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SUBJECT: Three-Dimensional Temperature History of a Multipass, Filled Weldment, Part 1

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ABSTRACT

A conceptual model is developed for the three-dimensional temperature history of a multipass, filled weldment. Property variations with temperature and phase change are included. A mathematical model and finite difference equations are derived from the conceptual model and a solution procedure for the equations is presented.

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1. SUMMARY

Finite difference equations were used to simulate the temperature history of a multipass, filled weldment. The conceptual model developed utilized a general physical description of the welding process in which the molten pool is assumed to contain two distinct regions, turbulent and laminar. Energy balances and boundary conditions are derived and approximated with the set of finite difference equations.

Input parameters are described and recommendations are made for implementation of the solution procedure.

2. INTRODUCTION

In welding thick plates, several passes of a welding head are required to join the plates. Each pass deposits filler metal and fuses the filler metal to base metal in the vicinity of the weld (Fig. 1). The residual stress, and therefore the strength of the weld, will depend on the temperature variation that occurs with each pass of the welding head.

Previous work has been divided into analytical and numerical approaches. Eyres, Rosenthal, and Sarjant and Slack(3, 5, 6) performed the classical analytical work. These authors assumed that isotherms, centered relative to the moving welding head, did not change with time (quasi-stationary case). The differential equations and boundary conditions describing the system were simplified to obtain an analytical solution. Although far from the weld, there was good agreement between calculated and experimental results; close to the weld there was significant deviation. This deviation was due in part to the assumptions that surface convection and radiation were negligible and that the physical properties of the filler and base metal were independent of temperature.

With the advent of high speed computers, a second approach, numerical solution of the partial differential equations, became attractive. Computer programs have been developed which treat the case of heat transfer from a moving welding head (7, 8). In all of these, agreement is reasonably good between the computer model and experimental results.

The purpose of this project was to develop the equations for a computer program that could calculate the three-dimensional temperature history of a multipass, filled weldment. To achieve this, a conceptual model for multipass welding was developed and the conceptual model was translated into a suitable mathematical model.

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MULTIPASS WELDMENT

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FIG.
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3. MODEL DEVELOPMENT

3.1 Physical Process

In multipass, filled welding, the thick steel plates are brought together with a V-butt (Fig. 2). An electrode is positioned over the V-notch and a 15-40 volt, 100-1000 amp arc melts the base and filler metal in its immediate vicinity. The arc moves along the V-notch, depositing filler metal and fusing it with base metal as it travels. Inert gas flows through an annulus around the electrode to cool it, provide gas for the arc plasma, and prevent oxidation of the weld surface. After the metal has been traversed, the weldment is allowed to cool to approximately 250°F. A second pass is then started to deposit additional filler metal and fuse it with part of the previous weld. This process is continued until the entire V-notch has been filled.

One of the main complications in modeling the temperature profile of a weld is that the physical properties of the metal vary with the microstructure, and this in turn is a function of both temperature and composition. For a given composition of base or filler metal, several microstructures are possible: martensite, ferrite, pearlite, and austenite. The microstructure depends on the composition of the material in question and its thermal history. The temperature induced variations cannot be ignored because of the wide range of temperatures (250 - 1500°C) during welding. The property variations due to composition changes are somewhat more difficult to consider. The filler and base metal are usually not of the same composition. Upon fusing, the filler and part of the base metal are intimately mixed, resulting in a weld zone composition which has some weighted average of the two initial compositions.

Energy is transferred to the molten pool by means of electrons emitted from the cathode and accelerated in the electric field. Part of this energy is lost through the ionization of the gas molecules resulting in a plasma. Heat is introduced into the weld mainly by this electron stream.

The weld zone can be divided into several regions. In the immediate vicinity of the arc, a molten pool of metal exists on the surface of the metal. The pool is usually 1-2-cm wide and 3-4-cm long. Its temperature is only a few degrees higher than the melting temperatures of the base metal. The pool loses heat by convection and radiation and by conduction to the surrounding solid metal.

Surrounding the molten pool is a region known as the heat affected zone (HAZ). In the HAZ the maximum temperature at any time was greater than the lowest microstructure transition temperature (730°C in carbon steel). Heat flows into the HAZ by conduction from the molten pool and by convection and radiation from the arc. Heat is transferred from the HAZ by conduction to the surrounding metal and by convection and radiation to the atmosphere. A third zone comprises all metal which is heated, but not

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WELD CROSS-SECTION

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contained in the first two zones. Heat enters by conduction from the HAZ and leaves by convection and radiation to the atmosphere.

Very little is known about the welding pool. There is evidence that uniform mixing of filler and base occurs throughout the pool (9). Apps and Milner determined that the effective thermal conductivity of molten metal under an arc is one to ten times greater than that of the remainder of the liquid metal (1). They interpreted this to mean that regions of varying turbulence existed in the molten pool. Lorentz mixing from the passage of current through the weld pool is the primary source of the turbulence (Appendix 6.4).

3.2 Conceptual Model

An estimate of the fraction of heat lost by each of the three modes, conduction through the solid, radiation from the surface, and convection from the surface, was made (Appendix 6.1). Since combined radiation and convection represents approximately 20% of the total heat loss, it is included in the model. Radiation and convection are considered at all surfaces except those underneath the arc and at the edges of the weld piece.

The molten pool is modeled as having two regions; one of highly turbulent mixing and one of laminar flow (Fig. 3). The thermal conductivity of the laminar region was assigned the thermal conductivity of stationary molten metal. In the turbulent region, energy transport was assumed to occur primarily through turbulent convection. Since the mixing velocity in the turbulent pool is on the order of 10 cm/sec (9), it can reasonably be assumed that the turbulent region is isothermal. Based on Apps data (Appendix 6.3), it was assumed that the turbulent zone extended 80% of the distance from the pool surface to the pool-solid interface (1).

To simplify the heat transfer equations, the welding head was assumed to move in discrete steps rather than continuously. The laminar zone was therefore assumed to be a stationary boundary layer of uniform thickness between the turbulent zone and solid metal.

Energy transfer from the arc to the pool was treated with a constant flux boundary condition. To facilitate the finite difference form of the heat transfer equations, the arc was assumed to have a square cross-section at the molten pool surface.

Instead of adding filler metal to the molten pool at a constant rate, it was assumed that a filler rod with a square cross-section and the length of the V-notch was in the notch at the start of a pass. The V-notch was approximated with a series of square-sided steps. Thus in the multipass operation, the V-notch is filled with successive additions of the square cross section filler metal fused to the base metal. The base and filler metal were assumed to have the same composition. The thermal properties of the

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MOLTEN POOL REGIONS

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metal are based on data for AISI-304 stainless steel. It was also assumed that the entire weld piece was at a uniform temperature at the start of each pass.

3.3 Mathematical Model

As shown in Fig. 4, two coordinate systems are used in the model: a coordinate system attached to the metal plate, the Reference coordinate system, with axes x , y , and z , and a coordinate system attached to the moving heat source, the Lagrangian coordinate system, with axes a , b , and c . All of the energy balances apply to the Lagrangian coordinate system. In single pass models, there is no need to introduce a reference coordinate system since any point in the solid can be specified in the Lagrangian system by specifying v and t . However, in a multipass weld model, the position of the welding bead and hence the origin of the Lagrangian system, will change from pass to pass. Therefore, the reference system is needed to relate the positions of the Lagrangian systems for each pass. The inter-relationship of the reference coordinate system and the Lagrangian system is:

$$\begin{aligned} L_1(0,0,0) &= R_1(x_1, y_1, z_1 + vt) \\ L_2(0,0,0) &= R_2(x_2, y_2, z_2 + vt) \\ L_3(0,0,0) &= R_3(x_3, y_3, z_3 + vt) \\ &\vdots \\ &\vdots \\ &\vdots \end{aligned} \quad (1)$$

where the subscripts refer to the first, second, and third pass of the welding head.

Since the arc is assumed to move in finite steps rather than with a continuous velocity, v , the arc is stationary over a fixed position for a time equal to that of the arc width divided by the velocity. An energy balance on a volume element within the metal then yields:

$$\rho C_p \frac{\partial T}{\partial t} = \frac{\partial}{\partial a} \left(k_s \frac{\partial T}{\partial a} \right) + \frac{\partial}{\partial b} \left(k_s \frac{\partial T}{\partial b} \right) + \frac{\partial}{\partial c} \left(k_s \frac{\partial T}{\partial c} \right) - \rho L f(T_m) \quad (2)$$

where the last term accounts for the heat of fusion and $f(T_m)$ is zero at all temperatures other than the melting temperature. The boundary conditions are:

$$-k_s \left(\frac{\partial T}{\partial a} \right)_{\text{surf}} = -k_s \left(\frac{\partial T}{\partial b} \right)_{\text{surf}} = -k_s \left(\frac{\partial T}{\partial c} \right)_{\text{surf}} = h(T_s - T_a) + \epsilon \sigma (T_s^4 - T_a^4) \quad (3)$$

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COORDINATE SYSTEMS
FOR MATHEMATICAL MODEL

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at all surfaces except those on the edges of the weld piece which have only convective losses and those in the V-notch adjacent to the arc which are assumed to be adiabatic. At the molten pool surface,

$$q_{\text{arc}} = \frac{Q_{\text{arc}}}{A_{\text{arc}}} = -k_s \left(\frac{\partial T}{\partial a} \right)_{\text{surf}} \quad (4)$$

for the area directly under the arc. The initial condition is:

$$T(a,b,c) = T_0 \quad (5)$$

where T_0 is the uniform temperature of the slab initially, and at the start of each pass.

Equation (2) is valid up to the melting temperature at which time turbulent mixing is assumed to eliminate temperature gradients within the volume element. An overall energy balance on the turbulent molten pool is then required to determine its temperature and to establish new boundary conditions for the adjacent, solid metal. The energy balance is

$$Q_{\text{arc}} - hA_p(T_p - T_m) = \rho C_p V_p \frac{dT_p}{dt} \quad (6)$$

where T_p is the pool temperature, T_m is the temperature at the pool edge, the melting temperature, and A_p and V_p are the pool area and volume respectively.

The boundary equation that couples Eq. (6) to the equation for the solid is

$$hA_p(T_p - T_m) - \rho L \frac{dV_p}{dt} = -k_s \iint_{A_p} \left(\frac{\partial T}{\partial n} \right) ds \quad (7)$$

The right side of Eq. (7) accounts for conduction through the solid from the pool boundary, and the second term on the left accounts for the growth of the pool.

To numerically solve these equations the heat transfer coefficient of Eqs. (6) and (7) is estimated from data compiled by Apps (1) and the assumption that the convective boundary layer is 10% of the largest dimension of the molten pool. This boundary layer accounts for the laminar flow region.

An implicit form of a finite difference equation for Eq. (2) is (4):

$$\begin{aligned}
(\rho C_p)_{i,j,k} \frac{T_{i,j,k}^{n+1} - T_{i,j,k}^n}{\Delta t} = & \frac{k_{i-\frac{1}{2},j,k}(T_{i-1,j,k}^{n+1} - T_{i,j,k}^{n+1})}{(\Delta a)^2} \\
& - \frac{k_{i+\frac{1}{2},j,k}(T_{i,j,k}^{n+1} - T_{i+1,j,k}^{n+1})}{(\Delta a)^2} + \frac{k_{i,j-\frac{1}{2},k}(T_{i,j-1,k}^{n+1} - T_{i,j,k}^{n+1})}{(\Delta b)^2} \\
& - \frac{k_{i,j+\frac{1}{2},k}(T_{i,j,k}^{n+1} - T_{i,j+1,k}^{n+1})}{(\Delta b)^2} + \frac{k_{i,j,k-\frac{1}{2}}(T_{i,j,k-1}^{n+1} - T_{i,j,k}^{n+1})}{(\Delta c)^2} \\
& - \frac{k_{i,j,k+\frac{1}{2}}(T_{i,j,k}^{n+1} - T_{i,j,k+1}^{n+1})}{(\Delta c)^2} - \rho L f(T_m) \quad (8)
\end{aligned}$$

where:

$$a = i\Delta a$$

$$b = j\Delta b$$

$$c = k\Delta c$$

and the conductivities $k_{i\pm\frac{1}{2}}$, $k_{j\pm\frac{1}{2}}$, and $k_{k\pm\frac{1}{2}}$ are mean conductivities between nodes i and $i+1$, j , and $i-1$, etc. Equation (8) would be solved for $T_{i,j,k}^{n+1}$ for each node, and the temperature profile obtained through an iterative procedure. However, the molten pool introduces an expanding irregular boundary that will require fractional node distances. When these are coupled with Eq. (8), the finite difference form is unwieldy. Since the density and heat capacity are weak functions of temperature, and the conductivity increases linearly with temperature (2), the simpler, explicit finite difference form for Eq. (2) will be used with the material properties evaluated at $T_{i,j,k}^n$. This has the form:

$$\begin{aligned}
\frac{T_{i,j,k}^{n+1} - T_{i,j,k}^n}{\Delta t} = & \left(\frac{k_s}{\rho C_p} \right)_{i,j,k}^n \left[\frac{T_{i-1,j,k}^n + T_{i+1,j,k}^n - 2T_{i,j,k}^n}{(\Delta a)^2} \right. \\
& + \frac{T_{i,j-1,k}^n + T_{i,j+1,k}^n - 2T_{i,j,k}^n}{(\Delta b)^2} + \frac{T_{i,j,k-1}^n + T_{i,j,k+1}^n - 2T_{i,j,k}^n}{(\Delta c)^2} \left. \right] - \frac{L f(T_m)}{C_p} \quad (9)
\end{aligned}$$

or,

$$\begin{aligned}
 T_{i,j,k}^{n+1} = & T_{i,j,k}^n + \left(\frac{k_s}{\rho C_p}\right)_{i,j,k}^n \Delta t \left[\frac{T_{i-1,j,k}^n + T_{i+1,j,k}^n - 2T_{i,j,k}^n}{(\Delta a)^2} \right. \\
 & + \frac{T_{i,j-1,k}^n + T_{i,j+1,k}^n - 2T_{i,j,k}^n}{(\Delta b)^2} + \frac{T_{i,j,k-1}^n + T_{i,j,k+1}^n - 2T_{i,j,k}^n}{(\Delta c)^2} \left. \right] \\
 & - \Delta t \frac{Lf(T_m)}{C_p} \quad (10)
 \end{aligned}$$

The finite difference form of the boundary condition depends on the node position relative to a metal surface. The equation for an element on the top surface of the base metal not under the arc is:

$$\begin{aligned}
 -k_s \frac{(T_{N,j,k}^n - T_{N,j-1,k}^n)}{\Delta a} &= h(T_{N,j,k}^n - T_a) + \epsilon \sigma (T_a^2 + T_{N,j,k}^{n2}) \\
 &\quad \cdot (T_a + T_{N,j,k}^n)(T_a - T_{N,j,k}^n) \\
 &= \dot{W}(T_{N,j,k}^n, T_a) \quad (11)
 \end{aligned}$$

Here $\dot{W}$ is a tabulated function of the surface and ambient temperatures (Appendix 6.2). Thus, the top surface temperature is:

$$T_{N,j,k}^n = T_{N,j-1,k}^n - \frac{\Delta a}{k_s} \dot{W}(T_{N,j,k}^n, T_a) \quad (12)$$

for

$$n > 0$$

$$T_{N,j-1,k}^n > T_0 \text{ and all } j,k \text{ except those under the arc}$$

The equation for the bottom surface has the same form but with $i = 0$ and a different function of $\dot{W}$ (Appendix 6.2).

Radiation is neglected for side surfaces of the base metal away from the arc. These surface temperatures are:

$$T_{i,0,k}^n = \frac{T_{i,1,k}^n - \frac{h \Delta b}{k_s} T_a}{(1 - \frac{h \Delta b}{k_s})} \quad (13)$$

for

$$0 \leq i \leq N$$

$$0 \leq k \leq N$$

$$T_{i,N,k}^n = \frac{T_{i,N-1,k}^n - \frac{h \Delta b}{k_s} T_a}{(1 - \frac{h \Delta b}{k_s})} \quad (14)$$

for

$$0 \leq i \leq N$$

$$0 \leq k \leq N$$

$$T_{i,j,0}^n = \frac{T_{i,j,1}^n - \frac{h \Delta c}{k_s} T_a}{(1 - \frac{h \Delta c}{k_s})} \quad (15)$$

for

$$0 \leq i \leq N$$

$$0 \leq j \leq N$$

$$T_{i,j,N}^n = \frac{T_{i,j,N-1}^n - \frac{h \Delta c}{k_s} T_a}{(1 - \frac{h \Delta c}{k_s})} \quad (16)$$

for

$$0 \leq i \leq N$$

$$0 \leq j \leq N$$

For nodes directly under the arc,

$$T_{N,j,k}^n = \frac{q_{\text{arc}} \Delta a}{k_s} - T_{N-1,j,k}^n \quad (17)$$

until $T_{N,j,k}^n = T_m$. The initial temperature condition for all nodes is

$$T_{i,j,k}^0 = T_0 \quad (18)$$

During each finite step of the welding head, the arc center is at the origin of the Lagrangian coordinate system. Any base metal surfaces closer than twice the arc width from this center point are arbitrarily chosen to be adiabatic.

With Eq. (10) and these boundary conditions, the computation of temperatures through the weld piece begins until node temperatures are equal to the melting temperature. Above the melting temperature, the finite difference form of Eq. (6) must be applied to determine the increase in the molten pool temperature. Determination of the pool area and volume for this equation is difficult, but they can be approximated through a summation of the area and volume of each node at the melting temperature. The coordinates of all points at the melting temperature are searched for the minimum and maximum values of the j and k indices (b and c dimensions) from the lowest value of i (the bottom of the pool) to the surface ($i = N$). For each value of i , the area is approximated by

$$A_p(i) = 2\Delta j(\Delta a \Delta b) + 2\Delta k(\Delta a \Delta c) \quad (19)$$

$$A_p(i_{\min}) = 2\Delta j(\Delta a \Delta b) + 2\Delta k(\Delta a \Delta c) + \Delta j \Delta k(\Delta b \Delta c) \quad (20)$$

where $\Delta j = j_{\max} - j_{\min}$ and $\Delta k = k_{\max} - k_{\min}$ for each i . The total surface area of the molten pool is:

$$A_p = \sum_{i=i_{\min}}^{i=N} A(i) \quad (21)$$

The pool volume is approximated by

$$V_p = \Delta a \sum_{i=i_{\min}}^{i=(N-1)} [\Delta j \Delta k]_i (\Delta b \Delta c) \quad (22)$$

With these approximations the finite difference form for Eq. (6) is:

$$T_{i,j,k}^{n+1} = T_{i,j,k}^n \left[1 - \frac{hA_p^n \Delta t}{\rho C_p V_p^n} \right] + \frac{T_m hA_p^n \Delta t}{\rho C_p V_p^n} + \frac{Q_{arc} \Delta t}{\rho C_p V_p^n} \quad (23)$$

for all $T_{i,j,k} > T_m$.

The actual boundary of the molten pool will not always be located at node points. Unsymmetrical boundary conditions will produce an unsymmetrical molten pool. To better account for the position of the pool boundary, fractional increments of the dimensions Δa , Δb , and Δc will be employed in the finite difference form of Eq. (7). These equations will be solved for the fractional increment values, so that they may be incorporated into the equation for heat transfer in the adjacent solid. All points with temperatures greater than or equal to the melting temperature must be searched, and those that lie at the boundary of the molten pool can be located. This is easily done by starting the search at i_{min} and searching each point in the jk plane.

For a point $T_{i,j,k}$ in the solid adjacent to the molten pool in one, two, or three directions, the fractional distances between it and the molten pool nodes along each coordinate axis are determined from the equations

$$h(T_{i\pm 1,j,k}^n - T_m) + \rho L \Delta a \frac{\Sigma_i^{n+1} - \Sigma_i^n}{\Delta t} = \frac{-k_s (T_m - T_{i,j,k}^n)}{\Sigma_j^n \Delta b} \quad (24)$$

$$h(T_{i,j\pm 1,k}^n - T_m) + \rho L \Delta b \frac{\Sigma_j^{n+1} - \Sigma_j^n}{\Delta t} = \frac{-k_s (T_m - T_{i,j,k}^n)}{\Sigma_j^n \Delta b} \quad (25)$$

$$h(T_{i,j,k\pm 1}^n - T_m) + \rho L \Delta c \frac{\Sigma_k^{n+1} - \Sigma_k^n}{\Delta t} = \frac{-k_s (T_m - T_{i,j,k}^n)}{\Sigma_k^n \Delta c} \quad (26)$$

In these equations, Σ_i , Σ_j , and Σ_k are fractions of the node spacings Δa , Δb , and Δc , respectively. The fractional distances are from the point in the solid, $T_{i,j,k}$, to the pool boundary at the melting temperature range from zero to one. Thus, $(1 - \Sigma_i)\Delta a$, etc. are the distances from a point in the molten pool to the boundary. Rearranging Eq. (24) for Σ_i gives,

$$\Sigma_i^{n+1} = \Sigma_i^n - \frac{\Delta t}{\mu \Delta a} \left[\frac{k_s (T_m - T_{i,j,k}^n)}{\Sigma_i^n a} + h(T_{i+1,j,k}^n - T_m) \right] \quad (27)$$

where $\Sigma_i^0 = G$. With Eqs. (25) and (26) rearranged in the same form, the modified equation for conduction in the solid can be written. For example, if the molten boundary is in the positive i, j, k directions and less than the internode distance away in each direction from node i, j, k , Taylor series expansions about $T_{i,j,k}$ yield:

$$T_m = T_{i,j,k} + \Sigma_i \Delta a \left(\frac{\partial T}{\partial a} \right)_{i,j,k} + \frac{(\Sigma_i \Delta a)^2}{2} \left(\frac{\partial^2 T}{\partial a^2} \right)_{i,j,k} \quad (28)$$

$$T_{i-1,j,k} = T_{i,j,k} - \Delta a \left(\frac{\partial T}{\partial a} \right)_{i,j,k} + \frac{(\Delta a)^2}{2} \left(\frac{\partial^2 T}{\partial a^2} \right)_{i,j,k} \quad (29)$$

$$T_m = T_{i,j,k} + \Sigma_j \Delta b \left(\frac{\partial T}{\partial b} \right)_{i,j,k} + \frac{(\Sigma_j \Delta b)^2}{2} \left(\frac{\partial^2 T}{\partial b^2} \right)_{i,j,k} \quad (30)$$

$$T_{i,j-1,k} = T_{i,j,k} - \Delta b \left(\frac{\partial T}{\partial b} \right)_{i,j,k} + \frac{(\Delta b)^2}{2} \left(\frac{\partial^2 T}{\partial b^2} \right)_{i,j,k} \quad (31)$$

$$T_m = T_{i,j,k} + \Sigma_k \Delta c \left(\frac{\partial T}{\partial c} \right)_{i,j,k} + \frac{(\Sigma_k \Delta c)^2}{2} \left(\frac{\partial^2 T}{\partial c^2} \right)_{i,j,k} \quad (32)$$

$$T_{i,j,k-1} = T_{i,j,k} - \Delta c \left(\frac{\partial T}{\partial c} \right)_{i,j,k} + \frac{(\Delta c)^2}{2} \left(\frac{\partial^2 T}{\partial c^2} \right)_{i,j,k} \quad (33)$$

Solving Eqs. (28) through (33) for the second derivative terms in each direction and substituting them into the constant coefficient form, the transient conduction equation yields:

$$\begin{aligned}
T_{i,j,k}^{n+1} = & T_{i,j,k}^n + \left(\frac{k}{\rho C_p}\right)^n \Delta t \left\{ \frac{2}{(\Delta a)^2} \left[\frac{T_m}{\Sigma_i^n(1+\Sigma_i^n)} + \frac{T_{i-1,j,k}^n}{(1+\Sigma_i^n)} - \frac{T_{i,j,k}^n}{\Sigma_i^n} \right] \right. \\
& + \frac{2}{(\Delta b)^2} \left[\frac{T_m}{\Sigma_j^n(1+\Sigma_j^n)} + \frac{T_{i,j-1,k}^n}{(1+\Sigma_j^n)} - \frac{T_{i,j,k}^n}{\Sigma_j^n} \right] \\
& \left. + \frac{2}{(\Delta c)^2} \left[\frac{T_m}{\Sigma_k^n(1+\Sigma_k^n)} + \frac{T_{i,j,k-1}^n}{(1+\Sigma_k^n)} - \frac{T_{i,j,k}^n}{\Sigma_k^n} \right] \right\} - \frac{\Delta t L f(T_m)}{C_p} \quad (34)
\end{aligned}$$

Other nodes around the molten pool boundary have the same form as Eq. (34) except that if the boundary is more than the internode distance away in any direction from $T_{i,j,k}$, the fractional scale factor equals one for that direction and T_m is replaced with the temperature of the solid at that next nearest node.

Equation (34) completes the set of equations necessary to determine the temperature history of the weld piece. To reiterate the solution procedure, the temperature at all nodes is calculated with Eq. (10), boundary conditions (12) through (17), and the initial temperature $T_{i,j,k}^0$ with an explicit solution procedure () until the melting temperature is reached. Equation (23) is then substituted for those nodes at the melting temperature and Eqs. (24) through (26) used to calculate the growth of the molten pool and the fractional distance factors Σ_i , Σ_j , and Σ_k . Equation (34) is substituted for nodes in the solid near the molten pool boundary utilizing the Σ factors. The explicit solution procedure is repeated with these substitutions and Eq. (10) for all nodes not in or near the molten pool or at the boundaries. Of course, the same boundary conditions are used for the nodes at the surfaces.

A significant limitation on the use of these or any other finite difference equations is the computer storage capacity and computation time. The equations have been written for simplicity assuming a constant element size throughout the slab. To decrease computation time, the element size should increase as the thermal gradients get smaller. Therefore, larger elements could be used for nodes far from the weld zone. The solution to the set of equations will be stable if the internode spacing and time increment satisfy the criteria (4):

$$0 < \frac{\alpha \Delta t}{(\Delta m)^2} \leq \frac{1}{2} \quad (35)$$

where Δm is the smallest internode distance.

Another way of saving computer space and time would be to do energy balances on one side of the notch only. Since the plate has line symmetry, points on either side of the notch would have similar temperature histories. Also a space and time saving method which was beyond the scope of this project is to use the finite element method of numerical analysis instead of the finite difference method. Indications from the literature (10) imply that the finite element method gives much more accurate solutions for the same number of elements and irregular configurations are modeled routinely.

4. RECOMMENDATIONS

1. The equations should be expanded to include base metals and filler metals of different properties and compositions.
2. The effect of filler cross-sectional shape on the accuracy of the temperature histories should be explored.
3. Computation time and storage space should be cut down by using different sized elements in different regions by considering only elements on one side of the weld plane of symmetry.
4. Finite elements should be considered for the numerical analysis instead of finite differences.

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6. APPENDIX

6.1 Radiative vs Convective Surface Heat Loss

An order of magnitude analysis for radiative and convective heat losses from the surfaces of the metal plates during welding follows.

Radiation

$$Q_{\text{rad}} = \sigma A \epsilon (T_s^4 - T_a^4) \quad (36)$$

In differential form,

$$dQ_{\text{rad}} = \sigma \epsilon (T_s^4 - T_a^4) dA \quad (37)$$

Therefore,

$$dQ_{\text{rad}} = C_{\text{rad}} (T_s^4 - T_a^4) dA \quad (38)$$

where:

$$C_{\text{rad}} = \sigma \epsilon$$

Convection

$$Q_{\text{conv}} = hA(T_s - T_a) \quad (39)$$

In differential form,

$$dQ_{\text{conv}} = h(T_s - T_a) dA \quad (40)$$

$$h = \frac{k}{D} \text{Nu} = \frac{k}{D} \lambda (\text{Gr Pr})^m \quad (41)$$

$$\text{Gr} = \left(\frac{D^3 \rho_f^2 g \beta_f \Delta T}{\mu_f^2} \right) \quad (42)$$

$$Pr = \left(\frac{C_p \mu_f}{k_f} \right) \quad (43)$$

λ = variable evaluated at film conditions

For an ideal gas:

$$\rho_f = \frac{(P)(M.W.)}{(R)(T_f)} \quad (44)$$

$$\beta_f = \frac{1}{T_f} \quad (45)$$

The local film temperature can be approximated as the mean between the local surface temperature and ambient temperature:

$$T_f = \frac{T_s + T_a}{2} \quad (46)$$

A combination of Eqs. (40), (42), (43), and (46) yields:

$$dQ_{conv} = C_{conv} \frac{(T_s - T_a)^{1+m}}{(T_s + T_a)^{3m}} dA \quad (47)$$

where:

$$C_{conv} = \frac{k}{D} \lambda \left[80^3 \left(\frac{P \times M.W.}{R} \right)^2 g \frac{C_p}{\mu_f k_f} \right]^m$$

The ratio of radiative heat loss to convective heat loss at any position on the surface will give the relative importance of each at that point.

$$\frac{dQ_{rad}}{dQ_{conv}} = \frac{C_{rad}}{C_{conv}} \frac{(T_s^4 - T_a^4)}{(T_s - T_a)^{1+m} / (T_s + T_a)^{3m}} \quad (48)$$

For turbulent convection off the upward facing surface of a flat plate, $\lambda = 0.14$ and $m = 1/3$. Therefore $C_{rad} = 1.43 \times 10^{-7}$ joule/m²-sec-°K⁴ ($\epsilon = 0.4$) and $C_{cond} = 5.57 \times 10^3$ joule/m²-sec-°K^{1/3}. This calculation shows

that convection cannot be neglected. Even if convection were neglected above 1500°K, the incurred error would be approximately 5%.

6.2 Boundary Conditions at the Upper or Lower Solid-Air Surface

At the surface of the metal, the heat loss occurs by thermal radiation and free convection.

$$\dot{W} = \dot{W}_{\text{rad}} + \dot{W}_{\text{conv}} \quad (49)$$

$\dot{W}_{\text{rad}}$ and $\dot{W}_{\text{conv}}$ are computed separately and then added to give the total heat loss. The heat loss due to radiation for a unit area is given by:

$$\dot{W}_{\text{rad}} = \sigma \epsilon (T_s^4 - T_a^4) = \sigma \epsilon (T_s^2 + T_a^2)(T_s + T_a) \Delta T \quad (50)$$

The emissivity, ϵ , is plotted in Fig. 5.

The heat loss due to free convection per unit surface area is given as:

$$\dot{W}_{\text{conv}} = h_{\text{conv}} \Delta T \quad (51)$$

where:

$$\frac{h_{\text{conv}} D}{k} = 0.14 \overbrace{\left[\left(\frac{\rho^2 g f D^3 \Delta T}{\mu} \right) \left(\frac{\mu C_p}{k} \right) \right]^{1/3}}^{\psi} \quad \text{if } \psi > 2 \times 10^7 \quad (52)$$

$$= 0.54 \left[\left(\frac{\rho^2 g f D^3 \Delta T}{\mu} \right) \left(\frac{\mu C_p}{k} \right) \right]^{1/4} \quad \text{if } \psi < 2 \times 10^7 \quad (53)$$

If we assume that air is an ideal gas and that β and ρ vary with film temperature = $[(T_s + T_{\text{am}})/2]$, then,

$$\beta = \frac{2}{T_s + T_{\text{am}}} \quad (54)$$

$$\rho = \frac{300}{T_s + T_{\text{am}}} (1.2928 \times 10^{-3}) = \frac{0.77568}{T_s + T_{\text{am}}}$$

By substituting these values, ψ is found to be larger than 2×10^7 for all temperatures of interest. Thus

$$h_{\text{conv}} = 0.14 \frac{k}{D} \left(\frac{\rho^2 g \beta D^3 \Delta T}{\mu} \frac{C_p}{k} \right)^{1/3} \quad (55)$$

$$\dot{W} = \dot{W}_{\text{rad}} + \dot{W}_{\text{conv}} = (h_{\text{conv}} + h_{\text{rad}}) \Delta T \quad (56)$$

$$= [\sigma \epsilon (T_s^2 + T_a^2) (T_s + T_a) + 0.14 \frac{k}{D} \left(\frac{\rho^2 g \beta D^3 \Delta T C_p}{\mu k} \right)^{1/3}] \Delta T \quad (57)$$

Upon substitution,

$$\dot{W} = \left[4.29 \times 10^{-2} \frac{(T_s - 300)^{1/3}}{T_s + 300} + 1.356 \times 10^{-12} \epsilon (T_s + 90,000) (T_s + 300) \right] \Delta T \quad (58)$$

Heat loss from the upper surface to the surroundings is given in Table 1.

Table 1. Heat Lost to the Surroundings at Upper Surface

T_s (°K)	$h_{\text{conv}} \times 10^{-5}$	$h_{\text{rad}} \times 10^{-5}$	$\dot{W}_{\text{conv}}$	$\dot{W}_{\text{rad}}$	$\dot{W}$
300	0	7.322	0	0	0
400	33.187	11.153	0.03	0.01	0.04
500	31.360	16.229	0.06	0.03	0.09
600	31.909	22.516	0.10	0.07	0.17
700	31.609	29.886	0.13	0.12	0.25
800	30.954	29.399	0.15	0.15	0.30
900	30.153	35.148	0.18	0.21	0.39
1000	29.301	65.329	0.21	0.46	0.67
1100	28.446	106.121	0.23	0.85	1.08
1200	27.613	164.937	0.25	1.48	1.73
1300	26.813	239.437	0.27	2.39	2.66
1400	26.050	340.248	0.29	3.74	4.03
1500	25.327	422.649	0.30	5.07	5.37
1600	24.642	513.887	0.32	6.75	7.07
1700	23.996	638.459	0.34	8.94	9.28

The heat loss from the lower surface by radiation (Table 2) is represented by the same equations as presented above. Based on the relative heat losses in the laminar range, it was assumed that the heat loss from the lower surface by force convection is half as that of the upper surface. Thus,

$$\dot{W} = [\sigma \epsilon (T_s^2 + T_a^2)(T_s + T_a) + 0.07 \frac{k}{D} (\frac{\rho^2 g \beta D^3 \Delta T C_p}{\mu k})^{1/3}] \Delta T \quad (59)$$

Table 2. Heat Loss at the Lower Surface

T_s (°K)	$h_{conv} \times 10^{-5}$	$h_{rad} \times 10^{-5}$	$\dot{W}_{conv}$	$\dot{W}_{rad}$	$\dot{W}$
300	0	7.322	0	0	0
400	16.593	11.153	0.02	0.01	0.03
500	15.680	16.229	0.03	0.03	0.06
600	15.954	22.516	0.05	0.07	0.12
700	15.804	29.886	0.07	0.12	0.19
800	15.477	29.399	0.08	0.15	0.23
900	15.076	35.148	0.09	0.21	0.30
1000	14.650	65.329	0.11	0.46	0.57
1100	14.223	106.121	0.12	0.85	0.97
1200	13.806	164.937	0.13	1.48	1.61
1300	13.406	239.437	0.14	2.39	2.53
1400	13.025	340.248	0.15	3.74	3.89
1500	12.663	422.649	0.15	5.07	5.22
1600	12.321	518.887	0.16	6.75	6.91
1700	11.998	638.459	0.17	8.94	9.11

6.3 Turbulent Zone Model

Although ample qualitative literature was found on welding puddles, little quantitative information was found. The single exception was the Apps and Milner article (1) on the behavior of metal under the arc. Their data are for lead with a 110-120-amp arc and are presented as plots of effective conductivity (Fig. 6). The turbulence occurs directly under the arc and in that portion of the molten pool within ~80% of the distance from the center of the arc (at the pool surface) to the edge of the molten pool.

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THERMAL CONDUCTIVITY OF MOLTEN POOL (4)

DATE 10-24-76	DRAWN BY TT	FILE NO. CEPS-X-241	FIG. 6
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6.4 Lorentz Mixing

Despite the importance of motion within the weld pool, very little is known about it. The force due to the welding arc occurs from two causes: the dynamic pressure of the arc plasma impinging on the pool surface and the Lorentz force caused by the interaction of the electric field and the welding pools inherent magnetic field. The primary source of motion is due to the Lorentz force, the gas only being important for a plasma jet not perpendicular to the surface. The Lorentz force causes a pressure buildup proportional to the current density. This sets up a pressure gradient which leads to fluid flow from the regions of high current density (high pressure) to regions of low current density.

6.5 Nomenclature

a	coordinate
Δa	internode spacing, cm
A	surface area, cm^2
A_{arc}	cross-sectional area of arc, cm^2
A_p	boundary surface area of molten pool
b	coordinate
Δb	internode distance, cm
c	coordinate
Δc	internode distance, cm
C_p	specific heat, cal/gm-°C
D	characteristic length, cm
$f(\tau_{\text{m}})$	single valued function for heat of fusion, 0 or 1
g	gravitational constant
Gr	Grashof number
h	heat transfer coefficient, cal/cm-sec-°C

h_{conv}	convective film transfer coefficient, cal/cm-sec-°C
h_{rad}	radiative coefficient, cal/cm-sec-°C
i	a direction index
j	b direction index
Δj	$j_{\text{max}} - j_{\text{min}}$
j_{max}	maximum value for j
j_{min}	minimum value for j
k	c direction index
Δk	$k_{\text{max}} - k_{\text{min}}$
k	thermal conductivity, cal/cm-sec-°C
L	heat of fusion, cal/gm
L_1, L_2, L_3	Lagrangian coordinates, first, second, and third pass
m	power in the correlation for convective heat transfer coefficient
Δm	smallest internode distance
n	time index
N	maximum distance index
Nu	Nusselt number
P	pressure, dynes/cm ²
Pr	Prandtl number
q_{arc}	arc flux, cal/cm ² -sec
Q_{arc}	total arc energy input, cal/sec
Q_{conv}	convective energy loss, cal/sec
Q_{rad}	radiative energy loss, cal/sec
R	gas constant
R_1, R_2, R_3	fixed coordinate points
t	time, sec

T	temperature, °C
T_a	ambient temperature, °C
T_f	film temperature, °C
T_m	melting temperature, °C
T_o	initial temperature, °C
T_p	molten pool temperature, °C
T_s	solid temperature, °C
v	welding heat velocity, cm/sec
V_p	volume of molten pool, cm ³
W	rate of heat loss by radiation and convection to the surroundings, cal/sec-cm ²
x	fixed coordinate
y	fixed coordinate
z	fixed coordinate
α	thermal diffusivity, cm ² /sec
β	thermal expansion coefficient, 1/°C
ϵ	emissivity
Σ_i	fractional distance factor, a direction, 0 to 1
Σ_j	fractional distance factor, b direction, 0 to 1
Σ_k	fractional distance factor, c direction, 0 to 1
λ	proportional constant in the convective heat transfer correlation
μ	viscosity, gm/cm-sec
ρ	density, gm/cm ³
σ	Stefan Boltzman constant, cal/sec-cm ² -C ⁴
ψ	product of Pr and Gr

Subscripts

a ambient
 s solid
 surf surface

6.6 Literature References

6.6.1 Cited References

1. Apps, R.L., and D.R. Milner, "A Note on the Behavior of Liquid Metal Under the Arc," Br. Weld. J., 10, 348 (July 1963).
2. British Iron and Steel Research Assn., "Physical Constants of Some Commercial Steels at Elevated Temperatures," Butterworths, London (1953).
3. Eyres, N.R., D.R. Hartree, J. Ingham, R. Jackson, R.J. Sarjant, and J.B. Wagstaff, "The Calculation of Variable Heat Flow in Solids," Phil. Trans. Royal Soc. London., 240 (Series A): 1-57 (1946).
4. Ozisik, M.N., "Boundary Value Problems of Heat Conduction," International Textbook, Scranton, Pa. (1968).
5. Rosenthal, D., "The Theory of Moving Sources of Heat and Its Application to Metal Treatments," Trans. ASME, pp. 849-866 (November 1946).
6. Sarjant, R.J., and M.R. Slack, "Internal Temperature Distribution in the Cooling and Reheating of Steel Ingots," J. Iron Steel Inst., 428-444 (August 1954).
7. Turner, W.D., and I.I. Siman-Tov, "HEATING5 - An IBM 350 Heat Conduction Program," ORNL-TM-4386 (April 1976).
8. Westby, O., "Temperature Distribution in the Work-Piece by Welding," Univ. of Norway (March 1968).
9. Woods, R.A., and D.R. Milner, "Motion in the Weld Pool in Arc Welding," Weld. J. (Research Supplement), 50(4), 163s-173s (April 1971).
10. Yalamanchili, R.V.S., and S.-C. Chu, "Stability and Oscillation Characteristics of Finite-Element, Finite-Difference, and Weighted-Residuals Methods for Transient Two-Dimensional Heat Conduction in Solids," Trans. ASME, J. Heat Transfer, 235-239 (May 1973).

6.4.2 Additional References

1. Adams, C.M., "Cooling Rates and Peak Temperatures in Fusion Welding," Welding J. (Research Supplement), 37(5), 210s-215s (May 1958).
2. Adams, C.M., Jr., "Heat Flow in Welding," in "Welding Handbook," C.A. Weisman, ed., 7th ed., pp. 80-98, Am. Welding, Soc., Miami.
3. Apps, R.L., and D.R. Milner, "Heat Flow in Argon-Arc Welding," Br. Weld. J., 2, 475-85 (October 1955).
4. Avdeev, M.V., "Analysis of Hydrodynamic Phenomena in the Weld Pool," Weld. Prod., 10, 1-3 (1973).
5. Baranov, M.S., B.A. Vershok, and I.N. Gainrikhs, "Analytical Evaluation of Integrals Used in Thermal Calculations in the Welding of Metals," Weld. Prod., 11, 9-12 (1974).
6. Barry, J.M., Z. Paley, and C.M. Adams, Jr., "Heat Conduction from Moving Arcs in Welding," Weld. J. (Research Supplement), 42(3), 97s-104s (March 1963).
7. Bird, R.B., W.E. Stewart, and E.N. Lightfoot, "Transport Phenomena," Wiley, New York (1960).
8. Bradstreet, B.J., "Effect of Surface Tension and Metal Flow on Weld Bead Formation," Weld. J. (Research Supplement), 47(7): 314s-322s (July 1968).
9. Brick, K.M., and A. Phillips, "Structure and Properties of Alloys," 1st ed., McGraw-Hill, New York (1942).
10. Buki, A.A., and V.S. Shtennikov, "Effect of Welding Conditions on the Transfer Coefficients of Alloying Elements," Weld. Prod. (6): 5-7 (1974).
11. Bykov, A.N., "A Statistical Model for Calculating Metallurgical Reactions in Welding," Weld. Prod., 4: 1-4 (1975).
12. Demyantsevich, V.P., and V.I. Matyukhin, "Determination of the Area of the Gas-Weld Pool Metal Interface," Weld. Prod. (1): 8-9 (1975).
13. Demyantsevich, V.P., *et al.*, "Temperature and Velocity Distributions in the Argon Plasma Jet," Weld. Prod. (10): 1-3 (1969).
14. Dusinberre, G.M., "Heat Transfer Calculations by Finite Differences," International Textbook, Scranton, Pa. (1961).
15. Friedman, E., "Thermomechanical Analysis of the Welding Process Using the Finite Element Method," Trans. ASME, J. Pressure Vessel Tech., 97(3), 206-213 (1975).

16. Hansen, M., and K. Anderko, "Constitution of Binary Alloys," 2nd ed., McGraw-Hill, New York (1958).
17. Harth, G.H., and W.C. Leslie, "A New Diagram for the Application of Welding Theories," Weld. J. (Research Supplement), 54(4), 124s-128s (1975).
18. Hibbitt, H.D., and P.V. Marcal, "A Numerical, Thermomechanical Model for the Welding and Subsequent Loading of a Fabricated Structure," Comput. Struct., 3, 1145-74 (1973).
19. Hull, F.C., "Effect of Delta Ferrite on the Hot Cracking of Stainless Steel," Weld. J. (Research Supplement), 46(9), 399s-409s (1967).
20. Jhaveri, P., W.G. Moffatt, and C.M. Adams, Jr., "The Effect of Plate Thickness and Radiation on Heat Flow in Welding and Cutting," Weld. J. (Research Supplement), 41(1), 12s-15s (1962).
21. Kovalev, I.M., *et al.*, "Effect of Movement of the Metal in the Weld Pool on Arc Stability and Weld Formation," Weld. Prod., 11, 5-7 (1974).
22. Kovalev, I.M., "Investigation of Currents of Liquid Metal in Argon-Arc Welding with a Non-Consumable Electrode," Weld. Prod., 9, 10-12 (1974).
23. Kublanov, V. Ya, and A.A. Erokhin, "The Arc Force on Molten Metal Pools," Weld. Prod., 5, 11-12 (1974).
24. Laboratory Development, "Thermophysical Study of 304 Stainless Steel Weldment," ORNL Internal Correspondence (July 25, 1974).
25. Leniukin, V.A., *et al.*, and G.I. Klenov and Y.A. Baiduganov, "Features of Weld Formation in Pulsed-Arc Welding with a Consumable Electrode," Weld. Prod., 2, 29-31 (1973).
26. Masumoto, I., I. Tamaki, and M. Kutsuna, "Hot Cracking of Austenitic Steel Weld Metal," J. Japan Weld. Soc., 41(11), 1306-1314 (1972).
27. McAdams, W.H., "Heat Transmission," McGraw-Hill, New York (1954).
28. Milner, D.R., G.R. Salter, and J.B. Wilkinson, "Arc Characteristics and Their Significance in Welding," Br. Weld. J., 7(2), 73-88 (February 1960).
29. Mueller, R.A., D.G. Howden, and F.B. Simmons, "HAZ Thermocycle and Structure Simulation," Weld. J. (Research Supplement), 52(9), 411s-416s (Sept. 1973).
30. Muraki, T., J.J. Bryan, and K. Masubuchi, "Analysis of Thermal Stresses and Metal Movement During Welding: Part I - Analytical Study," Trans. ASME, J. Eng. Tech., , 81-84 (January 1975).

31. Nestor, O.H., "Heat Intensity and Current Density Distributions at the Anode of High Current, Inert Gas Arcs," J. Appl. Phys., 33(5), 1638-1648 (1962).
32. Oliver, W.C., B.F. Oliver, and C.L. Lundin, "Ferrite in Austenitic Stainless Steels," ORNL Memo.
33. Paley, Z., and P.D. Hibbert, "Computation of Temperatures in Actual Weld Design," Weld. J. (Research Supplement), 54(11), 385s-392s (1975).
34. Patskevich, I.R., A.V. Zernov, and V.S. Serafimov, "Influence of a Longitudinal Magnetic Field on Fusion and Transfer of Electrode Metal," Weld. Prod., 7, 8-10 (1973).
35. Pavelic, V., R. Tanbakuchi, O.A. Uyehara, and P.S. Myers, "Experimental and Computed Temperature Histories in Gas Tungsten-Arc Welding of Thin Plates," Weld. J. (Research Supplement), 48(7), 296s-305s (1969).
36. Perry, R.H., and C.H. Chilton (eds.), "Chemical Engineers' Handbook," 5th ed., McGraw-Hill, New York (1973).
37. Price, P.H., and M.R. Slack, "Stability and Accuracy of Numerical Solutions of the Heat Flow Equation," Br. J. Appl. Phys., 3, 379-384 (1952).
38. Rykalin, N.N., *et al.*, "Mathematical Description of Thermal Processes in Resistance Welding," Weld. Prod., 3, 1-3 (1973).
39. Savage, W.F., "The Effect of Welding Variables on the Solidification Substructure and the Properties of Austenitic Stainless Steel Weld Metal," Research Proposal to the Atomic Energy Commission (1969).
40. Selyanenko, V.N., "Distribution of the Pressure of a D.C. Arc," Weld. Prod., 7, 4-6 (1974).
41. Slavin, G.A., N.D. Maslova, and T.V. Morozova, "Some Special Features of the Solidification of Liquid Pool Metal in Pulsed Arc Welding Using a Tungsten Electrode," Weld. Prod., 6, 7-9 (1973).
42. Slavin, G.A., *et al.*, "The Planning of a Multifactorial Experiment for Determining Weld Pool Geometry and Its Influence on Weld Structure in the Pulsed-Arc Welding of Heat-Resistant Materials," Weld. Prod., 9, 16-18 (1974).
43. Touloukian, Y.S., ed., "Thermophysical Properties of High Temperature Solid Materials," MacMillan, New York (1967).
44. Touloukian, Y.S., R.W. Powell, C.Y. Ho, and P.G. Klemens, "Thermophysical Properties of Matter," Vols. 1 and 7, IFI/Plenum, New York (1970).

45. White, J.W., "An Elementary Introduction to Finite Difference Equations," Lawrence Livermore Laboratory, Univ. of Calif. (May 3, 1976).

46. Wilkinson, J.B., and D.R. Milner, "Heat Transfer from Arcs," Br. Weld. J., 7, 115-128 (Feb. 1960).

47. Williams, R.K., and R.S. Graves, "Physical Property Results on Inconel 82 and 16-8-2 Stainless Steel Weldments," ORNL Intra-Laboratory Correspondence (July 9, 1975).