}} \sim \frac{M_\sigma^*}{M_\sigma} \sim \frac{f_\pi^*}{f_\pi}. \quad (16)$$

We need detailed examination in various approaches to confirm or exclude the above predictions.

(4) Hadron width:

Even if a particle is stable in the vacuum, it acquires a width at finite T due to the scattering of the particle with thermally excited pions. For example, pion and nucleon will turn into ρ and Δ respectively due to the collision with the thermal pions. Such process produces a positive width increasing with T . In the virial expansion,⁵¹ the width is related to the imaginary part of the forward $\pi - \pi$ and $\pi - N$ scattering amplitudes. In Fig.14(a), the width of pion (from ρ -resonance) and nucleon are shown in this approach. Note that the approximation breaks down above ~ 160 MeV.

As for the unstable particles such as ρ -meson, there are two competing processes i.e. the decay width due to $\rho \rightarrow 2\pi$ and the scattering width due to the process like $\rho + \pi \rightarrow a_1$. The latter width increases as T increases while the former depends on the change of the hadron masses: If ρ -meson mass decreases and pion mass stays constant as we have seen before, the decay width becomes small just because of the phase space factor.

In the case of the σ -meson, the phase space suppression dominates over the other processes. Therefore, although σ does not appear in the vacuum due to its huge width ($\Gamma_{\sigma \rightarrow 2\pi} \sim M_\sigma$), it becomes a sharp resonance near T_c as has been pointed out by Hatsuda and Kunihiro.²⁶ (See also the experimental effort to see this effect in the relativistic heavy ion collisions.⁵⁹) The half-width of the σ -meson at finite T is given in Fig.14(b) where the NJL model is used.

Fig.14(a): The width of pion and nucleon in the virial expansion approach.⁵¹

Fig.14(b): T -dependent half-width of the σ -meson.²⁶

(5) Hadron couplings:

The interactions among hadrons can be also T -dependent in principle, but they are not examined extensively so far. In the NJL model, the n -point hadron vertices generally become small as T increases (decoupling of hadrons) but model independent confirmation of this result is called for.²⁶ In Fig.15, T dependence of f_π , $g_{\pi N}$ and σ - π - π coupling ($\lambda_{\sigma\pi\pi}$) is shown in arbitrary unit.

There are some exceptions of this decoupling due to kinematical reason; One of the examples is the enhancement of the $\pi^0 \rightarrow 2\gamma$ decay rate at finite T where an extra mass scale m_q plays a significant role.⁶⁰

Fig.15: Coupling constants in the NJL model.²⁶

(5) $U_A(1)$ restoration:

The nature of the restoration of $U_A(1)$ symmetry, which is broken by the axial anomaly at $T = 0$, is still an open problem. This is related to the T dependence of the topological susceptibility χ_t and also the η' -meson mass. In the framework of the effective model, there have been several studies.^{61,53}

Up to now, we have considered the system with zero baryon number, where the chiral restoration and its signals can be seen only near T_c just because of the slow decrease of $\langle \bar{q}q \rangle_T$. This situation does change once one introduces the finite baryon number, which is the central topic of the next section.

4. QCD at finite density ($T=0$, $\rho \neq 0$)

4.1. Condensates at finite ρ

Let us consider the finite density medium with $T=0$. One of the main differences of this system from the ($T \neq 0, \rho=0$) system is the behavior of various condensates. For instance, the light quark condensate in the chiral limit decreases slowly at low temperature⁵⁰

$$\langle \bar{q}q \rangle_T / \langle \bar{q}q \rangle_0 = 1 - [(N_f^2 - 1)/N_f] T^2 / 12 f_\pi^2 + \dots, \quad (17)$$

while it decreases linearly at low density⁶²

$$\langle \bar{q}q \rangle_\rho / \langle \bar{q}q \rangle_0 = 1 - \rho \cdot \Sigma_{\pi N} / f_\pi^2 m_\pi^2 + \dots, \quad (18)$$

with $\Sigma_{\pi N} = (45 \pm 7) \text{ MeV}$ being the π -N sigma term.⁶³ Note that eqs.(17) and (18) are model independent relations. If one extrapolates the latter formula to normal nuclear matter density $\rho_0 = 0.17 \text{ fm}^{-3}$, the condensate decreases almost 34%. Accordingly, one can expect rapid change of hadron properties at relatively low densities. (Such changes can be seen only near T_c in the ($T \neq 0, \rho=0$) system.) Actually, there are implicit evidences of the partial restoration of chiral symmetry in finite nuclei.⁶⁴ In Fig. 16, the density dependence of the several condensates are shown in the symmetric nuclear matter. The light quark condensate decreases relatively faster than the gluon condensate and heavy quark condensate.

Fig.16: Condensates vs ρ/ρ_0 in medium; $S = \langle \bar{s}s \rangle_\rho / \langle \bar{s}s \rangle_0$, $U = \langle \bar{u}u \rangle_\rho / \langle \bar{u}u \rangle_0$ and $G = \langle G^2 \rangle_\rho / \langle G^2 \rangle_0$. As for the vacuum values, we took $\langle \bar{u}u \rangle_0 = \langle \bar{s}s \rangle_0 / 0.8 = (-250 \text{ MeV})^3$, $\langle (\alpha_s/\pi) G^2 \rangle_0 = (330 \text{ MeV})^3$, $M_N^0 = 800 \text{ MeV}$ (the nucleon mass in the chiral limit) and $y = 2\langle \bar{s}s \rangle_N / \langle \bar{u}u + \bar{d}d \rangle_N = 0.12$ (the OZI violation in nucleon) are used.⁶⁵

4.2. Hadrons at finite ρ

The QCD sum rule is one of the best ways to analyse the hadrons in medium (as far as the density is not too high), since it gives a direct relation between the condensates in medium with resonance parameters in a model independent way.

The recent analysis by Su H. Lee and myself⁶⁶ shows that the vector mesons (ρ , ω and ϕ mesons) change their properties rapidly at finite density.⁶⁷ In this channel, the same current correlation function $TJ_\mu(x)J_\nu(0)$ appears both in QCD sum rules and in the deep inelastic lepton-hadron scattering (DIS). In the low density medium, this operator not only picks up the vacuum part $\langle TJ_\mu(x)J_\nu(0) \rangle_0$ but also picks up the medium part $\langle TJ_\mu(x)J_\nu(0) \rangle_N \cdot \rho$. The latter is related to the parton distribution function of the nucleon in DIS. One can show that the most important medium condensates in the vector channel are

$$\langle \bar{q}\gamma_\mu D_\nu q \rangle_\rho \quad \text{and} \quad \langle \bar{q}q \rangle_\rho^2, \quad (19)$$

for $\rho - \omega$ meson and $m_s \langle \bar{s}s \rangle_\rho$ for ϕ -meson.

After carrying out the Borel improvement for the operator product expansion, one finally obtains the results in Fig. 17, where the curves are well approximated by the formula

$$M_V^*/M_V = 1 - C_V \cdot (\rho/\rho_0), \quad (20)$$

with $C_V \simeq 0.18(0.15y)$ for $\rho - \omega$ meson (ϕ meson) and y being the strangeness content of the nucleon ($y = 2\langle \bar{s}s \rangle_N / \langle \bar{u}u + \bar{d}d \rangle_N = 0.12 - 0.22$).⁶⁵ The result is remarkably well consistent with the other approach based on the trace anomaly in QCD by Brown and Rho.⁵⁸

Fig.17(a): Density dependent mass of $\rho - \omega$ meson in QCD sum rules.⁶⁶

Fig.17(b): Mass of ϕ -meson.⁶⁶ Dashed lines denote the $K^0 K^0$ and $K^+ K^-$ threshold.

4.3. Implications of in-medium modification of hadrons

Many implicit evidences of the dropping rho-omega mass in finite nuclei are extensively discussed in Ref.64 which include the K^+ scattering off medium-heavy nuclei, polarization observables in nucleon-nucleus scattering, enhancement of the spin-orbit part of the G-matrix element, and enhancement of tensor-force due to ρ -meson in medium. These phenomenological studies require $C_V \sim 0.15$ which is comparable to our value in eq.(20). In this sense, our result of the QCD sum rule supports such phenomenological idea.

In the relativistic heavy ion collisions, one can even expect to detect the modification of hadron properties by the 2π correlation from the decay of the scalar meson,^{26,59} the enhancement of the kaon production,⁶⁸ and also the leptonic decay of rho, omega and ϕ mesons,⁶⁹ and σ -meson.⁷⁰ In particular, only a few % change of the ϕ -meson mass can affect its decay property drastically, since m_ϕ is very close to the threshold of its main decay mode ($K^0 K^0$ and $K^+ K^-$).⁶⁹ Fig.17(b) suggests that ϕ -meson is going to be more stable in medium, if the kaon mass (m_K) does not change in medium as some of the effective models claim.⁵³ As a result, the leptonic yield $R = N_\phi/(N_\rho + N_\omega)$ will be enhanced in high density/temperature medium.

As we have already discussed, one can see the considerable change of $M_V^*(T)$ only near the critical point, while $M_V^*(\rho)$ has $O(20\%)$ change even at ρ_0 . Therefore, the finite density system (such as the finite nuclei or the high density region of the heavy ion collisions) will be a better place to see the effect of the partial restoration of chiral symmetry and changes of hadron properties in experiment.

4.4. Phase transition to quark matter

There have been proposed numbers of scenarios and various exotic phases as intermediate states between nuclear (or neutron) matter and quark matter. They include charged and neutral pion condensation,⁷¹ the kaon condensation,⁷² the parity doubling matter,⁷³ massive quark matter⁷⁴ and so on. Unfortunately, none of these phases is not confirmed yet since the place to test the idea has been so far confined in neutron stars (the mass-radius relation, pulsation frequencies and glitches, cooling of the surface temperature and so on).⁷⁵ Therefore, even a little information from the heavy ion collisions in laboratory is welcome.

Instead of going into the details of the above scenarios, let us get back to a primitive idea of the phase transition from nuclear matter to quark matter. The nucleon has a radius of order $R_N = \sqrt{\langle r^2 \rangle} \simeq 0.8\text{fm}$ and the inter nucleon distance in nuclear matter at $\rho_0 = 0.17\text{fm}^{-3}$ is $\sim 2.5\text{ fm}$. A simple geometrical argument tells us that close packing density ρ_{c1} and a complete overlapping density ρ_{c2} read

$$\rho_{c1} = (4\pi R_N^3/3)^{-1} \sim 3\rho_0 \quad , \quad \rho_{c2} \sim 6\rho_0 \quad , \quad (21)$$

where the mean nucleon distance $l_d \simeq 1.4\rho^{-1/3}$ becomes comparable to R_N . In higher densities than ρ_{c1} , one cannot neglect the internal structure of the nucleon, since the nucleons start to overlap ($l_d < 2R_N$). In other words, there is no a priori reason to believe in models based on the point-like structure of nucleon above this density and one has to consider the internal structure of nucleon.⁷⁶ As the recent development of the non-perturbative QCD tells us, nucleon can be described either as a 3-quark bound state with $R_N \sim 0.8\text{fm}$ or as a chiral soliton having the same size. The seeming difference between

the two pictures merely stems from the different choice of variables to describe the same physics. At extremely high ρ , the quark-gluon language is certainly a most effective way to describe the system. Then, as one decreases ρ toward ρ_{c1} , the non-perturbative physics starts to emerge, which is similar to the situation we have encountered in section 2. In the quark-gluon picture, the matter with density around $\rho_{c1} \sim \rho_{c2}$ will be a strongly correlated quark matter, while it is a system of pions with non trivial topology in the hadron picture.⁷⁷

Now there is another kind of interesting possibility without going into such strong coupling system above ρ_{c1} , which I will call multi-clustering scenario.⁷⁸ Suppose that one of the 6-quark channels can form compact ($\sim$ size of nucleon) and not too high mass ($\sim 2M_N$) state in medium. (It does not have to be stable in the free space). Then, instead of increasing the baryon density, one can form a low density bose gas composed of the 6-quark state. Namely, it occurs the localization of the quark density instead of distributing it to entire space. One of the candidate of such 6-quark state is the H-dihyperon whose mass is possibly not too higher than $2M_N$ even in the free space. One can also generalize the idea to a hierarchy of the similar phase transitions: nuclear matter $\rightarrow$ 6-quark matter $\rightarrow$ 12-quark matter $\rightarrow$ 18-quark matter $\rightarrow \dots$, which is an interesting alternative path toward quark matter. A similar scenario has been also proposed by Kagiya et al..⁷⁹

5. Conclusions

We have looked at the physics in the $T - \rho$ plane on the basis of various approaches; perturbative QCD, lattice QCD, effective theories and QCD sum rules. None of them is enough powerful to cover all the region of the phase diagram. However, recent studies are making their applicable range wider and we have already learned some interesting physics related to the non-perturbative nature of QCD.

(1) *Non-perturbative physics above T_c :*

The higher order analyses of the perturbation theory have shown that there exists non-perturbative physics even above T_c . The lattice data of the free energy and also the screening mass in the scalar-pseudoscalar channel seem to support the coexistence of perturbative and non-perturbative physics above T_c . There are several ideas about the physics behind them, which include the non-perturbative cutoff, the gluon condensate above T_c , color singlet soft modes above T_c and so on. Clear physical understanding of them are called for. In particular, the hadronic mode above T_c is a big challenging problem.

(2) *Chiral restoration and its associated phenomena:*

$\langle \bar{q}q \rangle_T$ for light quarks is a good order parameter near the chiral limit. There is little doubt about the existence of chiral transition at high T in the chiral limit and probably it is also true at high ρ . Like similar cases in condensed matter and nuclear physics, we can expect phenomena associated with this phase transition; in particular the modification of the elementary modes of excitations (hadrons) in medium. QCD sum rules and effective theories are suitable to calculate such modifications at least at relatively low T and ρ . It turns out that $\langle \bar{q}q \rangle_T$ and hence the hadron properties change much faster at finite ρ

than at finite T . This suggests a right place to see the effects in the relativistic heavy ion collisions: To see the quark-gluon plasma clearly, one should look at the high T central region, while to see the chiral restoration and its associated phenomena, one should look at the high density region. Lepton pairs from the vector mesons and the 2π correlation from the scalar meson are the useful probes in the latter case.

(3) *Transition from nuclear matter to quark matter:*

Nothing definite is known for this transition although many possible intermediate phases are proposed such as the pion and kaon condensation, the parity doubling matter, skyrmion crystal and so on. One of the interesting recent ideas is the multi-clustering scenario where the bose condensation of multi-quark states (such as H dihyperon) is formed to avoid the high nucleon density.

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