WEAK MATRIX ELEMENTS OF KAONS*

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ABSTRACT

We present results from the Wilson fermion part of the "Grand Challenge" weak matrix element project. A new procedure for correcting the chiral behavior of B_{LL}^{sd} , the K^0 - $\bar{K}^0$ "B parameter," is proposed and applied. On our largest lattice ($24^3 \times 40$ at $\beta = 6.0$), we get $\hat{B}_{LL}^{sd} = .86 \pm .11 \pm .05$, where the first error is statistical and the second is a measure of the systematic errors due to the procedure and to related finite-size effects. Results for the direct $K^+ \rightarrow \pi^+ \pi^0$ amplitude are also presented. There is some evidence for higher order chiral effects which may make these results compatible both with experiment and with the B_{LL}^{sd} computation. The status of the direct $K_s^0 \rightarrow \pi^+ \pi^-$ $\Delta I = 1/2$ amplitude is then discussed.

1. PREFACE:

The calculation of weak matrix elements on the lattice is a long-term effort. Considerable progress has been made, but difficulties still remain, especially in the kaon system. We assume that the reader is familiar with the "state of the art" that existed prior to this conference, and present only the new results from our group. For reviews see Refs. 1 and 2.

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2. THE K^0 - $\bar{K}^0$ “B PARAMETER” (B_{LL}^{sd}):

Recall that the amplitude $\langle \bar{K}^0 | (\bar{s}d\bar{s}d)_{LL} | K^0 \rangle$ for Wilson fermions shows “bad” chiral behavior.^{3,4,1} Let p_K and $p_{\bar{K}}$ be the meson 4-momenta. For p_K and $p_{\bar{K}}$ small, chiral perturbation theory (CPT_h) predicts the following behavior for the matrix element of the continuum operator $(\bar{s}d\bar{s}d)_{LL}^{\text{cont}}$:

$$\langle \bar{K}^0 | (\bar{s}d\bar{s}d)_{LL}^{\text{cont}} | K^0 \rangle = \gamma p_K \cdot p_{\bar{K}} + \dots, \quad (1)$$

where γ is a constant. For computational ease, one takes the mesons at rest in $\langle \bar{K}^0 | (\bar{s}d\bar{s}d)_{LL} | K^0 \rangle$, giving $p_K \cdot p_{\bar{K}} = m^2$, with m the meson mass. However, as $m \rightarrow 0$, the matrix element computed on the lattice clearly approaches a non-zero constant. (See, for example, Fig. 4 of Ref. 1.)

We believe the problem is caused by a failure of weak coupling perturbation theory to give correctly the coefficients of the dimension-6 operators which mix with $(\bar{s}d\bar{s}d)_{LL}$ on the lattice. With Wilson fermions, these operators can have different chiral properties from $(\bar{s}d\bar{s}d)_{LL}$; indeed their matrix elements go to constants as $m \rightarrow 0$. The mixing is calculated⁵ to $\mathcal{O}(g^2)$. However, it is known that there are large violations of asymptotic scaling for $\beta \approx 6.0$, so we have no reason to expect the $\mathcal{O}(g^2)$ results to be accurate. Large corrections may come from non-perturbative ($\mathcal{O}(e^{-1/g^2})$) or even simply $\mathcal{O}(g^4)$ contributions to the coefficients of “bad” operators. There are also other indications of perturbation-theory failures for currently accessible couplings.^{6,7}

We propose a non-perturbative solution to this problem which is related to — but quite different in detail from — one proposed in Ref. 4. In perturbation theory one does not calculate the true continuum operator $(\bar{s}d\bar{s}d)_{LL}^{\text{cont}}$ but a (possibly poor) approximation to it, $(\bar{s}d\bar{s}d)_{LL}^{\text{pert}}$. The expression for the latter is

$$(\bar{s}d\bar{s}d)_{LL}^{\text{pert}} \equiv z_+ \left((\bar{s}d\bar{s}d)_{LL}^{\text{latt}} + \sum_{i=1}^4 z'_i (\bar{s}d\bar{s}d)_i' \right), \quad (2)$$

where z_+ and z'_i are constants known⁵ to $\mathcal{O}(g^2)$, $(\bar{s}d\bar{s}d)_{LL}^{\text{latt}}$ is the naive lattice operator, and $(\bar{s}d\bar{s}d)_i'$ ($i = 1, 2, 3, 4$) are the four independent operators which can

mix with it:

$$\begin{aligned}
(\bar{s}d\bar{s}d)'_1 &= \bar{s}d\bar{s}d \\
(\bar{s}d\bar{s}d)'_2 &= \bar{s}\gamma_5 d\bar{s}\gamma_5 d \\
(\bar{s}d\bar{s}d)'_3 &= \bar{s}\sigma_{\mu\nu} d\bar{s}\sigma_{\mu\nu} d \\
(\bar{s}d\bar{s}d)'_4 &= \bar{s}\gamma_\mu d\bar{s}\gamma_\mu d - \bar{s}\gamma_\mu\gamma_5 d\bar{s}\gamma_\mu\gamma_5 d .
\end{aligned} \tag{3}$$

Here $(\bar{s}d\bar{s}d)'_4$ is just the parity-even part of the LR operator.

The $(\bar{s}d\bar{s}d)'_i$ have the behavior

$$\langle \bar{K}^0 | (\bar{s}d\bar{s}d)'_i | K^0 \rangle = \sigma_0^i + \sigma_1^i m^2 + \sigma_2^i p_K \cdot p_K + \dots , \tag{4}$$

where σ_0^i , σ_1^i , and σ_2^i are constants and $\dots$ represents higher order terms in m or p . The incorrectly subtracted contamination by $(\bar{s}d\bar{s}d)'_i$ makes $(\bar{s}d\bar{s}d)_{LL}^{\text{pert}}$ behave similarly:

$$\langle \bar{K}^0 | (\bar{s}d\bar{s}d)_{LL}^{\text{pert}} | K^0 \rangle = \gamma_0 + \gamma_1 m^2 + \gamma_2 p_K \cdot p_K + \dots \tag{5}$$

The procedure for "improving" $(\bar{s}d\bar{s}d)_{LL}^{\text{pert}}$ to give it the correct chiral behavior of (1) can now be stated:

- 1) Compute the matrix elements of $(\bar{s}d\bar{s}d)_{LL}^{\text{pert}}$ and $(\bar{s}d\bar{s}d)'_i$ for various values of the meson mass and then fit to the forms (4) and (5). The simplest way to separate the m^2 and $p_K \cdot p_K$ terms is to compute $\langle 0 | \Theta | K^0 K^0 \rangle$ as well as $\langle \bar{K}^0 | \Theta | K^0 \rangle$ for each operator Θ . Note that $p_K \cdot p_K$ changes sign for the crossed matrix element, while m^2 of course does not.
- 2) Choose any two of $(\bar{s}d\bar{s}d)'_i$ from (3). Adjust their coefficients z'_i in (2) to cancel γ_0 and γ_1 of (5), and then compute the corresponding "physical" value of γ_2 .
- 3) Use γ_2 from step 2) to compute the B_{LL}^{sd} parameter corresponding to the particular choices of $(\bar{s}d\bar{s}d)'_i$ under consideration.
- 4) Repeat 2) and 3) for all possible choices (6 in all).
- 5) Average the 6 results in 4) (weighted by the statistical errors associated with each choice) to compute a final B_{LL}^{sd} . The standard deviation of the 6 results may be taken as a measure of the systematic error associated with the procedure.

Note that this procedure cannot correct the overall renormalization factor z_+ . This is not believed to be particularly dangerous because: a) errors in z_+ are not magnified by the chiral behavior at low mass (unlike errors in z'_i), and b) much of the perturbative contribution to z_+ is cancelled by perturbative corrections to the pseudoscalar decay constant f in the ratio which defines B_{LL}^{sd} . Indeed, while

FIGURE 1

Raw B_{LL}^{sd} vs. $p_K \cdot p_{\bar{K}}$ at $\beta = 6.0$. Key: *octagon* $\rightarrow 24^3 \times 40$, 8 configs.; *cross* $\rightarrow 16^3 \times 40$, 19 configs. Negative values of $p_K \cdot p_{\bar{K}}$ correspond to $\langle 0 | (\bar{s}d\bar{s}d)_{LL} | K^0 \bar{K}^0 \rangle$. $a^{-1} = 1.75 \text{ GeV}$ is assumed.

z_+ may differ from 1 by 40% at $\beta \approx 6$, the corresponding diagonal coefficient in B_{LL}^{sd} differs from 1 by only 15%.

In extracting the coefficients in (4) and (5), one may choose to fit *vs.* m^2 or *vs.* $f^2 m^2$. The two are the same in lowest order CPTh (f is constant), but the latter takes into account some higher order effects, and indeed gives a considerably better fit. Nevertheless, our final central values come from the m^2 fits. We consider this the most natural, since the entire procedure is based on lowest order CPTh. Fortunately, the variation in the final results due to the type of fit is small, especially on the largest lattices. The differences are: 0.1% at $\beta = 6.0$, $24^3 \times 40$; 15% at $\beta = 6.0$, $16^3 \times 40$; 7% at $\beta = 5.7$, $16^3 \times 25$. This variation has been included in the final systematic error estimates.

Our method differs from that of Gavela *et al.*⁴ in that they do not take into account the $p_K \cdot p_K$ dependence of the $(\bar{s}d\bar{s}d)'$ (i.e., σ_2 in (4)). Despite this, our results would only change by 5 to 10% with their method if we use f^2m^2 fits, as they do. However their method does seem to be quite sensitive to the choice of fitting procedure: with m^2 fits, the results would change by $\approx 35\%$.

Fig. 1 shows some results for the “raw” (lattice normalized) B_{LL}^{sd} from our calculation at $\beta = 6.0$. Asymmetries between the $p_K \cdot p_K > 0$ and $p_K \cdot p_K < 0$ sides are presumably indications of finite size effects. The $p_K \cdot p_K < 0$ results have two mesons in the box at the same time, and therefore should be much more sensitive to the finite volume. This interpretation is confirmed by the comparison of the two volumes in Fig. 1. There is a very clear volume dependence for $p_K \cdot p_K < 0$; while for $p_K \cdot p_K > 0$ the different volumes are consistent within the errors. Note that on the $24^3 \times 40$ lattice the results are quite symmetric, indicating that finite size effects are more or less under control at this volume. Asymmetries between $p_K \cdot p_K > 0$ and $p_K \cdot p_K < 0$ are included in the final systematic error estimates.

Fig. 2 shows results for the “raw” lattice B_{LL}^{sd} at $\beta = 5.7$. Some asymmetry is present, but not as bad as for $\beta = 6.0$, $16^3 \times 40$. Note that the values here are considerably larger than those at $\beta = 6.0$. (The normalization here is the same as that in Fig. 1 to within 1%.) This is a clear indication that the $\beta = 5.7$ results are not scaling; even stronger evidence for this comes from the staggered fermion calculations.⁸

Table 1 shows our results for the continuum-normalized B_{LL}^{sd} at $\mu = 1\text{GeV}$. The systematic errors are obtained by adding in quadrature errors due to: a) variations from choices in subtraction operators (step 5 above), b) the difference between m^2 and f^2m^2 fits c) the asymmetry. The results are in agreement with those of the staggered group⁸. The agreement is particularly satisfying at $\beta = 6.0$, $24^3 \times 40$ where our errors are fairly small. (The errors in Ref. 8 are considerably smaller.) Note that we are continuing to analyze configurations at $\beta = 6.0$, $24^3 \times 40$. These configurations are shared with the staggered group under the *aegis* of the “Grand Challenge” project; the ones we have analyzed are a subset of those used in Ref. 8. The results are also consistent with those of Ref. 4. It appears that the lattice is close to definitive quenched approximation answer for B_{LL}^{sd} .

FIGURE 2

Same as Fig. 1, but for $\beta = 5.7$, $16^3 \times 25$ or $16^3 \times 32$; 32 configs. total. $a^{-1} = 1.0$ GeV is assumed.

TABLE 1

Results for B_{LL}^{sd} at scale $\mu = 1$ GeV. The first error is statistical; the second, systematic. With the choices $N_F = 3$, $\Lambda_{\text{QCD}}^{(3)} = 250$ MeV, the renormalization-group invariant $\hat{B}_{LL}^{sd}$ is obtained by multiplying these results by 1.26. (Variations in the choices give $\sim 10\%$ changes in this factor.)

β	size	configs.	$B_{LL}^{sd}(\mu = 1\text{GeV})$
6.0	$24^3 \times 40$	8	$.69 \pm .08 \pm .04$
6.0	$16^3 \times 40$	19	$.86 \pm .12 \pm .12$
5.7	$16^3 \times 25, 32$	32	$1.00 \pm .10 \pm .12$
6.1	$12^3 \times 33$	30	$.79 \pm .21 \pm .22$

3. THE DIRECT $K^+ \rightarrow \pi^+ \pi^0$ AMPLITUDE:

Fig. 3 shows our recent results for the direct $K^+ \rightarrow \pi^+ \pi^0$ amplitude with

FIGURE 3

The amplitude $K^+ \rightarrow \pi^+ \pi^0$ for all mesons at rest vs. m^2 . Key: same as for Figs. 1 and 2. Fits are to the form $\alpha_1 m^2 + \alpha_2 m^4$.

all mesons at rest, and Fig. 4 shows the physical amplitude obtained from Fig. 3 using lowest order CPT_h. For small m , CPT_h implies that the rest amplitude should go like m^2 . Fig. 3 is completely consistent with this; the fits have not-unreasonable higher order ($\mathcal{O}(m^4)$) contributions. We see no evidence for large $\mathcal{O}(a)$ contributions, as reported by Martinelli.⁹

Assuming that the amplitude in Fig. 4 is a constant, lowest order CPT_h immediately gives the result $B_{LL}^{3d}(\mu = 1\text{GeV}) = .69 \pm .06$. The consistency with the results of Table 1 is encouraging; however, the amplitude is clearly quite large compared to experiment. A possible way out is through higher order CPT_h corrections. Indeed, the data show a definite slope, especially on the two larger physical volumes. For example, at $\beta = 6.0, 24^3$, the slope (b in a $a + bm^2$ fit) is 3.9 ± 1.0 . Further, continuum results,¹⁰ as well the picture advocated by Sharpe,⁸ suggest

FIGURE 4

The physical $K^+ \rightarrow \pi^+ \pi^0$ amplitude *vs.* m^2 . Key: same as for Figs. 1 and 2.

large higher order corrections, especially in the form of final state interactions. The corrections found in Ref. 10 are of the right sign and roughly the right size to explain the discrepancy in Fig. 4. There is also internal evidence from our data, coming from the time dependence of the 2π state, for a repulsive final state interaction. However, this evidence is far from convincing or complete. In particular the effect seems decrease as m increases, which is not what one would expect for a higher order CPT_h effect.

4. THE DIRECT $K_s^0 \rightarrow \pi^+ \pi^-$ AMPLITUDE:

At the time of the Fermilab conference¹, there seemed to be a signal for the $\Delta I = 1/2$ $K \rightarrow \pi\pi$ amplitude on 16 configurations of the $\beta = 5.7, 16^3$ lattice. The signal was a ~ 1.5 to 2σ effect, and was roughly consistent with experiment. Since

FIGURE 5

The $\Delta I = 1/2$ $K_S^0 \rightarrow \pi^+\pi^-$ amplitude vs. pass number. The 16 configurations labelled "UCLA" were generated by us; the 16 labelled "LANL" were generated from an independent start by the staggered group; we analyzed both sets. The LANL configurations are displaced horizontally for clarity; the first of them is actually 2000 passes from the start. The average and $\pm 1\sigma$ values for all 32 configurations are shown.

that time, however, we have doubled the statistics at $\beta = 5.7$. Fig. 5 shows the amplitude on both sets of configurations. The signal has clearly gone away. There is no evidence, however, that the first 16 configurations were correlated and gave an underestimate of the errors.

At $\beta = 6.0$, a clear $\Delta I = 1/2$ signal does exist. However, there is another severe problem, as can be seen in Fig. 6. As t_{π^-} approaches t_{π^+} ($= 11$) from below, the amplitude is growing very rapidly. This may be due to the presence other states that can be made from 2 pions — presumably the σ and its excited states. The possibility of this kind contamination was first pointed out by the ELC group.¹¹ For typical lattice quark masses, $m_\sigma \sim 2m_\pi$, so there can be resonant enhancements

FIGURE 6

The $\Delta I = 1/2$ $K_s^0 \rightarrow \pi^+\pi^-$ amplitude vs. t_{π^-} for $\beta = 6.0$, $16^3 \times 40$, $k = .154$, 19 configurations. The π^+ is held fixed at $t_{\pi^+} = 11$. The data for other masses and lattice sizes at $\beta = 6.0$ behave similarly.

of the amplitude which could not occur in the real world. If one ignores the rapid rise and takes the final amplitude in Fig. 6 seriously, one gets very large results: ~ 10 times the experimental value. Thus this method of computing the $\Delta I = 1/2$ amplitude suffers from severe problems. The only cure known at present would be to work at much smaller quark mass; this is not feasible with current numerical technology.

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