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INVERSE PROBLEMS
IN
REMOTE SENSING

by

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Progress Report

on

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ABSTRACT

The objective of our research is to improve on presently used techniques for mapping soundspeed variations in the earth by monitoring the return from seismic probes. For "small" variations in the propagation speed, a linear integral equation has been derived for that soundspeed variation. The integral equation is sufficiently general to characterize the placement of sources and receivers. The integral equation has been solved in closed form for most of the source-receiver configurations of practical interest.

We have also confronted the problem of dealing with the imperfect and incomplete data obtained in the real world. We have implemented the realistically constrained solution on the computer for a source-receiver configuration commonly used in seismic profiling. This latter project has dominated much of our research effort over the recent contract period.

PROGRESS REPORT

The objective of our research is to improve upon presently used techniques for detecting soundspeed variations in an inhomogeneous medium by actively probing the medium and observing the field scattered from it to an array of receivers. Our method of directly solving for the soundspeed variation contrasts with indirect methods which either catalog solutions of the direct problem or which are based on solving propagation equations backward in time.

Over the past year we have made significant progress on all aspects of the research proposed in this contract. Undoubtedly, the most exciting development in our work has been the production of a computer program which implements our analytical result to produce a subsurface mapping from "realistic" data.

This computer program incorporates a number of advances. Firstly, it uses a closed form analytical solution of the inverse problem when data is observed only on one side (e.g., upper surface of the earth) of the medium to be mapped. Secondly, it uses a discrete and finite source-receiver configuration. Thirdly, it uses only observations in a realistic band-limited frequency range. Fourthly, the computer program requires only a weighted sum over the observations and thus uses modest computation time.

We have refrained from writing reports on the analytical aspects of our research pending such a computer demonstration of the applicability of our methods to real world data. Since a debugged program

has only been running for a short period of time, we are just now in the progress of writing research reports on our work. Below, we shall systematically describe our overall research program on inverse problems to date.

For a medium of variable propagation speed, we have derived an exact integral equation for the perturbation from some known reference level of speed. This integral equation is non-linear because it contains a product of that perturbation with the wave field in the interior of the medium. That field is unknown, also, but satisfies a similar integral equation. In addition, these integral equations contain parameters which serve to characterize any desired source-receiver configuration.

For perturbations small* enough to allow linearization of the system, we derive a linear integral equation for the perturbation in soundspeed. We have solved this integral equation for a number of source-receiver configurations that have application to detection of soundspeed variations in the earth. The common feature of all of these models is "one-sided" viewing. That is, we do not place any sources or receivers in the interior of the medium to be probed. We shall describe below a number of such source-receiver configurations for which we can obtain exact closed form analytic solutions to the linearized equation for the soundspeed perturbation. Of course, closed form solutions do not represent the limit of possibilities

* We have not developed an explicit estimate of the meaning of "small". However, Figure 8 demonstrates good accuracy with 20% soundspeed variation.

since one can exploit standard analytic and computer techniques to solve the integral equation in other cases. Nonetheless, it is significant that explicit solutions can be found for cases of practical and physical interest. We repeat for emphasis that our closed form solutions require only that the observed data be processed in a relatively elementary manner.

As a general remark, we note that to obtain a fully three dimensional mapping of soundspeed variation, one requires a two dimensional surface array of sources and receivers. If one assumes only two dimensional soundspeed variation (as is often done in seismic modelling), then only a linear source-receiver array is required. Finally, if the soundspeed varies only with depth, a single source-receiver pair suffices to determine this dependence.

Among the configurations for which we have obtained closed form solutions are the following. In the cases 1 - 4, below, we consider propagation in three dimensions.

1. Backscattered data-source and receiver at the same location.
 - a) A two dimensional source-receiver array over a medium with three dimensional soundspeed variation.
 - b) A linear source receiver array over a medium with soundspeed variation in depth and the direction of the array.
 - c) A point array and depth dependent soundspeed variation.
2. Sources and receivers everywhere on the upper surface over a three dimensional soundspeed variation.

3. Single source and planer receiver array (or vice versa, by reciprocity) over a three dimensional soundspeed variation.
4. Horizontal and vertical offset of source and receiver over a depth dependent soundspeed variation.

It is clear that many other simple configurations admit closed form solutions. Furthermore, as was shown in Cohen and Bleistein (1976) our methods extend to the full equations of elasticity and provide a simultaneous determination of the longitudinal and shear velocities although we have not, as yet, carried out the closed form inversions analagous to 1 - 4 above.

Earlier numerical test cases were carried out on the one dimensional analog of case 1c above. Here, a known acoustic wave is propagated into an inhomogeneous medium. The backscattered signal is calculated numerically. We then process this backscattered signal as dictated by our closed form solution to the one dimensional problem and compare the computed perturbation with the known perturbation. Figures 1 through 4 are examples of these one dimensional tests. The function $\alpha(z)$ represents twice the percentage change in the propagation speed. The solid lines represent the actual perturbation, while the circles represent the results of processing the backscattered field according to our formalism. The first two examples have piecewise constant perturbations while the latter two are closer to the situation that would model a sea bottom-sedimentary layer-seabed site.

The recently developed computer program mentioned at the beginning

of the progress report is a test of our method for configurations of the type 1(b). The first data that we used was produced with the help of the research group at Marathon Oil, to whom we are very grateful. They designed a subsurface configuration of velocity zones appropriate to a natural gas deposit. A two dimensional slice of that configuration is shown in Figure 5. Using their ray tracing program, the Marathon group generated backscattered data at eighty one stations spaced one hundred feet apart at the surface of the earth. They supplied us only this data, which is displayed in Figure 6; Figure 7 is our reconstruction of the subsurface medium. In performing this inversion we used a frequency range of 6 to 24 Hertz, which is typical for seismic profiling. In Figure 8, we show our estimates of various significant depths and velocities. The errors in the result vary between zero and four percent.

The implementation was carried out on the Burroughs 6700 time sharing system at the University of Denver. Defining the number of records as the product, (Number of geophones) x (Number of subsurface mapping points) x (Number of reflectors), we found that the running time was 5 ms/record. This must be regarded merely as an upper bound because of the unknown effects of "swapping" on a time sharing system. Even at this early time we are aware of means to substantially speed up the processing time.

We have begun generating backscattered data for other subsurface models and intend to carry out the reconstructions of these models with our program. Included among these models are simple synclines,

anticlines, tilted planes and multiplanar structures. In some of these cases, the generation of the synthetic backscattered data is itself a non-trivial process.

We are beginning to gain insight into the fundamental relationships between quantities such as geophone spacing, frequency response, sampling rate and resolution. Figure 9 is an example of a near violation of one of these fundamental relationships. This is a reconstruction of the same subsurface region as in Figure 7, however, the geophone spacing was too coarse for the higher frequency range employed in the processing. The onset of instability in the mapping is observed in the higher amplitude oscillations in the interior of the lens.

REFERENCE

Cohen, J.K., and Bleistein, N., (1976), An Inverse Method for Determining Small Variations in Propagation Speeds, SIAM J. Appl. Math., to appear.

FIGURE 1

FIGURE 2

FIGURE 3

FIGURE 4

ACTUAL MODEL

FIGURE 5

FIGURE 6

FIGURE 7

COMPUTED MODEL

FIGURE 8

FIGURE 9