CHARGE DISTRIBUTIONS FOR THE $^{86}\text{Kr} + ^{139}\text{La}$
SYSTEM AT 505, 610, AND 710 MeV

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Nuclear charge distributions for projectile-like and fission-fragment-like reaction products were measured as a function of angle and energy loss at each of the three bombarding energies $E_L = 505, 610, \text{ and } 710 \text{ MeV}$ for the $^{86}\text{Kr} + ^{139}\text{La}$ reaction. The charge distributions at most angles and energy losses are dominated by a peak centered close to the Z of the projectile. The variances of the Z distributions of this component exhibit a different dependence on energy loss at each of the three bombarding energies. A nucleon exchange model incorporating the effects of the Fermi motion and Pauli Principle is quite successful in accounting for the rate of energy loss per exchange deduced from the charge distribution data. This result suggests that nucleon exchange is the major mechanism for energy dissipation in these collisions. The data are also analyzed in the framework of a phenomenological model from which diffusion constants are obtained. For large energy losses a second peak appears in the charge distribution with a most probable Z equal to one-half the sum of the projectile and target Z. This component is attributed to fusion-fission. The magnitude of the fusion-fission cross section at 505 and 610 MeV is larger than TDHF predictions.

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I. Introduction

The Kr + La cross sections at 6 to 8 MeV/A exhibit interesting features^{1,2} that indicate reaction mechanisms intermediate between those for lighter systems, where fusion is important, and those for heavier systems, where the Coulomb barrier is so high that little interpenetration of the projectile and target ever occurs. The picture of the Kr + La reaction that has evolved is one in which high-partial-wave projectiles are scattered to relatively large angles, interacting little with the target. Projectiles of smaller impact parameters, on the other hand, interact for increasingly longer times, with increasing overlap of nuclear matter; the combined complex rotates toward more forward angles, with possible rotation past 0° . With the large range of interaction times for the rotating complex, such a system is ideal for studying the time scales and mechanisms for energy damping and charge/mass transfer. This reaction is one of the few krypton-induced reactions that has been studied for as many as three bombarding energies.

In a previous paper,² we have presented differential cross sections as a function of angle and energy loss, integrated over all reaction product nuclear charges. The energy spectra have two components at most angles. The partially damped component is peaked just forward of the grazing angle, and the fully damped component is forward peaked (stronger than $1/\sin\theta$). In this paper we present the Z-dependence of the cross sections, in order to study the mass transfer mechanism, and in particular to study the correlation between energy loss and mass transfer. With this additional Z identification of the reaction products, it is possible

to observe a small symmetric-mass-split contribution to the total cross section, which we identify as fusion-fission yield.

II. Experimental Method

Targets of $300 \mu\text{g}/\text{cm}^2$ $^{139}\text{LaF}_3$ on $25 \mu\text{g}/\text{cm}^2$ carbon backings were bombarded with ^{86}Kr beams of 505, 610, and 710 MeV from the Lawrence Berkeley Laboratory SuperHILAC. Beam currents were in the range of 10 to 50 enA. Reaction products were detected in a ΔE -E telescope consisting of two silicon surface-barrier detectors. The ΔE detector used for the 505 and 610-MeV measurements was $12 \mu\text{m}$ thick, and was collimated by a rectangular aperture subtending a solid angle of 0.52 msr and a full angle in the reaction plane of 0.76° ; the Z resolution (FWHM) was 1.3 units. For the 710-MeV measurements, the ΔE detector was $10 \mu\text{m}$ thick, the solid angle was 0.42 msr, and the full angle subtended was 0.68° ; the Z resolution was 1.2 units. Energy resolution for the total energy $E_{\text{total}} = \Delta E + E$ was about 12 MeV.

The energy calibrations of the ΔE and E detectors were determined with the aid of a pulser fed through a decade attenuator and charge terminator into the preamplifier input. The calibration of the attenuator output in terms of energy deposited in the detector was made with the positions of the $^{86}\text{Kr} + ^{139}\text{La}$ elastic peaks in the spectra. This calibration agreed with that determined with 5.5-MeV alpha particles from an ^{241}Am source to within a few percent.

Absolute cross sections were determined by normalizing to Coulomb scattering at forward angles. Elastic yields from a stationary monitor

detector placed at a forward angle were used to normalize the relative yields between runs. Pulsers were used to correct for dead time. The data were event-recorded on magnetic tape.

The raw data, in the form of two-dimensional plots of counts as a function of ΔE and total energy $E_{\text{total}} = \Delta E + E$, for various angles, were transformed to differential cross sections as a function of Z and energy loss. In converting from ΔE to Z , it was found that our energy calibration plus the energy losses from the tables of Northcliffe and Schilling³ did not predict the observed position of the elastic peak and its tail at lower energies. We observed energy losses $30 \pm 8\%$ higher than those of Northcliffe and Schilling at our highest ^{86}Kr energies, with the percentage disagreement decreasing with decreasing energies (agreement at about 100 MeV). Similar results for 3.8 and 5.2 MeV/A ^{84}Kr have been obtained by Bimbot et al.⁴ As an additional check, we placed weighed 4.1 and 7.8 mg/cm² aluminum foils in front of the detector telescope, and measured the loss in energy for 550 MeV ^{86}Kr ions; these losses were also about 30% above those predicted by Northcliffe and Schilling. To determine Z 's for the present data, a curve for the ratio of observed energy loss to Northcliffe energy loss, as a function of energy, was generated for $Z = 36$. Similar curves were generated from Fe ($Z = 26$) and Xe ($Z = 54$) calibration runs. These curves were used to correct Northcliffe values for other Z 's.

For each Z , observed values of counts as a function of total energy were converted to center-of-mass differential cross sections as a function of energy loss, $-Q$. The Q -values were determined from the usual two-body

relativistic kinematics formula, where it was necessary to estimate the mass number A_3 for the primary reaction product; this was done by assuming that the charge to mass ratio is the same as that for ^{86}Kr : $Z_3/A_3 = 36/86$, where Z_3 is the Z of the reaction product. With this same assumption the Jacobian for the lab to center-of-mass cross section transformation was also calculated.

The Q -value scale (for $Z = 36$) was corrected for neutron evaporation from the reaction product (in order to obtain the Q -value for the primary reaction product), assuming that the excitation energy is divided according to the masses of the reaction products.⁵⁻⁷

The data were finally corrected for contributions from elastic tails by using results from the most forward angle (at each bombarding energy), where the inelastic yield is small compared to the elastic yield. After a small correction for inelastic events, the forward-angle tail yield was parametrized as a function of Q and Z . A tail of this shape was subtracted from inelastic yields at other angles, normalizing the magnitude of the tail contribution according to the number of counts in the elastic peaks. (Errors of 30 to 50% were placed on the magnitude of the correction.)

III. Results

As an example, differential cross sections as a function of Z and energy loss are shown in Fig. 1 for the 31° (lab) data at 610 MeV bombarding energy. Several regions in the plot have been labeled. Elastic

events ($Z = 36$) are in region A. At small energy losses (region B) are the partially damped events; the charge distributions in this region are very narrow. At larger energy losses (region C), the Z distributions are much broader. These are the fully damped events; the kinetic energies of these reaction products extend below the Coulomb barrier for two spherical nuclei. In region D (where the Z 's correspond to those of a symmetric mass split of the compound system and the energy loss is large) are events which we interpret as a fusion-fission events (see Section V). Lanthanum is one of the heaviest targets for which this component is readily observable in Kr-induced reactions. Data for other angles and bombarding energies show similar increases of the charge distribution width as the energy loss increases, though the symmetric-mass events are not very prominent in the 710-MeV data.

One of the primary interests in this work is to study the widths of the Z distributions vs. energy loss, as a function of the scattering angle and particularly as a function of bombarding energy. Distributions in Z were generated from the two-dimensional data by integrating over Q -value bins 25 MeV wide. An example of these distributions, again for 31° at 610 MeV, is given in Fig. 2. Some of the scatter in the data results from the binning method involved in the transformation of the raw data. The Z distributions were fit with Gaussian distributions: one Gaussian for small energy losses, and two where the symmetric-mass cross sections were significant. The amplitudes, widths, and positions of the Gaussian distributions were free to vary, although the position

of the symmetric-mass Gaussian sometimes had to be constrained to be above a Z of 40 to 42 when the cross section for these events was small (the errors were increased to cover the resulting uncertainty).

Parameters describing the Gaussian peaked near $Z = 36$ are described in this section; those describing the symmetric-mass Gaussian are given in Section V. The mean values, $\bar{Z}$, and the variances, σ_Z^2 , for the Gaussians centered near $Z = 36$ are plotted as a function of lab angle in Figs. 3 through 5, for several Q -values at each of the three bombarding energies. It can be seen that the mean Z is very close to that of the projectile, $Z = 36$, for almost all angles, energy losses, and bombarding energies. Over the range of parameters covered by these plots, the minimum $\bar{Z}$ is 33, and the maximum is 38. The change in the observed Z as compared to the primary Z due to charged particle evaporation has been estimated to be less than one Z unit for most energy losses. The variances increase with increasing energy loss, and have very little dependence on angle except for the smallest energy losses. At a given bombarding energy, the variances first increase slowly with energy loss, and then increase more rapidly; the highest variances are observed for large energy losses at the highest bombarding energy.

This dependence is better illustrated in Fig. 6, where the energy loss is plotted as a function of the weighted angle-averaged variance for the three bombarding energies. The error bars include the variation with angle, as well as the statistical errors. For a given energy loss, the variance increases with decreasing bombarding energy. (The region for small energy losses is shown in better detail in Fig. 10.)

IV. Discussion of Dependence of Charge Distributions on Energy Loss

A. Comparison with Nucleon Exchange Model

An important mechanism for energy dissipation in peripheral collisions of heavy ions is nucleon exchange. If the excitation energies of the nuclei are modest so that the nucleon mean free path is comparable to the dimension of the system, then the energy transfer associated with exchange can be treated as a one-body interaction in which the exchanged nucleon's share of the relative kinetic energy of the two ions at the time of exchange is converted into excitation energy.⁸⁻¹⁰ In the simplest form of this model, where the Fermi motion and Pauli blocking are neglected, one obtains a total kinetic energy loss per exchange

$$\frac{dE_{\text{loss}}}{dN} = \frac{m}{\mu} (E_0 - E_B - E_{\text{loss}}) = \frac{m}{\mu} E_{\text{rel}} \quad (1)$$

where m is the nuclear mass, μ is the reduced mass of the system, E_0 is the initial c.m. kinetic energy, E_B is the interaction potential at the point of exchange, and E_{loss} is the relative kinetic energy lost up to the time of the exchange under consideration. E_B can be estimated from the Coulomb potential at the strong absorption radius. E_{rel} therefore is the available relative kinetic energy at the time of exchange.

Randrup¹¹ has recently extended the exchange model to include the effects of the Fermi motion of the constituent nucleons and of the Pauli blocking. This has been done using transport theory. In order to obtain an expression for the rate of energy loss per exchange analogous to Eq.

(1) one must employ the drift coefficient for energy loss, V_Q , and the

diffusion coefficient for the particle number, D_{AA} . The drift coefficient V_Q is given by

$$V_Q = N'_A \langle w^2 \rangle_F$$

where N'_A is the differential current of exchanged nucleons and $\langle w^2 \rangle_F$ is the average value of the square of the change of the intrinsic excitation energy per transferred nucleon. For peripheral collisions between nuclei with similar Fermi levels

$$\langle w^2 \rangle_F = \frac{1}{4} p_F^2 u^2$$

where p_F is the Fermi momentum of the exchanged nucleon and u is the relative velocity of the two nuclei at the time of exchange. The nucleon (mass) diffusion constant is given by

$$D_{AA} = N'_A \tau^*$$

with

$$\tau^* \approx \frac{1}{2} \langle w^2 \rangle_F^{1/2} \coth \frac{\langle w^2 \rangle_F^{1/2}}{2\tau}$$

where τ is the ordinary nuclear temperature. For our application,

where the average energy per nucleon of relative motion is larger than τ , $\coth \left(\frac{\langle w^2 \rangle_F^{1/2}}{2\tau} \right) = 1$ to within 10% for all energy losses to be considered.

This leads to

$$\tau^* \approx \frac{\langle w^2 \rangle_F^{1/2}}{2} = \frac{1}{4} p_F u \quad \text{and} \quad D_{AA} \approx \frac{1}{4} N'_A p_F u$$

From the transport theory definition of the drift and diffusion constants we have

$$\frac{dN}{dt} = \frac{d\sigma_A^2}{dt} = 2 D_{AA} \approx \frac{1}{2} N'_A p_F u$$

and

$$\frac{dE_{\text{loss}}}{dt} = \bar{V}_Q = \frac{1}{4} N'_A p_F^2 u^2$$

where we have assumed $N = \sigma_A^2$ for a random walk subject only to the limitation of the Pauli exclusion principle. The rate of energy loss per exchange is given by the ratio

$$\begin{aligned} \frac{dE_{\text{loss}}}{dN} &= \left(\frac{dE_{\text{loss}}}{dt} / \frac{dN}{dt} \right) = \frac{1}{2} p_F u = \frac{1}{2} (2m E_f)^{1/2} \left(2 \frac{E_{\text{rel}}}{\mu} \right)^{1/2} \\ &= \left(\frac{m}{\mu} E_f E_{\text{rel}} \right)^{1/2} \end{aligned} \quad (2)$$

Note that the nucleon current N'_A has dropped out of this expression, so that the result is not dependent on the model used to calculate the absolute number of exchanges.

Schröder et al.¹² have pointed out that the kinetic energy loss per exchange can be deduced from the experimental dependence of the variance of the charge distributions on total kinetic energy loss by differentiating a plot of σ_Z^2 vs. energy loss. This deduced energy loss per exchange $\frac{dE_{\text{loss}}}{dN}$ includes all energy losses whether due to particle exchange or other mechanisms. In order to extract this quantity one must assume a relationship between σ_Z^2 and the total number

of nucleons exchanged, a relationship that is unclear at this time. If proton exchange is treated as a random walk, then N_Z , the number of protons exchanged, is given by σ_Z^2 . Similarly the number of neutrons exchanged N_N is given by σ_N^2 . The variance of the mass distribution σ_A^2 can be written quite generally as

$$\sigma_A^2 = \sigma_N^2 + \sigma_Z^2 + 2 \sigma_N \sigma_Z \rho$$

where ρ is the correlation coefficient between neutron and proton exchange. If the neutron and proton exchanges have similar rates and are uncorrelated ($\rho = 0$) then the total number of nucleons exchanged N is given by $N = \sigma_A^2 = \sigma_N^2 + \sigma_Z^2 = \left(\frac{A}{Z}\right) \sigma_Z^2$. If, on the other hand, the neutron and proton exchanges are fully correlated ($\rho = 1$), maintaining at each step the equilibrated $\frac{A}{Z}$ ratio, then $N = \sigma_A^2 = \left(\frac{A}{Z}\right)^2 \sigma_Z^2$. There have been several recent studies¹³⁻¹⁶ where the widths of the mass and charge distributions were measured simultaneously. For bombarding energies close to the Coulomb barrier^{11,12,14} the σ_A^2 values are approximately equal to $\left(\frac{A}{Z}\right)^2 \sigma_Z^2$ for all energy losses. (An exception to this is the $^{208}\text{Pb} + ^{110}\text{Pd}$ system¹³ where σ_A^2/σ_Z^2 surprisingly seems to decrease with increasing energy loss.) At higher bombarding energies¹⁵ the σ_A^2 values start out close to $\left(\frac{A}{Z}\right) \sigma_Z^2$ for small energy losses and approach $\left(\frac{A}{Z}\right)^2 \sigma_Z^2$ for large energy losses. There is a fundamental problem in the deduction of the number of exchanges from these results. The observation of $\sigma_A^2 \approx \left(\frac{A}{Z}\right)^2 \sigma_Z^2$ for large energy losses does not necessarily imply that ALL of the exchanges were correlated; correlation may start to set in only during the later

stages of the collision as the relative velocity of the ions becomes small and the Pauli principle starts to play a larger role in determining the relative probabilities of neutron and proton exchanges. Eyal et al.¹⁴ have shown that a result close to $\sigma_A^2 \approx \left(\frac{A}{Z}\right)^2 \sigma_Z^2$ may also be obtained if a thermalized intermediate dinuclear complex is formed at some stage of the collision, assuming that the isobaric cross sections are proportional to the available density of states in the exit channel. In view of these results and the discussion of them, no unambiguous statement about the relationship of the σ_Z^2 values and the total number of exchanged nucleons can be made. For bombarding energies considerably above the barrier and for not too large energy losses $N \approx \left(\frac{A}{Z}\right)^2 \sigma_Z^2$ is probably a reasonable assumption; for larger energy losses and lower bombarding energies the situation is less clear. For purposes of illustration we will make the provisional assumption that $N \approx \left(\frac{A}{Z}\right)^2 \sigma_Z^2$.

We first compare our results with the simplest form of the exchange model (Eq. 1) neglecting Pauli blocking. We show in Fig. 7 the values of $\frac{dE_{\text{loss}}}{dN}$ deduced from our 610 and 710 MeV incident energy charge dispersion data. At 505 MeV the incident energy is too close to the Coulomb barrier to enable extraction of $\frac{dE_{\text{loss}}}{dN}$ values for positive E_{rel} values. The available relative energy is calculated from an interaction potential that neglects deformation effects, and therefore negative E_{rel} values can be obtained for large energy losses or low bombarding energies. If all energy loss processes have the same dependence on the relative kinetic energy as that for nucleon exchange (and if the relationship between the charge distribution variance and number of nucleons exchanged

doesn't change with energy) then a linear dependence of $-\frac{dE_{\text{loss}}}{dN}$ on E_{rel} is expected. A slope of 1 as indicated by the lower solid line is expected if $N = \frac{A}{Z} \sigma_Z^2$ as was assumed in deriving the values of the ordinate for the experimental data points. For fully correlated exchanges the apparent slope expected is $\frac{A}{Z}$ times larger as is also indicated in the figure. (The theoretical values of dE_{loss}/dN are of course independent of the relation between N and σ_Z^2 ; it is the experimentally deduced values of dE_{loss}/dN which depend on this relation. For simplicity however the experimental values are only plotted once, using the assumption $N = \left(\frac{A}{Z}\right) \sigma_Z^2$.) Although the experimental data exhibits an approximately linear dependence for large E_{rel} values, the observed slopes are much larger than the theoretical expectations, and are different at the two different bombarding energies. The discrepancy in the absolute value of the slopes of such curves for other systems at a single bombarding energy has been noted earlier,¹⁷ and attributed to non-exchange energy loss mechanisms.

We turn now to a comparison of our data with the extended exchange model incorporating the effects of Fermi motion and the exclusion principle. From Eq. (2) we see that $\frac{dE_{\text{loss}}}{dN}$ should vary linearly as the square root of the relative kinetic energy. Our data are plotted in Fig. 8 as a function of $\left(\frac{m}{\mu} E_f E_{\text{rel}}\right)^{1/2}$, with $E_f = 40$ MeV. The full and dashed curves give the absolute slopes expected from the extended exchange model with $N = \frac{A}{Z} \sigma_Z^2$ and with $N = \left(\frac{A}{Z}\right)^2 \sigma_Z^2$. We note that with this new model the data for the two bombarding energies lie on the same curve, and that the slope of the curve is consistent with the model of $N \approx \frac{A}{Z} \sigma_Z^2$ as was suggested in our earlier discussion. The agreement

between experiment and theory suggests that the major mechanism for energy dissipation is in fact nucleon exchange. This conclusion is in contrast to that of previous studies^{12,17} in which the effect of the exclusion principle was not taken into account. We have also compared some results from other studies using Kr projectiles with the extended exchange model. These comparisons are shown in Fig. 9. The Kr + Au data are from Moretto and Sventek¹⁸ as transformed by Schröder et al.,¹² and the Kr + Ho data is from Huizenga et al.¹⁹ The Kr + Er results are from Rudolf et al.²⁰ These results are also qualitatively consistent with the exchange model. The $^{84}\text{Kr} + ^{165}\text{Ho}$ results however indicate somewhat smaller rates of energy loss per exchange than the other studies, reflecting the fact that the charge distributions in that study are somewhat broader than those obtained in other studies.

B. Comparison with Randrup's Dinuclear Dynamics Model

In a previous section we have compared the rate of energy loss per nucleon exchanged, inferred from the dependence of the variance of the charge distribution on energy loss, with the behavior expected for an exchange model incorporating the effects of Fermi motion and Pauli blocking.¹¹ This comparison was made in such a way that the absolute number of exchanges cancelled out. The comparison therefore was insensitive to any assumptions about the nuclear dynamics and to the absolute number of exchanges for any given partial wave. In previous works^{2,21} we have shown that energy loss due to nuclear exchange when calculated in the frozen spheres approximation is incapable of accounting for the absolute magnitude of the energy loss and angular momentum trans-

fer in collisions between heavy nuclei. In this section we consider an improved model incorporating a neck degree of freedom. We use a program developed by Randrup²² to calculate the average values and the variances of the charge and mass distributions for a particular dynamical model. At large distances the two nuclei follow trajectories determined by the conservative and dissipative forces given by a proximity model for diffuse spherical nuclei. At close separations the dinuclear complex is represented by two spherical nuclides joined by a cylindrical neck.²³ The neck radius evolves dynamically as determined by the conservative forces, including those arising from the surface energy of the neck and from the additional proximity energy due to the neck and nuclear surfaces facing each other, and a dissipative force for the neck motion calculated from an expression for wall dissipation in a cylinder. The driving forces for particle exchange are determined from the Lysekil mass formula.²⁴ The equations of motion are integrated for a given partial wave, and the resulting energy loss, mean charge and mass, and variance of the mass and charge distributions are calculated. The resulting dependence of variances of the charge distributions on energy loss are displayed in Fig. 10. The model accounts fairly well for the observed behavior for small energy losses, but fails for large energy losses. The failure at larger energy losses can be traced to a failure of the calculation to account for the magnitude of the energy loss, as evidenced by comparing the calculated energy losses as a function of angle with the experimental Wilczynski plot. The calculation never predicts the fission-like energies.

observed for the fully relaxed events. This is to be expected as the shape parametrization does not allow proper treatment of deformation effects in the exit channel. This new calculation with the neck degree of freedom does, however, give appreciably larger energy losses than the earlier calculation employing the frozen spheres approximation. For the larger partial waves it removes much of the discrepancy (approximately a factor of two) between calculation and experiment that we observed previously.²

The calculated average value of the projectile-like fragment is close to $Z = 36$, with a tendency for Z to decrease several units for the largest energy losses, in agreement with the experimental observations. It is also interesting to note, in connection with our previous discussion on the relation between the variances of the mass and charge distributions, that the calculated ratio σ_A^2/σ_Z^2 is intermediate between $\frac{A}{Z}$ and $\left(\frac{A}{Z}\right)^2$ but closer to the former at the higher bombarding energies and moderate energy losses. For a given partial wave the ratio of the variances increases along the trajectory as the number of exchanges and the energy loss increase.

C. Comparison with TDHF Calculation

The only other theory for which a priori charge distribution calculations are available is the time-dependent mean-field theory. Extensive time-dependent Hartree-Fock (TDHF) calculations employing the axially symmetric rotating frame approximation have been made by Davies et al.^{25,26} for the Kr + La system. The calculated charge distribution variances, after correction for the contributions of the "filling approximation" to the widths, are nearly an order of magnitude smaller than the experi-

mental values. A similar result has been seen previously in TDHF calculations of charge and mass distributions for other systems²⁷⁻²⁹. Dasso et al.³⁰ have discussed the effect on mass distributions of inherent limitations in the TDHF formalism associated with the single-determinant nature of the wave function. Here too we find that the ratio of the calculated variances³¹ of the charge and mass distributions σ_A^2/σ_Z^2 , is closer to $\frac{A}{Z}$ rather than $\left(\frac{A}{Z}\right)^2$.

D. Extraction of Transport Theory Charge Diffusion Coefficients

A prescription for extracting charge diffusion coefficients from experimental data has been given by Riedel et al.³² In the transport theory picture of deeply inelastic collisions, two nuclei rotate in contact for an interaction time during which radial kinetic energy and relative angular momentum are damped, and mass is transferred. With interaction times τ_{int} derived from experimental data, experimental values of the charge diffusion coefficient D_Z may be extracted from the Z distributions. These coefficients can then be compared to those obtained for other systems using a microscopic quantum-statistical model³³ to predict the scaling between different systems with differing bombarding energies.

The first step in the analysis is to derive the interaction time as a function of deflection angle. A classical model is introduced in which the projectile reaches the nuclear interaction region along a Coulomb trajectory, and the composite system rotates through an angle determined from entrance and exit Coulomb angles and the deflection

function, which is parametrized by two quantities, β and δ . The parameter β describes the degree of angular focussing in the angular distribution, and δ describes the difference between the observed grazing angle and the Coulomb deflection angle for the grazing angular momentum. During the interaction time, radial kinetic energy is dissipated, relative angular momentum is lost, and the two fragments approach an asymptotic deformation α_o ; the relaxation times are τ_R , τ_ℓ , and τ_α , respectively. For each value of projectile energy and incident angular momentum, it is possible to calculate the interaction time, the deflection angle, and the final energy, angular momentum, and deformation of the reaction products.

The parameters β , δ , τ_R , τ_ℓ , τ_α , and α_o are to be determined from fits to the experimental angular and kinetic energy distributions. In the present analysis, we primarily use parameters consistent with those found to fit data from other reactions. We keep the values $\tau_R = 0.3 \times 10^{-21}$ sec, $\tau_\ell = 1.5 \times 10^{-21}$ sec, $\tau_\alpha = 5.0 \times 10^{-21}$ sec, and $\alpha_o = 0.35$ used by Riedel et al.³² This value of α_o describes the experimental final kinetic energies of the fully damped Kr + La products well.

The dependence of τ_{int} on the relaxation times is weak. We choose the parameters β and δ to fit the experimental angular distributions. Two quite different values of β might be chosen, depending on whether or not the fully damped products have orbited past 0° ; large values of β give orbiting to negative angles. (It is not possible to choose between these two possibilities merely by measuring the angular distributions.) We consider two sets of the para-

meters β : one set (case of orbiting) derived from the systematics of other reactions considered by Wolschin³⁴ and by Riedel et al.,³² which gives a small amount of orbiting at 505 MeV and larger amounts of orbiting at 610 and 710 MeV; and another set (case of no orbiting) where the deflection function dips to 0° , but not below. The first set qualitatively describes the experimental Kr + La angular distributions, but the second set does not reproduce the large fully-damped cross sections, as the minima in the deflection functions occur at very small ℓ values, where there is little yield. An alternative parametrization of the deflection function would be required to yield good fits to the angular distributions with no orbiting. Nevertheless, we extract D_Z coefficients for this second set of β 's in order to study the sensitivity of the results to the choice of β . The D_Z values extracted below are derived primarily from Z distributions of partially damped events; the Riedel parametrization is probably adequate in this region. With these parameters and the coupled equations given by Riedel et al.,³² it is then possible to derive the interaction times as a function of deflection angle.

With these interaction times, the relation

$$\sigma_Z^2 = 2D_Z \tau_{\text{int}}$$

may be used to extract the charge diffusion coefficients at each bombarding energy. Experimental σ_Z^2 values were extracted at each angle for the most probable energy losses, one for the partially damped and one

for the fully damped component, if both were present at the given angle. (This corresponds to studying the variation of σ_Z^2 along the ridge line of the Wilczynski plot.) A plot of σ_Z^2 versus τ_{int} shows a linear rise, followed by saturation of σ_Z^2 (i.e., D_Z decreases for small impact parameters). We consider here only the linear rise, and extract D_Z values from the slope of this line. We also use the l -dependent form factor introduced by Ayik et al.,³³ since the diffusion is not expected to be fully developed for the largest impact parameters. The extracted D_Z values are thus derived from Z distributions corresponding to intermediate l values: larger than those for which σ_Z^2 saturates, and smaller than those for which the form factor is significantly different from unity. The resulting values, along with the β and δ parameters for the orbiting case, are given in Table 1. The errors quoted include statistical errors and scatter in the data. The D_Z values for the case of no orbiting (with $\beta = 37, 50$, and 46 for bombarding energies of $505, 610$, and 710 MeV, respectively) are about 30% smaller.

These experimental D_Z coefficients may be compared to those obtained for other systems using scaling laws derived from microscopic transport theory.³³ In this picture, mass exchange is considered a transport phenomenon to be described by generalized master equations. Nucleons of the composite system may be excited or may be exchanged between the two nuclei, the transitions occurring between single-particle states lying within a few MeV of the mean Fermi energy of both fragments. The single-particle matrix elements are parametrized by a Gaussian distribution having strength γ and width Δ . The resulting equation for D_A is given by

$$D_A(A_1, \ell) \approx 0.21 \gamma \sqrt{\Delta} \left(\frac{E_\ell^*(A_1)}{A_1 + A_2} \right)^{1/4} \left(\frac{(A_1 A_2)^{1/3}}{A_1^{1/3} + A_2^{1/3}} \right)^2$$

for fragments with masses A_1 and A_2 . The quantity $E_\ell^*(A_1)$ is the local excitation energy of the composite system, and is evaluated for a mean angular momentum $\bar{\ell}$ between the critical and grazing angular momenta; complete damping of the radial kinetic energy is assumed. The parameter $\gamma \sqrt{\Delta} = 2.21$ has been chosen by Wolschin et al.³⁵ in an analysis of the Z distributions of the reaction 714 MeV $^{84}\text{Kr} + ^{165}\text{Ho}$.

We calculate the expected D_Z charge drift coefficients corresponding to these D_A values for the two extreme cases of uncorrelated and correlated neutron and proton exchanges:

$$D_Z^{\text{uncorr}} = (Z/A) D_A$$

$$D_Z^{\text{corr}} = (Z/A)^2 D_A$$

These values are given in Table 1. Because of the uncertainties in determining the deflection function, and because the parameter $\gamma \sqrt{\Delta}$ was originally chosen with the assumption of correlated transfer, it is not very meaningful to compare the absolute values of experimental and theoretical D_Z values. However, the relative values for the three bombarding energies should provide some insight. The theory predicts a weak dependence on the bombarding energy, as is observed for the 505 and 610 MeV data. However, the experimental value at 710 MeV shows a significantly greater increase than is predicted by the model.

Such an effect was also observed for the $^{86}\text{Kr} + ^{166}\text{Er}$ reaction at 515 and 703 MeV.³² It is not clear where this discrepancy originates. The problem could be in the procedure for extracting interaction times from the experimental data, or it could be that the theory for determining D_A is not adequate. Alternatively, it may be that over this range of bombarding energies, we are observing a transition from correlated to uncorrelated neutron and proton exchange. Further studies with mass distribution measurements and better determinations of deflection functions will be required for a better understanding of the diffusion constants.

V. The Fusion-Fission Component

The Z distributions at the more backward angles and for large energy losses exhibit a second component which we attribute to fusion-fission events. An example of the appearance of this peak at several energy losses at a given angle is shown in Fig. 2. The Z distributions have been fit by two Gaussians whose positions and widths were allowed to vary. The decomposition of the Z spectrum is indicated in the figure. In addition to the most intense peak with a most probable Z close to that of the projectile, another peak with a most probable Z close to that expected for symmetric fission fragments ($Z = 46.5$) becomes relatively more intense at larger energy losses.

We have integrated this component over energy loss to obtain the angular distributions shown in Fig. 11. The errors are rather large at forward angles because of the smallness of this component compared to the deeply inelastic component. At 710 MeV the Z distribution of the

deeply inelastic component is so broad that decomposition is only possible at backward angles and for the more negative Q -values. The angular distribution for fusion-fission events should exhibit an approximately $1/\sin\theta$ angular dependence over the angular range that we have investigated. We have indicated in the figure such a distribution normalized to the more reliable backward angle cross sections.

As a further check on the identification of these events as fusion-fission events, we have constructed histograms of the exit channel total kinetic energy (TKE) spectra for this component from the three most backward angles. These are shown in Fig. 12. We have also calculated the expected average fission fragment total kinetic energy on the basis of the Viola systematics,³⁶ making corrections for the effect of neutron emission. These expectations are shown by the dashed vertical lines in the figure. Within the uncertainties imposed by the small number of events and the crude binning, the observed energies are consistent with the fusion-fission mechanism. The observed energies however could also be consistent with the deeply inelastic or "quasi-fission" process. (b)

On the basis of a second distinct peak in the Z distributions at backward angles, a most probable Z characteristic of symmetric fusion-fission, an angular distribution qualitatively consistent with $1/\sin\theta$, and a most probable kinetic energy consistent with fission systematics, we have attributed the cross section for this component as due to fusion-fission. The angle-integrated cross sections assuming a normalization to $1/\sin\theta$ as shown in Fig. 11, are given in Table 2.

Also shown are the critical ℓ values ℓ_c deduced from these cross sections assuming a sharp cutoff model, where ℓ values between 0 and ℓ_c are assumed to lead to fusion.

These results can be compared with recent TDHF calculations for this system. Davies et al.^{25,26} have performed calculations using the axially symmetric, rotating frame approximation and find that the fusion threshold is between 610 and 660 MeV. Thus no fusion is expected at the two lowest energies studied. At 710 MeV the calculations predict an ℓ_c value of about 65, consistent with our observations. (It is interesting to note that the most recent calculations predict that at the still higher energies of 860 and 910 MeV, fusion does not occur for $\ell = 0$. This presumably is the low- ℓ non-fusion window predicted previously for lighter systems.) The calculated kinetic energies of the deeply inelastic events have also been compared with our previous experimental results and indicate that the calculations underestimate the energy loss for the lower ℓ values. More recent 3-D TDHF calculations³⁷ give better agreement with the experimental energy losses but still predict that there will be no fusion at the lowest energies.

VI. Summary

The dependence of Kr + La nuclear charge distributions on scattering angle and, in particular, on bombarding energy and energy loss have been studied. Most of the reaction cross section comes from partially or fully damped events, for which the Z distributions are peaked close to $Z = 36$ and have widths that increase with energy loss for a given bombarding energy and with bombarding energy for a given energy loss, but that are relatively independent of scattering angle.

The measured dependence of the variances, σ_z^2 , on energy loss has been compared with a nucleon exchange model (incorporating Fermi motion and the Pauli principle),^{8-11,22-23} and TDHF calculations.^{25,26} For the nucleon exchange model comparison, experimental values for the rate of energy loss per nucleon exchange are extracted from the variances (assuming uncorrelated proton and neutron exchange) and plotted as a function of the parameter $\left(\frac{m}{\mu} E_f E_{rel}\right)^{1/2}$ (see Eq. 2); the 610 and 710-MeV data then reduce to a common function, which is consistent with that deduced from the model. Such remarkable agreement with an a priori model confirms the importance of particle transfer as a mechanism for energy dissipation. This comparison has been done in a manner that does not depend on a knowledge of the absolute number of exchanges as a function of impact parameter. A dynamical model due to Randrup has been employed to calculate the dependence of the energy loss and charge variances on impact parameter. This dinuclear dynamics model describes the dependence of the variance on energy loss well for small energy losses, but fails at the larger energy losses. The variances calculated from TDHF theory are an order of magnitude smaller than those observed.

Charge diffusion coefficients for the three bombarding energies have been derived in the framework of a phenomenological transport model.³²⁻³⁵ The energy dependence of the experimental coefficients is significantly stronger than that derived in the model, but uncertainties in determining the deflection function and the degree of correlation between proton and neutron transfers make it difficult to pinpoint the source of the discrepancy.

At large energy losses and large scattering angles, a second component of the Z distribution is observed. As the mean Z is close to that of a symmetric mass split of the compound nucleus, the most probable energy loss is consistent with that expected from fission of the compound nucleus, and the angular distribution is qualitatively consistent with the function $1/\sin\theta$, we identify this component as fusion-fission. Integration gives total cross sections for this mechanism (found to be approximately the same at the three bombarding energies), and thus critical ℓ values. The TDHF calculations predict no fusion-fission below 710 MeV, but at 710 MeV are in agreement with experiment.

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TABLE 1. Experimental and Predicted Values for the
 $^{86}\text{Kr} + ^{139}\text{La}$ Charge Diffusion Coefficients
in Units of 10^{21} sec^{-1}

E_L (MeV)	β	δ	$\bar{\ell}$	D_Z^{exp}	D_Z^{uncorr}	D_Z^{corr}
505	47	0.10	153	2.2 ± 0.3	7.0	2.9
610	84	0.08	199	2.6 ± 0.4	8.1	3.4
710	115	0.17	225	6.7 ± 1.0	9.1	3.8

TABLE 2. Angle-Integrated Cross Sections for the Fusion-Fission Component

E_L (MeV)	$\sigma_{\text{fusion-fission}}$ (mb)	l_c ($\hbar$)
505	170 ± 50	66 ± 10
610	225 ± 50	83 ± 10
710	170 ± 50	77 ± 12

Figure Captions

- 1 Contours of triple-differential cross section $\frac{d^3\sigma}{d\sigma dE dZ} \left(\frac{\text{mb}}{\text{sr} \cdot \text{MeV} \cdot Z} \right)$ at $\theta_L = 31^\circ$ and $E_L = 610$ MeV.
- 2 Charge distributions for different energy losses at $\theta_L = 31^\circ$ and $E_L = 610$ MeV.
- 3 Mean values (right hand scale) and variances (left hand scale) for the charge distributions at $E_L = 505$ MeV.
- 4 Mean values (right hand scale) and variances (left hand scale) for the charge distributions at $E_L = 610$ MeV.
- 5 Mean values (right hand scale) and variances (left hand scale) for the charge distributions at $E_L = 710$ MeV.
- 6 Energy loss as a function of the weighted angle-averaged variance for each of the three bombarding energies.
- 7 Dependence of the rate of energy loss per exchange as a function of the available relative energy per nucleon. The dashed curves are to guide the eye. The full curves give the dependences expected for two assumptions about the relation between the number of exchanged particles and the charge distribution variances. The absolute value of the ordinate is for the assumption $N = \frac{A}{Z} \sigma_Z^2$. Rather than replotting all the experimental data points for the assumption $N = \left(\frac{A}{Z} \right)^2 \sigma_Z^2$, the theoretical prediction has been scaled appropriately.

Figure Captions (Continued)

- 8 Dependence of the rate of energy loss per exchange as a function of the square root of the product of the relative available energy per nucleon and the Fermi energy, $E_f = 40$ MeV.. The full and dashed curves give the expected behavior from the exchange model for two assumptions about the relation between the number of exchanged particles and the charge distribution variances. The absolute value of the ordinate is for the assumption $N = \frac{A}{Z} \sigma_Z^2$ (see caption to Fig. 7).
- 9 Dependence of the rate of energy loss per exchange as a function of the square root of the product of the relative available energy per nucleon and the Fermi energy. The data are from refs. 18, 19, and 20. See also caption to Fig. 7.
- 10 Comparison of experimental dependence of energy loss on charge distribution variance with results of Randrup's dynamical dinuclear exchange model. (Experimental data for the largest energy losses are not shown.)
- 11 Angular distributions for the fusion-fission component. The $1/\sin\theta$ curves are normalized to the data near 90° .
- 12 Kinetic energy spectra of events attributed to fusion-fission. The vertical dashed line is the expected fission energy, after neutron emission correction, assuming the Viola systematics.

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 7

Fig 8.

Fig. 6

Fig. 10 *Energy of ...*

Fig. 11

Fig. 12