The Effect of Flux Flow on Self-Field Instability

Lawrence Dresner

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CONTENTS

Abstract	v
1. Introduction	1
2. Basic Equations.	2
3. Instantaneous Delta-Function Source in an Infinite Medium. . . .	6
4. Linear Charging.	12
5. Neglect of Axial Diffusion Term.	16
6. Self-Field Stability	17
References	21

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ABSTRACT

Flux flow causes type II superconductors to develop resistance continuously rather than suddenly as transport current increases. This means that the distribution of current among the filaments in a composite conductor is determined not only by their inductive coupling but also by the longitudinal resistance they develop as they begin to carry current. The current distribution is calculated in two cases, taking flux flow into account: a composite clamped suddenly across a constant-current source and a composite charged with current at a uniform rate. The results of the latter problem are used to show that slowly charged conductors will be much more stable against self-field instability than is indicated by purely inductive calculations.

1. INTRODUCTION

In a recent paper,¹ Wilson noted that Duchateau and Turck's theory² of self-field instability seemed pessimistic when compared with measurement. In Wilson's view, the distribution of current density used by Duchateau and Turck is wrong because it fails to account for the progressive decay towards uniformity caused by flux flow. Duchateau and Turck calculate the current distribution in a conductor on the basis of purely inductive coupling between perfectly conducting strands, with the restriction that the current in a filament never exceeds its critical current. This is the same as the assumption that the resistivity of the superconductor jumps suddenly from zero to its full normal value at the critical current density.

In reality, flux flow causes type II superconductors to develop resistance continuously rather than suddenly as the transport current increases. This means that the distribution of the current among the filaments is determined not only by their inductive coupling but also by the longitudinal resistance they develop as they begin to carry current.

In the simplest case of an untwisted composite, the purely inductive calculation of Duchateau and Turck leads to a current distribution in which a sheath of outer filaments is saturated and the inner filaments carry no current at all. Wilson has estimated the time it would take this current distribution to decay, and he finds it to be short compared with the times generally anticipated for charging toroidal field coils. Consequently, the current distribution of Duchateau and Turck will never exist at all in a slowly charged magnet. This is the reason for Duchateau and Turck's overly pessimistic expectations, as set forth by Wilson.

The purpose of the present paper is twofold: first, to improve Wilson's simple estimates of the decay time, and second, to calculate the distribution of current among the filaments of a conductor being charged at a uniform rate.

2. BASIC EQUATIONS

Let us consider a composite conductor in the form of an infinite slab, the faces of which are perpendicular to the y-axis (see Fig. 1).

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Fig. 1. Sketch of the slab conductor showing the coordinate system.

Let the midplane of the slab be the (x,z) plane. The superconductor is distributed in thin sheets parallel to the faces and is equally spaced at intervals Δy . The n^{th} sheet carries a current i_n in each unit of its length along the z-axis; the current flows in the x direction.

Above the midplane, between sheets $n - 1$ and n , the self-field B of the sheet currents points in the z direction and is given in magnitude by

$$B = \mu_0 \sum_{k=1}^{n-1} i_k. \quad (1)$$

As the currents and field change with time, the rate of change of flux through the shaded rectangle ABCD is

$$\Delta x \Delta y \frac{\partial B}{\partial t} = \mu_0 \Delta x \Delta y \frac{\partial}{\partial t} \sum_{k=1}^{n-1} i_k. \quad (2)$$

The right-hand side of Eq. (2) gives the voltage around the path ABCD. Along the leg AB, the voltage drop is $i_n(\rho_n \Delta x / \eta \Delta y)$, where ρ_n is the flux-flow resistivity in the n^{th} sheet and is a function of the current i_n , and η is the volume fraction of superconductor. The voltage drop along the leg BC is $j(x + \Delta x) \rho_{cu} \Delta y$, where j is the current density flowing in the copper perpendicular to the superconducting sheets and towards the center line. Thus,

$$\mu_0 \Delta x \Delta y \frac{\partial}{\partial t} \sum_{k=1}^{n-1} i_k = i_n \left(\frac{\rho_n \Delta x}{\eta \Delta y} \right) - i_{n-1} \left(\frac{\rho_{n-1} \Delta x}{\eta \Delta y} \right) + j(x + \Delta x) \rho_{cu} \Delta y - j(x) \rho_{cu} \Delta y. \quad (3)$$

If Δx and Δy are small, we can write this as

$$\mu_0 \frac{\partial}{\partial t} \int_0^y i \, dy = \frac{1}{\eta} \frac{\partial}{\partial y} (i \rho) + \Delta y \rho_{cu} \frac{\partial j}{\partial x}, \quad (4)$$

where i is the current in the superconducting sheet at the position y , etc. Conservation of current demands that

$$\left(\sum_{k=1}^{n-1} i_k \right)_x + j \Delta x = \left(\sum_{k=1}^{n-1} i_k \right)_{x + \Delta x} \quad (5a)$$

or

$$\Delta y j = \frac{\partial}{\partial x} \int_0^y i \, dy. \quad (5b)$$

Then

$$\mu_0 \frac{\partial}{\partial t} \int_0^y i \, dy = \frac{1}{\eta} \frac{\partial}{\partial y} (i \rho) + \rho_{cu} \frac{\partial^2 i}{\partial x^2} \int_0^y i \, dy \quad (6a)$$

or, after we differentiate with respect to y ,

$$\mu_0 \frac{\partial i}{\partial t} = \frac{1}{\eta} \frac{\partial^2}{\partial y^2} (i \rho) + \rho_{cu} \frac{\partial^2 i}{\partial x^2}. \quad (6b)$$

In an ordinary ohmic medium, ρ may be taken to be constant, but of course we cannot make this assumption for superconductors. For superconductors, it is usually assumed that ρ is a step function of current density, jumping sharply from $\rho = 0$ to $\rho = \rho_{sc}$, the full normal resistivity, at the critical current density J_c (see the top half of Fig. 2). In strict point of fact, the transition from zero to full normal resistivity takes place rapidly but continuously, as shown schematically in the bottom half of Fig. 2. Here the width of the transition zone has been greatly exaggerated along the J -axis. The lower part of the ρ - J curve can be approximated reasonably well by

$$\rho = \rho_{sc} \left(\frac{J}{J_0} \right)^{m-1}, \quad (7)$$

where J_0 is a reference current density and m is a number around 30 or 40.³ We use this form henceforth.

If we denote by i_0 the current in one sheet when the current density has the value J_0 , and set $g = i/i_0$ and $\tau = \rho_{sc} t / \eta \mu_0$, we get

$$\frac{\partial g}{\partial \tau} = \frac{\partial^2}{\partial y^2} (g^m) + \left(\frac{\eta \rho_{cu}}{\rho_{sc}} \right) \frac{\partial^2 g}{\partial x^2}. \quad (8)$$

The ratio g is a dimensionless measure of the current distribution in the slab. The variable τ is a measure of the time with the dimensions of length squared.

In the problems we consider below, the last term on the right-hand side of Eq. (8) will always be negligible compared with the first term on the right-hand side, except very near the current leads. For this reason, we now turn our attention to the current distribution determined by the equation

$$\frac{\partial g}{\partial \tau} = \frac{\partial^2}{\partial y^2} (g^m). \quad (9)$$

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Fig. 2. Resistivity of the superconductor according to the critical-state model (top) and the flux-flow model (bottom).

After we solve Eq. (9) for the main situation we are interested in below, we verify that the neglect of the last term on the right-hand side of Eq. (8) is justified.

3. INSTANTANEOUS DELTA-FUNCTION SOURCE IN AN INFINITE MEDIUM

If m were equal to 1, Eq. (9) would be the ordinary diffusion equation. The solutions of the ordinary diffusion equation in an infinite medium for an instantaneous delta-function source $\delta(x,t)$ are shown schematically in Fig. 3 for three times, $\tau_1 < \tau_2 < \tau_3$. Now imagine the slab between $y = 0$ and $y = a$ to be half the conductor. The infinite-medium solutions are then good even for the finite slab for times short enough so that at

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Fig. 3. g -profiles (schematic) for the ordinary diffusion equation with an instantaneous delta-function source.

$x = a$, g is very small. The infinite-medium solutions are good at least for times $< \tau_3$. A similar conclusion holds when $m > 1$. But whereas we already know the shape of the g -profiles in Fig. 3 when $m = 1$ (Gaussian), we still have to calculate it when $m > 1$.

The instantaneous delta-function source $\delta(x,t)$ is of practical interest, for if we suddenly clamp a long superconductor across the ends of a constant-current power supply, the current at first lies at the outer surface. Hence we can use the infinite-medium solution to study at least the early stages of current redistribution.

The infinite-medium solution can be found by the method of similarity solutions described by Birkhoff⁴ and by Ames.⁵ If we set

$$g(y, \tau) = \tau^{-1/(m+1)} F \left[y / \tau^{1/(m+1)} \right] \quad (10)$$

and substitute into Eq. (9), we get the following ordinary differential equation for the function F of the variable $\xi = y / \tau^{1/(m+1)}$:

$$(m+1) \frac{d^2(F^m)}{d\xi^2} + \xi \frac{dF}{d\xi} + F = 0. \quad (11)$$

The boundary conditions which F must obey are

$$\left(\frac{dF}{d\xi} \right)_{\xi=0} = 0 \iff \left(\frac{\partial g}{\partial y} \right)_{y=0} = 0 \quad (12a)$$

and

$$F(\infty) = 0 \iff g(\infty, \tau) = 0. \quad (12b)$$

One further condition, which arises from the initial presence of a delta-function source and serves to determine the normalization of F , can be obtained as follows. We integrate Eq. (9) over all positive y and, using Eq. (12), find that

$$\frac{d}{d\tau} \int_0^\infty g \, dy = 0. \quad (13)$$

Thus

$$\int_0^{\infty} g \, dy = \text{constant} = \int_0^{\infty} F(\xi) \, d\xi. \quad (14)$$

We can integrate Eq. (11) once immediately:

$$(m + 1) \frac{d(F^m)}{d\xi} + \xi F = 0. \quad (15)$$

The right-hand side of Eq. (15), a constant of integration, must be zero in order to satisfy the boundary condition, Eq. (12a). A second integration gives

$$\frac{m(m + 1)}{m - 1} F^{m-1} + \frac{\xi^2}{2} = \frac{\xi_0^2}{2} \quad (16)$$

where $\xi_0^2/2$ is a constant of integration. Thus, $F = 0$ for $\xi > \xi_0$, while for $\xi < \xi_0$,

$$F(\xi) = \left[\frac{m - 1}{2m(m + 1)} (\xi_0^2 - \xi^2) \right]^{1/(m-1)}. \quad (17)$$

The constant ξ_0 is determined by the normalization, Eq. (14).

Equation (17) gives the profile of g at any instant of time; its shape is the counterpart of the Gaussian shape for the ordinary diffusion equation. The profile is shown in Fig. 4, where $F(\xi)/F(0)$ is plotted against $y/y_0 = \xi/\xi_0$ for $m = 20, 30$, and 40 . The shape of the diffusing current density distribution is almost rectangular for large m .

Next, we need to estimate how long it will take for the current distributions diffusing in from each boundary to reach the center line; beyond this time the infinite-medium solutions are no longer valid. From Eqs. (14) and (17) we obtain

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Fig. 4. The shape of the current density distribution for an instantaneous delta-function source.

$$\int_{\text{half slab}} g dy = \left[\frac{m-1}{2m(m+1)} \right]^{1/(m-1)} \int_0^{\xi_0} (\xi_0^2 - \xi^2)^{1/(m-1)} d\xi \quad (18a)$$

or

$$\bar{g}a = \left[\frac{m-1}{2m(m+1)} \right]^{1/(m-1)} \xi_0^{(m+1)/(m-1)} \int_0^1 (1-z^2)^{1/(m-1)} dz, \quad (18b)$$

where $z = \xi/\xi_0$, a is the half-thickness of the slab, and $\bar{g} = \int_0^a g dy/a$. At the time t_0 at which the current distributions reach the center line, $\xi_0 = a\tau_0^{-1/(m+1)}$. Substituting this value in Eq. (18b) and remembering that $t = \eta\mu_0\tau/\rho_{sc}$, we find

$$t_0 = \frac{\eta\mu_0 a^2}{\rho_{sc}} \frac{m-1}{2m(m+1)} \frac{\left[\int_0^1 (1-z^2)^{1/(m-1)} dz \right]^{m-1}}{\bar{g}^{(m-1)}}. \quad (19)$$

For values of m between 20 and 50, the numerator of the last fraction on the right-hand side of Eq. (19) is within 0.6% of 0.547.

In his paper, Wilson considered a cylindrical composite 3.5 mm in diameter operating at 50% of critical current. What he meant by the term "critical current" must be inferred from his statement that at 90% of critical current density the resistivity of the composite is 10^{-14} $\Omega\text{-m}$, and at 95% of critical current density the resistivity of the composite is 10^{-13} $\Omega\text{-m}$. From these data, we deduce that the critical current density corresponds to a resistivity of the composite of 8.90×10^{-13} $\Omega\text{-m}$ and $m = 43.6$. If we take $\rho_{sc}/\eta = 10^{-7}$ $\Omega\text{-m}$, then $g_c = J_c/J_0 = (8.90 \times 10^{-13}/10^{-7})^{1/42.6} = 0.761$. At 50% of critical current, $\bar{g} = 0.5 \times 0.761 = 0.381$. If we take $a = 1.75$ mm, corresponding to a 3.5-mm-thick slab, we find $t_0 = 1.64 \times 10^{11}$ sec or about 5200 years! Hence, the infinite-medium solution should be applicable for all practical purposes.

We can find $g(0,t)$ and the corresponding flux-flow resistivity of the composite from

$$g(0,t) = \left(\frac{t_0}{t}\right)^{1/(m+1)} \left[\frac{m-1}{2m(m+1)}\right]^{1/(m-1)} \left(\frac{\eta \mu_0 a^2}{\rho_{sc} t_0}\right)^{1/(m-1)} \quad (20)$$

Some results are shown in Table 1.

Table 1. Decay of the current distribution in Wilson's example

$t(\text{sec})$	$g(0,t)$	$\rho/\eta \text{ } (\Omega\text{-m})$
1	0.689	1.32×10^{-14}
10	0.655	1.47×10^{-15}
10^2	0.622	1.62×10^{-16}
10^3	0.591	1.80×10^{-17}
10^4	0.561	2.00×10^{-18}
10^5	0.533	2.21×10^{-19}
10^6	0.506	2.46×10^{-20}

These results agree roughly with Wilson's simple estimate of 5 sec to reach 95% of the critical current density. In fact, the decay is even somewhat faster, reaching about 90% of critical current density in 1 sec. What is most significant is that even though the current density falls slowly with time, the longitudinal flux-flow resistivity ρ/η falls rapidly with time. We may conjecture that if a conductor is charged slowly with time, the longitudinal flux-flow resistivity will approximate the value in the table corresponding to the charging time. If this conjecture is right, the stability of composite conductors against self-field flux jumps should be much improved over what is indicated by Duchateau and Turck.²

Therefore, in Sect. 4 we turn to the question of what the current distribution is in a magnet being charged slowly.

4. LINEAR CHARGING

The simplest case of gradual charging is the case of linear charging, in which the current in the conductor increases linearly with time. Again, we begin with infinite-medium similarity solutions, but this time we take

$$g(y, \tau) = \tau^{1/(m+1)} G \left[y / \tau^{m/(1+m)} \right]. \quad (21)$$

Substituting Eq. (21) into Eq. (9), we find that the as yet unknown function G obeys the ordinary differential equation

$$(m+1) \frac{d^2}{d\zeta^2} (G^m) + m\zeta \frac{dG}{d\zeta} - G = 0; \quad \zeta = y / \tau^{m/(m+1)}. \quad (22)$$

Furthermore, the solution in Eq. (21) has the desired property that

$$\int_0^\infty g dy = \tau \int_0^\infty G(\zeta) d\zeta, \quad (23)$$

i.e., that $\int_0^\infty g dy$ increases linearly with time.

If we integrate Eq. (22) over ζ , we find, after an integration by parts, that

$$\left(\frac{dG}{d\zeta} \right)_{\zeta=0} = \frac{- \int_0^\infty G d\zeta}{m [G(0)]^{m-1}}, \quad (24)$$

which serves as one of the conditions we use to select the solution of Eq. (22) we want. In deriving Eq. (24) we have used another condition, namely,

$$G(\infty) = 0. \quad (25)$$

Equation (22) is not as easy to solve as Eq. (11), and we have integrated

it numerically. Shown in Fig. 5 are plots of G vs ζ for $m = 20, 30$, and 40 , normalized to $G(0) = 1$. Values of G for any other normalization of G can be obtained by simultaneously transforming G and ζ according to

$$\left. \begin{aligned} G' &= \lambda G \\ \zeta' &= \lambda^{(m-1)/2} \zeta \end{aligned} \right\} 0 < \lambda < \infty, \quad (26)$$

a transformation that leaves Eqs. (22)-(25) unchanged. Again, notice the nearly rectangular shape of the current distribution.

Let us now continue further analysis of Wilson's problem. If we define t_1 as the total charging time, and define $\bar{g}$ by

$$\bar{g}a = \left(\int_0^\infty g \, dy \right)_{t=t_1}, \quad (27)$$

then

$$\int_0^\infty G(\zeta) \, d\zeta = \frac{\bar{g}a}{\tau_1}. \quad (28)$$

If y_1 is the distance the inward diffusing current distribution has penetrated towards the center from each edge at time t_1 , then

$$\zeta_0 = y_1 / \tau_1^{m/(m+1)}, \quad (29)$$

where ζ_0 is the value at which $G(\zeta)$ has its first root.

Direct numerical calculations show that

$$\left. \begin{aligned} \int_0^\infty G \, d\zeta &= 1 \\ \text{and} \\ \zeta_0 &= \frac{m+1}{m} \end{aligned} \right\} \text{when } G(0) = 1, \quad (30)$$

Fig. 5. The shape of the current density distribution for a linearly ramped current source.

so that

$$\left. \begin{aligned} \int_0^{\infty} G \, d\zeta &= \lambda^{(m+1)/2} \\ \text{and} \\ \zeta_0 &= \left(\frac{m+1}{m} \right) \lambda^{(m-1)/2} \end{aligned} \right\} \text{when } G(0) = \lambda. \quad (31)$$

Then

$$\left(\frac{m\zeta_0}{m+1} \right)^{2/(m-1)} = \lambda \left(\int_0^{\infty} G \, d\zeta \right)^{2/(m+1)} = \left(\frac{\bar{g}a}{\tau_1} \right)^{2/(m+1)}, \quad (32)$$

so that finally

$$\frac{y_1}{a} = \frac{m+1}{m} \bar{g}^{(m-1)/(m+1)} \left(\frac{\rho_{sc} t_1}{\eta \mu_0 a^2} \right)^{1/(m+1)}. \quad (33)$$

Furthermore ,

$$g(0, t) = \left(\frac{t}{t_1} \right)^{1/(m+1)} \bar{g}^{2/(m+1)} \left(\frac{\eta \mu_0 a^2}{\rho_{sc} t_1} \right)^{1/(m+1)}. \quad (34)$$

Table 2 shows some values computed using these formulas for Wilson's problem. The first column is the charging time in seconds. We see, as expected, that for long charging times, the longitudinal flux-flow resistivity attained is very small.

Table 2. Linear charging in Wilson's example

$t_1(\text{sec})$	y_1/a	$g(0, t_1)$	$\frac{\rho}{\eta} (\Omega\text{-m})$
1	0.511	0.762	9.61×10^{-13}
10	0.538	0.724	1.07×10^{-13}
10^2	0.567	0.688	1.18×10^{-14}
10^3	0.597	0.653	1.31×10^{-15}
10^4	0.628	0.620	1.45×10^{-16}
10^5	0.662	0.589	1.61×10^{-17}
10^6	0.697	0.559	1.79×10^{-18}

5. NEGLECT OF AXIAL DIFFUSION TERM

Using the solution obtained in the last section, we now study the question of the neglect of the axial diffusion term, i.e., the last term on the right-hand side of Eq. (8). We compare it to the radial diffusion term, which is the first term on the right-hand side of Eq. (8). To facilitate comparison, we first integrate Eq. (8) with respect to y over half the slab. Then we get

$$a \frac{\partial}{\partial \tau} \langle g \rangle = \left[\frac{\partial}{\partial y} (g^m) \right]_{y=0} + a \frac{\eta \rho_{cu}}{\rho_{sc}} \frac{\partial^2 \langle g \rangle}{\partial x^2} . \quad (35)$$

We can calculate the first term on the right-hand side using the results of Sect. 4 as follows:

$$\left[\frac{\partial}{\partial y} (g^m) \right]_{y=0} = \left[\frac{d}{d\zeta} (G^m) \right]_{\zeta=0} = - \int_0^\infty G d\zeta = - \frac{\bar{g}a}{\tau_1} . \quad (36)$$

Then the ratio of the axial to the radial diffusion term is

$$\left(\frac{\rho_{cu} t_1}{\mu_o} \right) \frac{\partial^2}{\partial x^2} \left(\frac{\langle g \rangle}{\bar{g}} \right) . \quad (37)$$

Now, because $\langle g \rangle \sim \bar{g}$, it follows that the ratio in Eq. (37) can be on the order of one or larger only over distances on the order of $(\rho_{cu} t_1 / \mu_0)^{1/2}$ or less. For $t_1 = 1000$ sec and $\rho_{cu} = 4 \times 10^{-10}$ Ω -m, this distance is only 0.564 m. Clearly then, axial diffusion is only important near the current leads, and may, in general, be neglected.

6. SELF-FIELD STABILITY

The usual method of calculating the self-field stability begins with the assumption of a small rise in the temperature of the conductor. This causes a decrease in the critical current density of the superconductor. In the strict critical-state model, current leaves the outer saturated filaments until the filaments are once again saturated. The current flowing in the copper creates a longitudinal electric field, which, acting upon the current in the filaments, causes dissipation. This dissipation is easily estimated using the "frozen-flux" approximation of Kremlev, Mints, and Rakhmanov.⁶

In the frozen-flux approximation, the current leaving the filaments remains in the copper of the outer sheath of saturated filaments. Hence the average current density there (averaged over a volume containing a few filaments) is constant and equal to $\eta J_c(T_b)$, where $J_c(T_b)$ is the critical current density at the ambient bath temperature T_b . After a small rise in temperature, ΔT , we have

$$\eta J_c(T_b) = \eta J_c(T_b + \Delta T) + E / \rho_{cu} , \quad (38)$$

where E is the electric field and the second term on the right-hand side is the current density in the copper. Then

$$E = -\eta \rho_{cu} \left(\frac{dJ_c}{dT} \right)_{T_b} \Delta T = \eta J_c(T_b) \rho_{cu} \frac{\Delta T}{T_0} , \quad (39)$$

where T_0 is defined by

$$T_0 = - \left(\frac{1}{J_c} \frac{dJ_c}{dT} \right)_{T_b} . \quad (40)$$

Since $E/\rho_{cu} \ll J_c$ if ΔT is small, the power density being dissipated in the sheath of saturated filaments is

$$\dot{Q} = E \eta J_c = (\eta J_c)^2 \rho_{cu} \frac{\Delta T}{T_0} \quad (41)$$

When the continuous development of flux-flow resistance is taken into account, there is always a small dissipative power density given by

$$\dot{Q} = (\eta J_c)^2 \rho_c \left(\frac{g(0,t)}{g_c} \right)^{m+1}, \quad (42)$$

where J_c is the same as in Eq. (41), i.e., the current density at which some specified flux-flow resistivity ρ_c occurs. If there is a small rise in temperature ΔT , the average current density remains fixed at J_c , but the flux-flow resistivity increases by a factor $(1 + \Delta T/T_0)^{m-1}$. [This is because $J_0 = J_c/g_c$ and g_c is fixed by the flux-flow resistivity that "defines" the critical current. Hence, $dJ_0/J_0 dT = dJ_c/J_c dT = -1/T_0$. If we assume that $J_c \sim (T_{cr} - T)/(T_{cr} - T_b)$, it follows that $T_0 = T_{cr} - T_b$.] If the increase in the flux-flow resistivity is not sufficient to make it comparable with the resistivity of copper, no current will spill over into the copper.

If we take a charging time of 1000 sec and take $\eta J_c = 5.6 \times 10^7 \text{ A-m}^{-2}$ (NbTi at 8 T, $\eta \sim 0.1$), we find the steady dissipation $\dot{Q}$ of $3.02 \text{ } \mu\text{W-cm}^{-3}$. If we take $\rho_{cu} = 4 \times 10^{-10} \text{ } \Omega\text{-m}$ and $T_0 = 5.6 - 4.2 = 1.4 \text{ K}$, then Eq. (41) is less than Eq. (42) only for $\Delta T < 3.37 \times 10^{-6} \text{ K}$. The dissipations of Eqs. (41) and (42) are plotted as functions of ΔT in Fig. 6. (From this example, it is clear that the critical-state model may overestimate by many orders of magnitude the dissipation following a slight temperature rise.) There seems to be no question that the self-field distribution of a slowly charged magnet is much more stable against small temperature perturbations than is indicated by the strict critical-state model.

On the other hand, because of the very rapid rise of $\dot{Q}$ for sufficiently large ΔT in the flux-flow model, the conductor cannot be stable against arbitrarily large temperature rises. To estimate how large a temperature rise we can stand, we have plotted the cooling curve $hP\Delta T/A = 2h\Delta T/R$ for a heat transfer coefficient of $h = 0.1 \text{ W-cm}^{-2}$. Here P is the perimeter

Fig. 6. The dissipation as a function of temperature rise according to the critical-state and flux-flow models.

of the conductor, A is its cross-sectional area, and R is its radius. The cooling curve and the heat production curve of the flux-flow model intersect at temperature increases of 2.64×10^{-6} K and 0.458 K. The former temperature increase gives the order of magnitude of the temperature excess of the conductor over the bath temperature due to the steady production of $3.02 \mu\text{W}\text{-cm}^{-3}$. The latter temperature gives the temperature excess at which heating first begins to exceed cooling. The conductor in this example is almost surely stable against perturbations on the order of 0.1 K when $t_1 = 1000$ sec.

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